

# Ring dynamics around non-axisymmetric bodies

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**Dense and narrow rings have been discovered recently around the small Centaur object Chariklo<sup>1</sup> and the dwarf planet Haumea<sup>2</sup>, while being suspected around the Centaur Chiron<sup>3</sup>. They are the first rings observed in the Solar System elsewhere than around giant planets. Contrarily to the latter, gravitational fields of small bodies may exhibit large non-axisymmetric terms that create strong resonances between the spin of the object and the mean motion of rings particles. Here we show that modest topographic features or elongations of Chariklo and Haumea explain why their rings are relatively far away from the central body, when scaled to those of the giant planets. Lindblad-type resonances actually clear on decadal time-scales an initial collisional disk that straddles the corotation resonance (where the particles mean motion matches the spin rate of the body). The disk material inside the corotation radius migrates onto the body, while the material outside the corotation radius is pushed outside the 1/2 resonance, where the particles complete one revolution while the body completes two rotations. Consequently, the existence of rings around non-axisymmetric bodies requires that the 1/2 resonance resides inside the Roche limit of the body, favoring fast rotators for being surrounded by rings.**

The adopted physical parameters of Chariklo and Haumea's systems are listed in Table 1. Contrarily to the case of the giant planets<sup>4</sup>, Chariklo and Haumea's rings are relatively far away from their hosts. The new rings are in fact located well outside the corotation (also called synchronous) orbit, and are near the classical Roche limit of the bodies, where fluid particles with ice density should accrete into satellites, see discussion later. Both peculiarities call for explanations.

Both Chariklo and Haumea have non-spherical shapes. Haumea is a triaxial ellipsoid<sup>2</sup> with principal semi-axes  $A > B > C$  and elongation  $\epsilon \sim 0.43$  (see definition in Table 1). Chariklo's shape is less constrained due to scarce observations. Extreme solutions<sup>5</sup> are a spherical Chariklo of radius  $R_{\text{sph}} = 129$  km with typical topographic features of heights  $z \sim 5$  km, or an ellipsoid with elongation  $\epsilon \sim 0.16$ .

In that context, Chariklo and Haumea's rings should be strongly coupled with the non-axisymmetric terms of their respective potentials. Relative to a spherical body of same mass, the two bulges contain masses of order  $\epsilon$ , i.e. substantial fractions of Chariklo and Haumea's masses. Even a 5-km topographic feature on Chariklo represents a mass anomaly  $\mu \sim (z/2R_{\text{sph}})^3 \sim 10^{-5}$  relative to the body, This is much larger than the mass of Janus (a small satellite that confines the outer edge of Saturn's main rings) with  $\mu \sim 3 \times 10^{-9}$ , or putative Saturnian mass anomalies<sup>6</sup>, with  $\mu < 10^{-12}$ .

We focus here on the angular momentum exchange between the body and a collisional disk that has settled into its equatorial plane, either due to an equatorial topographic feature, or an elongated shape. This said, we do not discuss the possible origins of the rings<sup>2, 7, 8, 9</sup> nor the influence of close encounters of Chariklo with giant planets, which are too rare to affect its rings<sup>10, 11</sup>.

Fig. 1 outlines as examples two possible configurations of Chariklo's dynamical environment, with four fixed points  $C_1, \dots, C_4$  near the corotation radius  $a_{\text{cor}} \sim (GM/\Omega^2)^{1/3} = R/q^{1/3}$ , where the adimensional

rotation parameter  $q$  is defined by

$$q = \frac{\Omega^2 R^3}{GM}, \quad (1)$$

$G$  being the gravitation constant,  $M$  the mass of the body,  $\Omega$  its spin rate, and  $R$  denoting either the radius  $R_{\text{sph}}$  of a sphere or the reference radius of the ellipsoid (Table 1).

In principle, the region around  $C_2$  or  $C_4$  may host ring arcs, but these points being potential maxima, arcs are unstable against dissipative collisions over time scales of some  $10^4$  years at most (see Methods). Moreover, for Chariklo's elongations larger than the critical value  $\epsilon_{\text{crit}} \sim 0.16$  (close to the actual estimated value), the points  $C_2$  and  $C_4$  are linearly unstable. Consequently, particles moving away from  $C_2$  or  $C_4$  rapidly collide with the body (Fig. 1), This problem is exacerbated in the case Haumea, because of its larger elongation,  $\epsilon \sim 0.43$ .

Particles with mean motion  $n$  and epicyclic frequency  $\kappa$  experience Lindblad Resonances (LRs) for

$$\kappa = m(n - \Omega), \quad m \text{ integer}. \quad (2)$$

The resonances occur either inside ( $m > 0$ ) or outside ( $m < 0$ ) the corotation radius (Fig. 1), assuming that the disk revolves in a prograde direction with respect to the spin of the body. Retrograde resonances are in general weaker<sup>14</sup> and would require a study of their own. Since  $\kappa \sim n$ , the relation above reads  $n/\Omega \sim m/(m-1)$ , referred to as a  $m/(m-1)$  LR. In a disk dense enough to support collective effects (self-gravity, pressure or viscosity), a  $m/(m-1)$  LR forces a  $m$ -armed spiral wave that receives a torque

$$\Gamma_m = \text{sign}(\Omega - n) \left( \frac{4\pi^2 \Sigma_0}{3n} \right) \left( \frac{GM}{\Omega R} \right)^2 \mathcal{A}_m^2. \quad (3)$$

This formula encapsulates in separate factors the sign of the torque, the physical parameters of the disk ( $n$  and its surface density  $\Sigma_0$ ) and of the perturber ( $M$ ,  $R$ ,  $\Omega$ ), and an intrinsic adimensional strength factor  $\mathcal{A}_m$ , see Methods. This generic formula applies in contexts as different as galactic dynamics<sup>15, 16</sup>, circum-stellar accretion disks<sup>17</sup>, proto-planetary disks<sup>18</sup> or planetary rings<sup>19, 20</sup>. Both the sign of the torque and its value are largely independent of the physics of the disk<sup>20</sup>, providing a robust estimation of  $\Gamma_m$  even without knowing the detailed processes at work.

Eq. (3) shows that the LRs cause the migration of the disk material away from the corotation. An annulus of width  $W$  and average radius  $a$  has most of its angular momentum  $H \sim 2\pi a W \Sigma_0 \sqrt{GMa} = 2\pi \Sigma_0 W (\Omega R^3/q)$  transferred to the body over a migration time scale

$$t_{\text{mig}} \sim \frac{H}{|\sum \Gamma_m|} = \frac{3q}{4\pi^2} \left( \frac{W}{R} \right) \left( \frac{T_{\text{rot}}}{\sum [(m-1)/m] \mathcal{A}_m^2} \right), \quad (4)$$

where  $T_{\text{rot}} = 2\pi/\Omega$  is the rotation period of the body. Note that the current angular momentum of Chariklo's rings is less than  $10^{-5}$  of that of the body<sup>1, 7</sup>. Even considering an initial disk one hundred times more massive, the reaction torque of the disk on the body has a negligible effect on Chariklo's rotation rate, with similar conclusions for Haumea.

We estimate  $t_{\text{mig}}$  for two annuli around Chariklo, one initially placed inside the corotation radius, and one placed outside. Fig. 2 shows that (i) a difference  $A - B$  as small as a kilometer ( $\epsilon \lesssim 0.01$ ) cause a rapid, decadal scale outward migration of the outer annulus; (ii) the resonances on the inner annulus are weaker, but  $t_{\text{mig}}$  remains geologically short ( $\lesssim$  Myr) for  $A - B \gtrsim 5$  km; (iii) even  $\sim 5$ -km topographic features are sufficient to induce migration time scales of a few Myr.

Numerical simulations can test those mechanisms. Global collisional codes have been run<sup>12</sup>, but with no torque appearing as the potentials considered were axisymmetric. Other local simulations do consider elongated bodies<sup>13</sup>, but not rotating, hampering again any torque. Here we performed numerical integrations using a simple Stokes-like friction acting on the particles,

$$\gamma_{\text{Stokes}} = -\eta \Omega \mathbf{v}_r, \quad (5)$$

where  $\mathbf{v}_r$  is the particle radial velocity and  $\eta$  is an adimensional friction coefficient. This friction dissipates energy while conserving angular momentum, thus being a good proxy for collisions at low computing cost. Fig. 3 shows results using  $\eta = 0.01$  (see Methods for the choice of this particular value). As mentioned earlier, the specific form of  $\gamma_{\text{Stokes}}$  and the value of  $\eta$  have little effects on the resonant torque  $\Gamma_m$ , when compared to more realistic situations including collisions and self-gravity.

We have checked numerically the dependence  $t_{\text{mig}} \propto \mathcal{A}_m^{-2}$  (Eq. 4). This permits to save computing time in the case of a mass anomaly by using  $\mu = 0.005$  (instead of  $\sim 10^{-5}$ ), hence speeding up migration time scales by a factor  $500^2 = 2.5 \times 10^5$ , an effect accounted for in the left panels of Fig. 3. In contrast, the integration shown in the right panels uses a realistic Chariklo's elongation  $\epsilon = 0.16$ , with no further corrections applied. Fig. 3 confirms our calculations, i.e. (i) the rapid infall of particles onto Chariklo's equator inside the corotation radius, (ii) the strong torques up to the 1/2 resonance, that pushes the disk material outwards.

A LR opens a cavity in the disk if  $\Gamma_m$  exceeds the viscous torque<sup>21</sup>  $\Gamma_\nu = 3\pi n a^2 \nu \Sigma_0$ , where the kinematic viscosity  $\nu = h^2 n$  is related to the ring thickness  $h$ , see Methods. From Eq. 3, we obtain

$$\left| \frac{\Gamma_m}{\Gamma_\nu} \right| \sim \frac{4\pi}{9q^{4/3}} \left( \frac{m-1}{m} \right)^{5/3} \left( \frac{R}{h} \right)^2 \mathcal{A}_m^2. \quad (6)$$

Using  $h = 10$  m (see Methods) and  $z = 5$  km we get  $|\Gamma_{-2}/\Gamma_\nu| \sim 3 \times 10^{-2}$  for  $m = -2$  (2/3 outer LR). Thus, a 5-km feature is too weak to open a cavity, but not by much owing to the steep dependence of  $\mathcal{A}_m^2 \propto \mu^2 \propto z^6$ . In contrast, the torque exerted by an ellipsoid with  $\epsilon = 0.16$  is overwhelming (by six orders of magnitude) at the 2/3 LR compared to  $\Gamma_\nu$ . Since  $\mathcal{A}_{-2} \propto \epsilon$  (see Methods), ellipsoids with  $A - B$  as small as 0.1 km are actually able to carve a cavity inside the 2/3 LR.

Chariklo and Haumea's elongations considered here are large enough to strongly perturb a ring near the 1/2 resonance, although no torque formula is available at that resonance in the ellipsoid case, because it is of second order nature (it must actually be noted 2/4 resonance, see Methods). This said, the final radius of the cavity depends on processes that are not considered here, since our friction law is an oversimplification of actual collisions. More importantly, accretion into satellites takes over as the Roche limit is approached, leading to complex ring-satellites interaction like shepherding. Nevertheless, our results show that either due to mass anomalies or body elongation, Chariklo and Haumea's rings cannot exist inside the 1/2 resonance radius  $a_{1/2} = 2^{2/3} a_{\text{cor}}$ , as observed.

In fact, the ring existence requires that a space exists between  $a_{1/2}$  and the Roche limit  $a_{\text{Roche}}$ , to prevent the ring accretion into satellites. From  $a_{\text{Roche}} \sim (3/\gamma)^{1/3} (M/\rho')^{1/3}$ , where  $\rho'$  is the density of the ring particles, and  $\gamma$  is a factor describing the particles<sup>22</sup>, the condition  $a_{1/2} < a_{\text{Roche}}$  reads

$$\gamma \rho' \lesssim \frac{3}{4} \frac{\Omega^2}{G}. \quad (7)$$

Thus, a non-axisymmetric body must rotate fast enough and/or the particles be underdense enough for a ring to exist. Although  $\gamma$  and  $\rho'$  are poorly known, we can consider the preferred value  $\gamma = 1.6$  that describes particles filling their lemon-shaped Roche lobes<sup>23</sup>, and  $\rho' \sim 450$  kg m<sup>-3</sup>, typical of the small moons orbiting near Saturn's rings<sup>24</sup>, and a good proxy of ring particle densities. Eq. (7) then requires rotation periods shorter than about 7 h, a condition met by both Chariklo and Haumea.

Our model predicts that the inner part of the disk may be deposited on the equator of the body, forming a ridge akin to that of the Saturnian satellite Iapetus. This ridge has been explained by the presence of a transient ring that rained down onto Iapetus' equator due to the torque from of a former subsatellite<sup>25, 26, 27</sup>. We offer the same explanation, except that the disk decay is now caused by the body itself. The infall time scales being of several years (Fig. 2), impact angles on the surface are very shallow, with impact velocities of a fraction km s<sup>-1</sup>, ensuring that the material piles up as a ridge

instead of forming craters. Future stellar occultations, predicted using accurate Gaia catalogs<sup>28</sup>, might detect such ridges on Chariklo, Haumea, or other new ringed objects.

In a broader and more speculative perspective, it is interesting to consider the orbital distribution of satellites of asteroids and Trans-Neptunian Objects. Supplementary Fig. 1 displays the histogram of the satellite orbital periods, expressed in units of the rotation periods of the primaries. Apart from a conspicuous peak corresponding to synchronous, tidally evolved orbits, this histogram indicates a clearing between the corotation radius and the outer 1/2 resonance, followed by a steady increase beyond this resonance. This distribution might be the signature of satellite formation proceeding from an initial collisional disk that has been pushed away by the resonant mechanism described here. □

## References

1. Braga-Ribas, F. *et al.* A ring system detected around the Centaur (10199) Chariklo. *Nature* **508**, 72–75 (2014).
2. Ortiz, J.L. *et al.* The size, shape, density and ring of the dwarf planet Haumea from a stellar occultation. *Nature* **550**, 219–223 (2017).
3. Ortiz, J.L. *et al.* Possible ring material around centaur (2060) Chiron. *Astron. Astrophys.* **576**, A18 (2015).
4. Esposito, L.W. Planetary rings. *Rep. Prog. Phys.* **65**, 1741–1783 (2002).
5. Leiva, R. *et al.* Size and Shape of Chariklo from Multi-epoch Stellar Occultations. *Astron. J.* **154**, 159 (2017).
6. Hedman, M.M. & Nicholson, P.D. More Kronoseismology with Saturn's rings. *Mon. Not. R. Astron. Soc.* **444**, 1369–1388 (2014).
7. Sicardy, B., El Moutamid, M., Quillen, A.C., Schenk, P., Showalter, M.R. & Walsh, K. in *Planetary ring systems* (eds Tiscareno, M. S. & Murray, C.D.) 135153 (Cambridge Univ. Press, 2018).
8. Pan, M. & Wu, Y. On the mass and origin of Chariklo's rings. *Astrophys. J.* **821**, 18 (2016).
9. Hyodo, R., Charnoz, S., Genda, H. & Ohtsuki, K. Formation of centaurs' rings through their partial tidal disruption during planetary encounters. *Astrophys. J. Letters* **828**, L8 (2016).
10. Araujo, R.A.N., Sfair, R. & Winter, O.C. The rings of Chariklo under close encounters with the giant planets. *Astrophys. J.* **824**, 80 (2016).
11. Wood, J., Horner, J., Hinse, T.C. & Marsden, S.C. The Dynamical History of Chariklo and Its Rings. *Astron. J.* **153**, 245 (2017).
12. Michikoshi, S. & Kokubo, E. Simulating the Smallest Ring World of Chariklo. *Astrophys. J. Letters* **837**, L13 (2017).
13. Gupta, A., Nadkarni-Ghosh, S. & Sharma, I. Rings of non-spherical, axisymmetric bodies. *Icarus* **199**, 97–116 (2018).
14. Morais, M.H.M. & Giuppone, C.A. Stability of prograde and retrograde planets in circular binary systems. *Mon. Not. R. Astron. Soc.* **424**, 52–64 (2012).
15. Goldreich, P. & Tremaine, S. The excitation of density waves at the Lindblad and corotation resonances by an external potential. *Astrophys. J.* **233**, 857–871 (1979).
16. Hopkins, P.F. & Quataert, E. An analytic model of angular momentum transport by gravitational torques: from galaxies to massive black holes. *Mon. Not. R. Astron. Soc.* **415**, 1027–1050 (2011).
17. Lin, D.N.C. & Papaloizou, J. Tidal torques on accretion discs in binary systems with extreme mass ratios. *Mon. Not. R. Astron. Soc.* **186**, 799–812 (1979).
18. Goldreich, P. & Tremaine, S. Disk-satellite interactions. *Astrophys. J.* **241**, 425–441 (1980).
19. Goldreich, P. & Tremaine, S. The dynamics of planetary rings. *Ann. Rev. Astron. Astrophys.* **20**, 249–283 (1982).
20. Meyer-Vernet, N. & Sicardy, B. On the physics of resonant disk-satellite interaction. *Icarus* **69**, 157–175 (1987).
21. Lynden-Bell, D. & Pringle, J.E. The evolution of viscous discs and the origin of the nebular variables. *Mon. Not. R. Astron. Soc.* **168**, 603–637 (1974).

22. Tiscareno, M.S., Hedman, M.M., Burns, J.A. & Castillo-Rogez, J. Compositions and Origins of Outer Planet Systems: Insights from the Roche Critical Density. *Astrophys. J. Letters* **765**, L28 (2013).
23. Porco, C.C., Thomas, P.C., Weiss, J.W. & Richardson, D.C. Saturn's small inner satellites: clues to their origins. *Science* **318**, 1602 (2007).
24. Thomas, P.C. Sizes, shapes, and derived properties of the saturnian satellites after the Cassini nominal mission. *Icarus* **208**, 395–401 (2010).
25. Ip, W.-H. On a ring origin of the equatorial ridge of Iapetus. *Geophys. Res. Letters* **33**, L16203 (2006).
26. Levison, H.F., Walsh, K.J., Barr, A.C. & Dones, L. Ridge formation and de-spinning of Iapetus via an impact-generated satellite. *Icarus* **214**, 773–778 (2011).
27. Dombard, A.J., Cheng, A.F., McKinnon, W.B. & Kay, J.P. Delayed formation of the equatorial ridge on Iapetus from a subsatellite created in a giant impact. *J. Geophys. Res.* **117**, E03002 (2012).
28. Camargo, J.I.B. *et al.* The future of stellar occultations by distant solar system bodies: Perspectives from the Gaia astrometry and the deep sky surveys. *Plan. Sp. Sc.* **154**, 59–62 (2018).
29. Johnson, W.R. Asteroids/TNOs with Satellites: Summary Data (2018).  
<http://www.johnstonsarchive.net/astro/astmoontable.html>
30. Balmino, G. Gravitational potential harmonics from the shape of an homogeneous body. *Celestial Mechanics and Dynamical Astronomy* **60**, 331–364 (1994).
31. Boyce, W. Comment on a Formula for the Gravitational Harmonic Coefficients of a Triaxial Ellipsoid. *Celestial Mechanics and Dynamical Astronomy* **67**, 107–110 (1997).
32. Dermott, S.F. & Murray, C.D. The dynamics of tadpole and horseshoe orbits. I. Theory. *Icarus* **48**, 1–11 (1981).
33. Schmidt, J. *et al.* Dynamics of Saturn's dense rings. in Saturn from Cassini-Huygens (eds Dougherty, M.K., Esposito, L.W. & Krimigis S.M.) 413–458 (Springer, 2009).
34. Murray, C.D. & Dermott, S.F. Solar system dynamics Cambridge University Press (1999).
35. Fornasier, S. *et al.* The Centaur 10199 Chariklo: investigation into rotational period, absolute magnitude, and cometary activity. *Astron. Astrophys.* **568**, L11 (2014).
36. Lellouch, E. *et al.* "TNOs are cool": a survey of the trans-Neptunian region. II. The thermal lightcurve of (136108) Haumea. *Astron. Astrophys.* **518**, L147 (2010).
37. Ragozzine, D. & Brown, M.E. Orbits and masses of the satellites of the dwarf planet Haumea (2003 EL<sub>61</sub>). *Astron. J.* **137**, 4766–4776 (2009).

Table 1 - Chariklo and Haumea's adopted parameters<sup>(a)</sup>

Parameters	Chariklo	Haumea
Rotation period, $T_{\text{rot}}$ (h) (refs. 35, 36)	7.004	3.915341
Mass $M$ (kg) (refs. 5, 37)	$6.3 \times 10^{18}$	$4.006 \times 10^{21}$
Rotational parameter $q^{(b)}$	0.226	0.268
Semi-axes $A \times B \times C$ (km) (refs. 5, 2)	$157 \times 139 \times 86$	$1161 \times 852 \times 513$
Reference radius $R^{(c)}$ (km)	115	712
Elongation parameter $\epsilon = (A - B)/R$	0.16	0.43
Height of topographic feature $z^{(d)}$ (km)	5	n.a.
Corotation radius $a_{\text{cor}}^{(e)}$ (km)	189	1104
Outer 1/2 (or 2/4) resonance radius $a_{1/2}^{(f)}$ (km)	300	1752
Classical Roche limit $a_{\text{Roche}}^{(g)}$ (km)	280	2400
Ring radii (km) (refs. 1, 2)	390 and 405	2287

<sup>(a)</sup> No error bars are considered here, the adopted parameters being representative of typical cases examined in the Main Text.

<sup>(b)</sup> See Eq. (1).

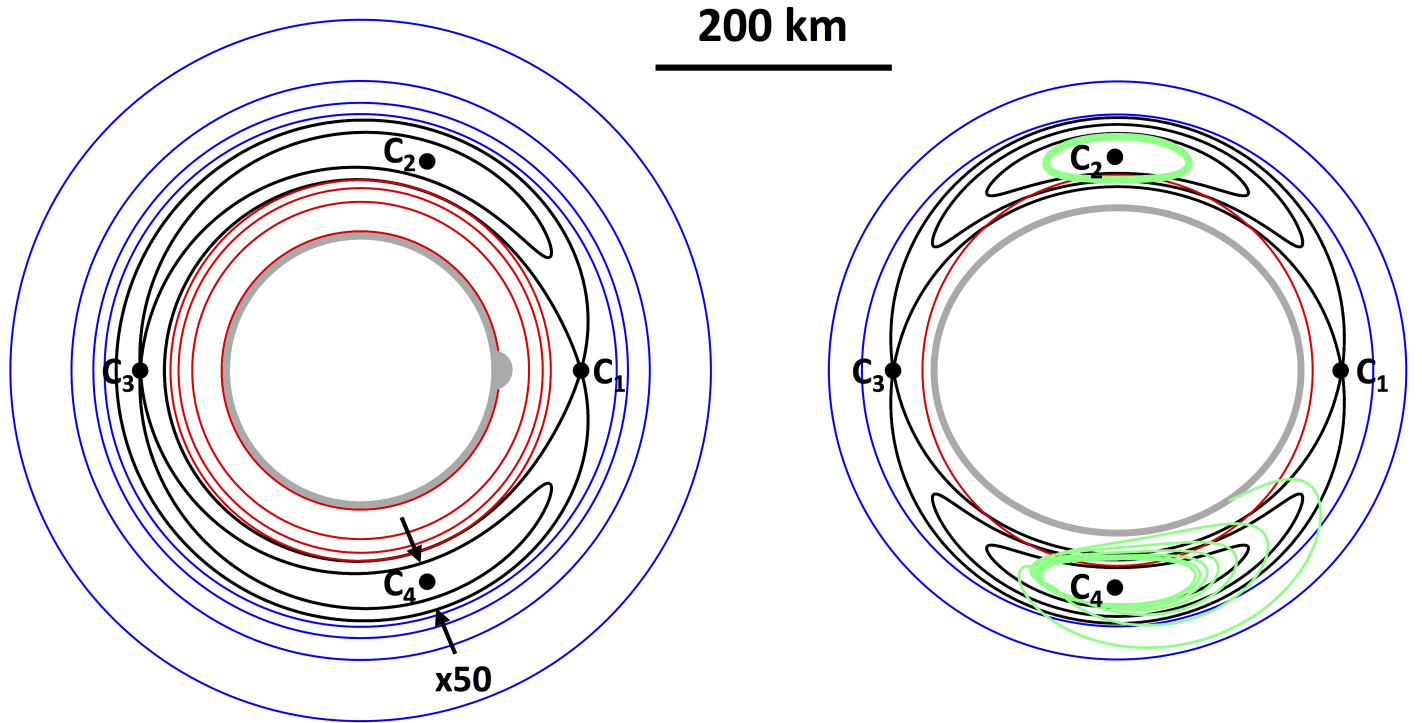
<sup>(c)</sup> Defined as  $R = \sqrt{3}(1/A^2 + 1/B^2 + 1/C^2)^{-1/2}$ , see Methods Eq. (12).

<sup>(d)</sup> Assuming a spherical body of radius  $R_{\text{sph}} = 129$  km (ref. 5). This corresponds to a mass anomaly  $\mu \sim (z/2R_{\text{sph}})^3 \sim 10^{-5}$ .

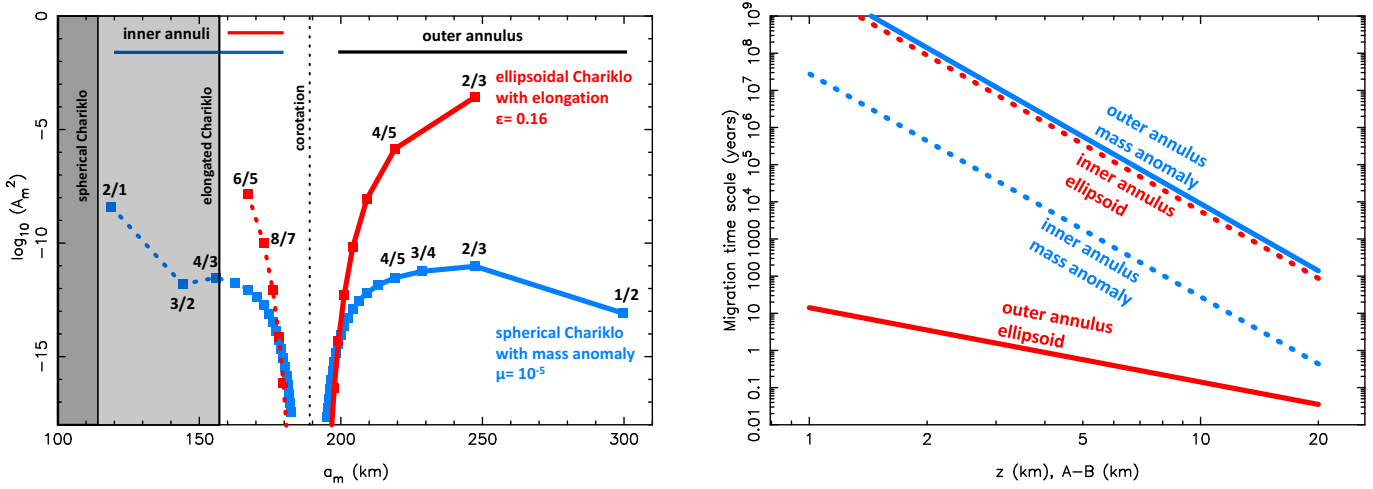
<sup>(e)</sup> Using  $a_{\text{cor}} = Rq^{-1/3}$ , from Eq. (1) and Kepler's third law.

<sup>(f)</sup> Using  $a_{1/2} = 2^{2/3}a_{\text{cor}}$ , from Kepler's third law.

<sup>(g)</sup> Using the classical expression  $a_{\text{Roche}} \sim (3/\gamma)^{1/3}(M/\rho')$ , with  $\gamma = 0.85$  and icy ring particles with density  $\rho' = 1000$  kg m<sup>-3</sup>. More realistic values of  $\gamma$  and  $\rho'$  are discussed in the text.

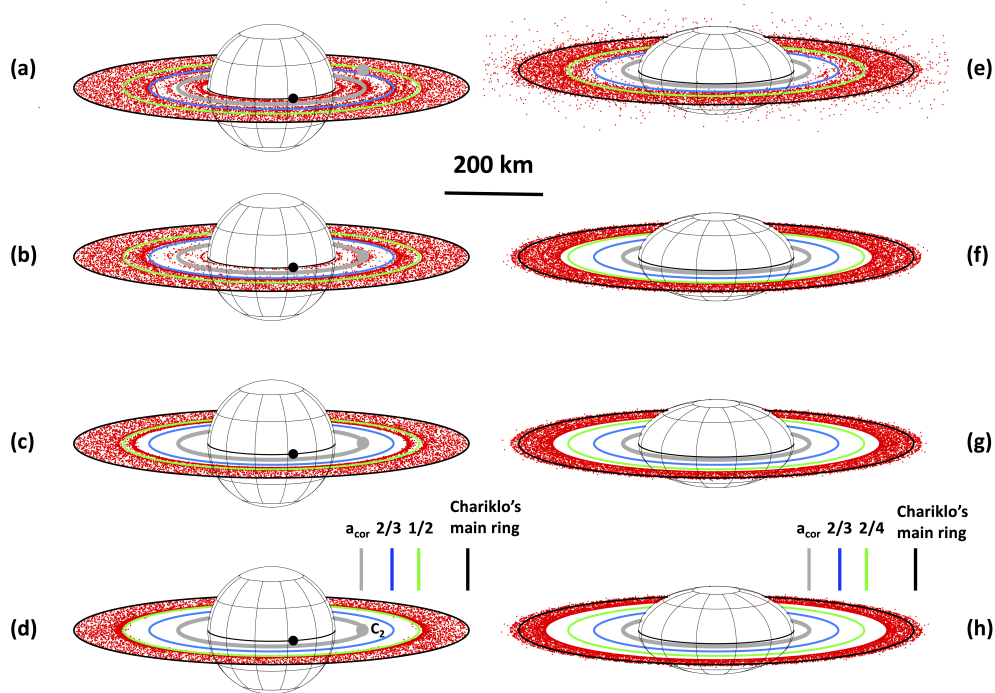


**Figure 1 | Corotation and Lindblad resonances around Chariklo.** In both panels (topographic feature on the left, elongated body on the right), the red (resp. blue) circles correspond to inner (resp. outer)  $m/(m - 1)$  Lindblad resonance (LR) radii with  $m > 0$  (resp.  $m < 0$ ), see Eq. (2). The black lines show isopotential curves in a frame corotating with Chariklo, and the gray lines outline the limb of the body. The dots  $C_1, \dots, C_4$  mark the corotation fixed points. The points  $C_2$  and  $C_4$  are local potential maxima and are linearly stable as long as the mass anomaly or the elongation of the body are not too large, see Methods. Left - A topographic feature of height  $z = 5$  km (gray half dome, not on scale) is sitting at the surface a body of radius  $R_{\text{sph}} = 129$  km, and corresponds to a mass anomaly  $\mu \sim 10^{-5}$ . For better viewing, the isopotential black lines have been radially stretched by a factor of 50 with respect to the corotation radius. A few inner ( $m = 2, 3, 4, 5$ ) and outer ( $m = -1, -2, -3, -4$ ) LR radii are shown. Right - The same for a Chariklo shape solution with elongation  $\epsilon = 0.16$ . The limb of the body and the isopotential lines are plotted on scale. Only LRs with  $m$  even are now allowed, the inner corresponding to  $m = 6$ , and the two outer ones corresponding to  $m = -2, -4$ . The green curve around  $C_2$  is typical of the widest possible closed orbit in the absence of friction. The green orbit around  $C_4$  is an example of escape, using initially the same orbit as around  $C_2$ , but with a radial friction coefficient  $\eta = 0.01$  (Eq. 5). The orbit then becomes rapidly unstable, yielding a collision with Chariklo.



**Figure 2 | Torques intensities at Lindblad Resonances around Chariklo and migration time scales.** Left - The adimensional coefficients  $\mathcal{A}_m^2$  providing the torque value at  $m/(m-1)$  Lindblad resonances (Eq. 3) vs. the resonant radii on each side of the corotation radius (dotted line). The values of  $\mathcal{A}_m$  are evaluated from Supplementary Table 1, using a Chariklo equatorial topographic feature of height  $z = 5$  km, corresponding to a mass anomaly  $\mu = 10^{-5}$  (blue squares), or a difference of semi-axes  $A - B = 18$  km (Table 1), corresponding to an elongation parameter  $\epsilon = 0.16$  (red squares). Note the steep decrease of the torques as the corotation radius is approached, due to the exponential decrease of  $\mathcal{A}_m^2$  as  $|m|$  increases, see Methods. The light gray region at left encloses Chariklo's largest semi-axis  $A = 157$  km, inside which particles collide with the body in the ellipsoidal case, while the dark gray region encloses Chariklo's radius  $R_{\text{sph}} = 129$  km in the spherical case (Table 1). Right - Solid lines: migration times (Eq. 4) of an outer annulus of width 100 km that extends outside the corotation (see left panel), either due to the topographic features (blue) of heights  $z$  or ellipsoids with various  $A - B$  (red). Dotted lines: the same for an inner annulus of width 20 km in the ellipsoidal case, and 60 km in the spherical case, see left panel.





**Figure 3 | Migration of ring particles around Chariklo.** The particles are submitted to Chariklo's gravitational field (topographic feature on the left, elongated body on the right), plus a radial Stokes-like friction with  $\eta = 0.01$  (Eq. 5). The radii of the corotation point  $C_2$  ( $a_{\text{cor}}$ ), the 2/3 and 1/2 outer Lindblad Resonances (LRs) between the particle mean motions and Chariklo's rotation period are marked at the bottom, together with the location of Chariklo's main ring<sup>1</sup> C1R. The left panels show the effect of an equatorial topographic feature (black dot) with mass  $\mu = 5 \times 10^{-3}$  relative to Chariklo. Initially, 701 particles are regularly placed between  $0.7a_{\text{cor}}$  and  $2.2a_{\text{cor}}$ . In all panels, each particle is plotted over twenty regular time steps spanning 40,000 years. Panel (a): after 40,000 years, the clearing of the corotation region is ongoing; (b) after  $2.5 \times 10^5$  years, some particles remain near  $C_2$ , while others are pushed outside the 2/3 LR; (c) after  $2.5 \times 10^6$  years, all the particles inside the corotation radius and near  $C_2$  have collided with Chariklo; (d) after  $6.3 \times 10^6$  years, all the remaining particles are now outside the 1/2 LR. Right panels: effect of an ellipsoid with elongation  $\epsilon = 0.16$ , displayed with its longest axis face on. The particles now start between  $1.1a_{\text{cor}}$  and  $2.2a_{\text{cor}}$  (particles inside  $1.1a_{\text{cor}}$  collide with Chariklo after a few days). Panel (e): after three months, most of the particles have been pushed outside the 2/3 LR; (f), (g) and (h): after one, five and twelve years, respectively, all the particles have either collapsed onto Chariklo, or continue their outward migration at decreasing pace outside of the 2/4 resonance. Note that time scales of same order (but shorter) would be obtained for particles orbiting around Haumea, which has a larger elongation  $\epsilon = 0.43$ .

## Methods

We calculate the potential outside a body in two simple cases: a topographic feature located at the equator of a spherical object and a homogeneous triaxial ellipsoid. Calculations are restricted to the equatorial plane of the body, where a collisional disk is expected to settle.

### Topographic feature

We consider a spherical body of mass  $M$ , radius  $R_{\text{sph}}$  and center  $\mathcal{C}$  with an equatorial topographic feature of mass  $\mu$  relative to the mass of the body, that rotates with period  $T_{\text{rot}}$  and angular velocity  $\Omega = 2\pi/T_{\text{rot}}$ . We denote  $O$  the center of mass of the body plus the topographic feature,  $\mathbf{r}$  the position vector of the particle, measured from the center of the body,  $r = \|\mathbf{r}\|$ ,  $\theta = L - L_A$ , where  $L$  is the true longitude of the particle, and  $L_A = \Omega t$  is the orientation angle of the topographic feature, counted from an arbitrary origin. Finally,  $\mathbf{R}$  is the vector that connects the center of mass  $O$  to the topographic feature and  $\Delta = \mathbf{r} - \mathbf{R}$ . The potential acting on the particle at position  $\mathbf{r}$  in a frame fixed at  $\mathcal{C}$  is:

$$U(\mathbf{r}) = -\frac{GM}{r} - \mu \frac{GM}{\Delta} + \Omega^2(\mathcal{C}\mathbf{O} \cdot \mathbf{r}), \quad (8)$$

where the last term is the indirect part stemming from the motion of  $\mathcal{C}$  around  $O$ . Using  $\mathcal{C}\mathbf{O} = \mu\mathbf{R}$  and the definition of the rotational parameter  $q$  (Main Text Eq. 1), we obtain

$$U(\mathbf{r}) = -\frac{GM}{r} - GM\mu \left[ \frac{1}{\Delta} - q \frac{\mathbf{R} \cdot \mathbf{r}}{R_{\text{sph}}^3} \right] = \quad (9)$$

$$-\frac{GM}{r} - \frac{GM}{2R_{\text{sph}}} \mu \left\{ \left[ \sum_{m=-\infty}^{+\infty} b_{1/2}^{(m)}(r/R_{\text{sph}}) \cos(m\theta) \right] - 2q \left( \frac{r}{R_{\text{sph}}} \right) \cos(\theta) \right\},$$

where the  $b_{1/2}^{(m)}$ 's are the classical Laplace coefficients.

### Homogeneous triaxial ellipsoid

We now consider a homogeneous triaxial ellipsoid of mass  $M$  and semi-axes  $A > B > C$ . The potential  $U(\mathbf{r})$  can again be expanded in a series in  $\cos(m\theta)$ , but with only even values of  $m$  to ensure the invariance of the potential under a rotation of  $\pi$  radians. Thus, posing  $m = 2p$ ,

$$U(\mathbf{r}) = \sum_{p=-\infty}^{+\infty} U_{2p}(r) \cdot \cos(2p\theta). \quad (10)$$

A closed form of  $U(\mathbf{r})$  outside the body, depending on  $A$ ,  $B$  and  $C$ , can be derived:

$$U_{2p}(r) = -\frac{GM}{r} \sum_{l=|p|}^{+\infty} \left( \frac{R}{r} \right)^{2l} Q_{2l,2|p|}, \quad (11)$$

where  $R$  is a reference radius defined by

$$\frac{3}{R^2} = \frac{1}{A^2} + \frac{1}{B^2} + \frac{1}{C^2} \quad (12)$$

and<sup>30, 31</sup>:

$$Q_{2l,2|p|} = \frac{3}{2^{l+2|p|}(2l+3)} \frac{(2l+2|p|)!(2l-2|p|)!l!}{(l+|p|)!(l-|p|)!(2l+1)!} \times \sum_{i=0}^{\text{int}\left(\frac{l-|p|}{2}\right)} \frac{1}{16^i} \frac{\epsilon^{|p|+2i}}{(|p|+i)!i!} \frac{f^{l-|p|-2i}}{(l-|p|-2i)!}, \quad (13)$$

The adimensional parameters  $\epsilon$  and  $f$  measure the elongation and oblateness of the body, respectively:

$$\epsilon = \frac{A-B}{R} \quad \text{and} \quad f = \frac{A-C}{R}, \quad (14)$$

with  $A' = \sqrt{(A^2 + B^2)}/2$ . The term  $Q_{2l,2|p|}$  is of order  $l$  in  $(\epsilon f)$ . For evaluating the effect of Lindblad resonances, it is enough to consider the term of lowest order in  $R/r$  in Eq. (11), corresponding to  $l = |p|$ . Defining the sequence  $S_{|p|} = Q_{2|p|,2|p|}/\epsilon^{|p|}$  and from  $m = 2p$ , we obtain

$$U(\mathbf{r}) = -\frac{GM}{r} \sum_{m=-\infty}^{+\infty} \left(\frac{R}{r}\right)^{|m|} S_{|m/2|} \epsilon^{|m/2|} \cos(m\theta) \quad (m \text{ even}), \quad (15)$$

where  $S_{|p|}$  is recursively given by

$$S_{|p|+1} = 2 \frac{(|p| + 1/4)(|p| + 3/4)}{(|p| + 1)(|p| + 5/2)} \times S_{|p|} \quad \text{with } S_0 = 1. \quad (16)$$

The potential (15) has been implemented in numerical schemes to integrate the motion of particles around an elongated body (adding the Stokes-like friction of Main Text Eq. 5). We have truncated the expansion of the potential above  $|m| > 10$ , which is justified by the fact that the resonance strength rapidly decreases as  $m$  increases (Fig. 2).

### Corotation resonance

The potential near the corotation radius  $a_{\text{cor}}$ , as observed in a frame corotation with the body, is

$$V(\mathbf{r}) = U(\mathbf{r}) - \frac{\Omega^2 r^2}{2} \sim -\frac{3}{2} \Omega^2 a_{\text{cor}}^2 \left(\frac{\Delta r}{a_{\text{cor}}}\right)^2 - \frac{GM}{a_{\text{cor}}} f(\theta) = -\frac{3}{2} \Omega^2 a_{\text{cor}}^2 \left(\frac{\Delta r}{a_{\text{cor}}}\right)^2 - \frac{\Omega^2 R^2}{q^{2/3}} f(\theta), \quad (17)$$

where the azimuthal function  $f(\theta)$  is given in Supplementary Table 1, and  $\Delta r = r - a_{\text{cor}} \ll a_{\text{cor}}$ . Examples of isopotential levels for the two cases examined here are displayed in Fig. 1. Note that the corotation points associated with the mass anomaly mimic the Lagrange points  $L_1, \dots, L_5$ , except that  $L_1$  and  $L_2$  have merged into a single saddle point  $C_1$  where the potential remains finite. Also, the points  $C_2$  and  $C_4$  are close to but not at 60 degrees from  $C_1$ . That angle actually depends on  $q$  (see Supplementary Table 1) and is close to 70 deg in the particular example displayed in Fig. 1.

Near  $a_{\text{cor}}$ , the particles follow the trajectory  $(3/8)(\Delta r/a_{\text{cor}})^2 + f(\theta) \sim \text{constant}$ . They are nothing else than the level curves of the potential in a frame corotating with the body (Fig. 1), except for a dilation by a factor two with respect to  $a_{\text{cor}}$  (ref. 32). The full radial width of the trajectory is then  $W_{\text{cor}} = 2\sqrt{8\Delta f/3}$ , where  $\Delta f = f_{\text{max}} - f_{\text{min}}$  is the total variation of  $f(\theta)$  over  $[0, 2\pi]$ .

For order of magnitude considerations, we note that  $\Delta f \sim \mu$  for the case of the mass anomaly. In the case of the ellipsoid, and for sake of estimation, we can simplify the expression (15) further by taking the lowest orders  $p = 0$  and  $|p| = 1$ , i.e. (noting that  $S_1 = 0.15$ )

$$V(\mathbf{r}) \sim -\frac{3}{2} \Omega^2 a_{\text{cor}}^2 \left(\frac{\Delta r}{a_{\text{cor}}}\right)^2 - \frac{3}{10} R^2 \Omega^2 \epsilon \cos(2\theta), \quad (18)$$

so that  $f(\theta) \sim (3/10)q^{2/3}\epsilon \cos(2\theta)$  and thus  $\Delta f \sim (3/5)q^{2/3}\epsilon$ , from which we derive

$$W_{\text{cor},\mu} \sim 4Rq^{-1/3} \sqrt{\frac{2}{3}\mu} \quad \text{and} \quad W_{\text{cor},\epsilon} \sim 4R \sqrt{\frac{2}{5}\epsilon}. \quad (19)$$

in each of the two cases examined here. For a typical Chariklo topographic feature ( $\mu \sim 10^{-5}$ ), we obtain a narrow corotation region with  $W_{\text{cor},\mu} \sim 2$  km only, while for  $\epsilon \sim 0.16$ ,  $W_{\text{cor},\epsilon} \sim 115$  km, meaning that the corotation region fills in all the space between  $a_{\text{cor}}$  and Chariklo's surface (Fig. 1).

If ring arcs are present near  $C_2$  and  $C_4$ , they should be destroyed by viscous spreading time scales  $t_{\text{spread}} \sim W_{\text{cor}}^2/\nu$ , where  $\nu$  is the kinematic viscosity. This quantity can be parametrized as  $\nu = h^2 n$ , where  $h$  typically represents, for a dense disk, the size of the largest particles, or equivalently, the thickness of the ring<sup>33</sup>. The local velocity field in Chariklo or Haumea's rings are comparable to those of Saturn<sup>1</sup>. Consequently, the collisional physics in those systems is expected to be similar<sup>7</sup>, i.e.  $h \sim$

10 meters (ref. 33). From the expressions of  $W_{\text{cor}}$  derived above, we obtain  $t_{\text{spread},\mu} \sim 2\mu(R/h)^2 T_{\text{rot}}$  for a mass anomaly  $\mu$ , and  $t_{\text{spread},\epsilon} \sim \epsilon(R/h)^2 T_{\text{rot}}$  for an ellipsoid. With  $\mu \sim 10^{-5}$ , we obtain very short escape times (a few years) of the arc material from the corotation region, if caused by a mass anomaly. The spreading time is longer, some  $10^4$  years, but still geologically short if the corotation is controlled by an ellipsoid with elongation  $\epsilon \sim 0.16$ .

The corotation points  $C_2$  and  $C_4$  are linearly unstable if the potential  $V(\mathbf{r})$  meets the condition

$$\left(4\Omega^2 + V_{xx} + V_{yy}\right)^2 \leq V_{xx}V_{yy} - V_{xy}^2, \quad (20)$$

where the indices  $x$  and  $y$  are short-hand notations for partial derivatives<sup>34</sup>.

For the classical  $L_4$  and  $L_5$  points (corresponding to  $q = 1$ ), this condition leads to the Gascheau-Routh criterium  $\mu > 0.0385\dots$ . For the cases examined here,  $q$  is smaller than but of order unity, so that the critical value of  $\mu$  remains close to 0.04. This value is safely avoided for Chariklo, as it would correspond to an unrealistic feature with  $z = 80$  km.

In the case of the ellipsoid, it is found from Eqs. (18) and (20) that  $C_2$  and  $C_4$  are unstable for:

$$\epsilon > \epsilon_{\text{crit}} \sim \frac{0.06}{q^{2/3}}. \quad (21)$$

Using  $q = 0.226$  for Chariklo implies  $\epsilon_{\text{crit}} \sim 0.16$ , which is close to Chariklo's adopted elongation (Table 1), making the points  $C_2$  and  $C_4$  marginally unstable, see Main Text. Haumea's elongation  $\epsilon = 0.43$  is well beyond the critical value, making  $C_2$  and  $C_4$  highly unstable.

### Lindblad resonances

A particle revolving around the central body is submitted to a potential of generic form

$$U(\mathbf{r}) = \sum_{-\infty}^{+\infty} U_m(r) \cdot \cos(m\theta) = \sum_{-\infty}^{+\infty} U_m(r) \cdot \cos[m(L - \lambda_A)]. \quad (22)$$

The quantities  $r$  and  $L$  can be expressed in terms of the keplerian elements of the particle,  $a$ ,  $e$ ,  $\lambda$  and  $\varpi$  (semi-major axis, eccentricity, mean longitude and longitude of periape, respectively). In doing so, terms with frequency  $j\kappa - m(n - \Omega)$  appear in the expansion of  $U(\mathbf{r})$ , where  $\kappa$  is the particle horizontal epicyclic frequency and  $j$  is a non negative integer. Each term for which

$$j\kappa \sim m(n - \Omega) \quad (j \text{ integer} > 0) \quad (23)$$

describes a resonance between the mean motion of the particle and the spin rate of the body. For bodies close to spherical, we have  $\kappa \sim n$ , and the condition above reads

$$\frac{n}{\Omega} \sim \frac{m}{m - j}, \quad (24)$$

referred to as an orbit-spin  $m/(m - j)$  resonance. In the case of an ellipsoid, the potential (15) contains only even terms of the form  $2p\theta$ , so that the only resonances encountered have the form

$$\frac{n}{\Omega} \sim \frac{2p}{2p - j}. \quad (25)$$

The classical d'Alembert's rule implies that the term responsible for the  $m/(m - j)$  resonance is of order  $e^j$ . Consequently, the strongest resonances are those with  $j = 1$ , and are classically referred to as first order Lindblad Eccentric Resonances, or simply Lindblad Resonances (LRs).

The corresponding terms in the expansion of  $U(\mathbf{r})$  are easily obtained by using the first order expansions,  $r \approx a - ae \cos(\lambda - \varpi)$  and  $L \approx \lambda + 2e \sin(\lambda - \varpi)$ . Introducing them into  $\sum_{-\infty}^{+\infty} U_m(r) \cdot \cos[m(L - \lambda_A)]$ , we obtain to first order in eccentricity

$$U(\mathbf{r}) = \sum_{k=-\infty}^{+\infty} U_k(a) \cdot \cos[k(\lambda - \lambda_A)] - e \sum_{m=-\infty}^{+\infty} A_m(a) \cdot \cos(\phi_m), \quad (26)$$

where  $\phi_m = m\lambda_A - (m-1)\lambda - \varpi$  is the resonant angle associated with the  $m/(m-1)$  LR and

$$A_m(a) = [2m + a(d/da)] U_m(a). \quad (27)$$

In order to separate the effects of the physical parameters of the body  $(\Omega, R, q)$  and the effects of the resonances *per se*, we define a new adimensional coefficient  $\mathcal{A}_m = -(q/\Omega^2 R^2) A_m$ , so that the potential can eventually be splitted into a corotation and a Lindblad resonance part,

$$U(\mathbf{r}) = \sum_{k=-\infty}^{+\infty} U_k(a) \cdot \cos[k(\lambda - \lambda_A)] + e \frac{\Omega^2 R^2}{q} \sum_{m=-\infty}^{+\infty} \mathcal{A}_m(a) \cdot \cos(\phi_m). \quad (28)$$

The coefficients  $\mathcal{A}_m$  are obtained from Eqs. 9, 15 and 27, and are listed in Supplementary Table 1. For large values of  $|m|$ ,  $\mathcal{A}_m$  has the exponential behaviour  $\mathcal{A}_m \propto K^{|m|}$  ( $K$  being a constant depending on the problem considered). Using Chariklo's parameters ( $q = 0.226$ ), we obtain asymptotically that  $\mathcal{A}_m \propto 0.54^{|m|} \mu \propto 0.54^{|m|} z^3$  in the mass anomaly case, and  $\mathcal{A}_m \propto (1.93\epsilon q^{2/3})^{|m/2|} = (0.72\epsilon)^{|m/2|}$  in the ellipsoid case. Note that  $m$  is even in the latter case and that the  $2/3$  LR is the strongest of all (Fig. 2), with  $\mathcal{A}_{-2} \propto \epsilon$ .

### Choice of the friction coefficient

Main Text Eq. (5) introduces an adimensional friction coefficient  $\eta$  that quantifies the drag applied to the ring particles in our numerical integrations. Note that we do not consider any other forces acting on the particles, such as radiation pressure or Poynting-Roberston (PR) drag. This is justified by the fact that both Chariklo and Haumea's rings probably contain mainly cm- to m-sized particles, which are stable against PR drag over hundreds of millions years<sup>1,7</sup>. This said, the choice of  $\eta$  is rather arbitrary as it does not enter in the expression of the torque  $\Gamma_m$  (Main Text Eq. 3). However, it does define the typical width  $\Delta a_m$  of the  $m/(m-1)$  LR, defined as the region over which most of the torque  $\Gamma_m$  is deposited around the resonance radius  $a_m$ . In order to be as realistic as possible about  $\Delta a_m$ , we choose the value of  $\eta$  to match the expected disk properties.

Following the formalism of ref. 20, the adimensional width  $\alpha = \Delta a_m/a_m$  of the resonance is determined by the dominant physical process at work in the disk, which can be self-gravity, viscosity or pressure. In the self gravity case,  $\alpha$  is given by

$$\alpha_G = \sqrt{\frac{2\pi|m-1|G\Sigma_0}{3m^2\Omega^2 a_m}}, \quad (29)$$

where  $G$  is the gravitational constant and  $\Sigma_0$  is the disk surface density. Using  $\Sigma_0 = 500\text{-}1000 \text{ kg m}^{-2}$  (ref. 7), a rotation period of 7 h (Table 1), we obtain a typical value  $\alpha_G \sim 2 \times 10^{-3}$ .

If viscosity prevails, then  $\alpha$  takes the form  $\alpha_\nu = [7\nu/(9|m|\Omega a_m^2)]^{1/3} = [7/(9|m-1|)]^{1/3} (h/a_m)^{2/3}$ . Taking  $h \sim 10 \text{ m}$  and a typical  $a_m \sim 250 \text{ km}$ , we obtain  $\alpha_\nu \lesssim 10^{-3}$ , with similar values if the disk is pressure-dominated. This shows that Chariklo's rings are likely to be dominated by self-gravity near LRs. Finally, the coefficient  $\alpha$  associated with a Stokes-like force as in Main Text Eq. (5) is  $\alpha_\eta = 2\eta/3|m|$ , so that  $\eta \sim 0.01$  provides a realistic estimation of the LR widths in Chariklo's rings. The same exercise can be performed for Haumea's rings, yielding smaller values of  $\eta$ , since both the spin rate  $\Omega$  and the radii  $a_m$  are larger in this case. However, the orders of magnitude remain the same and the main conclusions of this work are not altered.

### Higher order resonances

Besides the first order LRs considered in the Main Text (Eq. 2), higher order  $n/\Omega = m/(m-j)$  resonances appear, corresponding to  $j > 1$ . Being of order  $e^j$ , they are weaker than the LRs. Nevertheless, they may have significant effects in the ellipsoidal case, owing to the large values of Chariklo and Haumea's elongation parameters  $\epsilon$ .

In that case, combining d'Alembert's rule and Eq. (15), we see that a  $m/(m-j)$  resonance is of global order  $e^j \epsilon^{|m/2|}$  ( $m$  even). For instance, while the outer  $1/2$  (first order) LR appears in the case

of a mass anomaly, it only exists in its second order version 2/4 ( $m = -2, q = 2$ ) in the case of the ellipsoid. Similarly, the outer 1/3 LR appears in its second order version with a mass anomaly, but only in its fourth order version 2/6 ( $m = -2, q = 4$ ) when caused by an ellipsoid.

To our knowledge, no evaluation of the torque exerted at a  $m/(m-j)$  resonance with  $j > 1$  has been published. There are two reasons for that. First, the hydrodynamical equations describing the disk must be expanded to  $j^{\text{th}}$  order in the perturbations, a challenging task. Second, such resonances cause streamline self-crossings. It can be shown (Sicardy et al. 2018, in preparation) that near a  $m/(m-j)$  resonance, where  $m$  and  $j$  are *relatively prime*, a periodic resonant streamline has  $j$  braids with  $|m|(j-1)$  self-crossing points. This creates singularities in the hydrodynamical equations (shocks), even for vanishingly small perturbations, thus requiring new kinds of treatments.

This said, we see that although the 2/4 resonance (ellipsoid case) is of second order in the particle eccentricity, it does *not* induce self-crossing streamlines since the ratio 2/4 can be reduced to 1/2, resulting in  $|-1|(1-1) = 0$  self-crossing points. Still, as mentioned above, no expression of the resonance torque exists because of the second-order nature of that resonance. A general behaviour can nevertheless be sketched. At second order in eccentricity, Eq. (28) is replaced by

$$U(\mathbf{r}) = \sum_{k=-\infty}^{+\infty} U_k(a) \cdot \cos[k(\lambda - \lambda_A)] + e^2 \frac{\Omega^2 R^2}{q} \sum_{m=-\infty}^{+\infty} \mathcal{B}_m(a) \cdot \cos(2\phi_m), \quad (30)$$

where now  $\phi_m = [m\lambda_A - (m-2)\lambda - 2\varpi]/2$  (with  $m$  even). The expression of  $\mathcal{B}_m$  can be retrieved from ref. 34. It involves the operator

$$f_{45} = \frac{1}{8} \left[ (4m^2 - 5m) + 2(2m-1)a \frac{d}{da} + a^2 \frac{d^2}{da^2} \right] \quad (31)$$

that must be applied to each term of the expansion given in Eq. (15). This provides

$$\mathcal{B}_m(a) = -\frac{1}{4} S_{|m/2|} \epsilon^{|m/2|} \left[ (4m^2 - 5m) - 2(2m-1)(|m|+1) + (|m|+1)(|m|+2) \right] \left( \frac{R}{a} \right)^{|m|+1}, \quad (32)$$

which reduces to  $\mathcal{B}_{-2} = -2.55(R/a_{1/2})^3 \epsilon$ , where  $a_{1/2}$  is the radius of exact 1/2 resonance. The phase portrait of this resonance is found in various works (e.g. ref. 34). Posing  $X = e \cos(\phi_{2/4})$  and  $Y = e \sin(\phi_{2/4})$  ( $\phi_{2/4} = 2\lambda - \lambda_A - \varpi$ ), it can be shown that the origin  $(X, Y) = (0, 0)$  is always a fixed point. It is stable, except for a narrow interval of initial semi-major axes  $a_{1/2}(1 - 0.25\epsilon) \lesssim a \lesssim a_{1/2}(1 + 0.25\epsilon)$ , the coefficient  $\sim 0.25$  stemming from the particular values of  $R$  and  $q$  used here. In that interval (of width  $\sim 25$  km for Chariklo and  $\sim 375$  km for Haumea, from Main Text Table 1), the origin  $(X, Y) = (0, 0)$  is an unstable hyperbolic point, so that ring particles initially orbiting on those circular orbits periodically acquire orbital eccentricities of order  $e \sim \sqrt{0.25\epsilon}$ . This shows that a Chariklo with elongation  $\epsilon \sim 0.16$  forces large excentricities ( $e \sim 0.2$ ) at the second-order 2/4 resonance (see an example in Supplementary Fig. 2), while Haumea  $\epsilon \sim 0.43$  forces even larger values ( $e \sim 0.33$ ) that lead to collisions with the body. The 2/4 resonant zone is thus a highly perturbed region where no ring is expected to survive.

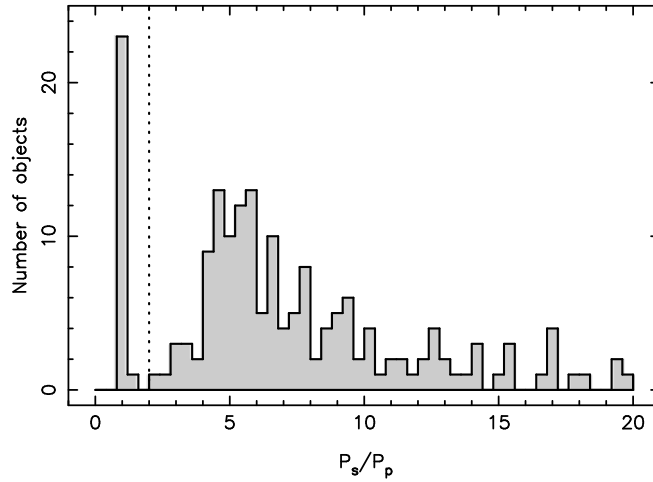
Turning to the second order 1/3 (mass anomaly) and fourth order 2/6 (ellipsoid) resonances, we see that it is the unique prograde resonant orbit with only one self-crossing point (corresponding to  $m = -1$  and  $j = 2$ , so that  $|m|(j-1) = 1$ ). Our numerical integrations show no significant effect of the 2/6 resonance on the particle motion, even with an elongation as high as  $\epsilon = 0.43$  (Haumea's case). This stems from the fourth-order nature of that resonance. It is noteworthy that both Chariklo and Haumea's rings are close to the 1/3 resonance configuration<sup>2,5</sup>, possibly leading to yet-to-be explicated more subtle confining effects of a narrow ring at that location. This makes further investigations (in particular using collisional codes) highly desirable.

Supplementary Table 1 - Resonance Coefficients.

Azimuthal variation $f(\theta)$ of the corotation potential (Methods Eq. 17)	
Mass anomaly	$q^{-1/6} \left( \frac{1}{\sqrt{q^{1/3} + q^{-1/3} - 2 \cos \theta}} - q^{1/2} \cos \theta \right) \cdot \mu$
Triaxial ellipsoid <sup>(a)</sup>	$2 \sum_{p=1}^{+\infty} q^{2p/3} S_p \epsilon^p \cos(2p\theta)$
Coefficients $\mathcal{A}_m(a)$ of the $m/(m-1)$ Lindblad resonances (Methods Eq. 28)	
Mass anomaly <sup>(b)</sup>	$\left\{ \left[ m + \frac{a}{2} \frac{d}{da} \right] b_{1/2}^{(m)}(a/R_{\text{sph}}) + q \left( \frac{a}{2R_{\text{sph}}} \right) \delta_{(m,-1)} \right\} \cdot \mu$
Triaxial ellipsoid <sup>(e)</sup> (with $m$ even)	$[2m - ( m  + 1)] S_{ m/2 } \left( \frac{R}{a} \right)^{ m +1} \cdot \epsilon^{ m/2 }$

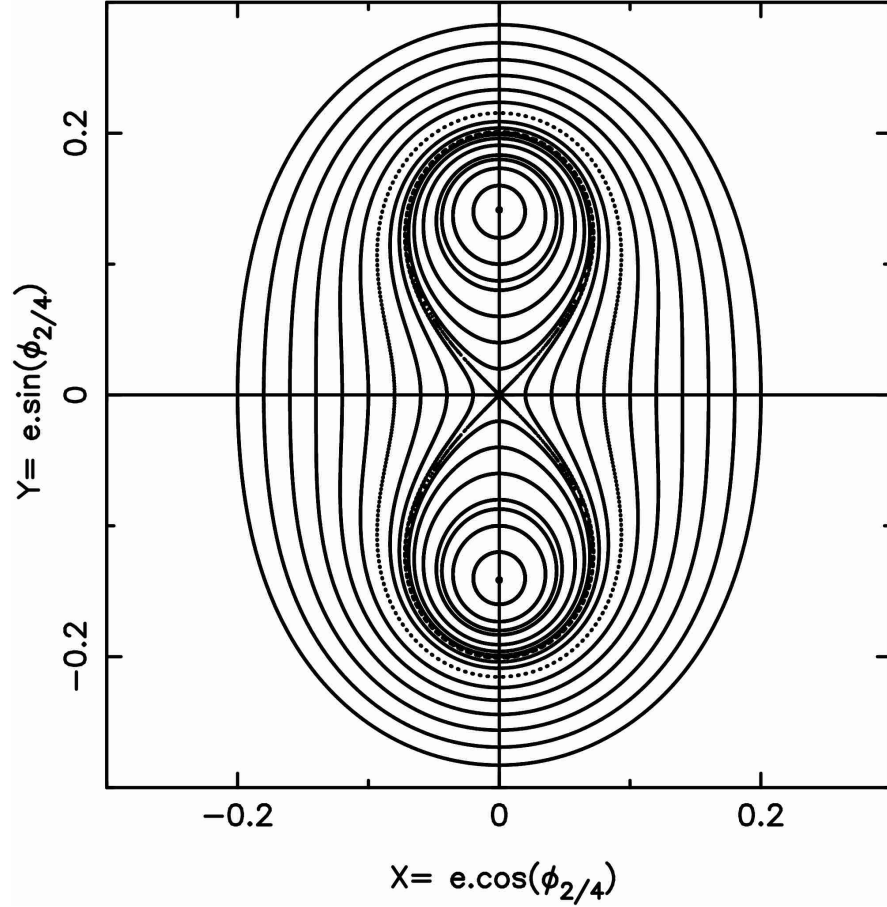
<sup>(a)</sup> The sequence  $S_p$  is defined by Eq. (16).

<sup>(b)</sup> Assuming a spherical body of radius  $R_{\text{sph}}$ . The terms  $b_{1/2}^{(m)}$  are the Laplace coefficients and  $\delta_{(m,-1)}$  is the Kronecker delta function.



**Supplementary Figure 1 | Distribution of orbital periods of satellites around asteroids and Trans-Neptunians Objects.** The orbital period  $P_s$  of 179 satellites known around binary or multiple asteroids and Trans-Neptunians Objects (taken from ref. 29 as of April 2018) are plotted in units of the rotation period  $P_p$  of their primaries. The resulting histogram of  $P_s/P_p$  shows a peak near unity, corresponding to tidally evolved systems, in which the primary rotates synchronously with the satellite orbital period. The vertical dotted line correspond to the outer 1/2 resonance, where the satellite completes one revolution while the primary completes two rotations. The steady increase of satellite presence beyond that resonance is in line with the model presented in the text, i.e. satellite formation in a primordial collisional disk that has been pushed outwards by the resonant torque of the 1/2 resonance.





**Supplementary Figure 2 | Phase portrait of the 2/4 outer spin-orbit resonance.** The phase portrait of the 2/4 resonance is shown for an ellipsoidal Chariklo with elongation  $\epsilon = 0.16$  (Table 1), with  $X = e \cos(\phi_{2/4})$  and  $Y = e \sin(\phi_{2/4})$ , where  $e$  is the particle eccentricity,  $\phi_{2/4} = 2\lambda - \lambda_A - \varpi$  is the resonant angle, and the various other angles are defined in the Methods. All the trajectories share the same Jacobi constant (see ref. 34 for details). This constant has been chosen so that the particle that starts at the origin  $(X, Y) = (0, 0)$  is at exact resonance, i.e. with semi-major axis  $a_{2/4} = a_{1/2}$ , see Main Text. The origin is then an unstable hyperbolic point that forces particles initially on a circular orbit to acquire high eccentricities of the order of  $e \sim 0.2$ , see Methods. This kind of topology occurs for a narrow semi-major axis range of  $a_{1/2}(1 - 0.25\epsilon) \lesssim a \lesssim a_{1/2}(1 + 0.25\epsilon)$  around the resonance.

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**Author Data and code availability.** All relevant data are available from the corresponding author on request. We have opted not to make our codes available as we cannot guarantee their correct performance on different computing platforms.