



Experimental validations of a tunable-lens-based visual demonstrator of multifocal corrections

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Abstract: The Simultaneous Vision simulator (SimVis) is a visual demonstrator of multifocal lens designs for prospective intraocular lens replacement surgery patients and contact lens wearers. This programmable device employs a fast tunable lens and works on the principle of temporal multiplexing. The SimVis input signal is tailored to mimic the optical quality of the multifocal lens using the theoretical SimVis temporal profile, which is evaluated from the through-focus Visual Strehl ratio metric of the multifocal lens. In this paper, for the first time, focimeter-verified on-bench validations of multifocal simulations using SimVis are presented. Two steps are identified as being critical to accurate SimVis simulations. Firstly, a new iterative approach is presented that improves the accuracy of the theoretical SimVis temporal profile for three different multifocal intraocular lens designs – diffractive trifocal, refractive segmented bifocal, and refractive extended depth of focus, while retaining a low sampling. Secondly, a fast focimeter is used to measure the step response of the tunable lens, and the input signal is corrected to include the effects of the transient behavior of the tunable lens. It was found that the root-mean-square of the difference between the estimated through-focus Visual Strehl ratio of the multifocal lens and SimVis is not greater than 0.02 for all the tested multifocal designs.

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1. Introduction

Ophthalmic corrections based on simultaneous vision [1] provide two or more optical powers, at the same time, to patients suffering from presbyopia [2], which is the age-related loss of crystalline lens accommodation. With multifocal contact and intraocular lens corrections, the incident light rays arriving from far and near distances are simultaneously imaged on the retina. Multifocal intraocular lenses (M-IOLs) are implanted in premium cataract surgery or to correct presbyopia in a refractive clear lens exchange. Multifocal contact lenses (M-CLs) are used to correct presbyopia in presbyopic patients, and also to control myopia progression in young myopes [3–5]. The state-of-the-art multifocal lenses can be divided into three major classes based on their energy distribution as a function of distance: bifocal, trifocal, and extended depth of focus (EDOF). Their design can be pure diffractive, pure refractive or hybrid diffractive-refractive. Further, refractive designs are either smooth or segmented radially or angularly.

The existence of several commercial designs and providers of M-IOLs and M-CLs makes it challenging for a surgeon or a clinician to determine the most appropriate multifocal lens for a patient [6–8]. Moreover, once the M-IOL is implanted during a cataract surgery, it is not easily reversible. Demonstrators of simultaneous vision allow a patient to experience multifocality prior to an intraocular surgery, with the help of adaptive optics technologies, thereby providing

a tool to choose a multifocal lens for their needs [9–13]. These instruments can also be used to aid M-CL selection. Simultaneous vision simulators based on a deformable mirror are limited to representing lenses with smooth surfaces [10, 11]. The devices using a spatial light modulator for simulating multifocality are affected by its pixelation, light efficiency limitations and monochromatic nature [12]. Most of these instruments are on-bench and bulky, and, in general, they have a small field of view with no naturalistic (indoor and outdoor) viewing experience.

SimVis is a portable, clinically-relevant, binocular and see-through simultaneous vision simulator, which is based on fast focus-tunable lenses. It uses the principle of temporal multiplexing to simulate multifocality [14]. The tunable lens has the ability to sweep across multiple foci at a rate faster than the flicker fusion threshold of human vision and hence provides seemingly static images to the human retina. A prerequisite to demonstrate any given multifocal design with SimVis is the determination of the corresponding amount of time to be spent by the tunable lens in each focusing distance (equivalently, the theoretical SimVis temporal profile). This is possible with the help of a metric-based evaluation [15]. The calculation uses the through-focus (TF) Visual Strehl (VS) ratio of a given multifocal phase map. In the case of M-IOLs, the multifocal phase maps can be extracted in a pseudophakic computer eye model [16] from the knowledge of the surface profile of the lenses. Alternately, the multifocal phase maps corresponding to the contact lenses can be evaluated from the optical power profiles [17]. In our earlier study [15], the time coefficients were obtained by fitting the TF VS ratio of the multifocal phase pattern with TF VS ratios of all possible monofocal lenses feasible with the tunable lens at any given instance [18]. We used a non-iterative least square fitting approach [15]. While it produced an accurate representation of the IOL, the undersampling procedure and optimal temporal sampling had to be adjusted to each multifocal design.

In this paper, a novel generalized method of evaluating the theoretical SimVis temporal profile is proposed using an iterative approach similar to stepwise regression. The SimVis time coefficients are iteratively adjusted such that they yield a reasonable match between the theoretical SimVis TF VS ratio and the TF VS ratio of the M-IOL. It is shown that the proposed method accurately evaluates the SimVis input signal for the three tested commercial designs while retaining a sufficiently low sampling. The generalized method allows an automatic convergence of the theoretical SimVis temporal profile to an optimal sampling for any given multifocal phase map. The effect of initial sampling is studied. Further, if this theoretical SimVis temporal profile is fed as input to the SimVis instrument, the electronic overshoot and the finite settling time (also called dynamic properties) of the tunable lens are expected to significantly affect the performance due to the short duration of the SimVis cycle. These transient characteristics of the tunable lens are measured using an in-house developed fast focimeter instrument [19] in terms of the step response function for a square input wave. The results of these experiments are used here to evaluate the final SimVis input signal that resulted in an optimal TF optical quality. These experiments validate the compensation of the dynamic behavior of the tunable lenses while demonstrating multifocal lenses with the SimVis device.

2. Method

The demonstration of a multifocal design by using temporal multiplexing involves the conversion of the spatially varying pupillary power distribution of a given multifocal design to a time-varying focus distribution in SimVis. Here, three different multifocal intraocular lens designs are tested—diffractive trifocal (D-T), refractive segmented bifocal (R-SB), and refractive EDof (R-EDof). Section 2.1 briefly introduces the metric-based evaluation of the theoretical SimVis temporal profile presented in [15] and points out the drawbacks of the non-iterative regression. Section 2.2 describes the temporal sampling constraints. Section 2.3 explains the proposed generalized formalism for improving the accuracy of SimVis in representing the multifocal phase patterns

while retaining a low sampling. The methods for evaluating the absolute time coefficients from the normalized time coefficients and for the generation of the SimVis input signal are presented in Sections 2.4 and 2.5. The method for compensating the transient response characteristics of the tunable lens is illustrated in Section 2.6. The specifications of the tunable lens used in this study are described in Section 2.7.

2.1. Metric-based evaluation of the theoretical SimVis temporal profile

The multifocal phase maps are evaluated in a pseudophakic computer eye model [16] and the corresponding TF VS ratios are obtained using the following expression [18]:

$$\text{VS ratio} = \frac{\iint_{-\infty}^{\infty} C(x', y') \cdot O(x', y') dx' dy'}{\iint_{-\infty}^{\infty} C(x', y') \cdot O_{DL}(x', y') dx' dy'}. \quad (1)$$

Here, $C(x', y')$ is the neural Contrast Sensitivity Function. $O(x', y')$ and $O_{DL}(x', y')$ are the Optical Transfer Functions (OTFs) of the multifocal phase map and a diffraction-limited phase.

A simple linear regression is used to obtain the theoretical SimVis temporal profile with large sampling (n) [15]. The TF VS ratio of a multifocal design, q (as a function of the axial coordinate, z , representing different defocus planes), is related to the TF VS ratios of 'n' monofocal lenses, $Q_i(z)$, feasible with the tunable lens as follows:

$$q(z) = k \cdot \sum_{i=1}^n t_i \cdot Q_i(z). \quad (2)$$

Here, k is a proportionality constant and the time coefficients are defined by t_i . These time coefficients, which comprise the theoretical SimVis temporal profile, determine the time duration that the tunable lens should spend at any given optical power to simulate the multifocal lens.

The TF VS ratio of SimVis, $q_s(z)$, can be theoretically estimated from the evaluated time coefficients, for a direct comparison with the TF VS ratio of a multifocal design, $q(z)$. The OTF of the theoretical SimVis temporal profile (Eq. 1) at a given distance, z , is calculated using a fast Fourier transform of $I'(z)$, which is a summation of the two dimensional intensity distribution, $I_i(z)$, of the 'n' monofocal lenses weighted with the evaluated time coefficients, t_i , and defined as follows:

$$I'(z) = \sum_{i=1}^n t_i \cdot I_i(z). \quad (3)$$

The error between the theoretical SimVis TF VS ratio and M-IOL TF VS ratio is not a part of the regression model (Eq. 2), and hence, the evaluated time coefficients are not necessarily optimal. The optical power correction is represented in terms of 'add' power or addition. In this paper, the monofocal lenses are sampled in the addition range from -2 D to 6 D in steps of 0.05 D, and consequently, $n = 161$.

Here, we define SimVis simulation error as a scalar quantity that is evaluated as the root-mean-square (RMS) of the absolute difference, $|q(z) - q_s(z)|$. In this paper, it is evaluated in the most relevant addition range, -1 D to 4.5 D. This error is sampling-dependent and no further optimization is possible in a non-iterative approach. The non-iterative regression is fast and performs well (Table 1) for designs with distinct and sharp far, near and intermediate peaks in the TF optical quality, for example, in the case of trifocal/bifocal designs. However, after being subject to undersampling, satisfactory results were not obtained (Table 1) [15].

2.2. Sampling of the theoretical SimVis temporal profile

In our earlier study that used a non-iterative evaluation of the theoretical SimVis temporal profile [15], the temporal sampling (number of non-zero time coefficients) was reduced to meet

the operational speed requirements of SimVis (> 25 Hz) while not correcting for the transient response characteristics of the tunable lens. This SimVis sampling reduction varied with the multifocal design. Thresholding and selective rejection criteria were applied to EDOF designs, and for bifocal and trifocal designs, SimVis sampling points to represent the M-IOL were chosen around the peaks of the TF optical quality curves. This non-iterative method is design dependent and does not allow generalization.

The iterative method proposed in this paper uses thresholding as the only undersampling criteria (in the first iteration alone). It is shown that the correction of the electronic overshoot is possible (Section 2.6) and hence, the transient response of the tunable lens is not the fundamental limiting factor that determines the sampling (the number of nonzero time coefficients in the theoretical SimVis temporal profile). However, the undersampling is necessary for two reasons. Firstly, the update resolution of the tunable lens cannot be lower than 0.1 ms, this limit is defined by the tunable lens response and device electronics. Secondly, as evident in Sections 2.6 and 3.3, the correction of the dynamic behavior involves increasing the number of focus transitions of the tunable lens in a single SimVis cycle, while going from the theoretical SimVis temporal profile to the final transient response characteristics corrected (or dynamics-corrected) SimVis input signal.

2.3. Iterative evaluation of the theoretical SimVis temporal profile

Stepwise regression involves the automated choice of independent variables in a step-by-step process. Here, the magnitude of a single SimVis time coefficient is manipulated in each step with the aim of decreasing the SimVis simulation error while not increasing the number of nonzero time coefficients beyond a predetermined sampling threshold (20, in this study). The first iteration uses the time coefficients obtained in a simple least square fitting process (Eq. 2). At the end of this iteration, the SimVis simulation error (Section 2.1) is evaluated. This metric is used as an explanatory variable to determine the addition location (add power), D_{max_err} , at which the absolute difference between the SimVis TF VS ratio and the M-IOL TF VS ratio is the largest. The time coefficient, t_{max_err} , at the corresponding addition location is corrected using t_a such that the new time coefficient at this addition is $t_{max_err} + t_a$. In this method, similar to a stepwise regression, a single time coefficient alone is adjusted/corrected in any given iteration. The adjustment to the time coefficient, t_a , is proportional to $\Delta q_a = q(z_a) - q_s(z_a)$, the difference between the TF VS ratio of the M-IOL (q) and SimVis (q_s) at that addition, (z_a), such that: $t_a = \beta \Delta q_a$. The proportionality constant, β , is always a positive real number. The magnitude of β is chosen such that t_{max_err} is corrected in small steps. Choosing a large value for ' β ' may lead to quicker convergence. However, the accuracy is compromised. If the new time coefficient is found to be less than zero (i.e. $t_{max_err} + t_a < 0$; this condition can occur when $\Delta q_a < 0$), then, its value is set to zero.

In the process of identifying D_{max_err} , addition locations with time coefficient value equal to zero are excluded. This exclusion criterion prevents an increase in the number of nonzero time coefficients and hence SimVis sampling. In the absence of this exclusion criterion, the iterations converge quickly, however, resulting in a large number of nonzero time coefficients. Consequently, making the SimVis undersampling process (Section 2.2) futile.

If the SimVis simulation error saturates after a sufficiently large number of iterations, addition locations with zero time coefficient values are included in the process of identifying D_{max_err} with the condition that the total number of nonzero time coefficients must be lower than or equal to the initial sampling threshold (set to 20 in this study).

2.4. Evaluating SimVis addressable time coefficients

The time coefficients obtained using Eq. 2 do not represent the true SimVis-addressable time coefficients [15]. The practically relevant values can be obtained by assuming that a single

SimVis cycle consumes a total of T_{total} ms. The sum of the normalized time coefficients (T_i) is directly proportional to the time taken by a single SimVis cycle and hence,

$$\alpha \sum_{i=1}^n T_i = T_{\text{total}} \text{ (ms)}. \quad (4)$$

Here, α is a time constant with units of ms. This constant multiplied by the normalized time coefficients, T_i , determines the actual time to be spent at each addition. The value of α can be estimated using Eq. 4 and in principle, it is different for each set of calculated time coefficients (with different initial sampling or with iteration number). Hence, it is important to use the estimated absolute time coefficients, αT_i , when comparing two sets of time coefficients. In this paper, T_{total} is chosen to be equal to 20 ms, so as to have a speed equal or better than the flicker fusion threshold of human vision.

2.5. Generating the SimVis input signal

The time coefficients are converted into an addressable electronic input signal for the SimVis instrument. The most instinctive method is to use an increasing wave, where the input signal starts with the least optical power and moves on to the highest. In other words, the tunable lens sweeps across the relevant foci starting from the least optical power to the highest within one SimVis cycle of 20 ms duration. Alternately, the input signal can be defined with a decreasing wave such that the tunable lens moves from the highest optical power to the lowest. These increasing or decreasing waves can produce large jumps in optical power, and potentially inducing greater errors due to the transient response characteristics of the lens.

In another strategy, the optical power increases from least optical power to the highest and comes back to the least optical power within a single SimVis cycle, called up-down wave. This strategy can be implemented in two ways. In the first method, all the time coefficients are divided by 2 and the decreasing wave (computed with the halved time coefficients) is followed by the increasing wave (computed with the halved time coefficients) in one single cycle. In the second up-down wave generation method, the alternate time coefficients and their corresponding focusing distances are distributed between the increasing and decreasing portion of the wave. Either way, this up-down wave is nearly symmetric (with the axis of symmetry near the center of the SimVis cycle) and does not have large jumps in optical power in comparison with the increasing wave.

2.6. Compensation of the tunable lens transient response characteristics

The settling time of the tunable lens to change from one optical power state to another is a finite value, as shown in Section 3.2. If the theoretical SimVis input signal is provided to the tunable lens, the behavior of the lens in replicating the input signal is limited by the resultant transient response function, $X(t)$. In mathematical terms, $X(t)$ is a convolution of any given input signal, $u(t)$, and the unit impulse response function, $h(t)$, as follows:

$$X(t) = u(t) \otimes h(t). \quad (5)$$

Here, we define the unit impulse response function, $h(t)$, as the time derivative of the step response function, $s(t)$, which is the transient response obtained when the input signal is a square wave with a magnitude of unity, and therefore,

$$h(t) = \frac{d}{dt} s(t). \quad (6)$$

A fast focimeter [19] allows the measurement of the step response function, $s(t)$, corresponding to an input square wave and $X(t)$ corresponding to any input signal, $u(t)$. The unit impulse response

function, $h(t)$ can be evaluated from Eq. 6. It is then possible to correct the input signal, $u(t)$, to obtain the dynamics-corrected SimVis input signal, $\hat{u}(t)$, such that, the corrected transient response, $\hat{X}(t)$ is nearly equal to the desired input signal, $u(t)$, and,

$$\hat{X}(t) = \hat{u}(t) \otimes h(t) \approx u(t). \quad (7)$$

As a strategy to compensate the transient response of the tunable lens, the final dynamics-corrected SimVis input signal, $\hat{u}(t)$, is evaluated from the theoretical SimVis input signal, $u(t)$, by adopting a step-by-step correction process in the time sequence. In each 0.1 ms step (going from 0 ms to 20 ms), a modification of the input signal is made by replacing it with a corrected value within a range of 5 D such that the RMS of the difference between the corrected transient response function, $\hat{X}(t)$, and the desired input signal, $u(t)$, is minimum. The tunable lens update resolution of 0.1 ms is determined by the limitations of the device electronics, and the range of optical powers used is limited by the tunable lens.

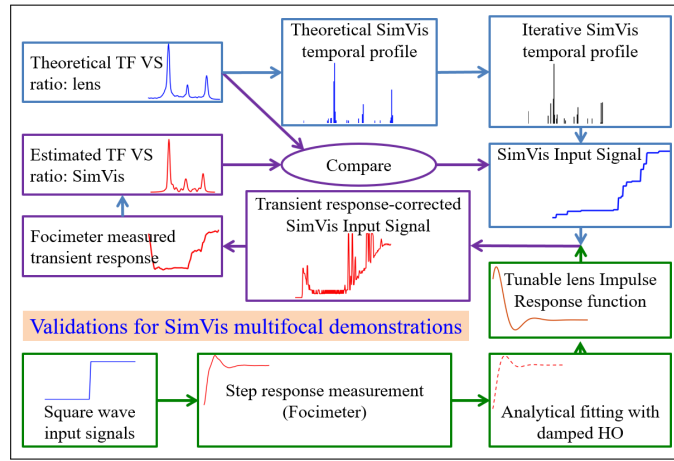


Fig. 1. Steps involved in the validations of the SimVis multifocal demonstrations using the fast focimeter.

To determine the unit impulse response function, $h(t)$, the transient response functions of the tunable lens are measured with the fast focimeter when 17 square waves of different magnitude (0.75 D to 4.75 D in steps of 0.25 D, starting at 0 D in every measurement) are provided to the tunable lens. The resultant step response functions are unit-normalized (to 1 D step) by dividing them with the magnitude of the input square waves. A mean unit step response function is then evaluated. An analytical expression is obtained by fitting the mean unit step response function with a damped harmonic oscillator function, $f(t)$, defined as follows,

$$f(t) = 1 - \frac{e^{-\tau\omega_n t}}{\sqrt{1 - \tau^2}} \cos(\omega_d t - \theta). \quad (8)$$

Here, the fitting parameter, τ is the damping ratio that determines the overshoot. ω_d and ω_n represent the magnitude and attenuation of the system response characteristics. The phase shift, which is close to zero, is determined by θ . To avoid uncertainties in determining the value of τ , a trust range can be set in the fitting method by estimating its value using the following expression,

$$\tau = \frac{-\ln(y_{\text{peak}} - 1)}{\sqrt{\pi^2 + [\ln(y_{\text{peak}} - 1)]^2}}. \quad (9)$$

Table 1. Theoretical SimVis simulation error (a dimensionless quantity): a comparison of two non-iterative methods (without undersampling and with undersampling to 20 nonzero time coefficients) and an iterative (below 20 nonzero time coefficients) method of generating the theoretical SimVis temporal profile for diffractive trifocal (D-T), refractive segmented bifocal (R-SB), and refractive extended depth of focus (R-EDOF) multifocal intraocular lenses (M-IOLs).

<i>M-IOL</i>	<i>Non-iterative</i>	<i>Non-iterative</i>	<i>Iterative</i>
	<i>No undersampling</i>	<i>Undersampling</i>	
D-T	0.013	0.022	0.013
R-SB	0.016	0.035	0.011
R-EDOF	0.010	0.030	0.009

Here, y_{peak} is the peak value of the measured mean unit step response function. Eqs. 8 and 9 assume that the step response saturates at 1 D. To further generalize the fit, the first term on the right-hand-side of Eq. 8, 1, can be replaced by a constant optical power, A . The unit impulse response function, $h(t)$ is estimated from the fitted mean unit step response function using numerical discrete time differentiation, and,

$$h(t) = \frac{f(t + \Delta t) - f(t)}{\Delta t} \quad (10)$$

where, $\Delta t = 0.1$ ms.

A summary of the methods and steps involved in the fast focimeter-based validations of the SimVis multifocal demonstrations is shown in Fig. 1.

2.7. Focus tunable lens and fast-focimetry measurements

An electrically driven focus tunable lens (Optotune, Model Number: EL-10-30-TC-VIS) is tested. The tunable lens has a clear aperture of 10 mm. In the fast focimeter [19], a prism is used to laterally shift the image of a slit, optically conjugated to the optotunable lens. A high speed camera (IL5SHSC; Fastec Imaging, USAIL5, 3846 fps) records the displacement of the slit as a function of the tunable lens input optical power.

The mean of the unit-normalized step response functions (Section 2.6) of the tunable lens is estimated from fast focimeter measurements. It is used to obtain an analytical expression by fitting (nonlinear least squares method) with a damped oscillatory response function (Eq. 8). The unit impulse response function, $h(t)$, which is obtained as the discrete time derivative of the mean unit step response function (Eq. 10), is used to predict the transient response, $X(t)$, of the tunable lens for any given input signal, $u(t)$ (Eq. 5). The unit impulse response function, $h(t)$, is also used to obtain a dynamics-corrected SimVis input signal, $\hat{u}(t)$, from a theoretical SimVis input signal, $u(t)$. The transient response of the tunable lens corresponding to dynamics-corrected and uncorrected SimVis input signal is again measured using the fast focimeter.

3. Results

3.1. Iterative theoretical SimVis temporal profile

The total number of nonzero time coefficients, without undersampling, after applying the least square fitting method (Eq. 2) in a non-iterative evaluation of the theoretical SimVis temporal profile are 81, 110, and 107 for the D-T, R-SB, and R-EDOF designs. The corresponding theoretical SimVis simulation errors are shown in column 2 of Table 1. With the application of

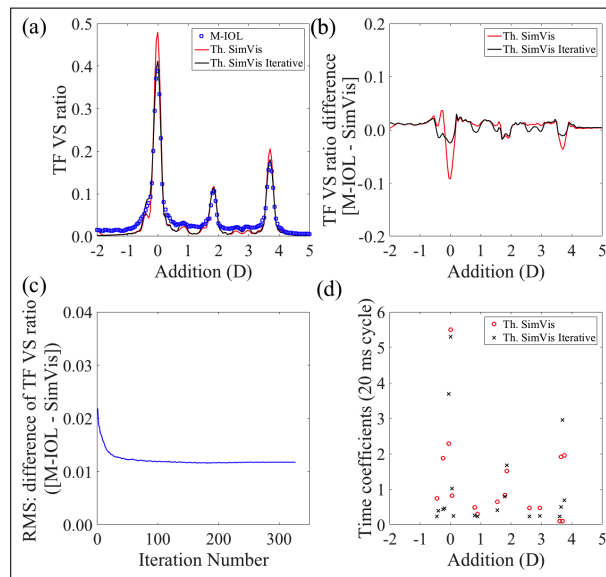


Fig. 2. Diffractional trifocal (D-T) intraocular lens: (a) Comparison of the TF VS ratio of the theoretical phase profile of the lens (blue squares) and the corresponding curves for the theoretical (Th.) SimVis temporal profile with (black line) and without (red line) the use of iterations. (b) Difference in the theoretical TF VS ratio of the lens and SimVis with (black line) and without (red line) iterative evaluation. (c) The progression of RMS difference between the TF VS ratio of the lens and iterative theoretical (Th.) SimVis temporal profile. (d) Comparison of the theoretical SimVis time coefficients in the first iteration (red circles) and at the end of 328 iterations (black crosses).

undersampling (thresholding) to 20 nonzero time coefficients, the theoretical SimVis simulation errors increase as expected (column 3 of Table 1).

As shown in Table 1, the proposed iterative approach produces a theoretical SimVis temporal profile such that the theoretical SimVis simulation error is lower or comparable to the non-iterative method with a large sampling. Comparisons of the TF VS ratio and the time coefficients in the non-iterative and iterative procedures are shown in Figs. 2, 3 and 4 for the three tested designs. Figures 2(a), 3(a) and 4(a) illustrate the significant improvement in the accuracy of evaluating the theoretical SimVis temporal profile with the proposed iterative method for the D-T, R-SB, and R-EDOF designs in terms of the TF VS ratio. The differences in the TF VS ratio with/without the iterative procedure and the TF VS ratio of the M-IOLs are shown in Figs. 2(b), 3(b) and 4(b).

The propagation of the theoretical SimVis simulation error with increasing iterations for the three designs is shown in Figs. 2(c), 3(c) and 4(c). The non-iterative case, which can be used as a reference, is given by the error value at iteration number 1. In the case of D-T design, in the process of identifying D_{max_err} (Section 2.3), the inclusion of addition locations with zero time coefficient values did not reduce the SimVis simulation error any further (Fig. 2(c)). However, in R-SB and R-EDOF designs, a large drop in the SimVis simulation error was found beyond 440 and 500 iterations respectively (Figs. 3(c) and 4(c)). Finally, a comparison of the absolute time coefficients with and without iterative calculations are shown in Figs. 2(d), 3(d), and 4(d). There is a significant dissimilarity between the time coefficients obtained in the iterative and non-iterative approaches.

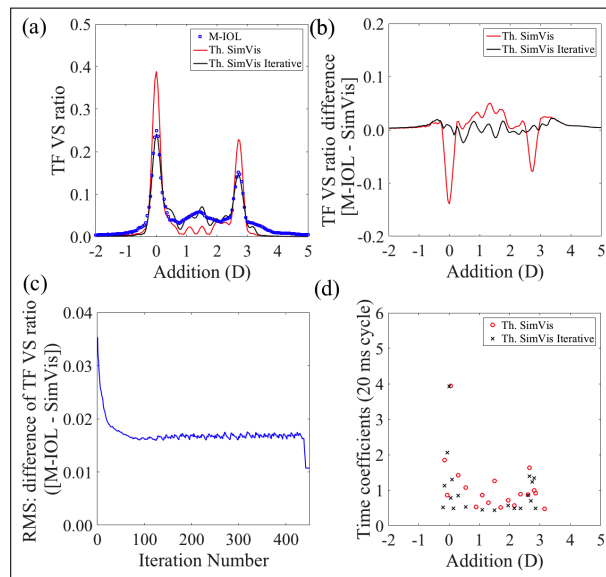


Fig. 3. Refractive segmented bifocal (R-SB) intraocular lens: (a) Comparison of the TF VS ratio of the theoretical phase profile of the lens (blue squares) and the corresponding curves for the theoretical (Th.) SimVis temporal profile with (black line) and without (red line) the use of iterations. (b) Difference in the theoretical TF VS ratio of the lens and SimVis with (black line) and without (red line) iterative evaluation. (c) The progression of RMS difference between the TF VS ratio of the lens and iterative theoretical (Th.) SimVis temporal profile. (d) Comparison of the theoretical SimVis time coefficients in the first iteration (red circles) and at the end of 446 iterations (black crosses).

3.2. Estimating the unit impulse response function

Focimeter-measured unit step response functions corresponding to 17 square input waves of different magnitude (0.75 D to 4.75 D in steps of 0.25 D, starting at 0 D for each measurement) were found to have little dependence on the magnitude of the input square waves (see Fig. 5), as evident from the low standard deviation [19]. Therefore, it was reasonable to assume a linear relationship between the magnitude of the square wave and the corresponding step response. It was found that the damping ratio, $\tau = 0.40$, after fitting the mean of the unit step response function with a damped oscillatory response function (Eq. 8). $\omega_d = 0.78$ and $\omega_n = 1.05$ represent the magnitude and attenuation characteristics of a tunable lens (Optotune, EL-10-30-TC). These parameters provided the best fit to the mean unit step response data (R-square = 0.99). y_{peak} (Eq. 9) was found to be equal to 1.29 D. The unit impulse response function was evaluated from the mean unit step response function (Eq. 10). The settling time, which can be defined as the time required for the fitted step response (thick blue line in Fig. 5) of the tunable lens to stay within 2% of the steady-state value (1 D), was found to be 9.56 ms from focimeter measurements. The focus measurements and the unit impulse response estimation were found to be highly repeatable [19].

3.3. Compensation of the transient response of the tunable lens

The SimVis input signal (blue line with fewer fluctuations) for the D-T M-IOL shown in Fig. 6(a) was evaluated from the theoretical SimVis temporal profile (Fig. 2(d)) obtained with the iterative method. The dynamics-corrected SimVis input signal (red line) with a range of 5 D is also shown. The increasing wave (Section 2.5) provided the best results for this design, and hence an example of this wave is shown here. In Fig. 6(b), a comparison of the theoretically predicted

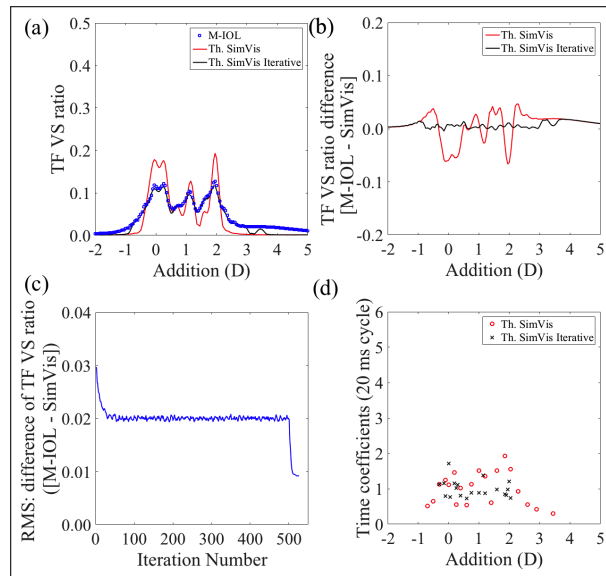


Fig. 4. Refractive extended depth of focus (R-EDOF) intraocular lens: (a) Comparison of the TF VS ratio of the theoretical phase profile of the lens (blue squares) and the corresponding curves for the theoretical (Th.) SimVis temporal profile with (black line) and without (red line) the use of iterations. (b) Difference in the theoretical TF VS ratio of the lens and SimVis with (black line) and without (red line) iterative evaluation. (c) The progression of RMS difference between the TF VS ratio of the lens and iterative theoretical (Th.) SimVis temporal profile. (d) Comparison of the theoretical SimVis time coefficients in the first iteration (red circles) and at the end of 527 iterations (black crosses).

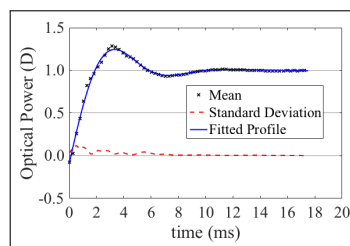


Fig. 5. The measured data of the mean unit step response of the tunable lens is indicated using crosses (black). The standard deviation of the unit step responses for the 17 different magnitude input square waves is shown with the dotted line (red). The fitted damped oscillator is shown with the thick line (blue).

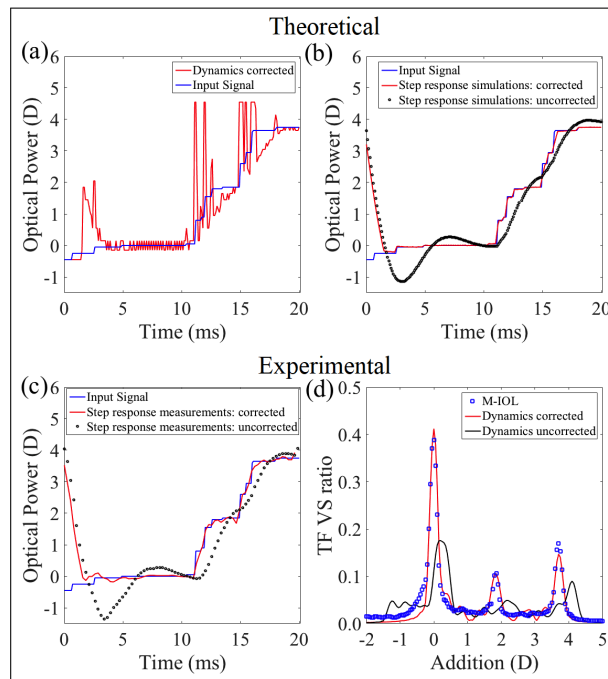


Fig. 6. Diffractive trifocal (D-T) intraocular lens: (a) Comparison of the input signal generated using the theoretical SimVis temporal profile after 328 iterations (blue line, Section 2.5) and the final dynamics-corrected SimVis input signal (red line, Section 2.6). (b) The theoretically estimated transient response functions corresponding to the dynamics-corrected (red line) and uncorrected (black circles) SimVis input signals. (c) Fast focimeter measured transient response corresponding to the dynamics-corrected (red line) and uncorrected (black circles) SimVis input signals. (d) Comparison of the SimVis TF VS ratio before (black line) and after (red line) the correction of the transient response (estimated using focimeter measurements).

Table 2. Experimental SimVis simulation error (a dimensionless quantity) from fast focimeter measurements – before and after the compensation of the transient response characteristics of the tunable lens – for diffractive trifocal (D-T), refractive segmented bifocal (R-SB), and refractive extended depth of focus (R-EDOF) multifocal intraocular lenses (M-IOLs).

<i>M-IOL</i>	<i>Without transient response correction</i>	<i>With transient response correction</i>
D-T	0.071	0.020
R-SB	0.033	0.014
R-EDOF	0.019	0.018

transient response corresponding to the dynamics-corrected and uncorrected SimVis input signals is shown with reference to the input signal. It was found theoretically that beyond 1.5 ms, the dynamics-corrected SimVis input signal (red line in Fig. 6(b)) provided a tunable lens response that matched well with the expected input signal (blue line in Fig. 6(b)). The experimental results agree with the theoretical predictions, as can be seen in Fig. 6(c), the transient response measurements using a fast focimeter for the dynamics-corrected and uncorrected SimVis input signals in comparison with the expected input signal (blue line). Finally, a comparison of the TF VS ratio with and without the correction of the transient response of the tunable lens is

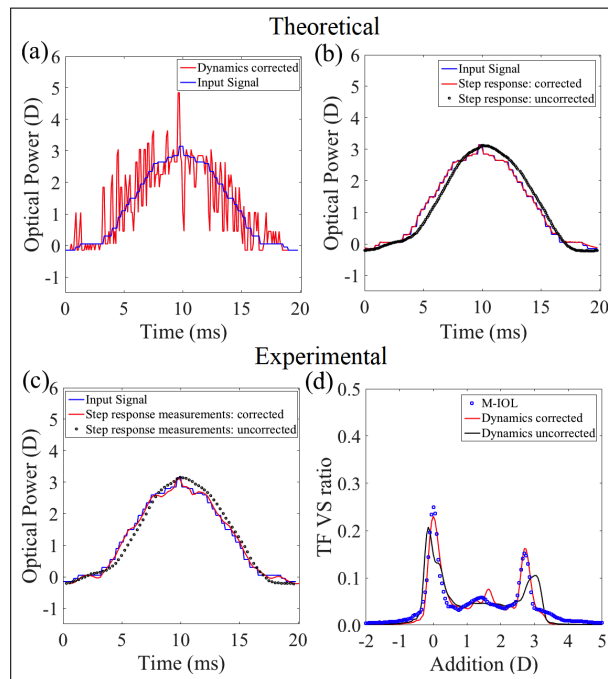


Fig. 7. Refractive segmented bifocal (R-SB) intraocular lens: (a) Comparison of the input signal generated using the theoretical SimVis temporal profile after 446 iterations (blue line, Section 2.5) and the final dynamics-corrected SimVis input signal (red line, Section 2.6). (b) The theoretically estimated transient response functions corresponding to the dynamics-corrected (red line) and uncorrected (black circles) SimVis input signals. (c) Fast focimeter measured transient response corresponding to the dynamics-corrected (red line) and uncorrected (black circles) SimVis input signals. (d) Comparison of the SimVis TF VS ratio before (black line) and after (red line) the correction of the transient response (estimated using focimeter measurements).

shown in Fig. 6(d). The SimVis TF VS ratio predicted from the dynamics-corrected transient response measurements matched closely the optical quality of the M-IOL. Graphs similar to those presented in Fig. 6 are shown for R-SB and R-EDOF M-IOLs in Figs. 7 and 8. In these two lenses, the up-down waves were used since they resulted in a smaller SimVis simulation error.

4. Discussion

The proposed iterative method of evaluating the theoretical SimVis temporal profile provides a strong platform for the practical demonstration of any multifocal design with SimVis. The fast focimeter device facilitates the measurement of the time characteristics of the tunable lens by using a fast camera and the prism-induced lateral separation of the addressed multiple foci. A high degree of linearity of the tunable lens response to square waves of different magnitude allowed the description of the tunable lens with a single system transfer function. The proposed step-by-step correction of the input signal, $u(t)$, to evaluate the dynamics-corrected SimVis input signal, $\hat{u}(t)$ is shown to be effective in terms of replicating the optical quality of three different multifocal lens designs - bifocal, trifocal and EDof. The transient response corrected input signal for the multifocal designs are pre-computed and stored in the SimVis device memory and therefore, these iterative computations do not introduce any delays during real visual demonstrations on the patients.

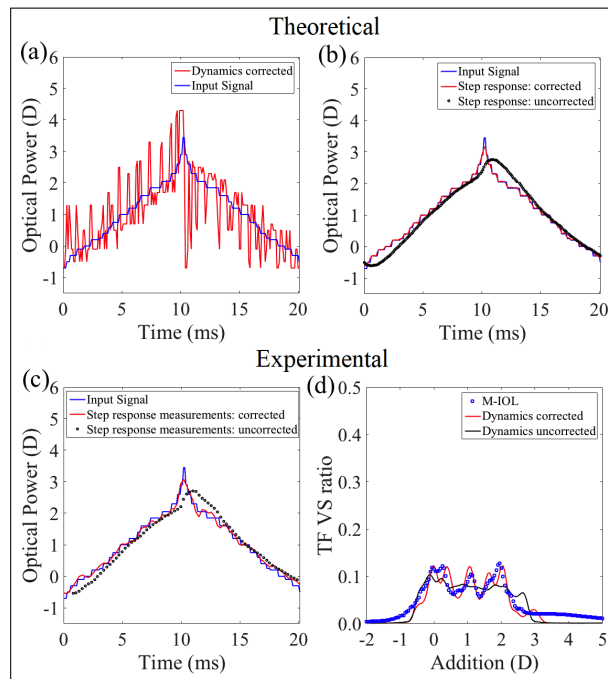


Fig. 8. Refractive extended depth of focus (R-EDOF) intraocular lens: (a) Comparison of the input signal generated using the theoretical SimVis temporal profile after 527 iterations (blue line, Section 2.5) and the final dynamics-corrected SimVis input signal (red line, Section 2.6). (b) The theoretically estimated transient response functions corresponding to the dynamics-corrected (red line) and uncorrected (black circles) SimVis input signals. (c) Fast focimeter measured transient response corresponding to the dynamics-corrected (red line) and uncorrected (black circles) SimVis input signals. (d) Comparison of the SimVis TF VS ratio before (black line) and after (red line) the correction of the transient response (estimated using fast focimeter measurements).

In the case of the D-T M-IOL, there is a mismatch of the input signal (blue line) and the dynamics-corrected transient response (red line) as can be seen in Figs. 6(b) (theoretical prediction) and 6(c) (fast focimeter measurements) in the duration range: 0 - 1.5 ms. This is reflected in a mismatch (although negligible) of the resultant TF VS ratio in the range -1 D to -0.25 D as can be seen in Fig. 6(d). It may be noted that for a lens with a lower near addition, this error will be smaller.

In the case of R-SB M-IOL, the small intermediate peak observed at 1.75 D in the TF VS ratio corresponding to the dynamics-corrected SimVis input signal (red line) in Fig. 7(d) matches with the small intermediate peak observed in the theoretical SimVis temporal profile (black line) obtained after 446 iterations as shown in Fig. 3(a). This shows the effectiveness of the correction of the transient response characteristics of the tunable lens.

A comparison of the SimVis simulation error with the iterative method (Table 1) and the dynamics-corrected SimVis simulation error measured by the focimeter (Table 2) shows that this error increases due to inaccuracies in the correction/measurement of the transient response of the tunable lens by 53.8%, 27.3% and 100% for the D-T, R-SB and R-EDOF M-IOLs respectively. These discrepancies between the theoretical predictions of the M-IOL (blue lines) and the measured dynamics-corrected transient response (red lines) in Figs. 6(d), 7(d), and 8(d) may be attributed to the focimeter measurement errors, fitting error of the mean unit step

response function (Eq. 8), the time resolution limitation of the focimeter measurements (0.26 ms), correspondingly different SimVis input signal sampling (0.1 ms) and the linearity assumption of the tunable lens characteristics (standard deviation in Fig. 5). Since the results of the focimeter measurements of the dynamics-corrected SimVis input signal and the RMS results shown in Table 2 are limited by the time resolution of the fast focimeter, the final SimVis demonstrations could be slightly better or worse than the validations with the fast focimeter.

As described in Section 2.2, although the SimVis device electronics allows up to 150 focus transitions in one SimVis cycle, undersampling is needed to reduce the number of nonzero time coefficients in the theoretical SimVis temporal profile. This is because the dynamics-corrected SimVis input signals require higher update frequency (as evident from the red lines in Figs. 6(a), 7(a) and 8(a)), and to accommodate the finer details in the dynamics-corrected SimVis input signal, it is necessary to limit the number of nonzero time coefficients in the theoretical SimVis temporal profile (Figs. 2(d), 3(d) and 4(d)).

The increasing and decreasing waves resulted in nearly the same TF optical quality (within 10% of the SimVis simulation error). Similarly, the two methods of generating the up-down waves also provided similar results. In the case of the trifocal design, the reason for the observed better results using the increasing wave (Table 2) in comparison to up-down wave (28% higher SimVis simulation error), can be attributed to its well-separated and distinct sharp peaks. Also, the near addition occurs at 3.5 D, making it difficult for the up-down wave to quickly move from far distance to near distance and back within a 20 ms cycle. The far and near distance peaks in the case of R-SB M-IOL are closer and less sharp. Thus, making them candidates for up-down waves.

It has to be noted that the tunable lens linearity – the step response function being linearly proportional to the magnitude of the input square wave – has largely simplified the evaluation of the impulse response function in the study. Step response measurements with small magnitude (≤ 0.5 D) square waves did not match well the mean unit step response and hence were excluded. If the case of tunable lens with nonlinear transient response characteristics or if the application demands greater accuracy with short focus transitions, it may be necessary to evaluate input square-wave magnitude-dependent impulse response functions to fully and accurately characterize the performance.

In this paper, for demonstration purposes, the VS ratio is evaluated using monochromatic light at a wavelength of 555 nm, where the spectral sensitivity of the human eye is the maximum. It was verified that the use of polychromatic TF VS ratio did not alter the evaluation of the theoretical SimVis temporal profiles in all M-IOL designs that do not involve longitudinal chromatic aberration (LCA) modulation [15]. However, the use of polychromatic SimVis temporal profile can improve [15], to a small extent, the replication of the TF VS ratio of the M-IOL that has the ability to correct the LCA [20] at specific distances.

The focus range over which the dynamics-corrected SimVis input signal should be computed is determined by the limitations of the tunable lens, driver electronics and the RMS difference between the input signal (for example, blue line in Fig. 6(b)) and the theoretically estimated dynamics-corrected transient response (red line in Fig. 6(b)). With improvements in tunable lens technology, it may be possible to reduce computational effort and further improve the accuracy of SimVis demonstrations.

It was found that a tunable lens with a different model number from the same manufacturer can have a different impulse response function. However, after characterization with the fast focimeter and dynamics correction, the final performance of the tunable lenses in terms of demonstrating different multifocal designs remained the same. The variability in the response of different tunable lens units from the same manufacturer necessitates a thorough calibration and measurement of the transient response for each lens before installation in the SimVis device. It has been reported that the surface tension of most tunable lenses is less sensitive to temperature

changes below a certain threshold (40°C) [21]. In this study, all the focimeter measurements were performed at room temperature.

In our future work, objective and subjective measurements in an adaptive optics-based visual psychophysics platform and clinical tests with the portable SimVis device will further validate the accuracy of the iterative evaluation of the theoretical SimVis temporal profile and the correction of the transient response characteristics proposed here. The fast focimeter has proven to be a powerful tool to evaluate the accuracy of replicating the desired TF optical quality. The methods developed here improve and validate the SimVis multifocal patterns. However, the accurate control of the tunable lens achieved with these methods, including the modeling of the transient response of the lens, can be directly used in many other applications using tunable lenses at high speed, for example, in depth-scan microscopy.

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References

1. M. J. Simpson, "The diffractive multifocal intraocular lens," *J. Cataract Refract. Surg.*, **1**(2), 115–121 (1989).
2. A. Glasser and M. C. Campbell, "Presbyopia and the optical changes in the human crystalline lens with age," *Vis. Res.*, **38**(2), 209–229 (1998).
3. D. Gatinel, C. Pagnouille, Y. Houbrechts, and L. Gobin, "Design and qualification of a diffractive trifocal optical profile for intraocular lenses," *J. Cataract Refract. Surg.*, **37**(11), 2060–2067 (2011).
4. E. S. Bennett, "Contact lens correction of presbyopia," *Clin. Exp. Optom.*, **91**(3), 265–278 (2008).
5. T. A. Aller, M. Liu, and C. F. Wildsoet, "Myopia Control with Bifocal Contact Lenses: A Randomized Clinical Trial," *Optom. Vis. Sci.* **93**(4), 344–352 (2016).
6. N. E. de Vries and M. M. A. N. Rudy, "Multifocal intraocular lenses in cataract surgery: literature review of benefits and side effects," *J. Cataract Refract. Surg.* **39**(2), 268–278 (2013).
7. R. Braga-Mele, D. Chang, S. Dewey, G. Foster, B. A. Henderson, W. Hill, R. Hoffman, B. Little, N. Mamalis, T. Oetting, D. Serafano, A. Talley-Rostov, Audrey, A. Vasavada, and S. Yoo, "Multifocal intraocular lenses: relative indications and contraindications for implantation," *J. Cataract Refract. Surg.*, **40**(2), 313–322 (2014).
8. M. A. Woodward, J. B. Randleman, and R. D. Stulting, "Dissatisfaction after multifocal intraocular lens implantation," *J. Cataract Refract. Surg.*, **35**(6), 992–997 (2009).
9. J. F. Bille, "Preoperative simulation of outcomes using adaptive optics," *J. Refract. Surg.*, **16**(5), S608–S610 (2000).
10. P. A. Piers, E. J. Fernandez, S. Manzanera, S. Norrby, and P. Artal, "Adaptive optics simulation of intraocular lenses with modified spherical aberration," *Invest. Ophthalmol. Vis. Sci.* **45**(12), 4601–4610 (2004).
11. K. M. Rocha, L. Vabre, N. Chateau, and R. R. Krueger, "Expanding depth of focus by modifying higher-order aberrations induced by an adaptive optics visual simulator," *J. Cataract Refract. Surg.* **35**(11), 1885–1892 (2009).
12. S. Manzanera, P. M. Prieto, D. B. Ayala, J. M. Lindacher, and P. Artal, "Liquid crystal Adaptive Optics Visual Simulator: Application to testing and design of ophthalmic optical elements," *Opt. Express* **15**(24), 16177–16188 (2007).
13. P. de Gracia, C. Dorronsoro, A. Sánchez-González, L. Sawides, and S. Marcos, "Experimental Simulation of Simultaneous Vision," *Invest. Ophthalmol. Vis. Sci.* **54**(1), 415–422 (2013).
14. C. Dorronsoro, A. Radhakrishnan, J. R. Alonso-Sanz, D. Pascual, M. Velasco-Ocana, P. Perez-Merino, and S. Marcos, "Portable simultaneous vision device to simulate multifocal corrections," *Optica* **3**(8), 918–924 (2016).
15. V. Akondi, C. Dorronsoro, E. Gamba, and S. Marcos, "Temporal multiplexing to simulate multifocal intraocular lenses: theoretical considerations," *Biomed. Opt. Express* **8**(7), 3410–3425 (2017).

16. V. Akondi, P. Pérez-Merino, E. Martínez-Enriquez, C. Dorronsoro, N. Alejandro, I. Jiménez-Alfaro, and S. Marcos, "Evaluation of the true wavefront aberrations in eyes implanted with a rotationally asymmetric multifocal intraocular lens," *J. Refract. Surg.* **33**(4), 257–265 (2017).
17. A. J. D. Águila-Carrasco, D. Monsálvez-Romín, E. Papadatou, "Optical quality of rotationally symmetrical contact lenses derived from their power profiles," *Contact Lens and Anterior Eye* **40**(5), 346–350 (2017).
18. L. N. Thibos, X. Hong, A. Bradley, and R. A. Applegate, "Accuracy and precision of objective refraction from wavefront aberrations," *J. Vis.* **4**(4), 329–351 (2004).
19. C. Dorronsoro, X. Barcala, E. Gamba, V. Akondi, L. Sawides, Y. Marrakchi, V. Rodríguez-Lopez, C. Benedi-Garcia, M. Vinas, E. Lage, and S. Marcos are preparing a manuscript to be called "Tunable lenses: dynamic characterization and fine-tuned control for high-speed applications."
20. M. Vinas, A. Gonzalez-Ramos, C. Dorronsoro, V. Akondi, N. Garzon, F. Poyales, S. Marcos, "In vivo measurement of longitudinal chromatic aberration in patients implanted with trifocal diffractive intraocular lenses," *J. Refract. Surg.* **33**(11), 736–742 (2017).
21. H. Zhang, H. Ren, S. Xu, and S. Wu, "Temperature effects on dielectric liquid lenses," *Opt. Express* **22**(2), 1930–1939 (2014).