Analysis of the efficiency of the Chor-Rivest cryptosystem implementation in a safe-parameter range

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Abstract

The Chor-Rivest cryptosystem, based on a high-density knapsack problem on a finite field $\mathbb{F}_{q^h}$, was broken by S. Vaudenay for $q \approx 200$, $h \approx 24$, and h admitting a factor s verifying a certain condition. A new set of parameters q and h , which prevent this cryptosystem against Vaudenay's attack, is presented and the computational aspects of its implementation in the Magma computational algebra system are analyzed.

Key words: Chor-Rivest cryptosystem, Knapsack problem, Finite fields, Magma software, Public Key Cryptography.

1. Introduction

The objective of cryptography is to assure the secrecy and confidentiality of communications and the goal of cryptanalysis is to break the security and privacy of such communications ([12, 13]). In particular, in Public Key Cryptography (PKC) each user has two keys: The public key, which is publicly known and it is used by the sender to encrypt a message; and the private key, which is kept in secret by the receiver and it is used by him to decrypt encrypted messages. In general, PKC bases its security on the computational intractability of some Number Theory problems, such as factorization problem, discrete logarithm problem and knapsack problem.

In 1985, Chor proposed a cryptosystem based on the knapsack problem (see [2, 3]). The last —and the only really efficient— attack to this system has been proposed by Vaudenay ([17]), but only for the parameters originally proposed.

The Chor-Rivest cryptosystem is based on the arithmetic of finite fields and it needs to compute discrete logarithms in order to determine the keys of the

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system. The Discrete Logarithm Problem (DLP) can be defined as follows: Given a prime integer p , a generator α of the cyclic group $\mathbb{Z}_p^*$, and an element $\beta \in \mathbb{Z}_p^*$, the DLP consists in finding an integer x , $0 < x \leq p-1$, such that $\beta = \alpha^x$. This problem is considered to be difficult because the best known algorithm for solving it is the number field sieve ([16]) which has a subexponential expected running time:

$$O\left(e^{((64/9)^{1/3}+o(1))(\ln p)^{1/3}(\ln \ln p)^{2/3}}\right).$$

The security of the cryptosystem depends on the knapsack problem but not on the discrete logarithm problem. In fact, if the DLP becomes tractable, then the Chor-Rivest cryptosystem is easier to implement, but not easier to break.

As other knapsack cryptosystems, Chor-Rivest cryptosystem is not a very popular public key cryptosystem. Some of its drawbacks are that it needs a large time for generating its keys and a big size of public keys.

Nevertheless, nowadays more knapsack-based cryptosystems are being proposed in order to consider new candidates in PKC (see, for example, [18] and the references therein). One of the main reasons argued by the authors to state that their cryptosystem is secure was that it resists the low-density attacks, due to the fact that its density is 1.2, which is bigger than 0.9408 (see [1, 4, 14]). Although the system resists low-density attacks, it was recently broken in [19] by means of a heuristic attack, which permits to recover the private key from the public key. This is an added reason to study the security of some of the most important knapsack proposed, as it is the case of Chor-Rivest cryptosystem.

Moreover, the recommendations for the most popular public key cryptosystems (RSA, ElGamal, etc.) suggest to increase the size of their parameters, due, basically, to the results in quantum computation and in the improvements of the computational time for some number theory problems (see [9, 10]). In this way, the recommended bitlength of the keys are around 2048-4096 (for example, nowadays there are millions of ID cards in Europe with keys of these sizes).

For these reasons, it is relevant and of interest to study safe alternative cryptosystems, and the Chor-Rivest cryptosystem may be one of them.

In spite of several attacks against Chor-Rivest cryptosystem have been proposed (see [2, 3]):

- low-density attacks.
- nothing known (by E. Brickell),
- known g and r ,
- known t and r ,
- known t (by O. Goldreich),
- known π and r (by A. Odlyzko),

none of them is efficient without knowing a part of the private key.

In [17] the Chor-Rivest cryptosystem was broken for the original proposed parameters, i.e., when the cryptosystem is defined over a finite field $\mathbb{F}_{q^h}$ with

$q \approx 200$ and $h \approx 24$. This attack is based on a weakness derived from the fact that the cryptosystem is insecure if the parameter h has a factor s verifying the following condition:

$$s \geq \sqrt{h + \frac{1}{4}} + \frac{1}{2}. \quad (1)$$

In order to avoid Vaudenay's attack, in [7] a new pair of parameters has been determined for using safely this cryptosystem. In fact, these parameters were computed in a suitable range guaranteeing its security and its computational feasibility; such parameters are $q = 409$ and $h = 17$. Only a new pair of values was obtained! This is clearly not useful for cryptographic applications.

Here, we present a new set of parameters q and h , which prevent Chor-Rivest cryptosystem against Vaudenay's attack. Moreover, the main computational aspects of its implementation in the Magma computational algebra system are analyzed. These results show that there are many (in fact, infinite) values of new parameters for which Vaudenay's attack are unfeasible, which is relevant from a cryptographic point of view. In addition, this new implementation is more efficient than that proposed in [6].

2. Preliminaries

A brief description (see [2, 3, 7] for the details), of the Chor-Rivest cryptosystem is as follows:

1. Choose a prime number q and an integer $h \leq q$ so that the DLP can be efficiently solved in the finite field $\mathbb{F}_{q^h}$. This fact is important to generate the keys because this generation needs to compute discrete logarithms in the group $\mathbb{F}_{q^h}^*$. The reason is that the DLP can be solve in a efficient way by using the Pohlig-Hellman algorithm if the order of the multiplicative group considered, $n = q^h - 1$, factorizes as a product of small prime factors ([12, 15]).
2. Select a random algebraic element of degree h over $\mathbb{F}_{q^h}$, t , and a random irreducible monic polynomial, $f(t)$, of degree h , such that $\mathbb{F}_{q^h} \approx \mathbb{F}_q[t]/(f(t))$.
3. Choose a random generator, g , of the group $\mathbb{F}_{q^h}^*$. Such generator can be chosen at random in $\mathbb{F}_{q^h}^*$ until it verifies $g^{(q^h-1)/l} \neq 1$ for all prime divisors, l , of $q^h - 1$.
4. Compute the discrete logarithms $\log_g(t + \alpha_i) = a_i$, $\forall \alpha_i \in \mathbb{F}_q$.
5. Generate a random permutation of q elements, $\pi: \{0, 1, \dots, q-1\} \rightarrow \{0, 1, \dots, q-1\}$ and compute $b_i = a_{\pi(i)}$.
6. Add a random noise, $0 \leq r \leq q^h - 2$, to obtain the elements of the knapsack: $c_i \equiv (b_i + r) \pmod{q^h - 1}$, $0 \leq i \leq q-1$, which are the public key.
7. The private key is formed by the values (t, g, π, r) .

The messages to be encrypted, M , are binary vectors of length q and weight h ([5]), i.e.,

$$M = (m_0, m_1, \dots, m_{q-1}), \quad m_i \in \{0, 1\}, \quad \sum_{i=0}^{q-1} m_i = h.$$

The ciphertext corresponding to a message M is:

$$E = \sum_{i=0}^{q-1} m_i c_i \pmod{q^h - 1}.$$

For decryption, M is obtained from E as follows:

- i. Compute $s \equiv E - hr \pmod{q^h - 1}$.
- ii. Obtain the polynomial of degree $h - 1$, given by $Q(t) = g^s \pmod{f(t)}$.
- iii. Determine the h roots, $\alpha_{\pi(i)}$, of $f(t) + Q(t)$ in $\mathbb{F}_q$, such that,

$$f(t) + Q(t) = \prod_{i \in I} (t + \alpha_{\pi(i)}).$$

- iv. Apply π^{-1} to the roots $\alpha_{\pi(i)}$ for recovering the coordinates of the original message, M , having the bit 1.

3. New and safer parameters

The first efficient attack against the Chor-Rivest cryptosystem for the original proposed parameters in [2, 3] (i.e., $q \cong 200$, $h \cong 24$), is developed in [17], assuming, in addition, h has a factor verifying the condition (1).

After that, some new parameters have been found, say $q = 409$, $h = 17$, keeping the safety of the cryptosystem and some important properties, like density, size of the public knapsack, etc. (see [7] for more information). Nevertheless, this pair of parameters, is too restrictive in order to develop practical implementations. For this reason, in the present study the restrictions considered in [7] have been relaxed in order to determine new pairs of the parameters (q, h) and thus to obtain a larger set of parameters for implementations of the cryptosystem, but keeping the same level of safety and avoiding Vaudenay's attack.

In order to avoid Vaudenay's attack we first need to determine those parameters h for which no divisor s verifies the condition (1). We obtain

Proposition 1. *If an integer h has no factor s verifying the inequality (1) then h is either a prime number or the square of a prime number.*

Proof. Assume h is not prime and let k be the least prime divisor of h . Hence, $h = s \cdot k$, $s \in \mathbb{Z}$, $s \geq k$. If $s = k$, the result follows. If $s > k$, then the inequality $s < \sqrt{h + \frac{1}{4} + \frac{1}{2}}$ implies $(s - \frac{1}{2})^2 < h + \frac{1}{4}$, and simplifying $s(s - 1) < h = s \cdot k$. Hence, $s - 1 < k$, thus leading us to $k < s < k + 1$, which is not possible. $\square$

The conditions for obtaining the new range of parameters are the following:

- (a) $2 \leq h \leq q$, q being a prime number and h being either a prime number or the square of a prime number.
- (b) $10^{35} < n = q^h - 1$, that is, the number of digits of n is $t(q, h) \geq 36$.
- (c) The bitlength of the public key for (q, h) satisfies $l(q, h) < 150000$.
- (d) The density, $d(q, h)$, for the knapsack is much bigger than 1, to avoid all known attacks to low-density knapsack cryptosystems.
- (e) The smoothness of n , $u(n)$, i.e., the greatest prime factor of n has at most 18 decimal digits: $u(n) \leq 18$.

These conditions are justified taking the following facts into account:

- First, Vaudenay's attack is not applicable if condition (a) is assumed. In fact, as q is a prime number and h is prime or the square of a prime number, h has no factors and the condition (1) has no sense.
- Condition (b) is less restrictive than that considered in [6], but the cryptosystem is still safe because no attack is known under the assumption of conditions (a) and (b).
- Condition (c) has been considered because of Lenstra's conjecture ([11]), which states that knowing the whole public knapsack could be a weakness for the system. Hence, the idea is to publish only a half (or less) of the public key. Note that the cryptosystem still works in this case.

Condition (d) remains unchanged as no outstanding improvements have been published in relation to the attacks against the value of the density of a knapsack. In fact, the improvement published in [8] considers that is possible to break a system if its density is < 1 . But this bound cannot be applied to the results obtained below (see Table 1).

- Finally, condition (e) is considered under the assumption that the number of bit operations per day of a standard PC is around 10^{14} . This value for the smoothness of n , $u(n) \leq 18$, has been obtained carrying out similar computations to those in [6].

In Table 1 the 44 pairs (q, h) verifying the conditions (a)-(e) are shown.

Table 1: Pairs of parameters (q, h) verifying the conditions (a)-(e).

h	q	$t(q, h)$	$l(q, h)$	$d(q, h)$	$u(n)$
13	547	36	64546	4.626	14
13	571	36	67949	4.796	13
13	577	36	68663	4.838	14
13	599	37	71281	4.994	13
13	601	37	72120	5.008	11
13	613	37	73560	5.092	15
13	631	37	75720	5.218	11
13	641	37	77561	5.288	17

Table 1: Pairs of parameters (q, h) verifying the conditions (a)-(e).

h	q	$t(q, h)$	$l(q, h)$	$d(q, h)$	$u(n)$
13	659	37	79739	5.413	16
13	683	37	83326	5.579	14
13	719	38	88437	5.828	15
13	757	38	93868	6.088	16
13	787	38	98375	6.292	15
13	797	38	99625	6.360	13
13	839	39	105714	6.644	11
13	877	39	111379	6.900	16
13	887	39	112649	6.967	16
13	941	39	120448	7.327	14
13	953	39	121984	7.407	16
13	967	39	123776	7.500	14
13	977	39	126033	7.566	16
13	1093	40	143183	8.329	13
13	1123	40	147113	8.524	16
17	127	36	14986	1.068	15
17	131	36	15589	1.095	15
17	167	38	20875	1.330	14
17	193	39	24897	1.495	17
17	233	41	30989	1.742	15
17	263	42	35768	1.924	15
17	277	42	37949	2.008	15
17	317	43	44697	2.244	17
17	331	43	47002	2.326	17
17	409	45	60123	2.773	9
17	433	45	64084	2.908	18
17	587	48	91572	3.754	14
17	643	48	101594	4.054	17
17	661	48	105099	4.150	16
23	173	52	29410	1.011	18
23	191	53	33234	1.095	18
23	199	53	34825	1.132	15
23	283	57	52921	1.510	16
23	563	64	118230	2.679	14
25	601	70	138230	2.604	17
25	613	70	141603	2.648	16

Table 2 shows the pairs of parameters for which the q discrete logarithms can be computed over in one day, approximately. This computation has been made by using the Magma implementation presented in §4, with a Pentium D-930 3.00 GHz, 2.00 GB RAM. In this Table, the column Total Time contains the number

of seconds of CPU time needed for the calculation of the q logarithms; while the column Average Time is filled with the average time, in seconds, required to calculate each of the q logarithms.

Table 2: Pairs (q, h) for which $\log_q(t + i)$, $0 \leq i \leq q - 1$, can be computed in 1 day, approximately.

h	q	Total Time	Average Time
17	409	198.109	0.484
13	601	3454.592	5.748
13	631	7308.658	11.583
13	839	14719.431	17.544
13	599	32918.392	54.956
13	797	77935.191	97.786
13	571	80912.053	141.702
13	577	88155.129	152.782
17	167	98539.430	590.056

4. Implementation with Magma software

Magma is a Computer Algebra System designed to solve problems in algebra, number theory, geometry and combinatorics that may involve sophisticated mathematics and which are computationally hard. Magma provides a mathematically rigorous environment which emphasizes structural computation. The kernel of Magma contains implementations of many of the important concrete classes of structure in five fundamental branches of algebra, namely group theory, ring theory, field theory, module theory and the theory of algebras. That is why we choose Magma, a more appropriate software than that used in [6]. Maple allows to develop mathematics with a more accessible and easy to use software, but Magma is a large, well-supported software package designed to solve computationally hard problems in algebra.

The implementation of Chor-Rivest cryptosystem with Magma software is composed by several procedures. The main procedures are related to the generation of the keys, and to encrypt and decrypt messages. We will present these procedures with detail. Moreover, there are other procedures necessary to transform the messages in a suitable way (see [2, 3]), to determine the generator of the multiplicative group, and to generate the permutation; but they are not an essential part of the cryptosystem implementation.

The public and private keys are generated by means of the procedure **keys** in the following way. Once they are obtained, the public and the private keys can be saved in two files for their later use.

First of all, the finite field of q elements, $\mathbb{F}_q$, is defined, and then an extension of it is considered. After defining the finite field and its extension, it is necessary to compute some discrete logarithms.

```

keys:=procedure(iq, ih)
// Initial parameters
n:=iq^ih-1; //order of the multiplicative group
Fq:=GF(iq); //base finite field
P<x>:=PolynomialRing(Fq);
f:=IrreduciblePolynomial(Fq, ih); //irreducible polynomial
F<t>:=ext<Fq|f>; //extension of the base field
g:=primitive(F); //generator of the group
permut:=permutation(iq); //permutation
r:=Random(iq^ih-2); //noise
b:=[]; c:=[]; logs:=[];
// Computation of discrete logarithms
for i in [0 .. iq-1] do
    logs[i+1]:=Log(g,t+i);
end for;
// Computation of the public key
for i in [1 .. iq] do
    b[i]:=logs[permut[i]+1];
    c[i]:=(b[i]+r) mod n;
end for;
end procedure;

```

The procedure for encrypting a message is as follows. First of all, the sender considers the text of his message, M , computes the length and the number of blocks of the message, transforms and divides the message into blocks of the same length, and transforms it in blocks of length q and weight h (see the corresponding algorithm in [2, 3]). Later, the sender encrypts the message and saves it in a file.

```

encrypt:=procedure(iq, ih)
// Initial parameters
inMens:=plaintext; N:=iq^ih-1; ltext:=#inMens;
lengt:=Ilog2(Binomial(iq,ih)); lblock:=Floor(lengt/8);
lmess:=lblock*Ceiling(ltext/lblock);
nblocks:=Floor(lmess/lblock); Scii:=[]; Sciic:=[];
// The message is transformed to ASCII and 0's are added if needed
Scara:=ElementToSequence(inMens);
for i in [1..#Scara] do
    Scii[i]:=StringToCode(Scara[i]);
end for;
for i in [1..lmess] do
    if i le ltext then
        Sciic[i]:=Scii[i];
    else
        Sciic[i]:=0;
    end if;
end for;

```



```

// Transformation of the message
messblock1:=[]: s in [1..nblocks]]; messblock:=[];
mess256:=[]; M:=[]; E:=[];
for i in [1..nblocks] do
    lim1:=(i-1)*lblock+1; lim2:=i*lblock; k:=1;
    for j in [lim1..lim2] do
        lim3:=lim2-lim1+j; messblock1[i][k]:=Sciic[j]; k:=k+1;
    end for;
end for;
//representation in base-256
for m in [1..nblocks] do
    aux:=0;
    for l in [1..lblock] do
        aux:=aux + messblock1[m][l]*256^(l-1);
    end for;
    mess256[m]:=Intseq(aux,10); aux2:=0;
    for n in [1..#mess256[m]] do
        aux2:=aux2+mess256[m][n]*10^(n-1);
    end for;
    messblock[m]:=aux2;
    // Transformation of the message into blocks of length  $q$  and weight  $h$ 
    // by using a procedure called Transformation
    M[m]:=Transformation(messblock[m],iq,ih); aux3:=0;
    // Encryption process
    for p in [1..inq] do
        aux3:=(aux3+M[m][p]*publicK[p]) mod N;
    end for;
    E[m]:=aux3;
end for;
end procedure;

```

The procedure for decrypting an encrypted message, which was saved in a file, is the following.

```

decrypted:=procedure(iq, ih)
    // Initial parameters
    cipherMess:=ciphertext; //encrypted text
    nblocks:=#cipherMess; N:=iq^ih-1; //length and number of blocks
    // Initialization
    sprime:=[]; Q:=[]; pol:=[]; aux:=[]; roots:=[]; Raux:=[];
    R:=[]; Paux:=[]; Pos:=[]; m:=[]; Me:=[];
    Mes:=[]; Message:="";
    // Decryption
    for i in [1..nblocks] do
        sprime[i]:=(cipherMess[i]-inh*r) mod N; Q[i]:=g^sprime[i];
        aux:=Eltseq(Q[i]); aux2:=P!aux;
        pol[i]:=Factorization(f+aux2); //factorization of the polynomial
    end for;
end procedure;

```

```

roots[i]:=Roots(f+aux2);//roots of the polynomial
m[i]:=[0:i in [0..inq]];
//positions of the 1's using the permutation
for j in [1..inh] do
    Raux[j]:=-roots[i][j][1];
    Paux[j]:=Position(permut,Raux[j]);
    m[i][Paux[j]]:=1;
end for;
// Transformation of integer numbers into binary ones (inverse
// transformation) by using the a procedure called ITransformation
Me[i]:=ITransformation(m[i],iq,ih);
Mes[i]:=Intseq(Me[i],256); //return to base-256 representation
// Recovering the original message
for k in [1..#Mes[i]] do
    Maux:=CodeToString(Mes[i][k]);
    Message:=Message cat Maux;
end for;
R[i]:=Raux; Pos[i]:=Sort(Paux);
end for;
end procedure;

```

5. Computational results

We have encrypted and decrypted several messages with different lengths in order to measure the running time needed in different cases. To do this, we have considered the pairs of parameters listed in Table 2, and the results obtained are shown in Table 3.

For these pairs of values, we have considered Lenstra's conjecture ([11]). Thus, we have made public different number of elements of the public key. This number of published elements is represented by PKN and the length of each block of the message is given in parentheses. In this sense, we have measured the time to encrypt and decrypt messages considering a part of the whole public knapsack. For each value of q we have considered a half part of them, then a quarter part, and finally the 10% of the public key.

The variables analyzed in this experimental computation are the following: Text length: TL (measured in KB); number of characters of the message: CM (adding 0's if needed); number of blocks of the message: BN; and the Encryption and Decryption Time (in seconds): ET and DT, respectively. The messages considered have 50 KB (around 49896 characters) and 192 KB (around 191727 characters).

Table 3: Experimental computational results for the pairs (q, h) listed in Table 2.

Parameters	PKN	TL	CM	BN	ET	DT
(409, 17)	205	50	49900	4990	4.250	14.859

Table 3: Experimental computational results for the pairs (q, h) listed in Table 2.

Parameters	PKN	TL	CM	BN	ET	DT
	(10)	192	191730	19173	16.078	59.093
(409, 17)	102	50	49896	7128	3.328	20.766
	(7)	192	191730	27390	12.938	81.875
(409, 17)	41	50	49892	12473	2.531	35.734
	(4)	192	191728	47932	9.281	138.797
(601, 13)	301	50	49896	5544	5.500	11.750
	(9)	192	191727	21303	21.172	46.922
(601, 13)	150	50	49896	7128	3.906	14.672
	(7)	192	191730	27390	14.718	58.218
(601, 13)	60	50	49895	9979	2.406	20.125
	(5)	192	191730	38346	8.859	79.234
(631, 13)	316	50	49896	5544	5.922	12.234
	(9)	192	191727	21303	22.437	48.953
(631, 13)	158	50	49896	7128	4.094	15.328
	(7)	192	191730	27390	15.593	60.859
(631, 13)	63	50	49895	9979	2.484	21.172
	(5)	192	191730	38346	9.312	83.078
(839, 13)	420	50	49900	4990	6.610	11.437
	(10)	192	191730	19173	25.390	45.640
(839, 13)	210	50	49896	6237	4.515	13.781
	(8)	192	191728	23966	17156	54.875
(839, 13)	84	50	49896	8316	2.562	18.016
	(6)	192	191730	31955	9.672	70.781
(599, 13)	300	50	49896	5544	5.562	11.953
	(9)	192	191727	21303	21.047	47.750
(599, 13)	150	50	49896	7128	3.921	14.953
	(7)	192	191730	27390	14.625	56.250
(599, 13)	60	50	49895	9979	2.360	20.485
	(5)	192	191730	38346	8.875	80.500
(797, 13)	399	50	49896	5544	8.093	12.562
	(9)	192	191727	21303	31.046	49.750
(797, 13)	199	50	49896	6237	4.218	13.640
	(8)	192	191728	23966	16.094	54.234
(797, 13)	80	50	49896	8316	2.500	17.844
	(6)	192	191730	31955	9.187	70.094
(571, 13)	286	50	49896	5544	5.140	11.656
	(9)	192	191727	21303	19.594	46.468
(571, 13)	143	50	49896	7128	3.625	14.594
	(7)	192	191730	27390	13.844	57.796
(571, 13)	57	50	49895	9979	2.328	19.875
	(5)	192	191730	38346	8.453	78.266

Table 3: Experimental computational results for the pairs (q, h) listed in Table 2.

Parameters	PKN	TL	CM	BN	ET	DT
(577, 13)	289	50	49896	5544	5.249	11.172
	(9)	192	191727	21303	19.797	44.860
(577, 13)	144	50	49896	7128	3.704	14.016
	(7)	192	191730	27390	13.968	55.578
(577, 13)	58	50	49895	9979	2.266	19.203
	(5)	192	191730	38346	8.703	75.234
(167, 17)	84	50	49896	7128	2.703	15.625
	(7)	192	191730	27390	10.156	62.016
(167, 17)	42	50	49892	12473	2.500	26.609
	(4)	192	191728	47932	9.359	104.281
(167, 17)	20	50	49891	49891	4.453	104.312
	(1)	192	191727	191727	16.438	406.844

If all the pairs of the parameters listed in Table 3 are analyzed, the best pair for the encryption process is the last one, namely (167, 17). In this case, the block length is the smallest one. This value depends only on the values of q and h and is calculated with the expression $\log_2 \binom{q}{h}$ (and divided by 8).

For the decryption process, the values are similar, and the best one is the pair (577, 13), with a block length equals to 10 bits.

Finally, as it could be observed, the decryption process is slower than the encryption process in all cases.

6. Conclusions

We have studied and implemented the Chor-Rivest cryptosystem with Magma software by using a larger range of safe parameters than those suggested in previous proposals. For these parameters no attack has been described. Moreover, we have presented a Magma implementation of this cryptosystem suitable for the set of these new parameters in an efficient way.

The implementation opens a door to the future in order to use realistically the Chor-Rivest cryptosystem, based on the fact that no attack has yet been proposed when the parameters q, h determined above, are used.

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