Preference shock and party consistency

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Abstract

I develop a two party electoral competition model in order to analyze the effects of an exogenous preference shock on the strategic policy choice of parties. I find that on the one hand, if the preference shock affects the issue that is currently most salient, then both parties strategically shift their policy choices in the direction of the shock, however the shifts are asymmetric: the policy shift of the party that is most favored by the preference shock is smaller. On the other hand, if the preference shock affects an issue that was not the most salient before and becomes salient because of the shock itself, then both parties strategically shift their policy choices towards the ideal point of the median voter of that issue. And again, the parties policy shift is asymmetric: the reaction of the party that is most favored by the preference shock is smaller. Finally, I show that the effects of a large policy shift that some parties perform when they are optimally reacting to a voters’ preference shocks may break the party’s internal equilibrium among its different factions, implying that exogenous preference shocks may cause important changes in a party system.

Keywords: preference shock, salience, party consistency.

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1 Introduction

When an exogenous shock affects the policy preferences of the voters, the equilibrium that competing political parties had attained is destroyed. Voters are expected to change their voting decisions and parties have to realign their positions in order to react optimally to their changes. The size of the policy shift that some parties have to make depends on the magnitude of the shift produced on the voters’ preferences, and thus, on the intensity of the shock. However, the optimal policy shift that a party will produce after a preference shock will not be the same for all parties.

Some parties may benefit from the preference shock. That will be the case when the shock drives the voters’ ideal points towards the ideal policy of the party. If the effects of the shock drive the ideal points of all voters closer to a given party’s ideal policy, then this party will enjoy an electoral advantage, since each vote has become cheaper in terms of ideological cost: in order to attract votes after the shock these parties can propose policy choices that are closer to their ideal points than the ones that they had to use before the shock. At the same time, some parties may be harmed by the shock. This happens whenever the shock drives the voters’ ideal points away from the ideal policy of the party. The shock is costly for these parties, because each single vote becomes more expensive in terms of ideological cost after the shock: in order to attract votes these parties have to propose policy choices that are further away from their ideal points than the ones that work for them before the shock. This observation allows me to build an argument that explains the asymmetry expected in the parties policy choice produced as a reaction to a voters’ preference shock.

It is reasonable to think that the policy preferences of the voters are often conditioned and even determined by the policy stands of the parties during campaigns, and by the policies implemented by incumbents. Indeed, it is in the interest of the parties to try to manipulate voters’ opinions through electoral campaigns, and media advertising. There is a literature devoted to the phenomenon of priming that includes a large number of experimental and empirical studies in psychology, political psychology and political science (Bartels 2006; Iyengar 1990; Iyengar and Kinder 1987; Iyengar, Kinder, and Peters 1982; Kahneman and Tversky 1979, 1981, 1984; Krosnick and Kinder 1990; Sheafer and Weimann 2005; for a critique, see also Lenz 2009). However, it is plausible to consider that there are some instances in which a shock that is exogenous to parties ideologies and to parties previous performances can change the voters’ preferences.

In order to provide evidence on the possibility of existence of exogenous shocks that affect voters preferences I will refer to a natural experiment described in Montalvo (2011 and 2012). Montalvo analyzes the effects on the result of an election of a shock on the voters’ preferences produced as a consequence of
a terrorist attack. His analysis focuses on the Spanish congressional election of March 2004. The special feature of this election comes from the fact that election day was scheduled by Sunday March 14th and the bombings in Madrid took place on Thursday March 11th. Polls taken before the bombings showed a clear advantage of the conservative party over the socialist party, however the actual vote of that election gave a clear victory to the socialist party. The important difference before the results of the polls taken before the attack and the actual voting results seemed to indicate that the voters might have changed their vote intention because of the bombings. Indeed, the conservative party, incumbent at that time, could be considered by many voters partly responsible of the attack, because several months before its government decided to join USA and UK in the war against terrorism in Iraq. Instead, in his electoral platform, the socialist leader proposed to withdraw the Spanish troops from Iraq in case of victory. Several papers have tried to analyze the relationship between the terrorist attacks and the election results using post election surveys. For instance Bali (2007) seem to imply that the bombings were decisive while and Lago and Montero (2005) find that they had no effect. Thus, the answers offered by the analysis of the surveys are inconclusive.

In order to show that the bombings were the cause of the change in the voters’ decision, Montalvo compares the evolution of the vote for congressional elections on election day with a control group of individuals that cast their vote before the terrorist attacks: residents abroad had cast their vote before March 7 at a Spanish consulate or by certified mail, thus they voted before the bombing took place. Montalvo shows that the results of the vote in this control group showed an advantage to the conservative party, as the predicted by the polls, and its turnout level was consistent with the normal trend in previous elections. Instead the vote cast on the election day, after the bombings, reversed the results predicted by the polls and showed a significant advantage for the socialist party over the conservative and in addition it showed a significant increase in the participation level, compared to previous congressional elections. With this natural experiment Montalvo demonstrates that the terrorist attacks were the reason that voters changed their political views and also the reason that induce voters to turnout in larger numbers. The change of political views in the case analyzed can be interpreted as a increase of preferences for more antiterrorist regulation or national security. The larger turnout can be interpreted as a result of an increase of the importance of the act of voting through the increase of the relevance of the policy dimension affected by the shock. Thus, this analysis shows that preference shocks that are exogenous to the parties are possible.

Following these conclusions I will assume that an exogenous preference shock affects the voters’ policy preferences on a given issue and their voting behavior in two different ways. On the one hand, it implies a shift of the voters ideal points. In the particular case of a terrorist attack it is plausible to think that
all voters may demand more national security. Thus, in the policy dimension affected by the shock, the ideal points of all voters change and they all change in the same direction. On the other hand, the voters may regard as more important and more relevant the policy dimension affected by the shock, and thus the salience of this policy dimension may increase after the shock.

I construct a model of two party electoral competition with policy motivated parties and sincere voters. I assume that there is a decisive voter whose ideal point is unknown to the parties and parties have beliefs about it that are common and common knowledge. In this model I introduce two assumptions that relate closely to the two effect described above: an exogenous shock on a given policy dimension produces a shift of the ideal points of all voters on that dimension in the same direction, and it also produces an increase of the salience of the policy dimension affected by the shock. I analyze the policy choices of two parties that compete in an election both before and after the shock takes place and compare them.

In order to disentangle the effects of each one of these two assumptions, I first analyze a one dimensional policy space model and consider only the effect of the shift of the voters’ ideal points on the policy proposals of the parties in equilibrium. Then I analyze a two dimensional policy space model in which I consider both effects: the policy preference shift and the issue salience increase.

The one dimensional model shows that in equilibrium parties that are policy motivated choose policies that are moderate compared to their ideal points. If there is no shock these policies are symmetric with respect to the expected location of the ideal point of the decisive voter (the median voter). These results are in line with the ones described in Calvert (1985) and Wittman (1977 and 1983). If the voters’ preferences are affected by a shock, as described above, the change in the policy choices of both parties is produced in the same direction, and it is also the direction in which the voters’ preferences changes. However, the resulting policy shifts are asymmetric because the magnitude of the policy change is different for each party. In fact, if all the voters’ ideal points move in the same direction then the ideal point of the decisive voter moves closer to one party ideal point and away from the other party ideal point. This implies that one party is favored by the shock while the other party is harmed. The party that is harmed by the shock has to produce a larger policy change in order to optimize his payoffs when competing for votes in an electoral contest, while the party favored by the shock has only to react optimally to his opponent change of strategy. Thus the policy choice of the party harmed by the shock will become more moderate than the policy he chose before the shock, while the policy of the party favored by the shock will be less moderate (closer to his ideal point) than the policy he chose before the shock.

In the two dimensional policy space model the policy choices of the two parties in equilibrium depend on the relative salience that the voters assign to the different issues. The more salient an issue the more moderate are the parties policy choices on that issue. In particular, if only one issue is salient, that is, if
voters base their voting decision only on the policy proposals of this issue, then the equilibrium policies on the salient issue coincide with the moderate ones found in the previous model, and the equilibrium policies on the non-salient issue coincide with the parties' ideal points. Indeed, there is no reason for the parties to compromise their policy positions on an issue if voters are not paying attention to it.

Now consider the effects of a shock that produces not only a shift in the voters’ preferences but also an important raise in the salience of the policy dimension affected by the shock. Here I will consider two possibilities. On the one hand, it is possible that the policy dimension affected by the shock was already the most salient before the shock was produced. In that case, the effects of the shock are only driven by the voters’ preference shift, and the results coincide with the ones obtained in the analysis of the one-dimensional model described before. On the other hand, it is possible that the dimension affected by the shock was not the most salient before the shock was produced, and it becomes the most salient as a consequence of the shock. In this case, the comparison between the policies chosen by the parties before and after the shock is a bit more complex. Before the shock, the policy dimension was not salient and thus both parties could implement their ideal points on that issue. Since when deciding their vote voters only take into account those policies proposed on issues that are salient, parties optimize their payoffs by implementing their ideal points on those issues that are not salient. After the shock, the policy dimension affected by the shock becomes the most salient and thus parties have to react by moderating their policies on this issue. The reactions of the parties in this case can be explained with an argument similar to the one built for the one-dimensional case. As before, the party that is harmed by the shock has to produce a moderate policy while the party that is favored by the shock does not have to moderate its policy so much. However, now the results obtained for the parties’ policy choices after the preference shock have to be compared to the parties’ ideal points on that dimension, since this was their optimal choice before the shock, when the issue was not salient. As a result I find that in this case, when the shock does not only produce a change in the voters’ policy preferences but also a change on which is the issue that voters consider as the most salient, both parties react by moderating their choices, that is, moving their policies towards the ideal point of the expected median voter. In addition, both parties are moderating their policies in an asymmetric way: in equilibrium the party harmed by the shock chooses a policy far away from his ideal point while the party favored by the shock can choose a policy relatively close to his ideal point.

Notice that in both cases, the parties react in an asymmetric manner to the voters’ preferences shocks. This result can be related to the empirical evidence provided by Plümper and Epifanio (2015). They analyze the change in the number of antiterrorist policies implemented by incumbents of different countries after a terrorist attack. They find that the increment of this number is much larger for leftist incumbents.
than for rightist incumbents. The formal arguments developed here offer a theoretical explanation for the empirical observation provided by Plümper and Epifanio.

The results also show that preference shocks affect the electoral balance between the two competing parties. By favoring one of the parties over the other one imposes different electoral costs over different parties: makes electoral competition much lighter for one of the parties and much more costly for the other one. Producing a large policy shift may be very costly for a party, because compromising ideological principles may cause internal frictions among the different factions within the party and may end up destroying the party’s internal equilibrium.

I illustrate this argument with an example based on the recent evolution of the Catalan party system. During the last few years there has been a shift in the voters’ policy preferences over the level of regional decentralization of power (with respect to the Spanish government) desired for Catalonia, and an increase of the salience of the sovereignty issue. This preference change has induced some parties to make drastic changes in their policy proposals. As a consequence, the last few electoral contests have shown the destruction of the two main political parties and therefore the end of a party system that had been stable for over thirty years.

The rest of the paper is organized as follows. In the next section I present the one dimensional model and I analyze the effects of a preference shift on the parties equilibrium policy choices, in section 3 I analyze the two dimensional model and I show the effects of a preference shift and an increase of issue salience on the parties equilibrium policy choices, in section 4 I illustrate the effects of a large policy shift on the party’s internal consistency and in section 5 I conclude with some final remarks.

2 Voters’ preference shift

This section describes the effects of a shock that produces a voters’ preference shift on a one dimensional model of two party electoral competition. The case analyzed here corresponds to a shock that affects the issue that was considered as the most salient for the voters even before the shock was produced. Thus, voters base their voting decision on the policies proposed on this single issue before and after the shock. In order to analyze this case a one dimensional policy space is sufficient.

2.1 The one dimensional model

There are two parties, A and B, that compete in an election. The policy space X is one dimensional and represented by the unit interval $X = [0, 1]$. Let $x \in X$ denote a policy position on the unique issue. Before the election, parties simultaneously choose policy platforms $x_A$ and $x_B$ respectively from the policy space
Without loss of generality, I assume that there is a unique decisive voter who has preferences over policies represented by a utility function \( u_m(x) = -(x - x_m)^2 \) where \( x_m \in X \) represents the voter’s ideal policy.

I assume that parties do not know the exact location of \( x_m \) and they have beliefs about it that are common knowledge and are represented by a probability distribution \( F : X \to [0, 1] \) with support over \( X \) and with probability mass function \( f : X \to [0, 1] \). If parties knew the exact value of \( x_m \), they could anticipate the probability with which the decisive voter votes for each party and they could use it to decide their optimal strategy. Since I assume that parties have beliefs about the value of \( x_m \), they can only anticipate the expected probability with which the decisive voter votes for each party.

Parties have policy preferences, just like voters. I assume that the ideal point of party \( A \) is 0 and the ideal point of party \( B \) is 1. The parties ideal points are common knowledge. Parties are policy motivated and each party maximizes his expected utility given by:

\[
U_A(x_A, x_B) = \pi_A \left( - (x_A)^2 + (1 - \pi_A) \left( - (x_B)^2 \right) \right)
\]

\[
U_B(x_A, x_B) = \pi_A \left( - (1 - x_A)^2 + (1 - \pi_A) \left( - (1 - x_B)^2 \right) \right)
\]

where \( \pi_A = \pi_A(x_A, x_B) \) denotes the probability of winning for party \( A \) and \( 1 - \pi_A \) denotes the probability of winning for party \( B \) as a function of the parties policy choices \( (x_A, x_B) \).

The game takes place in two stages. In the first stage, parties simultaneously choose positions in \( X \). As in the standard Downsian model, I assume that parties implement their announced positions if they win the election. In the second stage, the decisive voter votes for the party whose election would give him the higher utility, if elected. In case of indifference, the voter is assumed to vote for each party with probability equal to \( 1/2 \). Since the behavior of the voter is unambiguous in this model, I define an equilibrium of the game only in terms of the location strategies of the two parties in the first round.

Let \( (x_A, x_B) \in X^2 \) denote a pair of pure strategies for parties \( A \) and \( B \) respectively. I solve for the Nash equilibria of this game, that is, those pairs of pure strategies \( (x_A^*, x_B^*) \in X^2 \) such that \( U_A(x_A^*, x_B^*) \geq U_A(x_A, x_B^*) \) for all \( x_A \in X \) and \( U_B(x_A^*, x_B) \geq U_B(x_A^*, x_B) \) for all \( x_B \in X \).

### 2.2 Shock on the one dimensional policy space

Since I am interested in the effects of a shock on the voters’ preferences that produce biases on the policy preferences of the decisive voter I assume that the beliefs of the parties about the location of the ideal point of the decisive voter are represented by \( F(x) = F(x, a) \) where \( F(x, a) \) denotes a uniform distribution over a support equal to \([a, 1] \) with \( a \in [0, 1) \). Thus I represent the effect of the shock on the policy preferences
of the voters with the parameter $a \in [0,1)$. Before the shock occurs I assume that $a = 0$ that is, parties believe that the expected ideal point of the decisive voter is distributed uniformly over the unit interval. After the shock, parties believe that the expected ideal point of the decisive voter is distributed uniformly over the interval $[a, 1]$ with $a > 0$. This implies that voters’ ideal points have shifted towards the right hand side of the policy space. Thus the parties’ beliefs are represented by the following density function

$$f(x, a) = \begin{cases} 0 & \text{if } x \in [0, a) \\ \frac{1}{1-a} & \text{if } x \in [a, 1] \end{cases}.$$

For every pair of policy choices $(x_A, x_B) \in X^2$ I have that the probability of winning for party $A$ is given by

$$\pi_A(x_A, x_B) = \begin{cases} 0 & \text{if } \frac{x_A + x_B}{2} \in [0, a] \\ \frac{1}{2(1-a)} & \text{if } \frac{x_A + x_B}{2} \in (a, 1] \end{cases}$$

and the probability of winning for party $B$ is given by

$$\pi_B(x_A, x_B) = 1 - \pi_A(x_A, x_B) = \begin{cases} 1 & \text{if } \frac{x_A + x_B}{2} \in [0, a] \\ \frac{2-x_A-x_B}{2(1-a)} & \text{if } \frac{x_A + x_B}{2} \in (a, 1] \end{cases}.$$

See figures 1 and 2. Now I will compare the equilibrium policy choices of the parties when there is no shock, $a = 0$, and when there is a preference shock, $a > 0$. First I describe the equilibrium results in the absence of any shock.

**Proposition 1:** If $a = 0$ then the parties equilibrium strategies are $x^*_A(0) = \frac{1}{4}$ and $x^*_B(0) = \frac{3}{4}$. All proofs are relegated to the appendix.

This case is a direct application of the model of two party competition with policy motivated parties described in Calvert (1985) and Wittman (1977 and 1983). Both parties moderate their policies in a symmetric way, by choosing platforms halfway between their respective ideal points and the expected median voter ideal point, and they both win with equal probability. This result describes the parties optimal policy choices before the shock takes place. Next we analyze the effect of the shock on the parties’ policy choices in equilibrium.

**Proposition 2:** If $a > 0$ then the parties equilibrium strategies are $x^*_A(a) = \frac{9a-2+\sqrt{(2-9a)^2+32}}{16}$ and $x^*_B(a) = \frac{2+\sqrt{(2-9a)^2+32}}{3a}$.

This result shows that after the shock the parties’ policy choices in equilibrium are conditioned on the value of the parameter that represents the intensity of the shock, $a$. Figure 3 represents graphically the parties’ policy choices on the one dimensional policy space. After the shock is produced the expected ideal point of the decisive voter has shifted to the right side of the policy space. This produces an important change in the optimal strategy of party A, since now electoral competition has become much more costly in terms of policy compromise. Instead, party B has to face a much lighter electoral competition, since his ideal point has become more popular from the voters’ point of view. Indeed, when analyzing the
best response functions of both parties one observes that the best response function of party B does not change much after the shock. His final policy choice will change only because party A’s policy choice is conditioned by the magnitude of the shock, and thus party B reacts to a different strategy of his opponent. Since his opponent is forced to choose more moderate policies, party B can choose a policy closer to his ideal point without compromising his electoral success.

Notice that the policies chosen in equilibrium by the two parties are located on both sides of the expected ideal point of the decisive voter. Even though party A produces a larger move towards the expected ideal point of the decisive voter than party B, in equilibrium party B’s policy choice ends up being much closer to the expected ideal point of the decisive voter than party A’s. For larger values of the shock magnitude both parties’ policy choices move towards the right side of the policy space, closer to each other, and converge to 1 when the value of $\alpha$ approaches 1.

The probability of winning for party A is always smaller than the probability of winning for party B, and it decreases with the size of the shock. Similarly, the expected payoffs in equilibrium for party A are smaller than those of party B and they are decreasing with the size of the shock.

Finally, when I compare the equilibrium policy choices before and after the shock, that is, the results obtained in propositions 1 and 2, I find that the policy shift produced by party A is much larger than the shift produced by party B. Given that these two shifts are produced in the same direction on the policy space, it implies that the shock produces a double disadvantage on party A with respect to party B: party A has to produce a larger shift from his ideal point than party B’s shift and more important, party A has to shift his policy away from his ideal point, while party B can move his policy closer to his ideal point.

3 Voters’ preference shift and issue salience increase

This section describes the effects of a shock that produces a voters’ preference shift on one dimension and an increase of the salience of this dimension on a two dimensional model of two party electoral competition. The case analyzed here corresponds to a shock that affects an issue that was not considered as the most salient for the voters even before the shock was produced, and that becomes the most salient issue after the shock. Since the voting decision depends mostly on the parties policy choices on the most salient issue, in this case I will have that the voters’ decision is going to be based on different issues before and after the shock. Therefore, I am going to extend the previous model in order to consider a two dimensional policy space, in which each dimension represents a different issue.
3.1 The two dimensional model

There are two parties, $A$ and $B$, that compete in an election. The policy space $X$ is two dimensional and represented by the unit square $X = [0,1] \times [0,1]$. Let $(x, y) \in X$ denote a policy position on each one of the issues. Before the election, parties simultaneously choose policy platforms $(x_A, y_A)$ and $(x_B, y_B)$ respectively from the policy space $X$. Without loss of generality, I assume that there is a unique decisive voter who has preferences over policies represented by a utility function $u_m(x, y) = -\sigma (x - x_m)^2 - (1 - \sigma) (y - y_m)^2$ where $(x_m, y_m) \in X$ represents the voter’s ideal policy and $\sigma \in [0,1]$ denotes the relative salience of issue $x$ over issue $y$.

I assume that parties do not know the exact location of $(x_m, y_m)$ and they have beliefs about it that are common knowledge and are represented by a probability distribution $F : X \rightarrow [0,1]$ with support over $X$ and with probability mass function $f : X \rightarrow [0,1]$. If parties knew the exact value of $(x_m, y_m)$, they could anticipate the probability with which the decisive voter votes for each party and they could use it to decide their optimal strategy. Since I assume that parties have beliefs about the value of $(x_m, y_m)$, they can only anticipate the expected probability with which the decisive voter votes for each party.

Parties have policy preferences, just like voters. I assume that the ideal point of party $A$ is $(0,0)$ and the ideal point of party $B$ is $(1,1)$. The parties ideal points are common knowledge. parties are policy motivated and each party maximizes his expected utility given by:

$$U_A((x_A, y_A), (x_B, y_B)) = \pi_A \left[ - (x_A)^2 - (y_A)^2 \right] + (1 - \pi_A) \left[ - (x_B)^2 - (y_B)^2 \right]$$

$$U_B((x_A, y_A), (x_B, y_B)) = \pi_A \left[ - (1 - x_A)^2 - (1 - y_A)^2 \right] + (1 - \pi_A) \left[ - (1 - x_B)^2 - (1 - y_B)^2 \right]$$

where $\pi_A = \pi_A((x_A, y_A), (x_B, y_B))$ denotes the probability of winning for party $A$ and $1 - \pi_A$ denotes the probability of winning for party $B$ as a function of the parties policy choices $((x_A, y_A), (x_B, y_B))$.

As before, the game takes place in two stages. In the first stage, parties simultaneously choose positions in $X$. As in the standard Downsian model, I assume that parties implement their announced positions if they win the election. In the second stage, the decisive voter votes for the party whose election would give him the higher utility, if elected. In case of indifference, the voter is assumed to vote for each party with probability equal to 1/2. Since the behavior of the voter is unambiguous in this model, I define an equilibrium of the game only in terms of the location strategies of the two parties in the first round.

Let $((x_A, y_A), (x_B, y_B)) \in X^2$ denote a pair of pure strategies for parties $A$ and $B$ respectively. I solve for the Nash equilibria of this game, that is, those pairs of pure strategies $((x_A^*, y_A^*), (x_B^*, y_B^*)) \in X^2$ such that $U_A((x_A^*, y_A^*), (x_B^*, y_B^*)) \geq U_A((x_A, y_A), (x_B, y_B))$ for all $(x_A, y_A) \in X$ and $U_B((x_A^*, y_A^*), (x_B^*, y_B^*)) \geq U_B((x_A^*, y_A^*), (x_B, y_B))$ for all $(x_B, y_B) \in X$. 10
3.2 Shock on the two dimensional policy space

Here I am interested in the effects of a shock on the voters’ preferences that produce biases on the policy preferences of the decisive voter. I assume, as in the previous case, that the effect of the shock on the policy preferences of the voters is represented by the parameter \( \alpha \in [0, 1) \). In this case I assume that the beliefs of the parties about the location of the ideal point of the decisive voter are represented by \( \mathcal{F}(x, y) = F(x, y; a) \) where \( F(x, y; a) \) denotes a uniform distribution over a support equal to \([a, 1] \times [0, 1] \) with \( a \in [0, 1) \) and therefore the density function is given by

\[
f(x, y, a) = \begin{cases} 
0 & \text{if } x \in [0, a) \\
\frac{1}{1-a} & \text{if } x \in [a, 1] 
\end{cases}
\]

See figure 4. As in the previous case, before the shock occurs I assume that \( \alpha = 0 \) that is, parties believe that the expected ideal point of the decisive voter is distributed uniformly over the unit square. After the shock, when \( \alpha > 0 \), parties believe that the ideal points of voters have shifted towards the right side of the policy space on the dimension affected by the shock \((x)\) and they are remain uniformly distributed over the unit interval on the dimension that is not have been affected by the shock \((y)\).

In this case I have two parameters that are going to determine the parties’ equilibrium policy choices. On the one hand, there is the relative issue salience \( (\sigma) \). I assume that the value of \( \sigma \) before the shock is 0. This implies that before the shock the most salient issue is \( y \), and thus this is the only issue that voters take into account when deciding their vote before the shock. I assume that the value of \( \sigma \) after the shock is 1. This implies that after the shock the most salient issue is \( x \), and thus this is the only issue that voters take into account when deciding their vote before the shock. On the other hand, there is the effect of the shock on the voters’ policy preferences represented by the parameter \( \alpha \). Thus in this case I will compare the equilibrium policy choices of the parties when there is no shock, \( \alpha = 0 \) and \( \sigma = 0 \) to the equilibrium policy choices when there is a preference shock \( \alpha > 0 \) and \( \sigma = 1 \). First I describe the equilibrium results in the absence of any shock.

**Proposition 3:** If \( \sigma = 0 \) and \( \alpha = 0 \) the parties equilibrium strategies are \((x_A^*(a), y_A^*(a)) = (0, \frac{1}{4})\) and \((x_B^*(a), y_B^*(a)) = (1, \frac{3}{4})\).

Before the shock issue \( y \) is infinitely more salient than issue \( x \), thus the voting decision is only affected by the parties’ positions on issue \( x \). In this case, since parties are policy motivated, their optimal choices on the non salient issue coincides with their ideal points. On the salient issue they are going to moderate their policy choices, thus in equilibrium we obtain the solution predicted for the one dimensional case (proposition 1): both parties moderate their policies in a symmetric way, by choosing platforms halfway between their respective ideal points and the expected median voter ideal point.

Next, we introduce a preference shock that only affects the voters’ policy preferences but not their
relative issue salience.

**Proposition 4:** If $\sigma = 0$ then for any $a \in [0, 1]$ the parties equilibrium strategies are $(x_A^*(a), y_A^*(a)) = (0, \frac{1}{2})$ and $(x_B^*(a), y_B^*(a)) = (1, \frac{3}{4})$.

Notice that if the shock does not affect the voters’ relative issue salience, then the voting decision remains unaffected: after the shock voters only consider the parties’ positions on issue $y$ to decide their vote, as they did before the shock. Voters preferences are only changing with respect to an issue that they consider irrelevant for their voting decision. Since the voters’ preferences on the salient issue are not affected by the shock, then the parties’ optimality conditions are also unaffected, and the equilibrium obtained coincides with the one obtained in the absence of a shock.

Now we introduce a preference shock that affects both, the voters’ policy preferences and also the relative issue salience. Notice that as the value of the parameter that represents the relative issue salience increases, issue $x$ becomes more salient, and thus the decision of the voting decision is conditioned by the policy choices of the parties on both issues. As the value of $\sigma$ increases the voting decision shifts weights between issues: the utility that voters derive from policies implemented on issue $x$ becomes more important, and thus more relevant to decide to whom to give their vote, and this forces parties to moderate their positions also on that issue.

Thus if the shock affects the voters relative issue salience, after the shock parties will choose moderate policy position on the issue that has become salient due to the shock. In addition, after the shock voters’ preferences have shifted to the right side of the policy space on that issue. Therefore, the parties policy choices have to take into account both effects. Notice that as before party A suffers a disadvantage from the voters’ preference policy shift, because voters’ ideal points move away from party A’s ideal point, while party B obtains a strategic advantage since voters’ ideal points move closer to party B’s ideal point. The next result describes the equilibrium results after the shock is produced.

**Proposition 5:** If $\sigma = 1$ then for $a > 0$ the parties equilibrium strategies are $(x_A^*(a), y_A^*(a)) = \left( \frac{9a - 2 + \sqrt{(12 - 9a)^2 + 32}}{16}, 0 \right)$ and $(x_B^*(a), y_B^*(a)) = \left( \frac{2 + \frac{9a - 2 + \sqrt{(12 - 9a)^2 + 32}}{16}}{3}, 1 \right)$.

This result shows once more that after the shock the parties’ policy choices in equilibrium are conditioned on the value of the parameter that represents the intensity of the shock, $a$. Figure 5 represents graphically the parties’ policy choices on the two dimensional policy space. After the shock is produced the expected ideal point of the decisive voter has shifted to the right side of the policy space on the $x$ dimension and has remained unchanged on the $y$ dimension. Since $x$ has become the most salient issue after the shock, voters will base their decision only on the parties policy choices on issue $x$. Thus parties have to moderate their positions on issue $x$ exactly as if they were competing on a one dimensional model,
and they can propose their ideal points on the non-salient issue.

Finally, in order to compare the equilibrium policy choices before and after the shock I have to compare the equilibrium policy choices corresponding to no shock \((a = 0, \sigma = 0)\), with those corresponding to \((a > 0, \sigma = 1)\), that is a change in the voters policy preferences for issue \(x\) and an increase of the salience of issue \(x\). Since issue \(y\) becomes irrelevant in terms of the voting decision, parties are going to reverse their symmetric and moderate policy choices to their ideal points on that issue, and they will abandon their ideal points on the new salient issue and choose instead moderate policies in an asymmetric way: party A is forced to more moderate policies because of its electoral disadvantage.

In order to determine the effect of a preference shock on a given issue on the parties’ policy choices on that issue, we have to compare the equilibrium strategies before and after the shock on the issue that suffers the shock. The following corollary offers these results.

**Corollary:** If a shock produces a change from \((a = 0, \sigma = 0)\) to \((a > 0, \sigma = 1)\) then the parties’ equilibrium strategies on the dimension affected by the shock change from \((x_A^*, x_B^*) = (0, 1)\) to \((x_A^*, x_B^*) = \left(\frac{9a - 2 + \sqrt{(2 - 9a)^2 + 32}}{16}, \frac{9a - 2 + \sqrt{(2 - 9a)^2 + 32}}{16} \right)\).

This implies that on the issue affected by the shock both parties move their policy choices from their ideal points toward the ideal point of the expected decisive voter, thus both parties moderate their final policy choices on that issue. As before, the policy shift produced by party A is much larger than the shift produced by party B. However, in this case these two shifts are produced in opposite directions on the \(x\) dimension. This implies that both parties are suffering a disadvantage (of different intensities) from the shock, since both of them are forced to moderate their positions.

4 Party consequences of a large policy shift

The previous analysis shows that after a preference shock one of the parties is forced to produce a large change on his policy position away from its ideal point. Since a party internal consistency is based on a balance between the different ideological positions of the different factions, it is plausible to think that having to produce a sudden and large policy change might affect the equilibrium of forces inside a party.

In order to illustrate the consequences for a party of producing a large policy shift due to electoral reasons, I will describe how the effects of a preference shock observed recently in Catalonia have produced drastic changes in a party system that was stable for over 30 years.

The Catalan political debate has always been spread over two policy dimensions: the economic dimension and the sovereignty or decentralization dimension. On the economic dimension policy positions are defined as rightist or leftist depending on the amount of government intervention in the economy. The
sovereignty dimension is specific of countries with different levels of government (central, regional, municipal, etc...). On this dimension policy positions are defined according to the degree of decentralization of decision power among the different levels of government. Policy positions on the sovereignty issue range from full centralization (concentration of all the decision power on a unique level of government) to full decentralization (allocation of all the decision power to each one of the regional governments).

Until the first decade of the twenty first century political preferences of the Catalan society on the economic dimension covered most of its range: from extreme right to extreme left. However on the sovereignty dimension political preferences were rather moderate. There were claims for different degrees of decentralization but on the extremes these claims were very weak: demands for policies close to full centralization or to full independence were supported by a very small part of the population. Accordingly, the chosen policy positions of the political parties over these two issues were moderate on the sovereignty issue and covered the full range of policies on the economic issue.

Over the most of the Spanish democratic period (1980-2010) the Catalan party system appears as a stable system that contained five political parties. Convergència i Unió (CIU), a Catalan center-right coalition that stood for increasing decentralization had the largest electoral support. Partit Socialista de Catalunya (PSC), a Catalan center-left party with different decentralization claims over time was the second largest party. Two other parties had much smaller electoral supports, but they play significant roles in the governing coalitions that formed during the period: a Spanish rightist and centralist party, Partido Popular (PP) and a Catalan leftist and independentist party, Esquerra Republicana de Catalunya (ERC). Finally, Iniciativa per Catalunya-Verds (ICV) a leftist green party that mildly stood for decentralization had the smallest electoral support.

Given the features of the party system described above, the possibilities of majoritarian coalitions involve all kinds of cross ideological agreements among parties on both issues\(^1\). However, the governments that actually formed during this period were defined by the parties’ position on the economic issue. Some of these governments were minority governments with the support of external parties, some were governments with absolute majority, and others were proper coalition governments. But on the ideological front we only observe two kinds of governments: those supported by rightist parties (CIU and PP) and those supported by leftist parties (PSC, ERC, and ICV). There was never a government formed or supported by parties that shared the same political views on the sovereignty issue. This observation leads us to conclude that it was more costly for political parties to compromise their positions on the economic issue than on the sovereignty issue. They were willing to give up their ideological views on decentralization in order to become part of the governing coalition. The implication of this observation is that the salience

\(^1\)For a detailed analysis of the governments formed during this period in Catalonia see Aragones (2007).
of the sovereignty issue was clearly dominated by the salience of the economic issue during this period.

During the last few years, polls and electoral results have shown that the political preferences of the Catalan society have changed. In particular, increasing support for a position of full independence in numbers and in intensity of preferences has become a fact. This implies that on the one hand, the relevant policy space that parties now should cover has been enlarged: it contains extreme decentralization positions that are supported by increasing number of voters. On the other hand, the relative salience of the two issues has changed dramatically with the sovereignty issue becoming much more salient than the economic issue.

At the same time three new parties entered the political area. Ciutadans-Ciudadanos (C) claims no position on the economic issue but a strong position on the sovereignty issue for extreme centralization. Candidatura d’Unitat Popular (CUP) has a strong independentist and leftist position. And finally Podemos (P) holds a strong leftist position and a moderate position on the sovereignty issue.

Parties policy positions had to adapt to the new political environment. It is interesting to notice how different parties have used very different strategies to deal with the new preferences of the constituency.

On the one hand, we observe that a few parties have adapted in a very easy and natural way: some of them by not moving from their initial positions (C, PP, CUP, P, ICV) and others by reverting their compromised moderate policies to their original ideal points (ERC, originally defined independentist). However, the two largest parties have had a harder time to adapt to the new environment. PSC suffered severe internal party tensions that have driven it to break into several small factions that became new small parties holding leftist-independentist positions and a somewhat larger faction holding a leftist-centralist position. CIU first solved its internal tensions using two of its leaders to deliver two different claims: one would call for full independence and another one would maintain its initial moderate decentralization position. This strategy was easy to implement because CIU was in fact a long-lasting coalition of two different parties: Convergència Democràtica de Catalunya (CDC) and Unió Democràtica de Catalunya (UDC). Thus each one of the parties within the coalition held a different position on the sovereignty issue. However, the party was not able to hold this duality for long and ended up splitting into the two parties that originally formed the coalition. Thus, the effect of a voters preference shock is clearly mostly suffered by the two largest parties, which were parties that had to support a more complicated and perhaps fragile internal equilibrium of forces. The breakdown of the two major parties into different factions produced a drastic change in the existing Catalan party system.

Furthermore, recent electoral results have shown evidence of the lost of electoral support of the re-

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2For a detailed analysis of the evolution of the political preferences in Catalonia during this period see Guinjoan and Rodon (2016).
3For a detailed analysis of the evolution of the Catalan party system during this period see Aragones and Ponsati (2016).
maining factions of the two large parties. In the most recent Spanish congressional elections PSC moved from being the largest party in terms of electoral support to a residual fifth position. Similarly, the two parties that once formed CiU, the party with the largest support, lost a significant amount of electoral support: UDC was not able to obtain any representation, and CDC dropped to the fourth position in terms of electoral support. Thus, we can conclude that the drastic strategy change that these parties had to make in order to react to the voters’ preference shock has reduced their electoral relevance by destroying the internal consistency of their own factions.

5 Concluding remarks

I have shown by providing empirical evidence that exogenous shocks on voters’ policy preferences are possible. When these shocks occur they have important implications on the survival of political parties and on the stability of party systems. The shift of the voters’ political preferences produced by the shock induces parties to react in an asymmetric way, because while one party receives an advantage from the shock the other party suffers a disadvantage. The shock also produces a change in the relative salience of issues and therefore political competition induces parties to moderate their policies on an issue in which they could have otherwise implemented their ideal policies (because as in Ansolabehere and Puy (2015) we have that parties are not interested in competing on issues that are not relevant for the voters). The overall effect of a shock forces one of the parties, the disadvantaged one, to produce a large policy shift on the issue hit by the shock. I have illustrated how such a drastic policy change may destroy the internal consistency of the party.

The analysis presented relies on several assumptions. In particular, I assume that parties ideal points are not affected by the shock. This is a point that can hardly be proved nor disproved by empirical evidence. If they were assumed to be affected by the shock in the same way than voters’ preferences are, then none of the results presented would hold. However, if they were assumed to be less affected by the shock than voters’ preferences are, then the results would be qualitatively the same.

I have assumed that parties utility functions are quadratic. If they were assumed to be linear the results would not be as interesting, since in this case we have that some equilibrium policies are obtained as corner solutions, and thus the comparative statics analysis of the shock would not be as rich as the one provided by concave utility functions.

I have also assumed that parties are policy motivated. If parties were only office motivated, as in a standard model a la Downs (1954) then in equilibrium they would both always converge to the ideal point of the current expected median of the dimension that is most salient. In this case, the parties’
policy choices would change from \(((0,1/2),(1,1/2))\) before the shock to \(((1+a/2,0),(1+a/2,1))\) after the shock. Thus, the qualitative results of moderate policies and asymmetric reactions would still hold in this case. If instead parties were assumed to be both policy and office motivated, then the results presented here would be slightly modified by the fact that in this case both parties will have stronger incentives to moderate their policy choices.

Finally, I have assumed that parties assign the same weight to the utility loses produced on all issues, that is, all issues are considered equally salient and their relative issue salience is not affected by the shock. I think that this are reasonable assumptions since on the one hand a party represents an aggregated pool of different sensitivities and ideological views, and thus it is plausible to assume that in general it would suffer a utility cost from any policy loss on any issue. And, on the other hand, voters’ sensitivity is probably more likely to be affected by a shock, and thus voters are going to be reacting more intensity to a preference shock. However, it would be interesting to see how results might change if these assumptions were relaxed.
References


6 Appendix

6.1 Proofs

Proof of Proposition 1:
If $a = 0$ then $f(x, a) = 1$ for $x \in [0, 1]$. Thus, for every pair of policy choices $(x_A, x_B) \in X^2$ I have that the probability of winning for party $A$ is given by $\pi_A(x_A, x_B) = \frac{x_A + x_B}{2}$ and the probability of winning for party $B$ is given by $\pi_B(x_A, x_B) = 1 - \pi_A(x_A, x_B) = 1 - \frac{x_A + x_B}{2}$.

parties’ payoffs functions can be written as

\[
U_A(x_A, x_B) = -\frac{x_A + x_B}{2} (x_A)^2 - \left[1 - \frac{x_A + x_B}{2}\right] (x_B)^2
\]

\[
U_B(x_A, x_B) = -\frac{x_A + x_B}{2} (1-x_A)^2 - \left[1 - \frac{x_A + x_B}{2}\right] (1-x_B)^2
\]

Their first order conditions are:
\[
\frac{\partial U_A(x_A, x_B)}{\partial x_A} = -(x_A)^2 + (x_B)^2 - \frac{x_A + x_B}{2} x_A = 0
\]
\[
\frac{\partial U_B(x_A, x_B)}{\partial x_B} = (1-x_B)^2 - (1-x_A)^2 - \frac{x_A + x_B}{2} 2(1-x_B) + 2(1-x_B) = 0
\]

which imply that their reaction functions are:

\[
x_A(x_B) = \frac{x_B}{3}
\]

\[
x_B(x_A) = \frac{2+x_A}{3}
\]

Thus in equilibrium we must have: $x_A = \frac{1}{4}$ and $x_B = \frac{3}{4}$

with $x_A < x_B$, $x_A + x_B = 1$, $x_A - x_B = 1/2$,

and $U_A(x_A, x_B) = U_B(x_A, x_B) = -\frac{5}{16}$.

Proof of Proposition 2:
If $a > 0$ then $f(x, a) = \begin{cases} 
0 & \text{if } x \in [0, a) \\
\frac{1}{1-a} & \text{if } x \in [a, 1]
\end{cases}$ . Thus, for every pair of policy choices $(x_A, x_B) \in X^2$ I have that the probability of winning for party $A$ is given by $\pi_A(x_A, x_B) = \begin{cases} 
0 & \text{if } \frac{x_A + x_B}{2} \in [0, a] \\
\frac{x_A + x_B - 2a}{2(1-a)} & \text{if } \frac{x_A + x_B}{2} \in (a, 1]
\end{cases}$

and the probability of winning for party $B$ is given by $\pi_B(x_A, x_B) = 1 - \pi_A(x_A, x_B) = \begin{cases} 
1 & \text{if } \frac{x_A + x_B}{2} \in [0, a) \\
\frac{2-x_A-x_B}{2(1-a)} & \text{if } \frac{x_A + x_B}{2} \in (a, 1]
\end{cases}$.
In order to compute the probability of winning of the two parties we have to consider two cases:

a) those \( (\xi, \eta) \) such that \( \xi + \eta \leq \alpha \), that is, \( 2a < x_A + x_B \leq 2 \); and

b) those \( (\xi, \eta) \) such that \( \frac{x_A + x_B}{2} \in [0, a] \), that is, \( 0 \leq x_A + x_B \leq 2a \).

First suppose that we are in case a) and \( (\xi, \eta) \) are such that \( \frac{x_A + x_B}{2} \in (\alpha, 1] \). In this case, parties' payoffs functions are given by

\[
U_A(x_A, x_B) = \frac{x_A + x_B - 2a}{2(1-a)} \left[ -\left( x_A \right)^2 + \left( x_B \right)^2 \right] - (x_B)^2
\]

\[
U_B(x_A, x_B) = \frac{x_A + x_B - 2a}{2(1-a)} \left[ (1 - x_B)^2 - (1 - x_A)^2 \right] - (1 - x_B)^2
\]

and their first order conditions are:

\[
\frac{\partial U_A(x_A, x_B)}{\partial x_A} = \frac{1}{2(1-a)} \left[ -\left( x_A \right)^2 + \left( x_B \right)^2 \right] - \frac{x_A + x_B - 2a}{2(1-a)} 2x_A = 0
\]

\[
\frac{\partial U_A(x_A, x_B)}{\partial x_B} = \frac{1}{2(1-a)} \left[ (1 - x_B)^2 - (1 - x_A)^2 \right] - \frac{x_A + x_B - 2a}{2(1-a)} 2(1 - x_B) 2(1 - x_B) = 0
\]

which implies that their reaction functions are as follows. For party A

\[
x_A(x_B) = \frac{(2a-x_B)+\sqrt{4[(x_B-a)^2+x_B]}}{3}
\]

with

\[
x_A(x_B) \geq 0
\]

\[
x_A(x_B) \leq x_B \text{ if } x_R \geq a
\]

\[
\frac{\partial x_A(x_B)}{\partial x_B} \geq 0 \text{ if } x_R > a
\]

\[
\frac{\partial x_A(x_B)}{\partial a} \geq 0
\]

For party B

\[
x_B(x_A) = \frac{2+x_A}{3}
\]

with

\[
x_B(x_A) \leq 1
\]

\[
x_B(x_A) \geq x_A
\]

\[
\frac{\partial x_B(x_A)}{\partial x_A} \geq 0
\]

\[
\frac{\partial x_B(x_A)}{\partial a} = 0
\]

Next suppose that \( (x_A, x_B) \) are such that \( \frac{x_A + x_B}{2} \in [0, a] \), that is, \( 0 \leq x_A + x_B \leq 2a \).

Notice that if \( x_B \leq a \) we have that \( BR_A(x_B \leq a) = [0, x_B] \) because \( U_A(x_A, x_B) = -x_B^2 > -(\hat{A})^2 = U_A(\hat{x}_A, x_B) \) for all \( x_A \in [0, x_R] \) and for all \( \hat{x}_A \in (x_B, 1] \). Thus, parties payoff functions are given by

\[21\]
\[ U_A(x_A, x_B) = - (x_B)^2 \]
\[ U_B(x_A, x_B) = - (1 - x_B)^2 \]

If \( x_B > a \) we have that the only best response of party A is \( BR_A(x_B > a) = x_A(x_B) = \frac{(2a - x_B) + \sqrt{4(x_B - a)^2 + ax_B}}{3} \) and notice that \( x_A(x_B) + x_B \geq 2a \), because in this case the policy outcome is a convex combination of \( x_B \) and \( x_A(x_B) \) which leads to better payoffs for party A than any \( x_A \) such that \( 0 \leq x_A + x_B \leq 2a \) that produces \( U_A(x_A, x_B) = - (x_B)^2 \). Therefore, party A reaction function is given by:

\[
BR_A(x_B) = \begin{cases} 
\frac{(2a - x_B) + \sqrt{4(x_B - a)^2 + ax_B}}{3} & \text{if } x_B > a \\
[0, x_B] & \text{if } x_B \leq a 
\end{cases}
\]

Regarding B we have that if \( x_A \leq a \) then \( BR_B(x_A) = \max \{ 2a - x_A, \frac{2 + x_A}{3} \} \). Otherwise, if \( x_A > a \) then \( BR_B(x_A) = \frac{2 + x_A}{3} \). Therefore, party B reaction function is given by:

\[
BR_B(x_A) = \begin{cases} 
\frac{2 + x_A}{3} & \text{if } x_A > \frac{3a - 1}{2} \\
2a - x_A & \text{if } 2a - 1 < x_A < \frac{3a - 1}{2} \\
1 & \text{if } 2a - 1 > x_A 
\end{cases}
\]

Notice that for \( a < \frac{1}{3} \) we have that \( BR_B(x_A) = \frac{2 + x_A}{3} \), and for \( a < \frac{1}{2} \) we always have \( BR_B(x_A) < 1 \). See figure 6.

Thus, in equilibrium we will have that \( a < \frac{\sqrt{9a-1} + x_B}{2} \) and:

\[
x_A^* = \frac{9a - 2 + \sqrt{(2 - 9a)^2 + 32}}{16} \quad \text{and} \quad x_B^* = \frac{9a - 2 + \sqrt{(2 - 9a)^2 + 32}}{16}
\]

with

\[
\begin{align*}
\frac{\partial x_A^*}{\partial a} & > 0 \\
\frac{\partial x_B^*}{\partial a} & > 0 \\
\frac{\partial x_A^*}{\partial x_B} & = 3 \frac{\partial x_B^*}{\partial x_B} \quad \frac{\partial x_B^*}{\partial x_B} \\
x_A^* & \leq x_B^* \\
x_A^* & \leq \frac{1 + a}{2} \leq x_B^* \\
\lim_{a \to 1} x_A^* &= 1 \quad \lim_{a \to 1} x_B^* = 1 \\
2a & < x_A^* + x_B^* \leq 2 \\
\frac{\partial (x_A^* + x_B^*)}{\partial a} & = \frac{4 \partial x_A^*}{\partial a} > 0 \\
x_A^* + x_B^* & > 1 \\
\frac{1 + a - x_A^*}{\partial a} & > x_B^* - \frac{1 + a}{2} \\
\frac{\partial (x_B^* - x_A^*)}{\partial a} & = - \frac{2 \partial x_A^*}{3 \partial a} < 0
\end{align*}
\]
strategies are given by

\[ \pi_A = \frac{1+2x_A-3a}{3(1-a)} = \frac{6-15a+\sqrt{(2-9a)^2+32}}{24(1-a)} < \frac{1}{2} \]

\[ 1 - \pi_A = 1 - \frac{1+2x_A-3a}{3(1-a)} = \frac{2(1-x_A)}{3(1-a)} = \frac{18-9a+\sqrt{(2-9a)^2+32}}{24(1-a)} \]

\[ \frac{\partial \pi_A}{\partial a} < 0 \]

\[ \frac{\partial U_A(x_A,y_B)}{\partial a} < 0 \]

\[ \frac{\partial U_B(x_A,y_B)}{\partial a} > 0. \star \]

**Proof of proposition 3:**

If \( \sigma = 0 \) and \( a = 0 \) then the decisive voter’s utility function is given by:

\[ u_m(x,y) = -\sigma(x-x_m)^2 - (1-\sigma)(y-y_m)^2 = -(y-y_m)^2 \]

and the probability of winning for party A is given by

\[ \pi_A(\{x_A,y_A\}, \{x_B,y_B\}) = \frac{y_A+y_m}{2}. \]

Thus, the decisive voter’s problem is identical to the one analyzed in proposition 1. The parties’ optimization problem contains no restrictions on the \( x \) dimension, therefore they can implement their ideal points on this dimension with no electoral cost. The parties’ optimization problem with respect to the \( y \) dimension are identical to the ones analyzed in proposition 1. Therefore, the parties equilibrium strategies are given by \((x^*_A(a), y^*_A(a)) = (0, \frac{1}{2})\) and \((x^*_B(a), y^*_B(a)) = (1, \frac{3}{4})\). \star

**Proof of proposition 4:**

If \( \sigma = 0 \) and \( a > 0 \) then the decisive voter’s utility function is given by:

\[ u_m(x,y) = -\sigma(x-x_m)^2 - (1-\sigma)(y-y_m)^2 = -(y-y_m)^2 \]

and the probability of winning for party A is given by

\[ \pi_A(\{x_A,y_A\}, \{x_B,y_B\}) = \frac{y_A+y_m}{2}. \]

Once more the decisive voter’s problem is identical to the one analyzed in proposition 1. The parties’ optimization problem contains no restrictions on the \( x \) dimension, therefore they can implement their ideal points on this dimension with no electoral cost. The parties’ optimization problem with respect to the \( y \) dimension are identical to the ones analyzed in proposition 1. Therefore, the parties equilibrium strategies are given by \((x^*_A(a), y^*_A(a)) = (0, \frac{1}{2})\) and \((x^*_B(a), y^*_B(a)) = (1, \frac{3}{4})\). \star

**Proof of proposition 5:**

If \( \sigma = 1 \) and \( a > 0 \) then the decisive voter’s utility function is given by:

\[ u_m(x,y) = -\sigma(x-x_m)^2 - (1-\sigma)(y-y_m)^2 = -(x-x_m)^2 \]

and the probability of winning for party A is given by

\[ \pi_L(\{x_A,y_A\}, \{x_B,y_B\}) = \begin{cases} 0 & \text{if } \frac{x_L+x_B}{2} \in [0,a] \\ \frac{x_L+x_B-2a}{2(1-a)} & \text{if } \frac{x_L+x_B}{2} \in (a,1] \end{cases} \]

In this case the decisive voter’s problem is identical to the one analyzed in proposition 2. The parties’ optimization problem contains no restrictions on the \( y \) dimension, therefore they can implement their
ideal points on this dimension with no electoral cost. The parties’ optimization problem with respect to
the $x$ dimension are identical to the ones analyzed in proposition 2. Therefore, the parties equilibrium
strategies are given by: $(x^*_A(a), y^*_A(a)) = \left( \frac{9a-2+\sqrt{(2-9a)^2+32}}{16}, 0 \right)$

and

$(x^*_B(a), y^*_B(a)) = \left( \frac{2+\frac{9a-2+\sqrt{(2-9a)^2+32}}{3}}{3}, 1 \right)$.
Figure 1: density function

Figure 2: cdf

Figure 3: Equilibrium after the shock

Figure 4: density function
Figure 5: overall effect of the shock

\[ a = C \]
\[ \sigma = 0 \]

\[ a > C \]
\[ \sigma = 1 \]

Figure 6: best response functions

\[ a < \frac{1}{2} \]

\[ a > \frac{2}{3} \]