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A Model of Directed Consumer Search*

Marco A. Haan† José L. Moraga-González‡ Vaiva Petrikaitė§

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Abstract

We present a framework to study directed consumer search. Firms sell products with two attributes. One is readily observable, the other is observed only after visiting a firm. Search is directed as the order of search is influenced by the observable characteristics. Moreover, if prices are readily observable, firms also influence search direction by their choice of price. We show that when consumers observe prices before search, prices and profits are lower than when they do not. A lower price then not only retains more consumers, but is also more likely to attract them; the latter effect makes demand more elastic. When consumers observe prices before searching, prices decrease in search costs. Consumer surplus initially increases in search costs, but may ultimately decrease.

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†Department of Economics and Econometrics, University of Groningen. E-mail: m.a.haan@rug.nl.

‡Vrije Universiteit Amsterdam and University of Groningen. E-mail: j.l.moragagonzalez@vu.nl. Moraga is also fellow of the CEPR, Tinbergen Institute, and the PPSRC Center at IESE.

§Instituto de Análisis Económico (IAE-CSIC) and Barcelona GSE. E-mail: asvaiva@gmail.com

1 Introduction

Most of the consumer search literature assumes that consumers search randomly. In a standard model with symmetric firms, a consumer picks one firm at random, pays it a costly visit in order to learn its price and/or product characteristics, and on the basis of those decides whether to pay a costly visit to the next randomly selected not-yet-visited firm. And so on and so forth.

Assuming that search is random is reasonable when firm offerings are truly identical a priori. This is the case, for example, when products are differentiated but consumers are completely uninformed about the differences at the time they engage in search. Moreover, it requires that prices can only be learned after costly search.

In the real world, however, things are often different. First, it has become very easy for consumers to compare prices. There are many search engines that list all prices without requiring any search effort on behalf of the consumer. Second, it is also very easy to find at least some characteristics of a product without paying the firm a costly visit; yet, some product characteristics can only be discovered upon search. For example, a consumer that wants to buy a car can easily find all technical specifications in specialized magazines or on the Internet. However, she still has to make a costly visit to the dealer to be able to "kick the tyres" and take it for a test drive. Similarly, a consumer that wants to buy a new pair of jeans may find many details and pictures of the product online, which allows her to check whether she likes the design. But she still has to make a costly visit to the store[1] in order to try the jeans on and decide whether she likes their fit.

In these situations, consumer search will be directed and, more importantly, influenced by the prices the firms charge. The typical consumer will start searching the product that looks ex-ante more promising to her, either because some of the easily observable characteristics are more appealing, because the product is cheaper or both. And so on.

The main contribution of this paper is developing a tractable duopoly model for such a directed search. In Wolinsky's (1986) and Anderson and Renault's (1999) standard framework, unfortunately, allowing for price observability does not yield a tractable solution: an equilibrium in pure-strategies fails to exist, and it is extremely hard to characterise the mixed-strategy

[1]Or go through the hassle of ordering them online and returning them in case they do not fit. See Petrikaitė (2018).

equilibria. Some authors have addressed this problem by modifying the standard model in order to get sufficient tractability to be able to derive the mixed-strategy equilibrium. One such case is Armstrong and Zhou (2011). They present a Hotelling-type duopoly model where match utilities are negatively correlated. Consumers only observe match values after costly search and firms can advertise prices on a price comparison website. Consumers first visit the firm with the lowest price. By construction, upon learning that firm's match value they can immediately infer the match value the other firm offers. This feature makes it possible to explicitly characterize the mixed strategy equilibrium. Another case is Ding and Zhang (2018) who simplify the standard model of product differentiation by assuming that firms sell products that may or may not match the needs of consumers. The two-point probability distribution assumption allows for the computation of the mixed strategy equilibrium.

In this paper we propose an alternative solution: we build additional product differentiation into the standard Anderson and Renault's (1999) framework. In our model firms sell products with two product attributes that are horizontally differentiated. One attribute is observable without visiting the firm, while the other can only be discovered upon visiting the store and physically interacting with the product. Because of the ex-ante observable attributes, search is naturally directed. In addition, ex-ante product differentiation breaks the discontinuity that arises when firms announce different prices and this restores the existence of a pure-strategy equilibrium with observable prices. In this framework, we compare the price equilibrium when prices are observable before search, and when they are not. We also examine how search costs affect equilibrium prices, profits and consumer surplus in the two settings.

To the best of our knowledge, we have been the first to propose this approach.[2] Shen (2015) followed a similar strategy and embedded the Anderson and Renault (1999) framework into a Hotelling model. In his framework, however, the ex-ante characteristics are perfectly negatively correlated. More recently, Choi *et al.* (2017) study a model identical to ours but allowing for an arbitrary number of firms. To do this, they make use of a breakthrough independently developed by themselves and Armstrong (2017) that allows them to derive the demand of a firm without the need to specify the many search paths consumers may follow before they end up buying there (more details below). Without doing so, the n-firm case is not tractable.

[2]The first version of this paper dates back to 2011, see Haan and Moraga-González (2011).

Choi *et al.* (2017) formalize existence of equilibrium results that we conjectured earlier using numerical analysis, and extend some of our results by allowing for an outside option and for the distributions of observable and non-observable characteristics to be bounded. Moreover, they study the effects of providing consumers with more accurate product information before they search.

Our main results are as follows. We first provide a characterisation of the equilibrium price when search is directed by product features only. The equilibrium price can be computed in closed-form and smoothly converges to the equilibrium price in Anderson and Renault (1999) as the ex-ante product differentiation vanishes. Proving the existence of equilibrium turns out to be difficult for the same reasons as in their paper. In the Supplementary Appendix we provide evidence based on numerical simulations with Gumbel, Normal and uniform distributions for the observed and unobserved match values that the price equilibrium candidate is indeed an equilibrium.

We then provide a characterisation of the equilibrium price when search is directed by both the features of the products and the prices at which they sell. The equilibrium price can also be computed in closed-form in this case. Regarding the existence of equilibrium, we provide evidence based on numerical analysis with Gumbel distributions that the payoff may fail to be quasi-concave. Our numerical analysis reveals that, for a fixed search cost, a pure-strategy symmetric equilibrium fails to exist when the variance of the ex-ante observable product characteristics is small. This observation turns out to be quite general, as has now been proven by Choi *et al.* (2017). We also find in our numerical analysis that, if the dispersion of the ex-ante observable product characteristics is small, then the equilibrium still exists provided the search cost is either sufficiently small or sufficiently large.

A comparison of the two equilibrium prices yields a clear-cut and intuitive result: the equilibrium price is lower if consumers can observe prices before search. In that case, a lower price not only increases the probability that a consumer that visits a firm ends up buying there, but also the probability that a consumer visits the firm at all. This gives firms more of an incentive to lower prices.[3]

[3]Gomis-Porqueras *et al.* (2017) derive a result similar in spirit by comparing a model of posted prices with a model of first-price auctions with reserve prices in a directed search framework with capacity-constrained homogeneous product sellers.

Next, we study how the equilibrium prices vary with search costs. We find that if consumers can observe prices before search, equilibrium prices are likely to decrease in search costs. For this to occur, it suffices that the search cost is sufficiently high, but it holds more generally provided that the density of ex-ante observable match values is log-concave, a result that has also been proven independently by Choi *et al.* (2017). With higher search costs, consumers are less likely to walk away from the firms they visit. This gives firms more of an incentive to try to attract those consumers, which they can do by charging a lower price. More precisely, as search costs increase, consumers with a strong ex-ante preference for either firm (in terms of observable product characteristics) will be even less likely to also visit the other firm. That implies that competition is now for consumers with a weaker ex-ante preference. But as the measure of such indifferent consumers is necessarily higher,[4] competition becomes fiercer.

Our paper carries an important message for policymakers. Search costs are usually considered to be detrimental to consumers. We demonstrate that this conclusion is not valid when price information is readily available and consumers only have to search for product characteristics. Assuming that match values are drawn from uniform distributions, we show that consumer surplus is initially increasing in search costs, but may be decreasing if search costs are high enough. A higher search cost has two opposite effects on consumer surplus: on the one hand, it makes search more costly. On the other hand, it lowers prices. For small search costs, the price effect dominates. For high search costs, the other effect may be stronger.

2 Related literature

Our model builds on the literature on consumer search for differentiated products, pioneered by Wolinsky (1986) and Anderson and Renault (1999). Yet, different from those papers, we assume that firms are not visited at random. Other papers also drop that assumption, but in different contexts. Arbatskaya (2007) studies a model with otherwise homogeneous products and heterogeneous search costs, where search order is exogenously given. She finds that prices fall in the order of search: a consumer that walks away from a firm reveals that she has low search costs, giving the next firm an incentive to charge a lower price. Zhou (2011) finds the

[4] This is a property of the probability density function of the difference between two independently and identically distributed taste shocks.

opposite effect in a model with differentiated products. A consumer that walks away now reveals that she did not like that product much, which gives more market power to the next firm in line. A similar result is found in Armstrong *et al.* (2009), who study a search market with differentiated products where one firm is always visited first, while the other firms are sampled randomly if at all.

In Haan and Moraga-González (2011) firms can influence the order of search. In that paper, they do so by advertising. A firm that advertises more attracts a higher share of consumer visits. In equilibrium prices increase in search costs, but advertising also does, hence profits may decrease. Anderson and Renault (2017) modify their earlier paper by presenting a model in which asymmetric firms compete in prices and consumers search sequentially to discover prices and match values. They show that given a pre-set consumer search order, an equilibrium exists in which firms charge prices such that the marginal consumer surplus decreases in the order of search and therefore consumers search optimally. They characterize the socially optimal, profits maximizing and consumer surplus maximizing search order and show that the profits maximizing search order can be implemented via a second-price position auction.[5]

The result that search costs are procompetitive when prices are observable is quite intuitive and has also appeared in the papers aforementioned.[6] Specifically, the mixed-strategy models of Armstrong and Zhou (2011) and Ding and Zhang (2018) yield that average prices decrease as search costs rise. Choi *et al.* (2017) prove that this result is quite general in a setting like ours. In Shen (2015), by contrast, because match utilities are perfectly negatively correlated, equilibrium prices may actually increase, rather than decrease, in search costs. Garcia and Shelegia (2018) also find that equilibrium prices can decrease in search costs. In their paper, even though prices are not ex-ante observable by consumers, the dynamics are such that the current price of a firm does have a direct influence on the number of consumers that search the firm in the future. Our paper adds to these papers by showing that the price effect of higher search costs can be so strong so as to increase consumer surplus.

[5]Chen and He (2011) and Athey and Ellison (2011) also model the interaction between consumer search and firm bidding for positions in settings in which pricing does not play much of a role

[6]Relatedly, several papers have pointed out the role of price advertising to dispel consumers' fears that they will be held-up by the firms after they invest in costly search (see e.g. Butters, 1977; Anderson and Renault, 2006; Konishi and Sandfort, 2002; and Robert and Stahl, 1994). The higher the search cost, the lower the price that a firm will have to advertise to induce buyers to pay it a visit.

One of the main contributions of this paper is to provide a flexible framework to study directed search. In classic models of price competition, the role of prices is to attract consumers. A consumer that visits a firm always buys there, as she has full information on all products on offer. Models of random search allow consumers to shop around. In such models, lower prices do not attract consumers, but do help to retain them once they visit. In our model of directed search, prices serve both roles: they help to both attract and retain consumers.

Our framework thus allows firms to compete in prices despite the presence of search frictions. In this sense, it provides a natural setting to reflect on how typical business strategies intended to tempt consumers into the firms' stores (such as price and product advertising), perhaps to sell them alternative products (loss-leadership, add-on pricing), depend on the magnitude of search costs. Our model could also be used to study how much information platforms ought to provide consumers before they engage in search.

Our theoretical result that equilibrium prices will decrease when search costs fall is consistent with the findings in recent empirical work. Moraga-González *et al.* (2015) estimate demand and supply in a consumer search model using data from the Dutch automobile market. Prices of cars can easily be obtained from various sources. The authors find that reducing the costs of inspecting cars, for example by letting customers test-drive cars at their homes or workplaces, results in price increases for some models. Pires (2018) estimates a similar model with data from the liquid laundry detergent market in the US. He finds that if prices are observable, raising search costs results in lower prices of the detergents with larger market shares. Dubois and Perrone (2015) estimate a model of search for grocery products using household survey data. They also find products for which demand in the search model is more elastic than demand in the absence of search costs. Koulayev (2014) studies the market for hotel rooms using data from an online intermediary, and reports that removing search frictions results in smaller price elasticities for some hotels.

The remainder of this paper is structured as follows. In the next section, we set up the model. We solve for the equilibrium in Section 4, first for situations when prices are not observable before search, then when they are. Section 5 looks at the comparative statics: we compare the two scenarios and study the effect of an increase in search costs in both cases. In Section 6 we conduct welfare analysis for the case in which match values are uniformly distributed. Section 7 concludes. Most proofs are relegated to the Appendix.

3 The Model

Setup Consider a market where 2 single-product firms compete in prices to sell horizontally differentiated products to a unit mass of consumers.[7] Production costs are normalized to zero. A consumer incurs a search cost s if she visits a firm. Search is sequential and recall is costless. A consumer j who buys product i at price p_i obtains utility:

$$u_{ij} = \varepsilon_{ij} + \eta_{ij} - p_i. \tag{1}$$

The term $\varepsilon_{ij} + \eta_{ij}$ is the valuation of product i by consumer j, and can be interpreted as the match value between j and i. It consists of two components: the *observable* component η_{ij} reflects characteristics that can be readily observed, while the *opaque* component ε_{ij} reflects characteristics that can only be observed upon visiting the firm. For example, a consumer who wishes to buy a car can readily observe its design and exact specifications, so these would be part of the observable component η_{ij}.[8] Yet, before buying, she would probably like to "kick the tires" of the car and take it for a test drive to be able to evaluate her match with the car; this would be part of the opaque component ε_{ij}.

Adding an observable component η_{ij} differentiates our model from the canonical model of search with differentiated products in e.g. Wolinsky (1986) or Anderson and Renault (1999). It is this component that allows us to have *directed* search. When prices and distributions of opaque characteristics are equal across firms, a consumer will first visit the firm with the observable characteristic she likes most (see Weitzman, 1979). Moreover, the observable component η_{ij} also allows us to analyze cases in which prices are readily observable, so firms can

[7]Extending our model to the N-firms case is challenging. As will become clear later, because search is directed in our model, with two firms there are three different search paths a consumer may follow before buying from a firm i (namely, visit firm i first and buy there right away, visit firm i first then firm j and return to firm i to buy, and, finally, visit firm j first then i and buy there). As the number of firms goes up, more and more search paths have to be taken into consideration to compute the demand of a firm. Two recent papers, Armstrong (2017) and Choi *et al.* (2017), have made significant progress on this front but unfortunately their method is not applicable to the benchmark case in which, as in Wolinsky (1986) and Anderson and Renault (1999), prices are not observable before consumers start searching for a satisfactory product.

[8]Alternatively, she might learn these characteristics for all cars at some fixed cost (e.g. by acquiring a specialized magazine)- so the marginal search cost for finding the characteristics of a particular car are zero.

additionally direct search by adjusting their prices. In the standard framework with only an opaque component, a pure strategy equilibrium would then fail to exist.[9]

As in Anderson and Renault (1999), we assume that the utility of not buying is sufficiently negative such that all consumers buy in equilibrium. This allows us to compute the equilibrium price in closed form. Other than in welfare analysis, this assumption has no important bearing on our results. We will focus on symmetric pure-strategy Nash equilibria (SNE). For ease of exposition, we will omit the consumer-specific index j when doing so does not cause confusion.

Distribution functions Values of ε_{ij} and η_{ij} are private information of consumer j. It is common knowledge that ε_{ij} and η_{ij} are independently and identically distributed across consumers and firms with distribution functions $F(\varepsilon)$ and $G(\eta)$, respectively. Both F and G are continuously differentiable and the corresponding density functions are denoted $f(\varepsilon)$ and $g(\eta)$. We assume that f and g have full support. This assumption has the big advantage that demand functions do not have kinks, which greatly simplifies some parts of our analysis. Nevertheless, our main results do not depend on this assumption, as we show later when we consider the case in which both match values are distributed uniformly on closed intervals. Finally, in order to derive clear-cut comparative statics results, we assume that $1-F$ is log-concave.[10] The log-concavity of $1-G$, or of g, is not necessary for our results; however, when g is log-concave some of our proofs simplify substantially. We will explain later when the log-concavity of g can be invoked to make things simpler.

For our analysis, it will prove useful to define the difference between the observable components for both firms as $\Delta_\eta \equiv \eta_2 - \eta_1$. We denote the distribution function of Δ_η by Γ, and its

[9] This can be seen as follows. Suppose that in that case we had a symmetric equilibrium with $p^* > 0$. Both firms would then be visited first with equal probability. If firm i would slightly undercut p^*, however, all consumers would visit firm i first, and it would see a discontinuous increase in its demand. Hence, such a deviation would be profitable. It cannot be an equilibrium to have $p^* = 0$ either. Both firms would then make zero profits. If firm i deviated to a higher price, all consumers would visit the other firm first, but some would still prefer to buy from i, rendering the deviation profitable.

[10] This assumption is common in the consumer search literature. Log-concavity of $1-F$ is implied by the log-concavity of f, which is satisfied by for example the normal, logistic, extreme value and Laplace densities, which all have full support (see Bagnoli and Bergstrom, 2005). Besides helping obtain clear-cut comparative statics results, the log-concavity of $1-F$ ensures the existence of equilibrium in Wolinsky's (1986) monopolistic competition consumer search model; in the oligopoly model of Anderson and Renault (1999), the log-concavity of f is assumed.

density function by γ. Note that

$$\Gamma\left(\Delta_\eta\right) = \Pr\left(\eta_2 < \eta_1 + \Delta_\eta\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\eta_1+\Delta_\eta} dG(\eta_2)dG(\eta_1) = \int_{-\infty}^{\infty} G(\eta+\Delta_\eta)dG(\eta),$$

hence

$$\gamma\left(\Delta_\eta\right) = \int_{-\infty}^{\infty} g\left(\eta+\Delta_\eta\right) g\left(\eta\right) d\eta. \tag{2}$$

The density of Δ_η has the following properties:[11]

Lemma 1. *The random variable Δ_η has mean zero, and its density $\gamma(\Delta_\eta)$ is symmetric around zero and reaches its maximum value at zero. Moreover, if g is log-concave then γ is also log-concave.*

Proof. In Appendix. □

The fact that γ reaches its maximum value at zero follows from independence of the observable match values η_1 and η_2. It implies that the most frequent individuals do not have a strong ex-ante preference for either firm. As will be seen later, this property turns out to play a crucial role in our analysis. If we additionally assume that g is log-concave, then Lemma 1 implies that $\gamma(\Delta_\eta)$ is monotone increasing up to 0, reaches its maximum at 0, and decreases thereafter.

The consumer search rule Suppose we are in an equilibrium where all firms charge the same price p^*. Suppose a consumer has visited a firm, say i, has observed utility $\varepsilon_i + \eta_i - p^*$ and is contemplating whether to visit the other firm, k. Buying at firm k gives higher utility whenever $\varepsilon_k > x \equiv \varepsilon_i + \eta_i - \eta_k$. The expected gains of paying a costly visit to k are thus given by

$$h(x,s) \equiv \int_x^{\infty} \left(\varepsilon_k - x\right) dF\left(\varepsilon_k\right) - s. \tag{3}$$

Define the reservation value $\hat{x}$ as the solution to $h(\hat{x},s) = 0$. As the right-hand side of (3) is strictly decreasing in x, we have that the consumer buys product i without visiting firm k whenever $x > \hat{x}$, hence $\varepsilon_i > \hat{x} - \eta_i + \eta_k$. Otherwise, she does visit firm k.

[11]Note that γ is an autocorrelation function, so the first part of the lemma follows directly from the well-known properties of such a function, see e.g. Yarlagadda (2010), pg. 63. For completeness, we also provide a proof in the Appendix.

Following Weitzman (1979), a consumer that searches to maximize expected utility should first visit the firm where her reservation utility is the highest. With unobservable prices, the reservation utilities of consumer j at firms i and k are given by $\hat{x} + \eta_{ij} - p^*$ and $\hat{x} + \eta_{kj} - p^*$, respectively, where p^* is the equilibrium price consumers expect firms to charge. This implies that consumers for whom $\eta_i > \eta_k$ start their search at firm i. With observable prices, the reservation utilities are $\hat{x} + \eta_{ij} - p_i$ and $\hat{x} + \eta_{kj} - p_k$ and consumers for whom $\eta_i - p_i > \eta_k - p_k$ start their search at firm i.

4 Pricing

In this section, we present the analysis of equilibrium pricing in two versions of our model. We start with the benchmark case in which prices are not observable before search, as in Wolinsky (1986) and Anderson and Renault (1999). In this benchmark model, owing to the ex-ante observable product characteristics, search is directed; however, firms do not have the ability to influence the direction of search of a consumer. In the second part of this section, we study the case in which prices are readily observable before search. Due to this, in this second version of our model, firms can influence the search path of a consumer by modifying their prices.

4.1 Non-observable prices

We start by deriving a symmetric equilibrium price for the case in which consumers can only observe a firm's price after paying a visit to it. As mentioned above, the assumption that prices are not observable before search has been a constant in the traditional consumer search literature. Under this assumption, absent any form of firm differentiation, search is naturally random. In our model, however, search will be directed: because of the ex-ante observable characteristic, an individual consumer will start her search for an acceptable product at the firm that ex-ante offers her the highest utility.

Let p_N^* denote a symmetric equilibrium price. The subscript N indicates that prices are not observable before search. In order to derive a symmetric equilibrium price, we first compute the demand of a firm that deviates by charging a price different from the equilibrium price. After this, we take the first order condition (henceforth FOC) for profits maximization, apply symmetry and solve for a candidate symmetric equilibrium price. After this, we discuss the existence of equilibrium.

We start with the derivation of demand. Suppose firm 1 deviates from the tentative equilibrium price p_N^* by charging a price $p_1 \neq p_N^*$. For notational simplicity, let $\Delta_p \equiv p_N^* - p_1$. Consumers anyway expect both firms to charge p_N^*; hence their reservation utilities at firms 1 and 2 are equal to $\hat{x} + \eta_1 - p_N^*$ and $\hat{x} + \eta_2 - p_N^*$, respectively. Following Weitzman (1979), consumers for whom $\Delta_\eta < 0$ will first visit firm 1, while the other consumers will first visit firm 2.

Let $D_1^N(p_1, p_N^*; \hat{x})$ denote the total demand of firm 1, which depends on the deviation price p_1, the equilibrium price p_N^*, and search costs $\hat{x}$. This demand stems from the two distinct groups of consumers mentioned above. Let $q_{11}^N(p_1, p_N^*; \hat{x})$ denote the demand from consumers that first visit firm 1, and let $q_{21}^N(p_1, p_N^*; \hat{x})$ be the demand from those consumers who first visit firm 2. The subindexes 11 and 21 indicate the search path: the first subscript denotes where the consumer starts searching, and the second denotes where she ends up buying. Naturally,

$$D_1^N(p_1, p_N^*; \hat{x}) = q_{11}^N(p_1, p_N^*; \hat{x}) + q_{21}^N(p_1, p_N^*; \hat{x}). \tag{4}$$

In order to compute $q_{11}^N(p_1, p_N^*; \hat{x})$, we notice that some of the consumers who first visit firm 1 buy the product of firm 1 without visiting firm 2; the rest of the consumers who first visit firm 1 also visit firm 2. Consumers that first visit firm 1 and decide to buy without visiting firm 2 have $\Delta_\eta < 0$ (they first visit firm 1) and $\varepsilon_1 \geq \hat{x} + \Delta_\eta - \Delta_p$ (they find an ε_1 that does not make it worthwhile to visit firm 2). Those that first visit firm 1, then visit firm 2, but end up buying product 1 have $\Delta_\eta < 0$ (they first visit firm 1), $\varepsilon_1 < \hat{x} + \Delta_\eta - \Delta_p$ (they find it worthwhile to visit firm 2) and $\varepsilon_1 \geq \varepsilon_2 - \Delta_\eta - \Delta_p$ (they learn that firm 1 offers a better deal). Hence,

$$\begin{aligned} q_{11}^N(p_1, p_N^*; \hat{x}) &= \int_{-\infty}^{0} \left(1 - F\left(\hat{x} + \Delta_\eta - \Delta_p\right)\right) d\Gamma\left(\Delta_\eta\right) \\ &+ \int_{-\infty}^{0} \int_{-\infty}^{\hat{x}+\Delta_\eta-\Delta_p} F\left(\varepsilon - \Delta_\eta + \Delta_p\right) dF\left(\varepsilon\right) d\Gamma\left(\Delta_\eta\right). \end{aligned} \tag{5}$$

A consumer that first visits firm 2 has $\Delta_\eta \geq 0$. She decides to also visit firm 1 if $\varepsilon_2 < \hat{x} - \Delta_\eta$, as she expects firm 1 to charge the equilibrium price p_N^*. Upon observing the match value and price of firm 1, she buys product 1 if $\varepsilon_2 < \varepsilon_1 - \Delta_\eta + \Delta_p$. Hence,

$$\begin{aligned} q_{21}^N(p_1, p_N^*; \hat{x}) &= \int_0^\infty \int_{-\infty}^{\hat{x}-\Delta_p} F(\varepsilon - \Delta_\eta + \Delta_p)\, dF(\varepsilon)\, d\Gamma(\Delta_\eta) \\ &+ \int_0^\infty F(\hat{x} - \Delta_\eta)(1 - F(\hat{x} - \Delta_p))\, d\Gamma(\Delta_\eta), \end{aligned} \tag{6}$$

where the first term reflects the case that $\varepsilon_1 < \hat{x} - \Delta_p$, and the second term the case that $\varepsilon_1 \geq \hat{x} - \Delta_p$.

The profits of firm 1 are:

$$\pi_1^N(p_1, p_N^*) = p_1 D_1^N(p_1, p_N^*; \hat{x}). \tag{7}$$

Taking the FOC and imposing symmetry, we obtain:

Proposition 1. *When prices are not observable before search, if a SNE exists, the price is given by:*

$$p_N^* = \frac{1}{2\int_{-\infty}^{0}\left(f(\hat{x} + \Delta_\eta)(1 - F(\hat{x})) + 2\int_{-\infty}^{\hat{x}+\Delta_\eta} f(\varepsilon - \Delta_\eta)\, dF(\varepsilon)\right) d\Gamma(\Delta_\eta)}. \tag{8}$$

Per-firm demand is $D_i^{N*} = \frac{1}{2}$ *and the profits of the firms are* $\pi_i^{N*} = \frac{1}{2}p_N^*$, $i = 1, 2$.

Proof. In Appendix. □

Regarding the existence of equilibrium, we show in the Supplementary Appendix that the payoff function (7) is strictly concave in a neighbourhood of p_N^*. Yet, it may not be globally quasi-concave. In Wolinsky's (1986) model with infinitely many firms the log-concavity of $1 - F$ ensures the existence of equilibrium. Here, as in Anderson and Renault (1999), we have an oligopolistic market structure and, as explained above, an individual firm derives demand from different groups of consumers. Even if we make sure that each of the components of q_{11}^N and q_{21}^N is quasi-concave, aggregate demand may fail to be quasi-concave because the sum of quasi-concave functions need not be quasi-concave. Anderson and Renault (1999) show that the stronger condition that the density f is always non-decreasing suffices for the existence of equilibrium in their setting. Unfortunately, such a condition is incompatible with our full support assumption. We have nevertheless numerically checked the quasi-concavity of the payoff (7) for several distributions. In the Appendix we look at the cases in which η and ε both are normally distributed (Figure 4a), both follow a Gumbel distribution (Figure 4b), or both are uniformly distributed (Figure 7). In all these cases, the payoff (7) is quasi-concave.

4.2 Observable prices

In this section, we consider the case in which prices are readily observable. Hence, consumers do not need to visit a firm in order to learn its price. Because prices are observable before search, search is directed not only by the differences in observable characteristics but also by the price differences. That is, by its choice of price an individual firm can affect the share of consumers who choose to initiate their search at its premises.

Let p_A^* denote a symmetric equilibrium price for the case in which prices are advertised. The superscript A refers to the case in which prices are advertised. We derive a symmetric equilibrium price following the same steps as before. Suppose that a firm, say 1, deviates from the tentative equilibrium price p_A^* by charging a price $p_1 \neq p_A^*$. Let Δ_p now denote the difference in prices $p_A^* - p_1$. With observable prices, reservation utilities at firms 1 and 2 are $\hat{x} + \eta_1 - p_1$ and $\hat{x} + \eta_2 - p_A^*$, respectively. Hence, consumers for whom $\eta_1 - p_1 \geq \eta_2 - p_A^*$ (or $\Delta_\eta \leq \Delta_p$), will first visit firm 1, while the others will first visit firm 2. Let $D_1^A(p_1, p_A^*; \hat{x})$ denote total demand for firm 1.

As before, demand for firm 1 consists of two components. First, some consumers first visit firm 1 and also end up buying product 1. We denote demand from this source as $q_{11}^A(p_1, p_A^*; \hat{x})$. Second, there are consumers who visit firm 2 first, but choose to walk away from it to inspect product 1 and end up buying it. Demand from these consumers is denoted $q_{21}^A(p_1, p_A^*; \hat{x})$. Once again,

$$D_1^A(p_1, p_A^*; \hat{x}) = q_{11}^A(p_1, p_A^*; \hat{x}) + q_{21}^A(p_1, p_A^*; \hat{x}). \tag{9}$$

For the first group, we have:

$$\begin{aligned} q_{11}^A(p_1, p_A^*; \hat{x}) = &\int_{-\infty}^{\Delta_p} \left(1 - F\left(\hat{x} + \Delta_\eta - \Delta_p\right)\right) d\Gamma\left(\Delta_\eta\right) \\ &+ \int_{-\infty}^{\Delta_p} \int_{-\infty}^{\hat{x}+\Delta_\eta-\Delta_p} F\left(\varepsilon - \Delta_\eta + \Delta_p\right) dF\left(\varepsilon\right) d\Gamma\left(\Delta_\eta\right). \end{aligned} \tag{10}$$

This can be seen as follows. First, some consumers first visit firm 1, and decide to buy there without visiting firm 2. Such consumers necessarily have $\Delta_\eta \leq \Delta_p$ and $\varepsilon_1 \geq \hat{x} + \Delta_\eta - \Delta_p$. The first term of (10) reflects the joint probability of these events. Second, there are consumers who first visit firm 1, then decide to also visit firm 2, but do end up buying product 1. Such consumers have $\Delta_\eta \leq \Delta_p$, $\varepsilon_1 < \hat{x} + \Delta_\eta - \Delta_p$, and $\varepsilon_2 < \varepsilon_1 - \Delta_\eta + \Delta_p$. The second term of (10) reflects the joint probability of these events.

For the second group in (9), we have:

$$
\begin{aligned}
q_{21}^A(p_1, p_A^*; \hat{x}) = & \int_{\Delta_p}^{\infty} \int_{-\infty}^{\hat{x}} F\left(\varepsilon - \Delta_\eta + \Delta_p\right) dF\left(\varepsilon\right) d\Gamma\left(\Delta_\eta\right) \\
& + \int_{\Delta_p}^{\infty} F\left(\hat{x} - \Delta_\eta + \Delta_p\right)\left(1 - F(\hat{x})\right) d\Gamma\left(\Delta_\eta\right). \qquad (11)
\end{aligned}
$$

These consumers have $\Delta_\eta > \Delta_p$, $\varepsilon_2 < \hat{x} - \Delta_\eta + \Delta_p$, and $\varepsilon_2 < \varepsilon_1 - \Delta_\eta + \Delta_p$. The joint probability of these events is reflected in (11).

The payoff to firm 1 is:

$$
\pi_1^A(p_1, p_A^*; \hat{x}) = p_1 D_1^A(p_1, p_A^*; \hat{x}). \qquad (12)
$$

Taking the FOC and imposing symmetry, we obtain:

Proposition 2. *When prices are observable before search, if a SNE exists, the price is given by:*

$$
p_A^* = \frac{1}{4\int_{-\infty}^{0}\left[f\left(\hat{x} + \Delta_\eta\right)\left(1 - F\left(\hat{x}\right)\right) + \int_{-\infty}^{\hat{x}+\Delta_\eta} f\left(\varepsilon - \Delta_\eta\right) dF\left(\varepsilon\right)\right] d\Gamma\left(\Delta_\eta\right) + 2\gamma\left(0\right)\left(1 - F\left(\hat{x}\right)\right)^2}. \qquad (13)
$$

Per-firm demand is $D_1^{A*} = \frac{1}{2}$ *and per-firm profits are* $\pi_A^* = \frac{1}{2} p_A^*$.

Proof. In Appendix. □

Regarding the existence of the SNE, in a previous version of this paper we conjectured (and provided some numerical evidence) that the payoff (12) is quasi-concave provided that the dispersion of the ex-ante observable product features is sufficiently large (see the Supplementary Appendix). Choi *et al.* (2017) prove that our conjecture holds provided that f and g are log-concave.

This can be understood as follows. First, when the variation in η goes to zero, we are back to the case in which an equilibrium in pure strategies does not exist (cf. footnote 9).

Without observable characteristics, the decision what firm to visit first is solely based on price. Hence, slightly undercutting the competitor leads to a discrete increase in demand and hence is profitable. At the same time, charging price equal to marginal cost is not an equilibrium either: it is then a profitable defection to charge a much higher price to maximize profits from consumers that first visit the other firm but find out that they do not like its offer. By continuity, that still applies if there is relatively little variation. Also then, it would be profitable to defect from a tentative equilibrium by charging a price that maximizes profits from the walkaways of the other firm.

At the same time, we know that with zero search cost, we are back to the perfect information case of Perloff and Salop (1985) and a pure strategy equilibrium exists provided that f and g are log-concave. Together, these observations suggest that we need some ex-ante product differentiation to ensure the existence of a pure-strategy equilibrium – but also that the amount we need depends on the magnitude of search costs.

In Figure 1, we have plotted the region of parameters for which a pure-strategy Nash equilibrium fails to exist with observable prices. In the figure, we assume that both η and ε are distributed according to a Gumbel distribution with location parameter 0. Search cost is on the vertical axis; the scale parameter of η, denoted β_η, is on the horizontal axis.

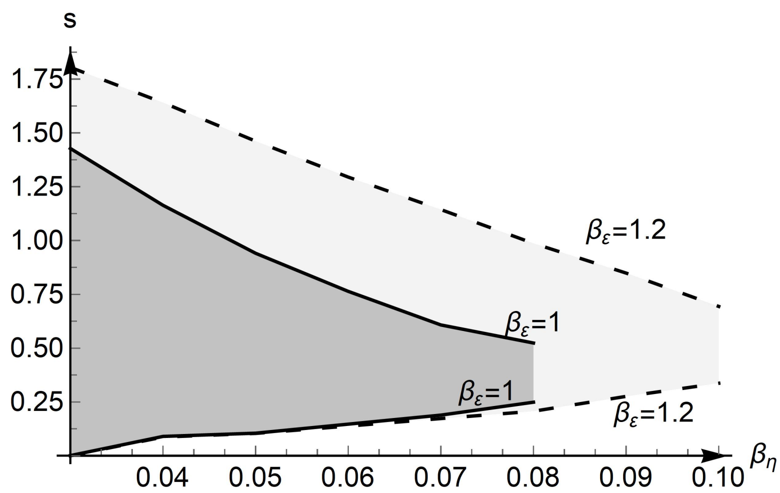


Figure 1: Region of parameters for which a pure-strategy equilibrium fails to exist.

As argued above, for a fixed search cost, sufficient variation in ex-ante observable product differentiation η ensures existence. We also see that for a fixed variance of η, the equilibrium exists if search cost is either sufficiently large or sufficiently low. This can be understood as follows.

If search costs are high, hardly any consumer walks away from the first firm that they visit. Hence, it is not profitable to defect from a tentative equilibrium by charging a higher price that maximizes profits from the other firm's walkaways - simply because there are too few to make that profitable. As a result, a pure strategy equilibrium exists.

With intermediate search costs, any consumer that walks away from a firm reveals to dislike its product. This gives the other firm monopoly power vis-a-vis those consumers. Hence, it is then profitable to defect from a tentative equilibrium by charging a higher price. Maximizing profits from the other firm's walkaways now yields a decent profit margin, and the number of walkaways is sufficiently high to make it worthwhile. Therefore, an equilibrium in pure strategies does not exist.

As search costs decrease, however, an increasing number of consumers checks out both firms. The act of walking away then becomes less informative. Consumers that do walk away may still have a decent match with that firm and hence may go back if the price at the second firm is too high. Hence, monopoly power vis-a-vis those consumers is now lower. In that case, it is not profitable to defect from a tentative equilibrium by defecting to a price that maximizes profits from walkaways, simply because the price increase from doing so is too low to make it worthwhile. Hence, a pure strategy equilibrium exists.

5 Comparison

In this section, we study the effects of price observability. We first look at the effect of price observability on the level of prices. Then we study how price observability bears on the relationship between prices and search costs.

5.1 The effects of price observability on equilibrium prices

Proposition 3. *Equilibrium prices are lower if consumers can observe prices before search, that is,* $p_N^* \geq p_A^*$.

Proof. In Appendix. □

In both models, a higher price decreases the probability that a consumer that visits a firm ends up buying there. When prices are observable before search, a higher price moreover decreases the probability that a consumer visits the firm at all. This second effect makes the demand for a firm's product more sensitive to price changes, as we will now show.

Note that $\pi_1^j = p_j^* \cdot D_1^j$, for $j \in \{N, A\}$, so the first-order condition implies $D_1^{j\prime} = -1/(2p_j^*)$. Using the prices in (8) and (13), we can then show that the price sensitivity of demand when prices are observable before search can be written as follows:

$$\left.\frac{\partial D_1^A}{\partial p_1}\right|_{p_1=p_2} = \underbrace{\left.\frac{\partial D_1^N}{\partial p_1}\right|_{p_1=p_2}}_{\text{non-price directed effect}} - \underbrace{\gamma(0)\left(1-F\left(\hat{x}\right)\right)}_{\text{direction effect 1st visits}} + \underbrace{\gamma(0)F\left(\hat{x}\right)\left(1-F\left(\hat{x}\right)\right)}_{\text{direction effect new 2nd visits}} \underbrace{-\left(1-F\left(\hat{x}\right)\right)\int_0^\infty f\left(\hat{x}-\Delta_\eta\right)d\Gamma(\Delta_\eta)}_{\text{direction effect existing 2nd visits}}. \tag{14}$$

The first term on the right-hand side is the effect of price changes on demand we also have in a model where prices are initially unobservable. This is the **non-price directed search effect**. Now consider the additional effects of having observable prices. At the margin, a firm that increases its price now sees its number of first visitors (i.e. the number of consumers for whom this is the first firm that they visit) decrease by $\gamma(0)$.[12] These consumers would have bought from firm 1 had they encountered a match value above $\hat{x}$, so sales through this channel decrease by $\gamma(0)\left(1-F\left(\hat{x}\right)\right)$. This is the **search direction effect through first visits**. Some of the $\gamma(0)$ consumers that now visit the other firm first, turn out not to like the product firm 2 offers, but do like that of firm 1. Hence, sales through this channel increase by $\gamma(0)F\left(\hat{x}\right)\left(1-F\left(\hat{x}\right)\right)$. This is the **search direction effect through new second visits**. Finally, after a price increase of firm 1, some consumers that visit firm 2 first decide to not visit firm 1 as they observe its higher price, whereas they would have visited firm 1 and ended up buying there had the price of firm 1 not been directly observable. This **search direction effect on existing second visits** is the last term in (14). The overall search direction effect on second visits is positive: combining the last two terms in (14) and using integration by parts, this effect can be shown to equal:

$$-\left(1-F\left(\hat{x}\right)\right)\int_0^\infty F\left(\hat{x}-\Delta_\eta\right)\gamma'(\Delta_\eta)d\Delta_\eta > 0, \tag{15}$$

[12] Those with $\Delta_\eta \geq \Delta_p$ now visit the rival firm first; their mass is $1-\Gamma\left(\Delta_p\right)$, so the marginal effect is $\gamma\left(0\right)$.

where the inequality is proven in the Appendix (see Lemma 3 and its proof).[13]

The total search direction effect thus depends on whether the negative effect through first visits outweighs the positive effect through second visits. Not surprisingly, that is indeed the case. Putting these two effects together yields:

$$-\left[1-F\left(\hat{x}\right)\right]\left[\gamma(0)+\int_{0}^{\infty}F\left(\hat{x}-\Delta_{\eta}\right)\gamma'(\Delta_{\eta})d\Delta_{\eta}\right]<0, \tag{16}$$

where the inequality is proven in the Appendix (see proof of Proposition 3).[14]

Hence, if consumers observe prices before search, a firm's individual demand is more sensitive to prices compared to a case in which they do not. As a result, equilibrium prices are lower.

5.2 The effects of an increase in search costs

We now consider the effect of an increase in search costs on equilibrium prices. First consider the case in which prices are unobservable before search. After simplification, price-sensitivity of demand D_1^N changes with the search cost as follows (details in the proof of Proposition 4 in the Appendix):

$$\frac{\partial}{\partial \hat{x}}\left(\left.\frac{\partial D_1^N}{\partial p_1}\right|_{p_1=p_2}\right)=-\int_{-\infty}^{0}\left[f'\left(\hat{x}+\Delta_{\eta}\right)\left(1-F\left(\hat{x}\right)\right)+f(\hat{x})f(\hat{x}+\Delta_{\eta})\right]d\Gamma\left(\Delta_{\eta}\right)<0, \tag{17}$$

where the inequality follows from log-concavity of $1-F$. A higher search cost (lower $\hat{x}$) thus implies a higher derivative $\partial D_1^N/\partial p_1$. As this derivative is negative, this in turn implies a lower price sensitivity of demand and higher equilibrium prices.

As in Wolinsky (1986) and Anderson and Renault (1999), equilibrium prices thus increase with search costs if prices cannot be observed before search. With higher search costs consumers that visit a firm are easier to retain. Hence firms will be inclined to charge higher prices.

Now assume consumers can observe prices before search. We then have two opposite effects. First, with an increase in search costs it is again easier for a firm to retain consumers, hence

[13] Note that this inequality follows directly if g is log-concave, in which case $\gamma'(\Delta_\eta)<0$ for $0<\Delta_\eta<\infty$. In the Appendix, we prove the inequality dispensing with this assumption.

[14] This follows directly with log-concavity of g, as

$$\gamma(0)+\int_{0}^{\infty}F\left(\hat{x}-\Delta_{\eta}\right)\gamma'(\Delta_{\eta})d\Delta_{\eta}>\gamma(0)+\int_{0}^{\infty}\gamma'(\Delta_{\eta})d\Delta_{\eta}=\gamma(0)+\gamma(\Delta_{\eta})|_{0}^{\infty}=0.$$

making demand less price sensitive. This is the impact on the non-price directed search effect. But second, anticipating that consumers are easier to retain, an increase in search costs makes it more appealing for a firm to attract those consumers. This is the impact on the search direction effect. From (14) and (16), we have:

$$\frac{\partial}{\partial \hat{x}}\left(\left.\frac{\partial D_1^A}{\partial p_1}\right|_{p_1=p_2}\right) = \underbrace{\frac{\partial}{\partial \hat{x}}\left(\left.\frac{\partial D_1^N}{\partial p_1}\right|_{p_1=p_2}\right)}_{\text{non-directed search effect}} - \underbrace{\frac{\partial}{\partial \hat{x}}\left\{(1-F(\hat{x}))\left[\gamma(0)+\int_0^\infty F(\hat{x}-\Delta_\eta)\,\gamma'(\Delta_\eta)d\Delta_\eta\right]\right\}}_{\text{search direction effect}}. \tag{18}$$

For the search direction effect, first note that:

$$-\frac{\partial}{\partial \hat{x}}\left\{(1-F(\hat{x}))\left[\gamma(0)+\int_0^\infty F(\hat{x}-\Delta_\eta)\,\gamma'(\Delta_\eta)d\Delta_\eta\right]\right\} = f(\hat{x})\left[\gamma(0)+\int_0^\infty F(\hat{x}-\Delta_\eta)\,\gamma'(\Delta_\eta)d\Delta_\eta\right] - (1-F(\hat{x}))\int_0^\infty f(\hat{x}-\Delta_\eta)\,\gamma'(\Delta_\eta)d\Delta_\eta. \tag{19}$$

We know from the proof of Proposition 3 that the term in square brackets is positive. The sign of the other term depends on $\int_0^\infty f(\hat{x}-\Delta_\eta)\,\gamma'(\Delta_\eta)d\Delta_\eta$. With g log-concave, $\gamma'(\Delta_\eta) < 0$ for $0 < \Delta_\eta < \infty$ so the integral is negative, rendering (19) positive. Using the same arguments as in the proof of Proposition 3, we can also establish that with f increasing up to $\hat{x}$ the integral is also negative (details in Appendix). Hence, with either g log-concave or f increasing up to $\hat{x}$, the directed search effect is positive: higher search costs, on this account, make demand more price sensitive.

The total effect in (18) is not a priori clear: the non-directed search effect and the search direction effect work in opposite directions. Plugging (17) and (19) into (18) and simplifying (see the Appendix for details), the total effect is given by:

$$\frac{\partial}{\partial \hat{x}}\left(\left.\frac{\partial D_1^A}{\partial p_1}\right|_{p_1=p_2}\right) = -2\,(1-F(\hat{x}))\int_0^\infty f(\hat{x}-\Delta_\eta)\,\gamma'(\Delta_\eta)\,d\Delta_\eta. \tag{20}$$

The sign of (20) depends on $\int_0^\infty f(\hat{x}-\Delta_\eta)\,\gamma'(\Delta_\eta)d\Delta_\eta$. We established above that this integral is negative if g is log-concave, or if f increases up to $\hat{x}$. Hence, under those conditions the search direction effect is stronger than the non-directed search effect. An increase in $\hat{x}$ increases the derivative of demand, making demand less price sensitive and hence increasing the equilibrium price. As a higher s implies a lower $\hat{x}$, higher search costs lower the equilibrium price.

We have thus established:

Proposition 4.

(a) If consumers do not observe prices before search, equilibrium prices increase in search costs, that is, $\partial p_N^*/\partial s > 0$.

(b) If consumers do observe prices before search, equilibrium prices decrease in search costs, that is, $\partial p_A^*/\partial s < 0$, if either of these conditions hold:

- *the density of observable match values g is log-concave,*
- *f is increasing up to $\hat{x}$.*

Proof. In Appendix. □

The first part of the Proposition extends the result of Anderson and Renault (1999) to the case in which consumers direct their search based on the ex-ante observable product attributes. The intuition is standard: as search costs increase, a firm has more market power over the consumers who pay it a visit, and therefore has incentives to raise its price.

The second part of the Proposition tells that prices may increase in search costs provided consumers observe them before they initiate search. We provide two conditions here. The first one is that the density of observable match values g is log-concave. Choi *et al.* (2017) also discovered this result independently.[15] The second condition is novel. The requirement that f is increasing up to $\hat{x}$ always holds for situations in which the search cost is high enough.[16]

The result can be understood as follows. As search costs increase, consumers with a strong ex-ante preference for either firm (in terms of observable product characteristics) will be less likely to also visit the other firm. That implies that the marginal consumer (indifferent between buying directly and walking away) now has a weaker ex-ante preference. By construction, the density of such consumers is higher, as the density function γ is highest at zero. Thus, as search costs increase, there are more consumers that are at the margin, making competition fiercer

[15]Their result is more general because they prove it for the case of n firms, and because they do not assume full support of the density of ex-ante observable matches.

[16]Log-concavity implies that f must be increasing for sufficiently low x. Thus, the condition is satisfied for sufficiently low $\hat{x}$, hence for sufficiently high s.

and pushing prices down. In other words, as search costs increase, the consumers that are now at the margin will have a weaker preference for one of the products in terms of observable product characteristics. But as the measure of such consumers is higher, competition will be fiercer.[17]

If prices are ex ante unobservable, this pro-competitive channel of higher search costs is cut off. With observable prices, higher search costs have two effects. On the one hand, they make it less likely that consumers walk away. That increases prices. On the other hand, they make it easier to induce the other firm's marginal consumers to walk away. That decreases prices. The second channel disappears if prices cannot be observed ex ante.

It is important to note that the conditions we need to impose to have equilibrium prices decrease in search costs are relatively weak. Log-concavity of the distributions of match values is routinely imposed in this literature. The other condition, that f is increasing up to $\hat{x}$, always holds for search costs sufficiently high. But even if these conditions are violated, the result may still hold. In fact, despite our best efforts, we have not been able to come up with a counterexample where, with observable prices, equilibrium prices are increasing in search costs.[18]

Of course, all this does require that η_1 are η_2 are i.i.d. draws from the same distribution. Once we drop that assumption, the result may change. In Shen (2015), for example, observable characteristics are given by a consumer's location on a Hotelling line, so match values are perfectly negatively correlated.[19] This implies that γ may no longer have its maximum at 0, violating Lemma 1. In fact, the author finds that under some conditions an increase in search costs increases prices for low enough s.[20]

[17]Alternatively, observe that as search costs are higher, unobservable characteristics have less of an effect on consumers' choices. Effectively, this makes them more homogeneous – which increases competition and lowers prices. Essentially, this is the intuition suggested by Choi *et al.* (2017).

[18]For example, take the case in which $g(\eta)$ follows a 50-50 mixture of normally distributed random variables, in particular $g(\eta) = 0.5\mathcal{N}(-2, 1) + 0.5\mathcal{N}(3, 1)$, and ε is drawn from a normal distribution. The density of $\gamma(\Delta_\eta)$ then has three peaks, while $f(\varepsilon)$ is decreasing for low search costs, so both conditions are violated. Yet, also in this case, we find in a numerical analysis that equilibrium prices are globally decreasing in search costs.

[19]The distribution of consumer locations is given by $H(\eta)$. In this case, $\gamma(\Delta_\eta) = h\left(\frac{1+\eta}{2}\right)$.

[20]When match values η_1 and η_2 are positively correlated, our result is still likely to hold. This can be seen as follows. Suppose that η_i is the sum of two random variables: $\eta_i = \zeta + \nu_i$, with $\zeta \sim G_1$ common for both firms and $\nu_i \sim G_2$ firm-specific. The common element ζ generates the positive correlation between η_1 and η_2. But ζ simply drops out when evaluating Δ_η, so our results still hold.

6 The case of uniform distributions

In this section we develop the case in which the match values η and ε are uniformly distributed. This exercise is useful for two reasons. First, it serves to illustrate that the infinite support assumption we made in Section 3 is not necessary for the main results of our paper. Second, it provides a simple framework for future applications. Third, the distributional assumption will enable us to derive additional comparative statics results. Most notably, we show that the magnitude of search costs affects whether prices are more sensitive to observable or opaque product differentiation. Moreover we show that, when prices are observable before search, an increase in search costs results in a higher consumer surplus provided these are not too high.

6.1 Equilibrium and product differentiation

Assume that $\eta \sim U\left[\beta - \bar{\eta}, \beta + \bar{\eta}\right]$ and $\varepsilon \sim U[\alpha - \bar{\varepsilon}, \alpha + \bar{\varepsilon}]$, so that:

$$\begin{array}{ll} G(\eta) = \frac{\eta - (\beta - \bar{\eta})}{2\bar{\eta}}; & g(\eta) = \frac{1}{2\bar{\eta}}; \\ F(\varepsilon) = \frac{\varepsilon - (\alpha - \bar{\varepsilon})}{2\bar{\varepsilon}}; & f(\varepsilon) = \frac{1}{2\bar{\varepsilon}}. \end{array} \tag{21}$$

In line with our discussion after Proposition 2 on the existence of equilibrium, we assume that $\bar{\eta}$ is not too small (see also footnote 9). An increase in β raises the mean of the observable characteristic η, while an increase in $\bar{\eta}$ raises the variance of η. Likewise, an increase in α raises the mean of the opaque characteristic ε and an increase in $\bar{\varepsilon}$ raises the variance of ε.

For our analysis we need the distribution of Δ_η rather than that of the individual η's. Hence, β will not affect the analysis. For ease of exposition, we set $\beta = 0$, so η is distributed on $[-\bar{\eta}, \bar{\eta}]$. We then have:

$$\begin{aligned} \Gamma(\Delta_\eta) &= \int_{\min\{\bar{\eta}, \bar{\eta} - \Delta_\eta\}}^{\bar{\eta}} dG(\eta_1) + \int_{\max\{-\bar{\eta}, -\bar{\eta} - \Delta_\eta\}}^{\min\{\bar{\eta}, \bar{\eta} - \Delta_\eta\}} \int_{-\bar{\eta}}^{\eta_1 + \Delta_\eta} dG(\eta_2) dG(\eta_1) \\ &= \begin{cases} \frac{1}{8\bar{\eta}^2}\left(2\bar{\eta} + \Delta_\eta\right)^2 & \text{if } \Delta_\eta \leq 0; \\ 1 - \frac{1}{8\bar{\eta}^2}\left(2\bar{\eta} + \Delta_\eta\right)^2 & \text{if } \Delta_\eta > 0, \end{cases} \end{aligned}$$

which in turn implies:

$$\gamma(\Delta_\eta) = \begin{cases} \frac{1}{4\bar{\eta}^2}\left(2\bar{\eta} + \Delta_\eta\right) & \text{if } \Delta_\eta \leq 0; \\ \frac{1}{4\bar{\eta}^2}\left(2\bar{\eta} - \Delta_\eta\right) & \text{if } \Delta_\eta \geq 0. \end{cases} \tag{22}$$

The full support assumption in Section 4 implies that consumers who happen to find a low enough match value ε at the first firm will continue search, regardless of the ex-ante observable product characteristics. For consistency, we also assume here that regardless of Δ_η a consumer may walk away from the firm she visits first. This requires that even with the highest possible Δ_η, she will still continue search if she finds the worst possible ε_1 at firm 1.[21] Hence, we need $\alpha - \bar{\varepsilon} < \hat{x} - 2\bar{\eta}$.

From (3), we now have:

$$h(x, s) = \int_x^{\alpha+\bar{\varepsilon}} \frac{(\varepsilon - x)}{2\bar{\varepsilon}} d\varepsilon.$$

Equating this to s and solving for the reservation match value yields:[22]

$$\hat{x} = \alpha + \bar{\varepsilon} - 2\sqrt{\bar{\varepsilon}s}. \tag{23}$$

The condition $\alpha - \bar{\varepsilon} < \hat{x} - 2\bar{\eta}$ then implies:

$$\bar{\eta} < \bar{\varepsilon} - \sqrt{\bar{\varepsilon}s}. \tag{24}$$

Hence, we need that the observable component is less noisy than the opaque one. Note that for (24) to be satisfied, it is necessary to have $s < \bar{\varepsilon}$.[23]

Plugging (23) into (8) and (13), using (22) and (21) and simplifying yields the equilibrium prices:

Lemma 2. *Assume that η_i and ε_i are uniformly distributed. The equilibrium prices corresponding to the Propositions 1 and 2 are given by:*

$$p_N^* = \frac{6\bar{\varepsilon}^2}{6\bar{\varepsilon} - 2\bar{\eta} - 3\sqrt{s\bar{\varepsilon}}} \quad \text{and} \quad p_A^* = \frac{3\bar{\varepsilon}^2\bar{\eta}}{3\bar{\varepsilon}\bar{\eta} + 3s\bar{\varepsilon} - \bar{\eta}^2}.$$

As $\bar{\eta} < \bar{\varepsilon}$ and $s < \bar{\varepsilon}$, these are positive. It is readily seen that when search costs are zero, the two prices coincide. As search costs increase, p_N^* increases while p_A^* decreases. Equilibrium prices do not depend on α: as α increases, both products become more attractive to the same extent. With a fully covered market, this does not affect pricing.

[21]Note that with infinite support this is always satisfied, as a consumer could always observe $\varepsilon_1 = -\infty$.

[22]Note that for this analysis to apply, we need that $\hat{x} < \alpha + \bar{\varepsilon}$, which implies $s < \bar{\varepsilon}$.

[23]If Assumption (24) is not satisfied, our demand expressions here would be slightly different from the ones in Section 4. We have explored the role of such an assumption and our qualitative results still hold.

Proposition 5. *Assume that η_i and ε_i are uniformly distributed. More product differentiation implies higher prices. With low search costs, product differentiation in the opaque characteristic has a stronger effect on prices, in the sense that $\frac{\partial p^*}{\partial \bar{\varepsilon}} > \frac{\partial p^*}{\partial \bar{\eta}}$. With high search costs, product differentiation in the observable characteristic has a stronger effect on prices, in the sense that $\frac{\partial p^*}{\partial \bar{\eta}} > \frac{\partial p^*}{\partial \bar{\varepsilon}}$. This holds both when prices are observable before search, and when they are not.*

Proof. In Appendix. □

As search cost increases, consumers are less likely to observe all opaque characteristics. As a result these start playing a smaller role on prices. For sufficiently high search cost, prices are more sensitive to observable product characteristics.

6.2 Consumer surplus

We now study the effect of an increase in search costs on consumer surplus. We focus on the case in which consumers observe prices before search. This is the interesting case: as search costs increase, equilibrium prices decrease rendering the net effect on consumer welfare ambiguous. More precisely, an increase in the cost of search has three effects on consumer surplus. First, it implies less search in equilibrium, lowering the expected quality of the match between consumers and the products they end up buying. Second, it makes each visit more costly, directly lowering consumer surplus. Third, it lowers prices, which is good for consumers. Hence, the net effect is unclear.

Write consumer surplus as:

$$CS = M - S \cdot s - p_A^*,$$

where M is the match utility and S the expected number of firms sampled. From the envelope theorem, we immediately have:[24]

$$\frac{\partial CS}{\partial s} = -S - \frac{\partial p_A^*}{\partial s}. \tag{25}$$

[24] More precisely, for given prices p_A^* and search costs s, we can rewrite the consumer's decision problem as deriving the optimal number of expected searches S (rather than the optimal cutoff $\hat{x}$) such that

$$S^*(s) = \arg\max_S U(S) \equiv M(S) - S \cdot s$$

From the envelope theorem, we then immediately have $\partial U/\partial s = -S^*$.

Consider S, the expected number of searches that a consumer engages in. Every consumer searches at least once. Some consumers search twice; those that visit firm 1 first and find an ε_1 that is too low (so $\Delta_\eta < 0$, $\varepsilon_1 < \hat{x} + \Delta_\eta$), and those that visit firm 2 first and find an ε_2 that is too low (so $\Delta_\eta > 0$, $\varepsilon_2 < \hat{x} - \Delta_\eta$). By symmetry we thus have:

$$\begin{aligned} S &= 1 + 2\int_{-\infty}^{0}\int_{-\infty}^{\hat{x}+\Delta_\eta} dF(\varepsilon)d\Gamma\left(\Delta_\eta\right) = 1 + 2\int_{-2\bar{\eta}}^{0}\int_{\alpha-\bar{\varepsilon}}^{\hat{x}+\Delta_\eta}\left(\frac{1}{2\bar{\varepsilon}}\right)\left(\frac{1}{4\bar{\eta}^2}\left(2\bar{\eta}+\Delta_\eta\right)\right) d\varepsilon d\Delta_\eta \\ &= \frac{12\bar{\varepsilon} - 2\bar{\eta} - 6\sqrt{s\bar{\varepsilon}}}{6\bar{\varepsilon}}. \end{aligned}$$

As match utility M clearly increases in search costs s, aggregate search costs $S \cdot s$ incurred by consumers also do.[25] As noted, the equilibrium price decreases in search cost. Putting these effects together, we can show that:

Proposition 6. *Assume that consumers observe prices before search, and that η_i and ε_i are uniformly distributed. Then:*

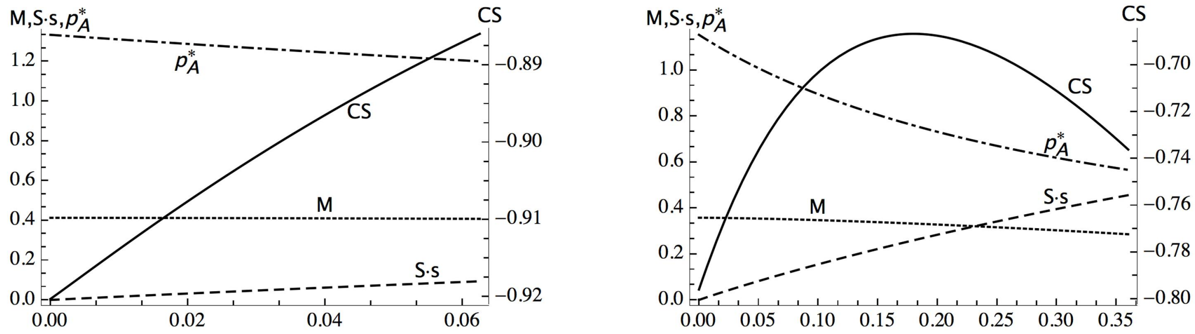
(a) Consumer surplus is increasing in search costs for low enough s.

(b) There exists a $\bar{\eta}_o \in (0, \bar{\varepsilon})$ such that, for sufficiently high s and $\bar{\eta} < \bar{\eta}_o$, consumer surplus is decreasing in search costs. With $\bar{\eta} > \bar{\eta}_o$, consumer surplus is increasing in search costs for high s. In particular, $\bar{\eta}_0 \approx 0.576\bar{\varepsilon}$.

Proposition 6 implies that when search costs are sufficiently small, the price effect dominates and consumer surplus increases in search cost. When $\bar{\eta}$ is not too small, consumer surplus also increases in search cost when search costs are sufficiently large. By contrast, when $\bar{\eta}$ is relatively small, consumer surplus decreases in search costs when search costs are large.

These observations suggest that, for $\bar{\eta}$ large, consumer surplus is increasing in s over the admissible interval, while for $\bar{\eta}$ small, it is first increasing and then decreasing. It is difficult to analytically study the behaviour of consumer surplus for the entire range of admissible values, but numerical simulations reported in Figure 2 suggest that these patterns hold true. In Figure 2, M, $S \cdot s$ and p_A^* are measured on the left-hand vertical axes; consumer surplus on the right-hand vertical axes. Search costs are on the horizontal axes. In Figure 2a, $\bar{\varepsilon} = 1$ and $\bar{\eta} = 0.75$.

[25] As $\frac{\partial(M - S \cdot s)}{\partial s} = -S$; see previous footnote.



(a) ε and η uniform; $\bar{\varepsilon} = 1$, $\bar{\eta} = 0.75$.

(b) ε and η uniform; $\bar{\varepsilon} = 1$, $\bar{\eta} = 0.4$.

Figure 2: Consumer surplus (gross economic surplus, search costs and price)

Consumer surplus is then monotonically increasing in search cost for all admissible values. In this case, match utility remains fairly constant and price falls more than incurred search costs. By contrast, Figure 2b shows that for $\bar{\eta} = 0.4$ consumer surplus is non-monotonic in search cost, first increasing and then decreasing.

7 Conclusion

In this paper we have presented a consumer search model where firms sell products with various characteristics, some observable, others unobservable before search. As consumers are more inclined to visit a firm where they like the observable characteristics, search is directed. In our model firms can also influence the direction of search by adjusting prices since consumers prefer to visit firms whose prices are lower.

We have shown that price observability leads to lower prices and profits. When prices are observable before search, a lower price not only retains more consumers, but is also more likely to attract them. Also, with price observability equilibrium prices and profits *decrease* in search costs. With higher search costs consumers are less likely to walk away, hence firms are more eager to attract them in the first place. More precisely, as search costs increase, consumers at the margin will be those that are relatively indifferent between the observable product characteristics of the two firms. But as the measure of such indifferent consumers is necessarily higher, competition will be fiercer.

Of course, our analysis begs the question of whether firms would have an incentive to make their prices public. In an earlier version of this paper (Haan *et al.*, 2015), we looked at this issue. Suppose that firms can choose whether to advertise their prices at some relatively low advertising cost. It can be shown that an equilibrium in which firms choose to not advertise does not exist, whereas an equilibrium where prices are advertised always exists. Hence, price advertising is a prisoner's dilemma.

Our paper has an important message for policymakers. Search costs are usually considered to be detrimental to consumers. We demonstrate that this conclusion is not valid when price information is readily available and consumers only have to search for product characteristics. If that is the case, higher search costs are likely to benefit consumers through lower prices.

Appendix

Proof of Lemma 1.

Because η_1 and η_2 follow the same distribution, $E(\Delta_\eta) = 0$. For symmetry, note that:

$$\begin{aligned}\gamma\left(-\Delta_\eta\right) = \int_{-\infty}^{\infty} g\left(\eta - \Delta_\eta\right) dG\left(\eta\right) &= \int_{-\infty}^{\infty} g\left(\eta\right) dG\left(\eta + \Delta_\eta\right) \\ &= \int_{-\infty}^{\infty} g\left(\eta + \Delta_\eta\right) dG\left(\eta\right) = \gamma\left(\Delta_\eta\right).\end{aligned}$$

To show that $\gamma(\Delta_\eta)$ reaches its maximum value at zero, notice first that:

$$\gamma(0) = \int_{-\infty}^{\infty} g(\eta)^2 d\eta.$$

Consider now the integral

$$\int_{-\infty}^{\infty} \left[g(\eta) - g(\Delta_\eta + \eta)\right]^2 d\eta > 0, \tag{26}$$

where the inequality follows from the fact that the integrand is positive. We can write (26) as:

$$\int_{-\infty}^{\infty} \left[g(\eta) - g(\Delta_\eta + \eta)\right]^2 d\eta = \int_{-\infty}^{\infty} g(\eta)^2 d\eta + \int_{-\infty}^{\infty} g(\Delta_\eta + \eta)^2 d\eta - 2\int_{-\infty}^{\infty} g(\eta) g(\Delta_\eta + \eta) d\eta > 0. \tag{27}$$

Notice that, by changing variables,

$$\int_{-\infty}^{\infty} g(\Delta_\eta + \eta)^2 d\eta = \int_{-\infty}^{\infty} g(\Delta_\eta)^2 d\eta.$$

Using this in (27) and simplifying gives:

$$\gamma(0) = \int_{-\infty}^{\infty} g(\eta)^2 d\eta > \int_{-\infty}^{\infty} g(\eta) g(\Delta_\eta + \eta) d\eta = \gamma(\Delta_\eta), \text{ for all } \Delta_\eta,$$

which implies that $\gamma(\Delta_\eta)$ has a maximum at zero.

Finally, define $\phi(\eta, \Delta_\eta) \equiv g\left(\eta + \Delta_\eta\right)$. If we assume that $g(\eta)$ is log-concave in η, we immediately have that $\phi(\eta, \Delta_\eta)$ is log-concave in η and Δ_η, hence $\phi(\eta, \Delta_\eta) g(\eta)$ is log-concave in η and Δ_η. With $\gamma\left(\Delta_\eta\right) = \int \phi(\eta, \Delta_\eta) g(\eta) d\eta$, the Prékopa–Leindler inequality immediately implies that $\gamma\left(\Delta_\eta\right)$ is logconcave. ■

Proof of Proposition 1.

After taking the first derivative of the payoff (7) and applying symmetry, the FOC for firm 1 simplifies to:

$$\begin{aligned}
\int_{-\infty}^{0}\left(1-F\left(\hat{x}+\Delta_\eta\right)+\int_{-\infty}^{\hat{x}+\Delta_\eta}F\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right) && - \\
p_N^*\int_{-\infty}^{0}\left(f\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)+\int_{-\infty}^{\hat{x}+\Delta_\eta}f\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right) && + \\
\int_{0}^{\infty}\left(F\left(\hat{x}-\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)+\int_{-\infty}^{\hat{x}}F\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right) && - \\
p_N^*\int_{0}^{\infty}\left(\int_{-\infty}^{\hat{x}}f\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right) && = 0.
\end{aligned} \tag{28}$$

Because of the symmetry of $\gamma\left(\Delta_\eta\right)$ we can state that:

$$\begin{aligned}
p_N^*\int_{0}^{\infty}\left(\int_{-\infty}^{\hat{x}}f\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right) &= p_N^*\int_{0}^{\infty}\left(\int_{-\infty}^{\hat{x}-\Delta_\eta}f\left(\varepsilon+\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right) \\
&= p_N^*\int_{-\infty}^{0}\left(\int_{-\infty}^{\hat{x}+\Delta_\eta}f\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right).
\end{aligned}$$

Note also that integration by parts gives:

$$\begin{aligned}
F\left(\hat{x}-\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)+\int_{-\infty}^{\hat{x}}F\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right) &= F\left(\hat{x}-\Delta_\eta\right)-\int_{-\infty}^{\hat{x}}F\left(\varepsilon\right)dF\left(\varepsilon-\Delta_\eta\right) \\
&= F\left(\hat{x}-\Delta_\eta\right)-\int_{-\infty}^{\hat{x}-\Delta_\eta}F\left(\varepsilon+\Delta_\eta\right)dF\left(\varepsilon\right).
\end{aligned}$$

Then, because of the symmetry of $\gamma\left(\Delta_\eta\right)$:

$$\begin{aligned}
&\int_{0}^{\infty}\left(F\left(\hat{x}-\Delta_\eta\right)-\int_{-\infty}^{\hat{x}-\Delta_\eta}F\left(\varepsilon+\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right) = \\
&\quad\int_{-\infty}^{0}\left(F\left(\hat{x}+\Delta_\eta\right)-\int_{-\infty}^{\hat{x}+\Delta_\eta}F\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right).
\end{aligned}$$

Using these remarks, the FOC (28) simplifies to:

$$\frac{1}{2}-p_N^*\int_{-\infty}^{0}\left(f\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)+2\int_{-\infty}^{\hat{x}+\Delta_\eta}f\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right)=0.$$

Solving for p_N^* gives the expression (8) in the proposition. ■

Proof of Proposition 2.

We first show how to obtain the equilibrium price in (13). After taking the derivative of the payoff (12) with respect to p_1 and setting $p_1 = p_A^*$, we obtain the following equation:

$$\begin{aligned}
\int_{-\infty}^{0} \left(1 - F\left(\hat{x} + \Delta_\eta\right) - p_A^* f\left(\hat{x} + \Delta_\eta\right)\left(1 - F\left(\hat{x}\right)\right)\right) d\Gamma\left(\Delta_\eta\right) & \qquad + \\
\int_{-\infty}^{0} \int_{-\infty}^{\hat{x}+\Delta_\eta} \left(F\left(\varepsilon - \Delta_\eta\right) - p_A^* f\left(\varepsilon - \Delta_\eta\right)\right) dF\left(\varepsilon\right) d\Gamma\left(\Delta_\eta\right) & \qquad + \\
\int_{0}^{\infty} \left(1 - F\left(\hat{x}\right)\right)\left(F\left(\hat{x} - \Delta_\eta\right) - p_A^* f\left(\hat{x} - \Delta_\eta\right)\right) d\Gamma\left(\Delta_\eta\right) & \qquad + \\
\int_{0}^{\infty} \int_{-\infty}^{\hat{x}} \left(F\left(\varepsilon - \Delta_\eta\right) - p_A^* f\left(\varepsilon - \Delta_\eta\right)\right) dF\left(\varepsilon\right) d\Gamma\left(\Delta_\eta\right) - \gamma\left(0\right) p_A^* \left(1 - F\left(\hat{x}\right)\right)^2 & = 0. \qquad (29)
\end{aligned}$$

Integration by parts yields:

$$\begin{aligned}
\int_{-\infty}^{\hat{x}} F\left(\varepsilon - \Delta_\eta\right) dF\left(\varepsilon\right) &= F\left(\hat{x} - \Delta_\eta\right) F\left(\hat{x}\right) - \int_{-\infty}^{\hat{x}} F\left(\varepsilon\right) dF\left(\varepsilon - \Delta_\eta\right) \\
&= F\left(\hat{x} - \Delta_\eta\right) F\left(\hat{x}\right) - \int_{-\infty}^{\hat{x}-\Delta_\eta} F\left(\varepsilon + \Delta_\eta\right) dF\left(\varepsilon\right).
\end{aligned}$$

Moreover, we can state that:

$$\int_{-\infty}^{\hat{x}} p_A^* f\left(\varepsilon - \Delta_\eta\right) dF\left(\varepsilon\right) = \int_{-\infty}^{\hat{x}} p_A^* f\left(\varepsilon\right) dF\left(\varepsilon - \Delta_\eta\right) = \int_{-\infty}^{\hat{x}-\Delta_\eta} p_A^* f\left(\varepsilon + \Delta_\eta\right) dF\left(\varepsilon\right).$$

Finally, because of the symmetry of $\gamma\left(\Delta_\eta\right)$:

$$\int_{-\infty}^{0} f\left(\hat{x} + \Delta_\eta\right) d\Gamma\left(\Delta_\eta\right) = \int_{0}^{\infty} f\left(\hat{x} - \Delta_\eta\right) d\Gamma\left(\Delta_\eta\right)$$

and

$$\begin{aligned}
\int_{-\infty}^{0} \int_{-\infty}^{\hat{x}+\Delta_\eta} \left(F\left(\varepsilon - \Delta_\eta\right) - p_A^* f\left(\varepsilon - \Delta_\eta\right)\right) dF\left(\varepsilon\right) d\Gamma\left(\Delta_\eta\right) = \\
\int_{0}^{\infty} \int_{-\infty}^{\hat{x}-\Delta_\eta} \left(F\left(\varepsilon + \Delta_\eta\right) - p_A^* f\left(\varepsilon + \Delta_\eta\right)\right) dF\left(\varepsilon\right) d\Gamma\left(\Delta_\eta\right).
\end{aligned}$$

Using these observations, equation (29) can be simplified to:

$$\begin{aligned}
\int_{-\infty}^{0} \left(1 - 2p_A^* f\left(\hat{x} + \Delta_\eta\right)\left(1 - F\left(\hat{x}\right)\right) - \int_{-\infty}^{\hat{x}+\Delta_\eta} \left(2p_A^* f\left(\varepsilon - \Delta_\eta\right)\right) dF\left(\varepsilon\right)\right) d\Gamma\left(\Delta_\eta\right) - \\
\gamma\left(0\right) p_A^* \left(1 - F\left(\hat{x}\right)\right)^2 = 0. \qquad (30)
\end{aligned}$$

Solving for p_A^* gives the expression in (13). ■

Proof of proposition 3.

We start by showing the following preliminary result:

Lemma 3.

$$\int_0^\infty F(\hat{x} - \Delta_\eta)\,\gamma'(\Delta_\eta)\,d\Delta_\eta < 0. \tag{31}$$

It is straightforward to show that this inequality holds under the log-concavity of g because then $\gamma'(\Delta_\eta) < 0$ for all $\Delta_\eta > 0$. Here we show that it holds for any g.

For arbitrary g, the density $\gamma(\Delta_\eta)$ need not be monotone decreasing on $[0, \infty)$. Nevertheless, from Lemma 1, we know that γ has a global maximum at 0, so on the interval $[0, \infty)$ it first decreases and then it may increase and decrease several times, finally converging to zero (because it is a density). The idea of the proof can be seen in Figure 3.

In this graph we plot a density $\gamma(\Delta_\eta)$ that first decreases, then increases and then decreases. The integral in (15) can be split into a sum of integrals corresponding to the sub-intervals where γ' changes sign. In every sub-interval where γ decreases, γ' first takes on value zero, then decreases and eventually increases to take again on value zero; in every sub-interval where γ increases, the opposite, that is, γ' first takes on value zero, then increases and then decreases till zero. Because F is increasing, $F(\hat{x} - \Delta_\eta)$ decreases in Δ_η.

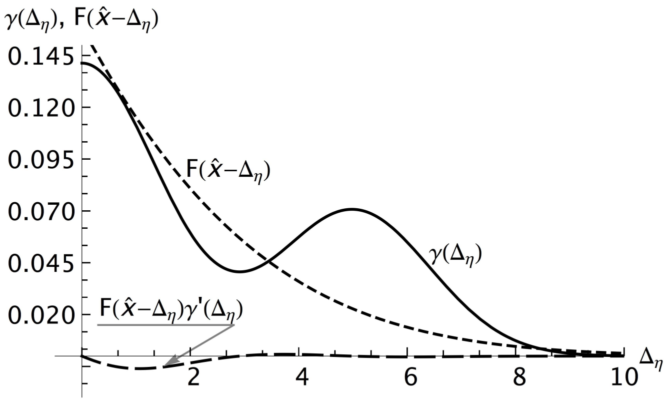


Figure 3: Non log-concave g and inequality (15).

The properties of F and γ' can be exploited to prove inequality (15) as follows. Suppose, without loss of generality, that $\gamma(\Delta_\eta)$ first decreases on $(0, \tilde{\Delta}_\eta)$, then increases on $(\tilde{\Delta}_\eta, \hat{\Delta}_\eta)$, then decreases on $(\hat{\Delta}_\eta, \check{\Delta}_\eta)$, increases again on $(\check{\Delta}_\eta, \bar{\Delta}_\eta)$, and decreases thereafter. As will become clear, if we prove that (31) holds for such a density, then we can use the same proof no matter how many times γ turns increasing.

For such a density we can write:

$$\int_0^{\infty} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta = \int_0^{\tilde{\Delta}_\eta} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta + \int_{\tilde{\Delta}_\eta}^{\hat{\Delta}_\eta} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta$$
$$+\int_{\hat{\Delta}_\eta}^{\check{\Delta}_\eta} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta + \int_{\check{\Delta}_\eta}^{\bar{\Delta}_\eta} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta + \int_{\bar{\Delta}_\eta}^{\infty} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta. \tag{32}$$

Notice that, because F is everywhere increasing and γ' first decreases and then increases we have:

$$\int_0^{\tilde{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta < F(\hat{x}-\tilde{\Delta}_\eta)\int_0^{\tilde{\Delta}_\eta} \gamma'\left(\Delta_\eta\right)d\Delta_\eta = F(\hat{x}-\tilde{\Delta}_\eta)\left[\gamma(\tilde{\Delta}_\eta)-\gamma(0)\right].$$

Likewise, we can write that:

$$\begin{aligned}
&\int_{\tilde{\Delta}_\eta}^{\hat{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta < F(\hat{x}-\tilde{\Delta}_\eta)\left[\gamma(\hat{\Delta}_\eta)-\gamma(\tilde{\Delta}_\eta)\right],\\
&\int_{\hat{\Delta}_\eta}^{\check{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta < F(\hat{x}-\check{\Delta}_\eta)\left[\gamma(\check{\Delta}_\eta)-\gamma(\hat{\Delta}_\eta)\right],\\
&\int_{\check{\Delta}_\eta}^{\bar{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta < F(\hat{x}-\check{\Delta}_\eta)\left[\gamma(\bar{\Delta}_\eta)-\gamma(\check{\Delta}_\eta)\right],\\
&\int_{\bar{\Delta}_\eta}^{\infty} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta < 0.
\end{aligned}$$

Using these inequalities in (32) and simplifying, we have that:

$$\int_0^{\infty} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta < F(\hat{x}-\tilde{\Delta}_\eta)\underbrace{\left[\gamma(\hat{\Delta}_\eta)-\gamma(0)\right]}_{<0} + F\left(\hat{x}-\check{\Delta}_\eta\right)\underbrace{\left[\gamma(\bar{\Delta}_\eta)-\gamma(\hat{\Delta}_\eta)\right]}_{<0,\text{ or }>0}.$$

The sign of the RHS of this expression is negative. This is obvious when $\left[\gamma(\bar{\Delta}_\eta)-\gamma(\hat{\Delta}_\eta)\right]<0$. Otherwise, for situations when $\left[\gamma(\bar{\Delta}_\eta)-\gamma(\hat{\Delta}_\eta)\right]>0$, we have that:

$$\begin{aligned}
&F(\hat{x}-\tilde{\Delta}_\eta)\left[\gamma(\hat{\Delta}_\eta)-\gamma(0)\right] + F\left(\hat{x}-\check{\Delta}_\eta\right)\left[\gamma(\bar{\Delta}_\eta)-\gamma(\hat{\Delta}_\eta)\right]\\
&< F(\hat{x}-\tilde{\Delta}_\eta)\left[\gamma(\hat{\Delta}_\eta)-\gamma(0)+\gamma(\bar{\Delta}_\eta)-\gamma(\hat{\Delta}_\eta)\right] = F(\hat{x}-\tilde{\Delta}_\eta)\left[-\gamma(0)+\gamma(\bar{\Delta}_\eta)\right]<0.
\end{aligned}$$

This completes the proof of Lemma 3. It is obvious that this proof can be replicated for a density γ that goes up and down an arbitrary number of times. ■

To prove Proposition 3, we need to show that inequality (16) holds. For this, we need that:

$$\gamma(0)+\int_0^{\infty} F\left(\hat{x}-\Delta_\eta\right)\gamma'(\Delta_\eta)d\Delta_\eta>0. \tag{33}$$

As in the proof of Lemma 3, suppose, without loss of generality, that $\gamma\left(\Delta_\eta\right)$ first decreases on $(0,\tilde{\Delta}_\eta)$, then increases on $(\tilde{\Delta}_\eta,\hat{\Delta}_\eta)$, then decreases on $(\hat{\Delta}_\eta,\check{\Delta}_\eta)$, increases again on $(\check{\Delta}_\eta,\bar{\Delta}_\eta)$, and decreases thereafter. As will become clear, if we prove that (33) holds for such a density, then we can use the same proof no matter how many times γ goes up and down.

Recall that the integral in (33) can be written as in (32). Notice now that:

$$\begin{aligned}
&\int_0^{\tilde{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta > F(\hat{x})\left[\gamma(\tilde{\Delta}_\eta)-\gamma(0)\right],\\
&\int_{\tilde{\Delta}_\eta}^{\hat{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta > F(\hat{x}-\hat{\Delta}_\eta)\left[\gamma(\hat{\Delta}_\eta)-\gamma(\tilde{\Delta}_\eta)\right],\\
&\int_{\hat{\Delta}_\eta}^{\check{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta > F(\hat{x}-\hat{\Delta}_\eta)\left[\gamma(\check{\Delta}_\eta)-\gamma(\hat{\Delta}_\eta)\right],\\
&\int_{\check{\Delta}_\eta}^{\bar{\Delta}_\eta} F(\hat{x}-\Delta_\eta)\gamma'\left(\Delta_\eta\right)d\Delta_\eta > F(\hat{x}-\bar{\Delta}_\eta)\left[\gamma(\bar{\Delta}_\eta)-\gamma(\check{\Delta}_\eta)\right],\\
&\int_{\bar{\Delta}_\eta}^{\infty} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta > -F(\hat{x}-\bar{\Delta}_\eta)\gamma(\bar{\Delta}_\eta),
\end{aligned}$$

where the inequalities follow from the fact that F is increasing and γ decreasing or increasing depending on the support of the integrals.

Using these inequalities in the integral in (33) and simplifying gives:

$$\begin{aligned}
&\gamma(0)+\int_0^{\infty} F\left(\hat{x}-\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta\\
&>\gamma(0)+F(\hat{x})\left[\gamma(\tilde{\Delta}_\eta)-\gamma(0)\right]+F(\hat{x}-\hat{\Delta}_\eta)\left[\gamma(\check{\Delta}_\eta)-\gamma(\tilde{\Delta}_\eta)\right]-F(\hat{x}-\bar{\Delta}_\eta)\gamma(\check{\Delta}_\eta)\\
&>\gamma(0)+F(\hat{x})\left[\gamma(\tilde{\Delta}_\eta)-\gamma(0)\right]+F(\hat{x}-\hat{\Delta}_\eta)\left[\gamma(\check{\Delta}_\eta)-\gamma(\tilde{\Delta}_\eta)-\gamma(\check{\Delta}_\eta)\right]\\
&>\gamma(0)+F(\hat{x})\left[\gamma(\tilde{\Delta}_\eta)-\gamma(0)-\gamma(\tilde{\Delta}_\eta)\right]=\gamma(0)(1-F(\hat{x}))>0,
\end{aligned} \tag{34}$$

where, in the second inequality we have used the fact that $F(\hat{x}-\hat{\Delta}_\eta)>F(\hat{x}-\bar{\Delta}_\eta)$ and in the last inequality that $F(\hat{x})>F(\hat{x}-\hat{\Delta}_\eta)$.

This completes the proof Proposition 3. It is obvious that this proof can be replicated for a density γ that goes up and down an arbitrary number of times. ■

Proof of proposition 4.

(a) We prove that p_N^* is increasing in s. For this we take the derivative of the denominator of p_N^* given in Proposition 1 with respect to $\hat{x}$. This gives:

$$\begin{aligned}
\frac{\partial}{\partial \hat{x}}\left(\int_{-\infty}^{0}\left(f\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)+2\int_{-\infty}^{\hat{x}+\Delta_\eta} f\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right)d\Gamma\left(\Delta_\eta\right)\right) &= \\
\int_{-\infty}^{0}\left(f'\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)+f\left(\hat{x}\right)f\left(\hat{x}+\Delta_\eta\right)\right)d\Gamma\left(\Delta_\eta\right) &= \\
\int_{-\infty}^{0} f\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)\left(\frac{f'\left(\hat{x}+\Delta_\eta\right)}{f\left(\hat{x}+\Delta_\eta\right)}+\frac{f\left(\hat{x}\right)}{1-F\left(\hat{x}\right)}\right)d\Gamma\left(\Delta_\eta\right). & \qquad (35)
\end{aligned}$$

Because $1-F\left(\varepsilon\right)$ is log-concave:

$$\frac{f\left(\hat{x}\right)}{1-F\left(\hat{x}\right)} \geq \frac{f\left(\hat{x}+\Delta_\eta\right)}{1-F\left(\hat{x}+\Delta_\eta\right)}, \text{ for all } \Delta_\eta \in (-\infty, 0] \text{ (with equality at } \Delta_\eta = 0).$$

Then:

$$\frac{f'\left(\hat{x}+\Delta_\eta\right)}{f\left(\hat{x}+\Delta_\eta\right)}+\frac{f\left(\hat{x}\right)}{1-F\left(\hat{x}\right)} \geq \frac{\left(1-F\left(\hat{x}+\Delta_\eta\right)\right)f'\left(\hat{x}+\Delta_\eta\right)+f^2\left(\hat{x}+\Delta_\eta\right)}{f\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}+\Delta_\eta\right)\right)} \geq 0,$$

where the last inequality follows from the log-concavity of $1-F$. This implies that the integral in (35) is positive. As a result, the denominator of the price increases in $\hat{x}$ so the price decreases in $\hat{x}$ and increases in s.

(b) Regarding the behaviour of the equilibrium price p_A^* with respect to search cost, taking the derivative of the denominator of (13) with respect to $\hat{x}$ gives:

$$\begin{aligned}
&\frac{\partial}{\partial \hat{x}}\left(2\int_{-\infty}^{0}\left[f\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right)+\int_{-\infty}^{\hat{x}+\Delta_\eta} f\left(\varepsilon-\Delta_\eta\right)dF\left(\varepsilon\right)\right]d\Gamma\left(\Delta_\eta\right)+\gamma\left(0\right)\left(1-F\left(\hat{x}\right)\right)^2\right) \\
&= 2\left(1-F\left(\hat{x}\right)\right)\left[\int_{-\infty}^{0} f'\left(\hat{x}+\Delta_\eta\right)d\Gamma\left(\Delta_\eta\right)-\gamma\left(0\right)f\left(\hat{x}\right)\right] \\
&= 2\left(1-F\left(\hat{x}\right)\right)\left[-\int_{-\infty}^{0} f\left(\hat{x}+\Delta_\eta\right)\gamma'\left(\Delta_\eta\right)d\Delta_\eta\right],
\end{aligned}$$

which is equal to expression (20) in the main text. The arguments that follow this expression in the main text complete the proof. ■

Proof of Lemma 2.

Plugging (23) into (8), using (22) and (21) and simplifying yields:

$$\begin{aligned}\frac{1}{p_N^*} =&2\int_{-2\bar{\eta}}^{0}\left[\frac{1}{2\bar{\varepsilon}}\left(\sqrt{\frac{s}{\bar{\varepsilon}}}\right)+\frac{2\bar{\varepsilon}-2\sqrt{\bar{\varepsilon}s}+\Delta_\eta}{2\bar{\varepsilon}^2}\right]\left(\frac{2\bar{\eta}+\Delta_\eta}{4\bar{\eta}^2}\right)d\Delta_\eta\\ =&2\int_{-2\bar{\eta}}^{0}\left(\frac{2\bar{\varepsilon}+\Delta_\eta-\sqrt{s\bar{\varepsilon}}}{2\bar{\varepsilon}^2}\right)\left(\frac{2\bar{\eta}+\Delta_\eta}{4\bar{\eta}^2}\right)d\Delta_\eta=\frac{6\bar{\varepsilon}-2\bar{\eta}-3\sqrt{s\bar{\varepsilon}}}{6\bar{\varepsilon}^2}.\end{aligned}$$

Isolating p_N^* gives the expression in the main text.

For the equilibrium price when consumers observe prices before search, using (13), we have:

$$\frac{1}{p_A^*}=4\int_{-2\bar{\eta}}^{0}\left[\frac{1}{2\bar{\varepsilon}}\left(\frac{\bar{\varepsilon}+\alpha-\hat{x}}{2\bar{\varepsilon}}\right)+\int_{\alpha-\bar{\varepsilon}}^{\hat{x}+\Delta_\eta}\left(\frac{1}{2\bar{\varepsilon}}\right)^2 d\varepsilon\right]\gamma\left(\Delta_\eta\right)d\Delta_\eta+\frac{1}{\bar{\eta}}\left(\frac{\bar{\varepsilon}+\alpha-\hat{x}}{2\bar{\varepsilon}}\right)^2. \tag{36}$$

Plugging (23) into (36) yields

$$\begin{aligned}\frac{1}{p_A^*} =&4\int_{-2\bar{\eta}}^{0}\left[\frac{1}{2\bar{\varepsilon}}\left(\sqrt{\frac{s}{\bar{\varepsilon}}}\right)+\int_{\alpha-\bar{\varepsilon}}^{\hat{x}+\Delta_\eta}\left(\frac{1}{2\bar{\varepsilon}}\right)^2 d\varepsilon\right]\left(\frac{2\bar{\eta}+\Delta_\eta}{4\bar{\eta}^2}\right)d\Delta_\eta+\frac{s}{\bar{\eta}\bar{\varepsilon}}\\ =&4\int_{-2\bar{\eta}}^{0}\left[\frac{1}{2\bar{\varepsilon}}\left(\sqrt{\frac{s}{\bar{\varepsilon}}}\right)+\frac{2\bar{\varepsilon}-2\sqrt{\bar{\varepsilon}s}+\Delta_\eta}{4\bar{\varepsilon}^2}\right]\left(\frac{2\bar{\eta}+\Delta_\eta}{4\bar{\eta}^2}\right)d\Delta_\eta+\frac{s}{\bar{\eta}\bar{\varepsilon}}\\ =&4\int_{-2\bar{\eta}}^{0}\left(\frac{2\bar{\varepsilon}+\Delta_\eta}{4\bar{\varepsilon}^2}\right)\left(\frac{2\bar{\eta}+\Delta_\eta}{4\bar{\eta}^2}\right)d\Delta_\eta+\frac{s}{\bar{\eta}\bar{\varepsilon}}=\frac{3\bar{\varepsilon}-\bar{\eta}}{3\bar{\varepsilon}^2}+\frac{s}{\bar{\eta}\bar{\varepsilon}}.\end{aligned}$$

Isolating p_A^* gives the expression in the main text.

Details about the existence of equilibrium in this case with uniform distributions are provided in the Supplementary Appendix. ■

Proof of Proposition 5.

The derivatives of prices with respect to $\bar{\varepsilon}$ and $\bar{\eta}$ are given by

$$\frac{\partial p_N^*}{\partial\bar{\varepsilon}}=\frac{3\bar{\varepsilon}\left(12\bar{\varepsilon}-8\bar{\eta}-9\sqrt{s\bar{\varepsilon}}\right)}{\left(2\bar{\eta}-6\bar{\varepsilon}+3\sqrt{s\bar{\varepsilon}}\right)^2};\qquad \frac{\partial p_N^*}{\partial\bar{\eta}}=\frac{12\bar{\varepsilon}^2}{\left(6\bar{\varepsilon}-3\sqrt{s\bar{\varepsilon}}-2\bar{\eta}\right)^2};$$

$$\frac{\partial p_A^*}{\partial\bar{\eta}}=\frac{3\bar{\varepsilon}^2\left(\bar{\eta}^2+3s\bar{\varepsilon}\right)}{\left(3\bar{\varepsilon}\bar{\eta}+3s\bar{\varepsilon}-\bar{\eta}^2\right)^2};\qquad \frac{\partial p_A^*}{\partial\bar{\varepsilon}}=\frac{3\bar{\varepsilon}\bar{\eta}\left(3\bar{\varepsilon}\bar{\eta}+3s\bar{\varepsilon}-2\bar{\eta}^2\right)}{\left(3\bar{\varepsilon}\bar{\eta}+3s\bar{\varepsilon}-\bar{\eta}^2\right)^2}.$$

The derivatives $\frac{\partial p_N^*}{\partial\bar{\eta}}$ and $\frac{\partial p_A^*}{\partial\bar{\eta}}$ are clearly positive. Also note that $\frac{\partial p_N^*}{\partial\bar{\varepsilon}}>0$, since with $\bar{\eta}<\bar{\varepsilon}-\sqrt{\bar{\varepsilon}s}$ we have $12\bar{\varepsilon}-8\bar{\eta}-9\sqrt{s\bar{\varepsilon}}>4\bar{\varepsilon}-\sqrt{s\bar{\varepsilon}}>0$ as $s<\bar{\varepsilon}$. Finally, from $\bar{\eta}<\bar{\varepsilon}$ we immediately have $\frac{\partial p_A^*}{\partial\bar{\varepsilon}}>0$.

Next, note that $\partial p_N^*/\partial\bar{\varepsilon}$ is greater than $\partial p_N^*/\partial\bar{\eta}$ if and only if

$$3\bar{\varepsilon}\left(12\bar{\varepsilon}-8\bar{\eta}-9\sqrt{s\bar{\varepsilon}}\right)-12\bar{\varepsilon}^2=3\bar{\varepsilon}\left(8(\bar{\varepsilon}-\bar{\eta})-9\sqrt{s\bar{\varepsilon}}\right)>0.$$

The expression is positive for small search cost ($s\to 0$). However, when s is large ($s\to\bar{\varepsilon}$), it is negative. For advertised prices, we have that $\partial p_A^*/\partial\bar{\varepsilon}$ is greater than $\partial p_A^*/\partial\bar{\eta}$ if and only if

$$3\bar{\eta}\bar{\varepsilon}(3s\bar{\varepsilon}+\bar{\eta}(3\bar{\varepsilon}-2\bar{\eta}))-3\bar{\varepsilon}^2\left(\bar{\eta}^2+3s\bar{\varepsilon}\right)=3\bar{\varepsilon}(\bar{\varepsilon}-\bar{\eta})\left(2\bar{\eta}^2-3s\bar{\varepsilon}\right)>0.$$

Also this expression is positive when search cost is small ($s\to 0$) but becomes negative when search cost is large ($s\to\bar{\varepsilon}$). ■

Proof of Proposition 6.

From (25),

$$\frac{\partial CS}{\partial s}=-\frac{12\bar{\varepsilon}-2\bar{\eta}-6\sqrt{s\bar{\varepsilon}}}{6\bar{\varepsilon}}+\frac{9\bar{\varepsilon}^3\bar{\eta}}{\left(3\bar{\varepsilon}\bar{\eta}+3s\bar{\varepsilon}-\bar{\eta}^2\right)^2} \tag{37}$$

For $s\to 0$, we then have

$$\left.\frac{\partial CS}{\partial s}\right|_{s\to 0}=\frac{54\bar{\varepsilon}^4\bar{\eta}-(12\bar{\varepsilon}-2\bar{\eta})\left(3\bar{\varepsilon}\bar{\eta}-\bar{\eta}^2\right)^2}{6\bar{\varepsilon}\left(3\bar{\varepsilon}\bar{\eta}-\bar{\eta}^2\right)^2}$$

Write $\bar{\eta}=\lambda\bar{\varepsilon}$ for some $\lambda\in(0,1)$. The numerator then equals$2\lambda\varepsilon^5\left(27-\lambda\left(6-\lambda\right)\left(3-\lambda\right)^2\right)>0$. Hence, consumer surplus is increasing in s for $s=0$.

For the upper bound on s, note that we need $\bar{\eta}<\bar{\varepsilon}-\sqrt{\bar{\varepsilon}s}$, hence $s\to\left(\bar{\varepsilon}-\bar{\eta}\right)^2/\bar{\varepsilon}$. If we plug this in into (37):

$$\left.\frac{\partial CS}{\partial s}\right|_{s\to(\bar{\varepsilon}-\bar{\eta})^2/\bar{\varepsilon}}=-\frac{6\varepsilon+4\eta}{6\bar{\varepsilon}}+\frac{9\bar{\varepsilon}^3\bar{\eta}}{\left(3\varepsilon^2-3\varepsilon\eta+2\eta^2\right)^2}$$

For this to be increasing, we thus need

$$-\left(6\varepsilon+4\eta\right)\left(3\varepsilon^2-3\varepsilon\eta+2\eta^2\right)^2+54\bar{\varepsilon}^4\bar{\eta}>0.$$

Again use $\bar{\eta}=\lambda\bar{\varepsilon}$ so this requires

$$2\varepsilon^5\left(-8\lambda^5+12\lambda^4-6\lambda^3-27\lambda^2+63\lambda-27\right)>0.$$

Solving numerically, this is true if and only if $\lambda > 0.57609$, which establishes the result. ■

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SUPPLEMENTARY APPENDIX TO

A Model of Directed Consumer Search

Marco A. Haan

José L. Moraga-González

Vaiva Petrikaitė

June 2018

On the existence of equilibrium when consumers do not observe prices before search (cf. Proposition 1).

We now show that the payoff function (7) is locally concave in a neighborhood of the equilibrium price. To do this, we first compute the second derivative of the demand for firm 1's product $D_1(p_1, p_N^*)$ with respect to p_1:

$$\begin{aligned}\frac{\partial^2 D_1}{\partial p_1^2} &= \int_{-\infty}^{0}\left(-f'\left(\hat{x}+\Delta_\eta - p_N^* + p_1\right)\left(1-F\left(\hat{x}\right)\right) + \int_{-\infty}^{\hat{x}+\Delta_\eta - p_N^*+p_1} f'\left(\varepsilon - \Delta_\eta - p_1 + p_N^*\right) dF\left(\varepsilon\right)\right.\\ &\quad \left. - f\left(\hat{x}\right) f\left(\hat{x}+\Delta_\eta - p_N^* + p_1\right)\right) d\Gamma\left(\Delta_\eta\right)\\ &\quad - \int_{0}^{\infty}\left(f\left(\hat{x}-\Delta_\eta\right) f\left(\hat{x}-p_N^*+p_1\right) - \int_{-\infty}^{\hat{x}-p_N^*+p_1} f'\left(\varepsilon-\Delta_\eta - p_1 + p_N^*\right) dF\left(\varepsilon\right)\right) d\Gamma\left(\Delta_\eta\right).\end{aligned}$$

Evaluating this derivative at $p_1 = p_N^*$ gives:

$$\begin{aligned}\left.\frac{\partial^2 D_1}{\partial p_1^2}\right|_{p_1=p_N^*} &= \int_{-\infty}^{0}\left(-f'\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right) + \int_{-\infty}^{\hat{x}+\Delta_\eta} f'\left(\varepsilon-\Delta_\eta\right) dF\left(\varepsilon\right)\right.\\ &\quad \left. - f\left(\hat{x}\right) f\left(\hat{x}+\Delta_\eta\right)\right) d\Gamma\left(\Delta_\eta\right)\\ &\quad - \int_{0}^{\infty}\left(f\left(\hat{x}-\Delta_\eta\right) f\left(\hat{x}\right) - \int_{-\infty}^{\hat{x}} f'\left(\varepsilon-\Delta_\eta\right) dF\left(\varepsilon\right)\right) d\Gamma\left(\Delta_\eta\right). \qquad (38)\end{aligned}$$

Applying the integration by parts theorem to the last integral of (38) we obtain:

$$\begin{aligned}f\left(\hat{x}-\Delta_\eta\right) f\left(\hat{x}\right) - \int_{-\infty}^{\hat{x}} f'\left(\varepsilon-\Delta_\eta\right) dF\left(\varepsilon\right) &= f\left(\hat{x}-\Delta_\eta\right) f\left(\hat{x}\right) - \int_{-\infty}^{\hat{x}} f\left(\varepsilon\right) df\left(\varepsilon-\Delta_\eta\right) =\\ \int_{-\infty}^{\hat{x}} f\left(\varepsilon-\Delta_\eta\right) df\left(\varepsilon\right) &= \int_{-\infty}^{\hat{x}} f'\left(\varepsilon\right) dF\left(\varepsilon-\Delta_\eta\right) = \int_{-\infty}^{\hat{x}-\Delta_\eta} f'\left(\varepsilon+\Delta_\eta\right) dF\left(\varepsilon\right),\end{aligned}$$

where we have used the fact that $f(-\infty) = 0$.

Therefore, because of the symmetry of $\gamma(\Delta_\eta)$, (38) simplifies to:

$$\left.\frac{\partial^2 D_1}{\partial p_1^2}\right|_{p_1=p_N^*} = -\int_{-\infty}^{0} \left(f'(\hat{x}+\Delta_\eta)(1-F(\hat{x})) + f(\hat{x})\, f(\hat{x}+\Delta_\eta)\right) d\Gamma(\Delta_\eta).$$

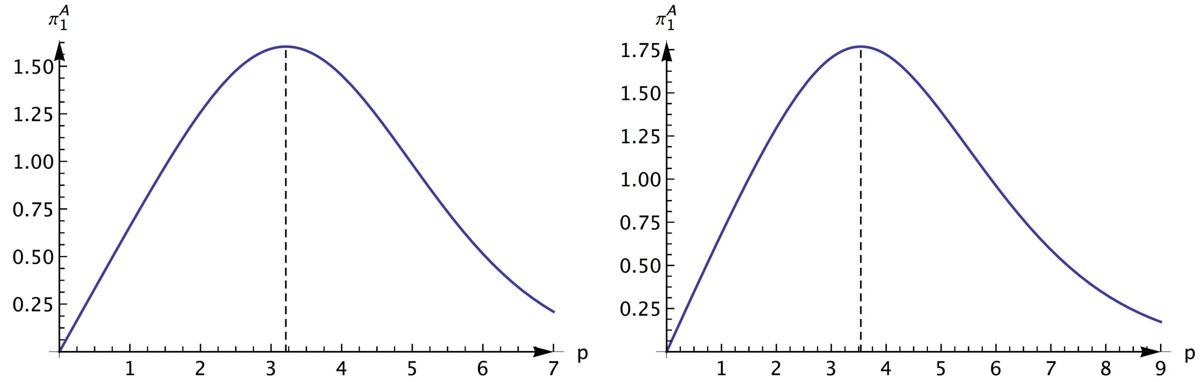
The expression under the integral is positive because

$$\begin{aligned}
f'(\hat{x}+\Delta_\eta)(1-F(\hat{x})) + f(\hat{x})\, f(\hat{x}+\Delta_\eta) &= \\
(1-F(\hat{x}))\, f(\hat{x}+\Delta_\eta)\left(\frac{f'(\hat{x}+\Delta_\eta)}{f(\hat{x}+\Delta_\eta)} + \frac{f(\hat{x})}{1-F(\hat{x})}\right) &\geq \\
(1-F(\hat{x}))\, f(\hat{x}+\Delta_\eta)\left(\frac{f'(\hat{x}+\Delta_\eta)}{f(\hat{x}+\Delta_\eta)} + \frac{f(\hat{x}+\Delta_\eta)}{1-F(\hat{x}+\Delta_\eta)}\right) &= \\
(1-F(\hat{x}))\left(\frac{f'(\hat{x}+\Delta_\eta)(1-F(\hat{x}+\Delta_\eta)) + f^2(\hat{x}+\Delta_\eta)}{1-F(\hat{x}+\Delta_\eta)}\right) &\geq 0,
\end{aligned}$$

where the two inequalities follow from log-concavity of $1-F$. Therefore, we conclude that the demand at the point $p_1 = p_2 = p_N^*$ is concave, which implies that the second order derivative of the payoff function (7) at this point is negative and π_1 attains a local maximum.

Unfortunately, local concavity of the payoff is not sufficient to guarantee the existence of equilibrium. The problem is that the payoff function (7) may not be globally quasi-concave. Providing sufficient conditions under which it is quasi-concave is very difficult because the demand function consists of the sum of four double integrals. The difficulty is that even if each of these integrals is log-concave, it is not guaranteed that the sum will be quasi-concave.

Anderson and Renault (1999) provide a useful discussion in their Appendix and derive conditions for existence. We will not attempt to extend their results to our setting with additional product heterogeneity. Instead, in order to shed some light on the global quasi-concavity of the payoff (7), we rely on numerical simulations.



(a) ε and η standard normal; $\hat{x} = 0.5$

(b) ε and η Gumbel distrib. with location 0 and scale 1; $\hat{x} = 0.5$.

Figure 4: Quasi-concavity of payoff functions (non-observable prices).

In Figure 4 we plot the payoff for the cases in which the distributions of match values are Normal and Gumbel. As the graphs clearly show, the payoffs are nicely quasi-concave, suggesting that an equilibrium exists for these often used density functions.[26] ■

On the existence of equilibrium when consumers observe prices before search (cf. Proposition 2).

We first show that the payoff function (12) is locally concave around the equilibrium price p_A^*. It is readily seen that the second derivative of $D_1^A(p_1, p_A^*)$ with respect to p_1 is

$$\begin{aligned}\frac{\partial^2 D_1^A}{\partial p_1^2} = &\int_{-\infty}^{p_A^*-p_1} \Big(-f'(\hat{x} + \Delta_\eta - p_A^* + p_1)(1 - F(\hat{x})) + \int_{-\infty}^{\hat{x}+\Delta_\eta-p_A^*+p_1} f'(\varepsilon - \Delta_\eta - p_1 + p_A^*)\, dF(\varepsilon) \\ &- f(\hat{x}) f(\hat{x} + \Delta_\eta - p_A^* + p_1)\Big)\, d\Gamma(\Delta_\eta) \\ &+ \int_0^\infty \left(f'(\hat{x} - \Delta_\eta - p_1 + p_A^*)(1 - F(\hat{x})) + \int_{-\infty}^{\hat{x}} f'(\varepsilon - \Delta_\eta - p_1 + p_A^*)\, dF(\varepsilon) \right) d\Gamma(\Delta_\eta) \\ &+ \gamma'(p_A^* - p_1)(1 - F(\hat{x}))^2 .\end{aligned}$$

[26]For the case in Section 6 in which the match values η and ε are uniformly distributed, the payoff is also quasi-concave (see Figure 7).

Setting $p_1 = p_A^*$ and simplifying gives:

$$\begin{aligned}
\left.\frac{\partial^2 D_1^A}{\partial p_1^2}\right|_{p_1=p_A^*} &= \int_{-\infty}^{0} \Big(-f'\left(\hat{x}+\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right) + \int_{-\infty}^{\hat{x}+\Delta_\eta} f'\left(\varepsilon-\Delta_\eta\right) dF\left(\varepsilon\right) \\
&\quad - f\left(\hat{x}\right) f\left(\hat{x}+\Delta_\eta\right)\Big) d\Gamma\left(\Delta_\eta\right) \\
&\quad + \int_{0}^{\infty} \left(f'\left(\hat{x}-\Delta_\eta\right)\left(1-F\left(\hat{x}\right)\right) + \int_{-\infty}^{\hat{x}} f'\left(\varepsilon-\Delta_\eta\right) dF\left(\varepsilon\right) \right) d\Gamma\left(\Delta_\eta\right). \qquad (39)
\end{aligned}$$

Integrating by parts, we establish that:

$$\begin{aligned}
\int_{-\infty}^{\hat{x}} f'\left(\varepsilon-\Delta_\eta\right) dF\left(\varepsilon\right) &= \int_{-\infty}^{\hat{x}} f\left(\varepsilon\right) df\left(\varepsilon-\Delta_\eta\right) \\
&= f\left(\hat{x}\right) f\left(\hat{x}-\Delta_\eta\right) - f^2\left(-\infty\right) - \int_{-\infty}^{\hat{x}} f\left(\varepsilon-\Delta_\eta\right) df\left(\varepsilon\right) \\
&= f\left(\hat{x}\right) f\left(\hat{x}-\Delta_\eta\right) - f^2\left(-\infty\right) - \int_{-\infty}^{\hat{x}-\Delta_\eta} f'\left(\varepsilon+\Delta_\eta\right) dF\left(\varepsilon\right).
\end{aligned}$$

Then, because of the symmetry of $\gamma\left(\Delta_\eta\right)$, we simplify (39) to:

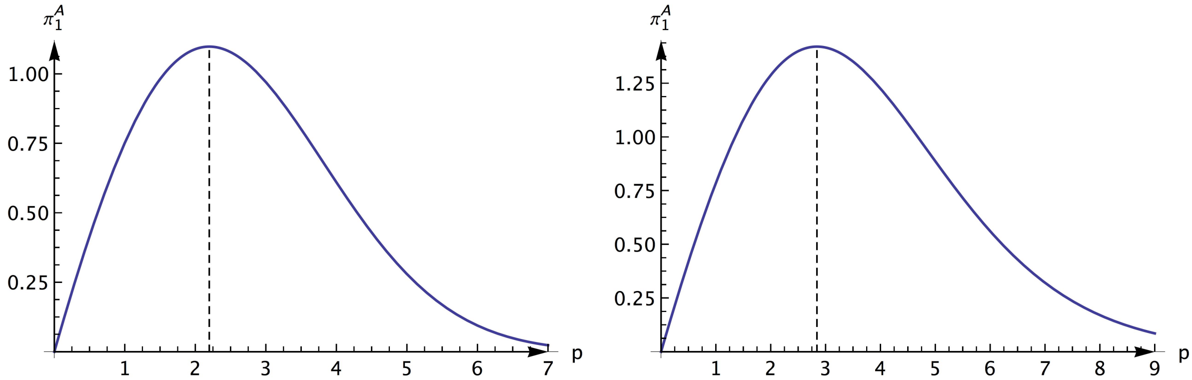
$$\int_{-\infty}^{0} \left(-f^2\left(-\infty\right)\right) d\Gamma\left(\Delta_\eta\right) < 0.$$

Since the second derivative of demand is negative in a neighborhood of p_A^*, we conclude that the demand function is concave in p_1 at the equilibrium point which implies that the payoff is locally concave.

As before, we will not attempt here to provide general conditions for existence of equilibrium but instead we will check numerically that the equilibrium exists for some common distributions of match values. As explained in the main text, we know that for existence of equilibrium there must be sufficient variation in the observable characteristic η. If we allow firm prices to be observable a pure-strategy SNE fails to exist in Anderson and Renault (1999). Our fix for this problem is to introduce additional heterogeneity in the model, namely the observable characteristic η. Obviously, we need sufficient heterogeneity for otherwise we would have the same problem of non-existence of equilibrium.

In Figure 5 we have plotted the payoff function (12) when the distributions of match values are normal (Figure 5a) and Gumbel (Figure 5b). In this Figure we have chosen the variance of η's sufficiently high and clearly the payoff is quasi-concave and therefore the equilibrium price

(13), indicated by the dashed vertical line, is indeed an equilibrium.

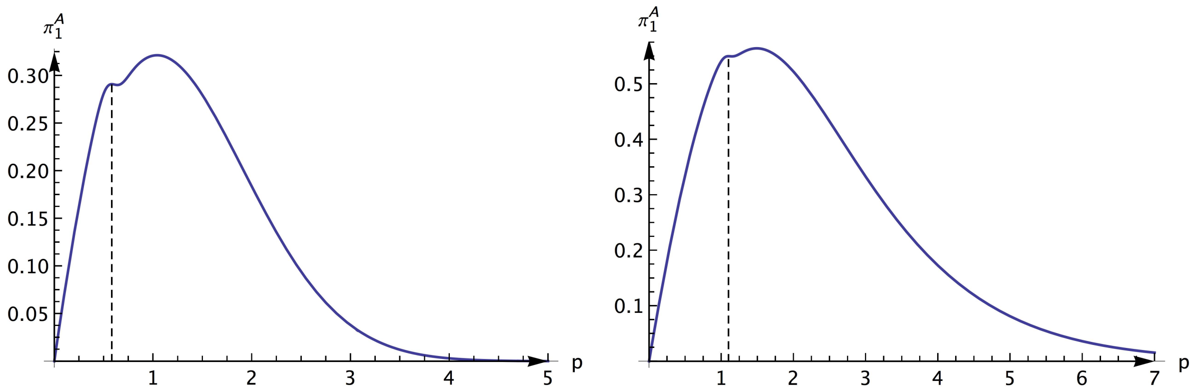


(a) ε and η: standard normal distrib.; $\hat{x} = 0.5$.

(b) ε and η: Gumbel distributions with location parameter 0 and scale parameter 1; $\hat{x} = 2$.

Figure 5: Quasi-concavity of payoff functions (observable prices)

To illustrate the non-existence of an SNE in pure strategies, Figure 6 plots cases in which there is little ex-ante heterogeneity across products.



(a) ε: standard normal distrib.; η normal distrib. with $\mu_\eta = 0$ and $\sigma_\eta = 0.05$; $\hat{x} = 0.5$.

(b) ε: Gumbel with location 0 and scale 1; η: Gumbel with location 0 and scale 0.05; $\hat{x} = 1.6$.

Figure 6: Non-quasi-concavity of payoff functions (observable prices)

As we can see, the tentative equilibrium price (13) indicated by the dashed vertical line is lower and then an individual firm has an incentive to deviate to a higher price, despite selling to fewer consumers. ■

The case of uniform distributions.

Here we illustrate that the payoff function (7) is quasi-concave when the match values are uniformly distributed. Because the supports of ε and η are closed intervals, the payoff function of a firm depends on the magnitude of its deviation price p. For instance, if $p > \bar{\varepsilon} + \hat{x} - \Delta_\eta + p^*$ and $\Delta_\eta \leq \hat{x} + p^*$, then the probability that a consumer starts searching from firm 2 and arrives at firm 1 equals zero. We have identified seven intervals in which p could be. In these intervals, the expressions of the payoff are different. The bounds of the intervals are depicted by the dashed lines in Figure 7. As the plot reveals, the payoff function is nicely quasi-concave.

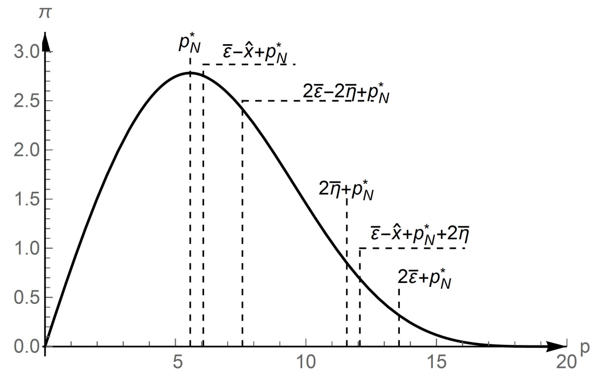


Figure 7: Payoff function with uniform distributions (non-observable prices). Parameters are $\bar{\varepsilon} = 4$, $\bar{\eta} = 3$, $\hat{x} = 3.5$, $\alpha = 0$.

If prices are observable and match values are uniformly distributed, the equilibrium payoff (12) has a similar shape. Again, because the match values are distributed in closed intervals, the magnitude of the deviation price affects the expression of the payoff. As a result, we have identified five intervals in which the deviation price p could be, and we have obtained a differently looking payoff function for every interval.[27] The bounds of the intervals are depicted by dashed lines in Figure 8. ■

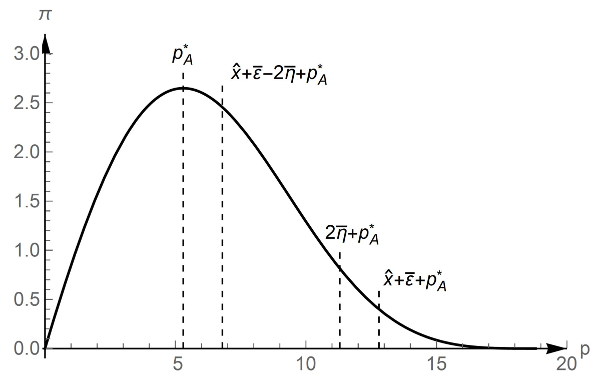


Figure 8: Payoff function with uniform distributions (observable prices). Parameters are $\bar{\varepsilon} = 4$, $\bar{\eta} = 3$, $\hat{x} = 3.5$, $\alpha = 0$.

[27]The exact expressions of the payoff function are available from authors upon request.