Digital Implementation of the 2-compartmental Pinsky-Rinzel Pyramidal Neuron Model

E. Rahimian, S. Zabihi, M. Amiri, and B. Linares-Barranco

Abstract — It is believed that brain-like computing system can be achieved by the fusion of electronics and neuroscience. In this way, the optimized digital hardware implementation of neurons, primary units of nervous system, play a vital role in neuromorphic applications. Moreover, one of the main features of pyramidal neurons in cortical areas is bursting activities which has a critical role in synaptic plasticity. The Pinsky-Rinzel model is a nonlinear 2-compartmental model for CA3 pyramidal cell which widely used in neuroscience. In this paper, a modified Pinsky-Rinzel pyramidal neuron model is proposed by replacing its complex differential equations with piecewise linear approximation. Next, a digital circuit is designed for the simplified model to be able to implement on a low-cost digital hardware, such as field-programmable gate array (FPGA). Both original and proposed models are simulated in MATLAB and next digital circuit simulated in VIVADO are also compared to show that obtained results are in good agreement. The presented circuit advances preceding designs with regards to the ability to replicate essential characteristics of different firing activities including bursting and spiking in the compartmental model. This new circuit has various applications in neuromorphic engineering such as developing new neuro-inspired chips.

Index Terms — Digital implementation, Pinsky-Rinzel Model, bursting pattern, pyramidal neuron, piecewise-linear approximation, brain.

I. INTRODUCTION

The brain consists of a great number of building blocks which is called neuronal cells and they are synaptically interconnected [1]-[3]. Recent studies in computational neuroscience and computational intelligence have shown a strong tendency towards a better understanding of the principles of brain information processing and the details of neuronal signal transmission [4]. Such research is motivated by the desire to obtain a more comprehensive picture of brain capabilities in information processing and at the same time to investigate how this understanding could be used to improve traditional information processing systems. This supports to build and extend the advanced neuromorphic processing systems. In recent years, inspiring from brain, researchers have proposed digital and analog electronic systems [5]-[8], such as a digital architecture having self-repairing mechanism to increase the system [9]-[14]. Moreover, several digital circuits focused on neuron-astrocyte interactions and synaptic plasticity are also developed by other researchers [3], [8], [15]-[17].

Understanding the behavior of primary building blocks of nervous system, neurons, plays a vital role in neuromorphic fields [4]. The recurrent connectivity within the CA3 network of the hippocampus, supports a strong computational capacity which can mediate behavioral processes including pattern completion [18], [19]. At the cellular level, CA3 pyramidal neurons show various firing patterns, ranging from single action potentials to complex bursts. There are several models describe neuron dynamics using nonlinear differential equations [6]. Traub proposed a 19-compartment model of a CA1 pyramidal neuron to simulate fundamental features of a CA1 neuron [20]. However, a more simplified model is required to reduce the computational demand in order to extend the model and create a network to investigate the effects of key parameters on network dynamics. By reducing a complex 19-compartment cable model [20], Pinsky and Rinzel suggested a new compartmental neuron model [21]. Next, using Pinsky-Rinzel model, Kepecs and Wang proposed a two-compartment model for producing bursting and spiking dynamics [22]. In the latter, soma and axon are modeled in one compartment and only necessary ionic currents for producing spikes has been considered. The second compartment which is dendrite, includes a persistent sodium and a slowly activating potassium currents. These two currents are able to generate bursting dynamics [22], [23].

Firing activity of neurons play a significant role in various applications such as processing information in the brain. Understanding neural computation needs knowledge about how neurons switch from one firing dynamic to another [24], [25]. On the other hand, direct hardware implementation holds out the promise of faster emulation due to the fact that it is inherently faster than software and also the operation is more parallel. Therefore, efficient digital implementation of neuron models such as Pinsky-Rinzel model has several applications in neuromorphic fields, motivating to implement networks having good stability and high dynamic range [26]-[32]. Analog hardware implementation of neuromorphic circuits has some

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bursting. The simplified Pinsky-Rinzel neuron model is explained by the following equations:

\[
\begin{align*}
C_m \frac{dV_s}{dt} &= -I_{Na} - I_K - I_{\text{leak}} + I_{\text{link}}\frac{P}{1-P} + I_{\text{soma}} \tag{1} \\
C_m \frac{dV_d}{dt} &= -I_{Na} - I_{KS} - I_{\text{leak}} - I_{\text{link}}\frac{P}{1-P} + I_{\text{dendrite}} \tag{2}
\end{align*}
\]

Where \( V_s \) and \( V_d \) are the somatic and dendrite membrane potentials (in mV) measured with respect to a reference potentials of -60 mV. The currents applied to the soma and dendrite are \( I_{\text{soma}} \) and \( I_{\text{dendrite}} \), respectively. In this paper \( I_{\text{dendrite}} \) is assumed to be zero [22]. \( P \) is the proportion of the cell area taken by soma and \( C_m \) is Membrane capacitance. The values of parameters are given in Table I. The ionic currents are given by:

\[
\begin{align*}
I_{Na} &= g_{Na} m^3 h (V_s - E_{Na}) \tag{3} \\
I_K &= g_K n^4 (V_s - E_K) \tag{4} \\
I_{\text{leak}}(V_s) &= g_L \cdot (V_s - E_L) \tag{5} \\
I_{\text{leak}}(V_d) &= g_L \cdot (V_d - E_L) \tag{6} \\
I_{\text{link}} &= g_e \cdot (V_d - V_s) \tag{7} \\
I_{Na} &= g_{Na} q(V_d - E_{Na}) \tag{8} \\
I_{KS} &= g_{KS} q(V_d - E_K) \tag{9}
\end{align*}
\]

where \( E_L, E_{Na}, \) and \( E_K \) are Equilibrium potentials, and \( g_{Na}, g_K, g_L, g_{Na}, g_{KS} \) are conductances, which their values are given in Table I. The kinetic equations for the gating variables \( h, n, \) and \( q \) have the form

\[
\frac{dy}{dt} = \phi_y \left( \frac{y_\text{m}(V_s) - y}{\tau_y(V_s)} \right) \tag{10}
\]

where \( y=h, n, q \), and \( \phi_y \) are dynamic gate temperature scaling factors, which values are presented in Table I. Gate steady state and time constant equations are given by:

\[
\begin{align*}
y_{\infty} &= \frac{\alpha_y}{\alpha_y + \beta_y}, \quad \tau_y = \frac{1}{\alpha_y + \beta_y} \tag{11} \\
q_{\infty}(V_d) &= \frac{1}{200} \exp\left(-\left(V_d + 55\right)/30\right) \tag{12} \\
r_{\infty}(V_d) &= \left(1+\exp\left(-\left(V_d + 57.7\right)/7.7\right)\right) \tag{13} \\
l_{\infty}(V_d) &= \frac{1}{200} \exp\left(-\left(V_d + 55\right)/30\right) \tag{14} \\
m_{\infty} &= \alpha_m/(\alpha_m + \beta_m) \tag{15}
\end{align*}
\]

Finally, rate constant equations are:
III. Digital Implementation

A. The proposed pricewise linear model

In this section, a piecewise-linear model is presented for the simplified P-R model. First, the continuous time equations, (1) and (2), will be discretized using Euler method which yields (22) and (23). In these equations, dt, the discretizing step size, is set to 0.01 (ms).

\[
V_s[n+1] = V_s[n] + \frac{dt}{C_m} \cdot (-I_{Na}[n] - I_K[n] - I_{Leak}[n]) + \frac{I_{Na}[n]}{P} + I_{soma}[n] \tag{22}
\]

\[
V_d[n+1] = V_d[n] + \frac{dt}{C_m} \cdot (-I_{Na}[n] - I_{Ks}[n] - I_{Leak}[n]) - \frac{I_{Na}[n]}{(1-P)} + I_{dendrite}[n]. \tag{23}
\]

Our digital implementation is based on fixed point operation and depending on the precision which is needed; we consider 32 bits for registers (1 bit for sign, 8 bits for integer part and 23 bits for fractional part). In order to discretize (10), it is simplified as follow:

\[
y = y_s - (y_s - y_i) \cdot \exp(-t \cdot \phi) \tau
\]

where \(y = h, n, \) and \(q\). By discretizing (24), we can get

\[
y[n+1] = y_s[n] - (y_s[n] - y_i[n]) \cdot \exp(-dt \cdot \phi) \tau[n]. \tag{25}
\]

To obtain \(V_s\), individual currents should be calculated. We start with \(I_{Na}\), which has the following discrete equation

\[
I_{Na}[n+1] = g_{Na} \cdot m^s[n] \cdot h[n] \cdot (V_s[n] - E_{Na}). \tag{26}
\]

For \(I_{Na}[n+1]\), we substitute \(y\) from (25) into \(h\), to get \(h[n]\), (27), which its steady and transient responses are separated in (28).

\[
\alpha_h(V_s) = 0.07 \exp(-V_s + 47)/20 \tag{16}
\]

\[
\beta_h(V_s) = 1/\left(\exp(-0.1(V_s + 17)) + 1\right) \tag{17}
\]

\[
\alpha_m(V_s) = -0.01(V_s + 34)/\left(\exp(-0.1(V_s + 34)) - 1\right) \tag{18}
\]

\[
\beta_m(V_s) = 0.125 \exp(-V_s + 44)/80 \tag{19}
\]

\[
\alpha_i(V_s) = -0.1(V_s + 31)/\left(\exp(-0.1(V_s + 31)) - 1\right) \tag{20}
\]

\[
\beta_i(V_s) = 4 \exp(-(V_s + 56)/18). \tag{21}
\]

<table>
<thead>
<tr>
<th>Region of (V_s) (mV)</th>
<th>(A_i)</th>
<th>(B_i)</th>
</tr>
</thead>
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<tr>
<td>[-70, -54.3]</td>
<td>-1.9666</td>
<td>-0.0814</td>
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<td>(-22.9, -7.1]</td>
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<td>0.0011</td>
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<tr>
<td>(-7.1, 8.6]</td>
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<td>0.0018</td>
</tr>
<tr>
<td>(8.6, 40]</td>
<td>-0.0273</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

TABLE III

<table>
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<th>Region of (V_s) (mV)</th>
<th>(A_i)</th>
<th>(B_i)</th>
</tr>
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<tbody>
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<td>1.0087</td>
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<td>-0.3159</td>
<td>0.9827</td>
</tr>
<tr>
<td>(-28.8, -1.3]</td>
<td>-0.7171</td>
<td>0.9712</td>
</tr>
<tr>
<td>(-1.3, 12.5]</td>
<td>-0.2566</td>
<td>0.9717</td>
</tr>
<tr>
<td>(12.5, 40]</td>
<td>-0.0444</td>
<td>0.9691</td>
</tr>
</tbody>
</table>

\[
h[n+1] = h[n] \left(1 - \exp\left(-\frac{dt \cdot \phi_i}{\tau[n]}\right)\right)
\]

\[
+ h[n] \cdot \exp\left(-\frac{dt \cdot \phi_i \cdot (\alpha_i[n] + \beta_i[n])}{\tau[n]}\right) \tag{27}
\]

where \(h[n]\) and \(h_2[n]\) are described as follows:

\[
h_i[n] = \left(\frac{\alpha_i[n]}{\alpha_i[n] + \beta_i[n]}\right) \cdot \left(1 - \exp\left(-dt \cdot \phi_i \cdot (\alpha_i[n] + \beta_i[n])\right)\right) \tag{29}
\]

\[
h_i[n] = h[n] \cdot \exp\left(-dt \cdot \phi_i \cdot (\alpha_i[n] + \beta_i[n])\right) \tag{30}
\]

One of the bottlenecks in digital hardware implementation is the presence of nonlinear equations, such as exponential functions in (29) and (30). Using look up tables, which is one of the traditional methods, to approximate these nonlinear functions need many comparators and several registers for saving data. In this paper, by using a piecewise linear approximation, nonlinear parts are eliminated and replaced with first order functions which preserve the dynamical characteristics of the original nonlinear systems. Hence, it is possible to implement the simplified P-R neuron model in a digital platform. An exhaustive search algorithm is utilized to find the best parameter values of the line segments which lead to the minimum error. As shown in Fig. 2(b), \(h_i\) is approximated with 6 lines, too.

Range of \(V_s\) is small in comparison to range of \(h_i\). Therefore, we multiply \(h_i\) by 8 in order to obtain good accuracy, and then approximate it by 6-line segments, as shown in Fig. 2(a). Finally, to obtain the exact values of \(h_i\), it is shifted to right for 3 digits.

In our digital implementation, a memory register stores the outputs such as current, somatic and dendrite voltages. The value of \(A_i\) and \(B_i\), with respect to the input is listed in Table II and Table III. Given linear approximation, the functions are implemented by using simple blocks such as adder, shifter and so on; therefore, the cost of hardware implementation reduces remarkably. Corresponding line parameters of \(h_i[n]\) and \(h_2[n]\) are listed in Table II and Table III, respectively.
Afer computing $h[n]$, we should approximate the $I_{Na}$ function. For this purpose, substituting (26) into the (31) and (32), yields

$$I_{Na}[n+1] = g_{Na} \cdot m_{h}^{3}[n] \cdot (h[n] + h[n] \cdot h_{2}[n]) + (V[n] - E_{Na}) \cdot (V[n] - E_{Na}) \cdot (V[n] - E_{Na})$$  \hspace{1cm} (31)

$$I_{Na}[n+1] = I_{Na}[n] + h[n] \cdot I_{Na2}[n]$$  \hspace{1cm} (32)

Since $I_{Na2}[n]$ is negligible compared to $(h[n] \cdot I_{Na2}[n])$ it can be ignored in (32) as shown in Fig. 3(a). Hence we can obtain:

$$I_{Na}[n+1] = h[n] \cdot I_{Na2}[n]$$  \hspace{1cm} (33)

Because we consider the integer part of numbers, it is necessary to scale $I_{Na2}[n]$ (dividing by 8), and then we approximate $I_{Na2}[n]$ by 9-line segments due to its highly nonlinearity, as shown in Fig. 3(b). At the end, we shift $I_{Na2}$ left by 3 digits to obtain the exact values. The parameter values of the resulting lines are listed in Table IV.

Next, $I_{K}$ must be computed. In this way, $n$ is substituted in (25); therefore, (34) is obtained which steady and transient states are as follows:

$$n[n+1] = n_{s}[n] \left[ 1 - \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right) \right] + n[n] \cdot \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right)\hspace{1cm} (34)$$

$$n[n+1] = n_{t}[n] + n[n] \cdot n_{s}[n]$$  \hspace{1cm} (35)

In (34) and (35), $n_{s}[n]$ and $n_{t}[n]$ are

$$n_{s}[n] = \left( \alpha_{s}[n] / (\alpha_{s}[n] + \beta_{s}[n]) \right) \cdot \left[ 1 - \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right) \right]$$  \hspace{1cm} (36)

$$n_{t}[n] = n[n] \cdot \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right) \left( \alpha_{s}[n] + \beta_{s}[n] \right)$$  \hspace{1cm} (37)

$n_{s}[n]$ and $n_{t}[n]$ are approximated with 4-line and 3-line segments, respectively, as shown in Fig. 4. The parameter values of the resulted lines are given in Table V and Table VI, respectively. After calculating $n[n]$, $f_{K}[n]$ will be discretized as

$$I_{K}[n+1] = g_{K} \cdot \left( \alpha_{K}[n] \right) \cdot (V[n] - E_{K}) \cdot (V[n] - E_{K}) \cdot (V[n] - E_{K})$$  \hspace{1cm} (38)

Similarly, $I_{Na+K}$ and $I_{Na}$ will be discretized

$$I_{Na+K}[n] = g_{K} \cdot (V[n] - E_{K})$$  \hspace{1cm} (39)

To calculate $V[d][n+1]$, the dynamical equation of the $I_{Na+K}$ (8), should be discretized, consequently, it is approximated by 8-line segments, as presented in Fig. 5. The parameter values of the $I_{Na+K}$ are given in Table VII. The $I_{Na+K}$ current can be written as follows:

$$I_{Na+K}[n+1] = A_{K} \cdot V_{d}[n] + B_{K}$$  \hspace{1cm} (41)

Calculating $k_{K}[n+1]$ is similar to $I_{Na+K}[n+1]$. Using (25), we can obtain $q[n]$ which steady and transient states are

$$q[n+1] = q[n] \left[ 1 - \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right) \right] + q[n] \cdot \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right)\hspace{1cm} (42)$$

$$q[n+1] = q[n] + q[n] \cdot q[n]$$  \hspace{1cm} (43)

$$q[n] = \left( \alpha_{q}[n] / (\alpha_{q}[n] + \beta_{q}[n]) \right) \cdot \left[ 1 - \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right) \right]$$  \hspace{1cm} (44)

$$q[n] = q[n] \cdot \exp \left( - \frac{dt \cdot \phi_{s}}{\tau_{s}[n]} \right) \left( \alpha_{q}[n] + \beta_{q}[n] \right)$$  \hspace{1cm} (45)

Range of $q_{1}$ is small in comparison to range of $V_{d}$. Therefore, we multiply $q_{1}$ by 16 in order to reach higher accuracy, and then approximate it by 6-line segments, as shown in Fig. 6(a). Finally, in order to obtain the exact values of $q_{1}$, it is shifted to right for 4 digits. Also, $q_{2}$ is approximated by 6-line segments,
as shown in Fig. 6(b). Next, we can approximate the $I_{KS}$. For this purpose, we can substitute (9) into (46) and (47).

$$I_{KS}[n+1] = g_{KS} \cdot (q[n] + q[n] \cdot q_2[n]) \cdot (V_d[n] - E_k)$$ \hfill (46)

$$I_{KS}[n+1] = I_{KS1}[n] + q[n] \cdot I_{KS2}[n]$$ \hfill (47)

$I_{KS1}[n]$ can be ignored since its value is negligible compared to $(q[n]*I_{KS2}[n])$, as shown in Fig. 7(a). Therefore we can obtain:

$$I_{KS}[n+1] = q[n] \cdot I_{KS2}[n]$$ \hfill (48)

$I_{KS2}[n]$ is approximated by 1-line segment too, as shown in Fig. 7(b). Table X shows the parameter values of this line.

**B. The proposed digital circuit**

To overcome the problems of analog fabrication without sacrificing area and power budgets, in this paper, a digital implementation is used. In this way, the proposed piecewise linear model enables us to design architecture for implementation on the FPGA. The required steps to get the response of individual function are showed in Fig. 8, 9, 10, and 11, respectively. As mentioned earlier, we consider 32 bits for registers (1 bit for sign, 8 bits for integer part and 23 bits for fractional part). Therefore, inputs and the corresponding outputs of our multipliers, adders and subtractions are 32 bits.

We used FPGA for hardware implementation. By providing an array of logic components that can be easily configured in a desired mode, FPGAs are reconfigurable devices for implementation of digital systems, [8]. Recent studies investigated that FPGA provide an appropriate platform for designing neuromorphic systems [29]. The digital circuit was simulated and synthesized using VIVADO, resulting a maximum clock frequency of 318.173 MHz. The resource utilization of the FPGA implementations is summarized in Table XI.

**C. Error calculation**

Error between the proposed models and the original P-R model is defined as Root Mean Square Error (RMSE), and calculated by (49). Furthermore, we calculate normalized RMSE (NRMSE) as (50). In (50), $Y_{real}$ defines the original
values of the function, and \( Y_{\text{approx}} \) is the corresponding approximated value. Considering Table XII, which lists the RMSE and NRMSE for piecewise linear approximations of different functions, the dynamic performance of the system is preserved.

\[
\text{RMSE}(Y_{\text{real}}, Y_{\text{approx}}) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (Y_{\text{real}} - Y_{\text{approx}})^2} \quad (49)
\]

\[
\text{NRMSE}(Y_{\text{real}}, Y_{\text{approx}}) = \frac{\text{RMSE}}{Y_{\text{max}} - Y_{\text{min}}} \quad (50)
\]

IV. SIMULATION RESULTS

In this section, software and hardware implementation results are discussed. Indeed, the primary objective is to examine the feasibility of FPGA implementation of the model and to show that hardware can reproduce the model responses. By adjusting \( P, I_{\text{soma}} \) and \( g_c \), different patterns of simplified Pinsky-Rinzel model can be obtained. In the first set of simulations, bursting activities is examined. In simulation shown in Fig. 12, \( g_c=1 \), \( p=0.15 \) and for (a),(b),(c) \( I_{\text{soma}}=3 \) and for (d),(e),(f) \( I_{\text{soma}}=19.7 \).
Fig. 12. The time response of the neuron membrane potential of soma ($V_s$) and dendrite ($V_d$) components in mV. (a), (d) MATLAB simulations of the original simplified P-R model. (b), (e) MATLAB simulations of the proposed piece-wise linear model. (c), (f) VIVADO simulation of the designed digital circuit. In these simulations $(g_e, p) = (1, 0.15)$ and in (a), (b), (c) $I_{soma} = 3$ and in (d), (e), (f) $I_{soma} = 19.7$. The first and the third panels show the bursting activities of somatic voltage. The second and the forth panels show the bursting activities of dendrite voltage.

Fig. 13. The time response of the neuron membrane potential of soma ($V_s$) and dendrite ($V_d$) components in mV. (a),(d) MATLAB simulations of the original simplified P-R model. (b),(e) MATLAB simulations of the proposed piecewise linear model. (c), (f) VIVADO simulation of the designed digital circuit. In these simulations $p = 0.15$ and in (a) $(g_e, I_{soma}) = (1, 23)$, in (b), (c) $(g_e, I_{soma}) = (1,34)$ and in (d), (e), (f) $(g_e, I_{soma}) = (5,3)$. The first and the third panels show the bursting activities of somatic voltage. The second and the forth panels show the bursting activities of dendrite voltage.
Fig. 14. The time response of the neuron membrane potential of soma ($V_s$) and dendrite ($V_d$) components in mV. (a), (d) MATLAB simulations of the original simplified P-R model. (b), (e) MATLAB simulations of the proposed piecewise linear model. (c), (f) VIVADO simulation of the designed digital circuit. In these simulations ($g_c$, $p$) = (0.1, 0.15) and in (a), (b), (c) $I_{soma}$=7 and in (d), (e), (f) $I_{soma}$=20. The first and the third panels show spiking and bursting activities of somatic voltage. The second and the forth panels show spiking and bursting activities of dendrite voltage.

Fig. 15. Phase plane of original and proposed model. (a), (b), (c) show the phase plane of original simplified P-R model. (d), (e), (f) show the phase plane of the proposed digital circuit. The dynamics of network is preserved in the proposed circuit.
In Fig. 12, the first and the third panels show the bursting activities of somatic compartment (blue color) and the second and the fourth panels display the bursting activities of dendrite compartment (red color). Fig. 12(a), (d) illustrates the MATLAB simulations of the membrane potential of original simplified P-R model, and (b), (e) are the MATLAB simulations of the proposed piece-wise linear model, and (c), (f) are the VIVADO simulation of the designed digital circuit. Comparing different panels, it is evident that both piecewise linear model and digital circuit produce similar responses to original biophysical P-R model. Additionally, by increasing the injection current to soma (i.e., $I_{soma}=19.7$), the firing frequency is also increased and tonic bursting activities can be appeared in digital circuit.

Next, we examine the spiking activities in the digital circuit of the 2-compartment model. Fig. 13 shows the spiking activities of the simplified P-R neuron model. For simulation illustrated in Fig. 13, $p = 0.15$ and for (a) $g_c = 1$, $I_{soma} = 23$, for (b), (c) $g_c = 1$, $I_{soma} = 34$, for (d), (e), (f) $g_c = 5$, $I_{soma} = 3$. In Fig. 13, the first and the third panels show the bursting/spiking behavior of somatic compartment (blue color) and the second and the forth panels display the bursting/spiking activities of dendrite compartment (red color). As can be seen, the results of MATLAB simulation of the membrane potential of original simplified P-R model which is shown in Fig. 13 (a), (d) not only have good agreement with the MATLAB simulations of the proposed piecewise linear model illustrated in (b), (e) but also with the VIVADO simulation of the digital circuit shown in (c), (f).

In the third set of simulations, we investigate the effect of variation of $g_c$, which couple the somatic and dendritic components. When somatic and dendritic compartments are strongly coupled (e.g. $g_c = 5$), the spikes will be generate as shown in Fig. 13 (d), (e), (f). On the other hand, when $g_c$ is weak (e.g. $g_c=0.1$), the burst entirely change and the dendritic component cannot follow somatic part, showing a small hump during a burst, as shown in the second panel of Fig. 14 (a), (b), (c). In Fig. 14 (d), (e), (f), a larger current is injected to the soma, thus the bursting patterns disappears and spiking activities are modulated in frequency.

The phase plane, $(V_s-h), (V_s-n), (V_d-q)$, of the simplified P-R neuron model (a) , (b), (c) and for the proposed digital circuit (d), (e), (f) are shown in Fig. 15. As can be observed, although there are also some quantitative differences, the general shape and the qualitative behavior of trajectories are similar. Regarding the obtained results, one can conclude that the dynamical characteristics of the original system are preserved by the designed circuit.

Computing RMSE and NRMSE, the reliability of the proposed model is supported. Table XIII shows the RMSE and NRMSE between biophysical P-R and proposed piecewise linear models in MATLAB, and Table XIV presents the RMSE and NRMSE between simulations of digital circuit in VIVADO with the original model in MATLAB.

Considering Fig. 12, 13, 14, and 15, the digital circuit maintains the essential properties of its biological counterpart in different conditions. Considering the important criteria from the hardware point of view such as scaling up the designed circuit, reducing the digital implementation cost and keeping low power operation while obtaining results similar to the biophysical model of P-R model, the proposed circuit produces acceptable results.
V. Conclusion

Design and effective implementation of spiking neural network in hardware is now an active domain of research and several procedures have been proposed for designing and construction of brain like processing systems for conventional computing. The focus of this paper was to propose a piecewise linear model and a digital circuit for the simplified Pinsky-Rinzel neuron model to verify that both proposed linearized and digital circuit can produce similar responses as the biophysical model does. This work can be considered as a basic step towards creating adaptable and bio-plausible hardware-based neural network using simplified Pinsky-Rinzel neuron model. The results of the FPGA implementation are in agreement with those Matlab and Vivado simulations. Future works aim at extending this model and creating a neural population model and constructs a spiking neural network in digital domain. These issues will be addressed in future study.

REFERENCES


