Digital Implementation of the 2-compartmental Pinsky-Rinzel Pyramidal Neuron Model

E. Rahimian, S. Zabihi, M. Amiri, and B. Linares-Barranco

Abstract— It is believed that brain-like computing system can be achieved by the fusion of electronics and neuroscience. In this way, the optimized digital hardware implementation of neurons, primary units of nervous system, play a vital role in neuromorphic applications. Moreover, one of the main features of pyramidal neurons in cortical areas is bursting activities which has a critical role in synaptic plasticity. The Pinsky-Rinzel model is a nonlinear 2-compartmental model for CA3 pyramidal cell which widely used in neuroscience. In this paper, a modified Pinsky-Rinzel pyramidal neuron model is proposed by replacing its complex differential equations with piecewise linear approximation. Next, a digital circuit is designed for the simplified model to be able to implement on a low-cost digital hardware, such as field-programmable gate array (FPGA). Both original and proposed models are simulated in MATLAB and next digital circuit simulated in VIVADO are also compared to show that obtained results are in good agreement. The presented circuit advances preceding designs with regards to the ability to replicate essential characteristics of different firing activities including bursting and spiking in the compartmental model. This new circuit has various applications in neuromorphic engineering such as developing new neuro-inspired chips.

Index Terms— Digital implementation, Pinsky-Rinzel Model, bursting pattern, pyramidal neuron, piecewise-linear approximation, brain.

I. INTRODUCTION

THE brain consists of a great number of building blocks which is called neuronal cells and they are synaptically interconnected [1]-[3]. Recent studies in computational neuroscience and computational intelligence have shown a strong tendency towards a better understanding of the principles of brain information processing and the details of neuronal signal transmission [4]. Such research is motivated by the desire to obtain a more comprehensive picture of brain capabilities in information processing and at the same time to investigate how this understanding could be used to improve traditional information processing systems. This supports to build and extend the advanced neuromorphic processing systems. In recent years, inspiring from brain, researchers have proposed digital and analog electronic systems [5]-[8], such as a digital

architecture having self-repairing mechanism to increase the system [9]-[14]. Moreover, several digital circuits focused on neuron-astrocyte interactions and synaptic plasticity are also developed by other researchers [3], [8], [15]-[17].

Understanding the behavior of primary building blocks of nervous system, neurons, plays a vital role in neuromorphic fields [4]. The recurrent connectivity within the CA3 network of the hippocampus, supports a strong computational capacity which can mediate behavioral processes including pattern completion [18], [19]. At the cellular level, CA3 pyramidal neurons show various firing patterns, ranging from single action potentials to complex bursts. There are several models describe neuron dynamics using nonlinear differential equations [6]. Traub proposed a 19-compartment model of a CA1 pyramidal neuron to simulate fundamental features of a CA1 neuron [20]. However, a more simplified model is required to reduce the computational demand in order to extend the model and create a network to investigate the effects of key parameters on network dynamics. By reducing a complex 19-compartment cable model [20], Pinsky and Rinzel suggested a new compartmental neuron model [21]. Next, using Pinsky-Rinzel model, Kepecs and Wang proposed a two-compartment model for producing bursting and spiking dynamics [22]. In the latter, soma and axon are modeled in one compartment and only necessary ionic currents for producing spikes has been considered. The second compartment which is dendrite, includes a persistent sodium and a slowly activating potassium currents. These two currents are able to generate bursting dynamics [22], [23].

Firing activity of neurons play a significant role in various applications such as processing information in the brain. Understanding neural computation needs knowledge about how neurons switch from one firing dynamic to another [24], [25]. On the other hand, direct hardware implementation holds out the promise of faster emulation due to the fact that it is inherently faster than software and also the operation is more parallel. Therefore, efficient digital implementation of neuron models such as Pinsky-Rinzel model has several applications in neuromorphic fields, motivating to implement networks having good stability and high dynamic range [26]-[32]. Analog hardware implementation of neuromorphic circuits has some

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Fig. 1. Two-compartmental model of pyramidal neurons of CA1. The dendrite compartment contains currents that are responsible for bursting behavior and somatic compartment includes currents which cause spiking activity.

limitation such as noise, drift, mismatch and the problem of accuracy of the model [33]-[35]. Therefore, digital implementation with field-programmable gate array, (FPGA) is more reliable and robust [36]-[38]. Recent advances in FPGA technology provide flexibility required for algorithm exploration while meeting performance, power and size constraints. The development of these new technologies to mimic the neuron-biological process has many applications including emulating large scale spiking neural networks.

In this paper, we propose a digital circuit for the simplified Pinsky-Rinzel model (Kepecs and Wang model). First, in order to achieve an optimized implementation in FPGA, the nonlinear differential equations are simplified by a piecewise-linear approximation. Then, the proposed linear model is simulated in MATLAB and the acquired results are compared with the original biophysical model. Next, a digital circuit is designed for the proposed model and is simulated in VIVADO simulation environment. Finally, using several simulations in different conditions, it is demonstrated that the digital circuit follow the dynamical performance of the biophysical Pinsky-Rinzel neuron model simulated in MATLAB and the results are in good agreements.

The rest of the paper is organized as follows: in Section 2, the mathematical model of the simplified Pinsky-Rinzel neuron model is explained. The proposed piecewise-linear model and the digital circuit are described in Section 3. In Section 4, the simulation results are discussed. Finally, Section 5 concludes the paper.

II. THE SIMPLIFIED PINSKY-RINZEL MODEL

Kepecs and Wang simplified the Pinsky-Rinzel (P-R) neuron model, a pyramidal cell with two compartments in CA region [21], [22]. Soma and axon in one compartment include ionic channels for spike generation. Dendrite in the other compartment produces bursting dynamics. Fig. 1 shows that the soma-axon compartment is coupled to the dendrite compartment. I_{Na} and I_K are the considered currents for somatic compartment and the dendritic compartment includes slow potassium I_{KS} and a persistent sodium I_{NaP} currents [22].

This model includes only the minimal biophysical mechanisms necessary to study the phenomenon of complex

TABLE I
PARAMETER VALUES FOR SIMPLIFIED PINSKY-RINZEL MODEL

Quantity	Symbol	Value	Quantity	Symbol	Value
Equilibrium Potentials			Membrane Capacitance		
	E_L	-65 mV		C_m	1 $\mu F/cm^2$
	E_{Na}	+55 mV	Compartment Coupling		
	E_K	-90 mV		p	0.15
Dynamic Gate Temperature Scaling Factors			Conductances		
	ϕ_q	1		g_L	0.18 mS/cm ²
	ϕ_h	3.33		g_{Na}	55 mS/cm ²
	ϕ_n	3.33		g_K	20 mS/cm ²
				g_c	1 mS/cm ²
				g_{NaP}	0.12 mS/cm ²
				g_{KS}	0.7 mS/cm ²

bursting. The simplified Pinsky-Rinzel neuron model is explained by the following equations:

$$C_m \frac{dV_s}{dt} = -I_{Na} - I_K - I_{Leak} + \frac{I_{link}}{P} + I_{soma} \quad (1)$$

$$C_m \frac{dV_d}{dt} = -I_{NaP} - I_{KS} - I_{Leak} - \frac{I_{link}}{(1-P)} + I_{dendrite} \quad (2)$$

Where V_s and V_d are the somatic and dendrite membrane potentials (in mV) measured with respect to a reference potentials of -60 mV. The currents applied to the soma and dendrite are I_{soma} and $I_{dendrite}$, respectively. In this paper $I_{dendrite}$ is assumed to be zero [22]. P is the proportion of the cell area taken by soma and C_m is Membrane capacitance. The values of parameters are given in Table I. The ionic currents are given by:

$$I_{Na} = g_{Na} m_{\infty}^3 h \cdot (V_s - E_{Na}) \quad (3)$$

$$I_K = g_K n^4 \cdot (V_s - E_K) \quad (4)$$

$$I_{leak}(V_s) = g_L \cdot (V_s - E_L) \quad (5)$$

$$I_{leak}(V_d) = g_L \cdot (V_d - E_L) \quad (6)$$

$$I_{link} = g_c \cdot (V_d - V_s) \quad (7)$$

$$I_{NaP} = g_{NaP} l_{\infty}^3 \cdot (V_d - E_{Na}) \quad (8)$$

$$I_{KS} = g_{KS} q \cdot (V_d - E_K) \quad (9)$$

where E_L , E_{Na} , and E_K are Equilibrium potentials, and g_{Na} , g_K , g_L , g_c , g_{NaP} , g_{KS} are conductances, which their values are given in Table I. The kinetic equations for the gating variables h , n , and q have the form

$$\frac{dy}{dt} = \phi_y \left(\frac{y_{\infty}(V_s) - y}{\tau_y(V_s)} \right) \quad (10)$$

where $y=h, n, q$, and ϕ_y are dynamic gate temperature scaling factors, which values are presented in Table I. Gate steady state and time constant equations are given by:

$$y_{\infty} = \frac{\alpha_y}{\alpha_y + \beta_y}; \tau_y = \frac{1}{\alpha_y + \beta_y}, \text{ where } (y=h, n) \quad (11)$$

$$q_{\infty}(V_d) = 1 / (1 + \exp(-(V_d + 36)/6.5)) \quad (12)$$

$$\tau_q(V_d) = \frac{200}{\exp(-(V_d + 55)/30) + \exp((V_d + 55)/30)} \quad (13)$$

$$l_{\infty}(V_d) = 1 / (1 + \exp(-(V_d + 57.7)/7.7)) \quad (14)$$

$$m_{\infty} = \alpha_m / (\alpha_m + \beta_m). \quad (15)$$

Finally, rate constant equations are:

Fig. 2. (a) 6-line segments approximation for $h_1(V_s)$. (b) 6-line segments approximation for $h_2(V_s)$

$$\alpha_h(V_s) = 0.07 \exp(-(V_s + 47)/20) \quad (16)$$

$$\beta_h(V_s) = 1/(\exp(-0.1(V_s + 17)) + 1) \quad (17)$$

$$\alpha_n(V_s) = -0.01(V_s + 34)/(\exp(-0.1(V_s + 34)) - 1) \quad (18)$$

$$\beta_n(V_s) = 0.125 \exp(-(V_s + 44)/80) \quad (19)$$

$$\alpha_m(V_s) = -0.1(V_s + 31)/(\exp(-0.1(V_s + 31)) - 1) \quad (20)$$

$$\beta_m(V_s) = 4 \exp(-(V_s + 56)/18). \quad (21)$$

III. Digital Implementation

A. The proposed piecewise linear model

In this section, a piecewise-linear model is presented for the simplified P-R model. First, the continuous time equations, (1) and (2), will be discretized using Euler method which yields (22) and (23). In these equations, dt , the discretizing step size, is set to 0.01(ms).

$$V_s[n+1] = V_s[n] + \frac{dt}{C_m} \cdot (-I_{Na}[n] - I_K[n] - I_{Leak}[n] + \frac{I_{link}[n]}{P} + I_{soma}[n]) \quad (22)$$

$$V_d[n+1] = V_d[n] + \frac{dt}{C_m} \cdot (-I_{NaP}[n] - I_{KS}[n] - I_{Leak}[n] - \frac{I_{link}[n]}{(1-P)} + I_{dendrite}[n]). \quad (23)$$

Our digital implementation is based on fixed point operation and depending on the precision which is needed; we consider 32 bits for registers (1 bit for sign, 8 bits for integer part and 23 bits for fractional part). In order to discretize (10), it is simplified as follow:

$$y = y_\infty - (y_\infty - y_0) \cdot \exp(-(t \cdot \phi)/\tau) \quad (24)$$

where $y = h, n$, and q . By discretizing (24), we can get

$$y[n+1] = y_\infty[n] - (y_\infty[n] - y[n]) \cdot \exp(-(dt \cdot \phi)/\tau[n]). \quad (25)$$

To obtain V_s , individual currents should be calculated. We start with I_{Na} , which has the following discrete equation

$$I_{Na}[n+1] = g_{Na} \cdot m_\infty^3[n] \cdot h[n] \cdot (V_s[n] - E_{Na}). \quad (26)$$

For $I_{Na}[n+1]$, we substitute y from (25) into h , to get $h[n]$, (27), which its steady and transient responses are separated in (28).

TABLE II
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 6-LINE SEGMENTS
APPROXIMATION OF $8 * h_1(V_s) = A_1 * V_s + B_1$

Region of V_s (mV)	A_1	B_1
[-70, -54.3]	-1.9666	-0.0814
(-54.3, -38.6]	-0.8715	-0.0220
(-38.6, -22.9]	-0.4056	-0.0040
(-22.9, -7.1]	-0.1824	0.0011
(-7.1, 8.6]	-0.0829	0.0018
(8.6, 40]	-0.0273	0.0013

TABLE III
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 6-LINE SEGMENTS
APPROXIMATION OF $h_2(V_s) = A_2 * V_s + B_2$

Region of V_s (mV)	A_2	B_2
[-70, -56.3]	0.2272	1.0087
(-56.3, -42.5]	0.0160	0.9968
(-42.5, -28.8]	-0.3159	0.9827
(-28.8, -1.3]	-0.7171	0.9712
(-1.3, 12.5]	-0.2566	0.9717
(12.5, 40]	-0.0444	0.9691

$$h[n+1] = h_\infty[n](1 - \exp(-(dt \cdot \phi_h)/\tau_h[n])) \quad (27)$$

$$+ h[n] \cdot \exp(-(dt \cdot \phi_h)/\tau_h[n])$$

$$h[n+1] = h_i[n] + h[n] \cdot h_2[n] \quad (28)$$

where $h_1[n]$ and $h_2[n]$ are described as follows:

$$h_1[n] = \left(\frac{\alpha_h[n]}{\alpha_h[n] + \beta_h[n]} \right) \quad (29)$$

$$\cdot (1 - \exp(-dt \cdot \phi_h \cdot (\alpha_h[n] + \beta_h[n])))$$

$$h_2[n] = h[n] \cdot \exp(-dt \cdot \phi_h \cdot (\alpha_h[n] + \beta_h[n])). \quad (30)$$

One of the bottlenecks in digital hardware implementation is the presence of nonlinear equations, such as exponential functions in (29) and (30). Using look up tables, which is one of the traditional methods, to approximate these nonlinear functions need many comparators and several registers for saving data. In this paper, by using a piecewise linear approximation, nonlinear parts are eliminated and replaced with first order functions which preserve the dynamical characteristics of the original nonlinear systems. Hence, it is possible to implement the simplified P-R neuron model in a digital platform. An exhaustive search algorithm is utilized to find the best parameter values of the line segments which lead to the minimum error. As shown in Fig. 2(b), h_2 is approximated with 6 lines, too.

Range of V_s is small in comparison to range of h_1 . Therefore, we multiply h_1 by 8 in order to obtain good accuracy, and then approximate it by 6-line segments, as shown in Fig. 2(a). Finally, to obtain the exact values of h_1 , it is shifted to right for 3 digits.

In our digital implementation, a memory register stores the outputs such as current, somatic and dendrite voltages. The value of A_i and B_i with respect to the input is listed in Table II and Table III. Given linear approximation, the functions are implemented by using simple blocks such as adder, shifter and so on; therefore, the cost of hardware implementation reduces remarkably. Corresponding line parameters of $h_1[n]$ and $h_2[n]$ are listed in Table II and Table III, respectively.

Fig. 3. (a) Comparison between term1 and term2 of I_{Na} function, (b) 9-line segments approximation for $I_{Na2}(V_s)$

Fig. 4. (a) 4-line segments approximation for $n_1(V_s)$. (b) 3-line segments approximation for $n_2(V_s)$.

After computing $h[n]$, we should approximate the I_{Na} function. For this purpose, substituting (26) into the (31) and (32), yields

$$I_{Na}[n+1] = g_{Na} \cdot m_{\infty}^3[n] \cdot (h_1[n] + h[n] \cdot h_2[n]) \cdot (V_s[n] - E_{Na}) \quad (31)$$

$$I_{Na}[n+1] = I_{Na1}[n] + h[n] \cdot I_{Na2}[n]. \quad (32)$$

Since $I_{Na1}[n]$ is negligible compared to $(h[n] \cdot I_{Na2}[n])$ it can be ignored in (32) as shown in Fig. 3(a). Hence we can obtain:

$$I_{Na}[n+1] \approx h[n] \cdot I_{Na2}[n]. \quad (33)$$

Because we consider 8 bits for the integer part of numbers, it is necessary to scale $I_{Na2}[n]$ (dividing by 8), and then we approximate $I_{Na2}[n]$ by 9-line segments due to its highly nonlinearity, as shown in Fig. 3(b). At the end, we shift I_{Na2} left by 3 digits to obtain the exact values. The parameter values of the resulting lines are listed in Table IV.

Next, I_K must be computed. In this way, n is substituted in (25); therefore, (34) is obtained which steady and transient states are as follow:

$$n[n+1] = n_{\infty}[n] \left(1 - \exp\left(-(\Delta t \cdot \phi_n)/\tau_n[n]\right)\right) + n[n] \cdot \exp\left(-(\Delta t \cdot \phi_n)/\tau_n[n]\right) \quad (34)$$

$$n[n+1] = n_1[n] + n[n] \cdot n_2[n]. \quad (35)$$

$$\begin{aligned} \text{In (34) and (35), } n_1[n] \text{ and } n_2[n] \text{ are} \\ n_1[n] = (\alpha_n[n] / (\alpha_n[n] + \beta_n[n])) \\ \cdot (1 - \exp(-\Delta t \cdot \phi_n \cdot (\alpha_n[n] + \beta_n[n]))) \end{aligned} \quad (36)$$

$$n_2[n] = n[n] \cdot \exp(-\Delta t \cdot \phi_n \cdot (\alpha_n[n] + \beta_n[n])). \quad (37)$$

$n_1[n]$ and $n_2[n]$ are approximated with 4-line and 3-line segments, respectively, as shown in Fig. 4. The parameter

TABLE IV
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 9-LINE SEGMENTS
APPROXIMATION OF $I_{Na2}(V_s)/8 = A_3 \cdot V_s + B_3$

Region of V_s (mV)	A_3	B_3
[-70, -48.0]	-0.4610	-0.0323
(-48.0, -40.7]	-10.4597	-0.5122
(-40.7, -33.3]	-51.7974	-2.1933
(-33.3, -18.7]	-129.1278	-4.7710
(-18.7, -11.3]	-87.4004	-3.9920
(-11.3, -4]	-25.5956	-3.2916
(-4, 3.3]	16.9626	-3.1214
(3.3, 10.7]	41.0090	-3.2015
(10.7, 40]	60.4787	-3.4092

TABLE V
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 4-LINE SEGMENTS
APPROXIMATION OF $n_1(V_s) = A_4 \cdot V_s + B_4$

Region of V_s (mV)	A_4	B_4
[-70, -48]	0.0500	0.0037
(-48, -26]	0.1489	0.0085
(-26, -4]	0.2604	0.0114
(-4, 40]	0.3179	0.0116

TABLE VI
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 3-LINE SEGMENTS
APPROXIMATION OF $n_2(V_s) = A_5 \cdot V_s + B_5$

Region of V_s (mV)	A_5	B_5
[-70, -42.5]	0	0.9942
(-42.5, -15]	-0.1514	0.9877
(-15, 40]	-0.2865	0.9857

values of the resulted lines are given in Table V and Table VI, respectively. After calculating $n[n]$, $I_K[n]$ will be discretized as $I_K[n+1] = g_K \cdot (n[n])^4 \cdot (V_s[n] - E_K)$. (38)

Similarly, I_{leak} and I_{link} will be discretized

$$I_{leak}[n] = g_L \cdot (V[n] - E_L); \quad \text{where } V = V_s, V_d \quad (39)$$

$$I_{link}[n] = g_c \cdot (V_d[n] - V_s[n]). \quad (40)$$

To calculate $V_d[n+1]$, the dynamical equation of the I_{NaP} , (8), should be discretized, consequently, it is approximated by 8-line segments, as presented in Fig. 5. The parameter values of the I_{NaP} are given in Table VII. The I_{NaP} current can be written as follow:

$$I_{NaP}[n+1] = A_6 \cdot V_d[n] + B_6 \quad (41)$$

Calculating $I_{KS}[n+1]$ is similar to $I_{Na}[n+1]$. Using (25), we can obtain $q[n]$ which steady and transient states are

$$\begin{aligned} q[n+1] = q_{\infty}[n] \left(1 - \exp\left(-(\Delta t \cdot \phi_q)/\tau_q[n]\right)\right) \\ + q[n] \cdot \exp\left(-(\Delta t \cdot \phi_q)/\tau_q[n]\right) \end{aligned} \quad (42)$$

$$q[n+1] = q_1[n] + q[n] \cdot q_2[n] \quad (43)$$

$$\begin{aligned} q_1[n] = (\alpha_q[n] / (\alpha_q[n] + \beta_q[n])) \\ \cdot (1 - \exp(-\Delta t \cdot \phi_q \cdot (\alpha_q[n] + \beta_q[n]))) \end{aligned} \quad (44)$$

$$q_2[n] = q[n] \cdot \exp(-\Delta t \cdot \phi_q \cdot (\alpha_q[n] + \beta_q[n])) \quad (45)$$

Range of q_1 is small in comparison to range of V_d . Therefore, we multiply q_1 by 16 in order to reach higher accuracy, and then approximate it by 6-line segments, as shown in Fig. 6(a). Finally, in order to obtain the exact values of q_1 , it is shifted to right for 4 digits. Also, q_2 is approximated by 6-line segments,

Fig. 5. 8-line segments approximation for $I_{NaP}(V_d)$

Fig. 6. (a) 6-line segments approximation for $q_1(V_d)$. (b) 6-line segments approximation for $q_2(V_d)$.

Fig. 7. (a) Comparison between term1 and term2 of $I_{KS}(V_d)$ function. (b) 1-line segment approximation for $I_{KS2}(V_d)$

as shown in Fig. 6(b). Next, we can approximate the I_{KS} . For this purpose, we can substitute (9) into (46) and (47).

$$I_{KS}[n+1] = g_{KS} \cdot (q_1[n] + q[n] \cdot q_2[n]) \cdot (V_d[n] - E_K) \quad (46)$$

$$I_{KS}[n+1] = I_{KS1}[n] + q[n] \cdot I_{KS2}[n] \quad (47)$$

$I_{KS1}[n]$ can be ignored since its value is negligible compared to $(q[n] \cdot I_{KS2}[n])$, as shown in Fig. 7(a). Therefore we can obtain:

$$I_{KS}[n+1] \approx q[n] \cdot I_{KS2}[n] \quad (48)$$

$I_{KS2}[n]$ is approximated by 1-line segment too, as shown in Fig. 7(b). Table X shows the parameter values of this line.

B. The proposed digital circuit

To overcome the problems of analog fabrication without sacrificing area and power budgets, in this paper, a digital implementation is used. In this way, the proposed piecewise linear model enables us to design architecture for implementation on the FPGA. The required steps to get the

TABLE VII
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 8-LINE SEGMENTS
APPROXIMATION OF $I_{NaP}(V_d) = A_6 \cdot V_d + B_6$

Region of V_d (mV)	A_6	B_6
[-70, -63.5]	-0.5103	-0.0362
(-63.5, -57.1]	-2.1664	-0.1414
(-57.1, -44.1]	-4.4365	-0.2709
(-44.1, -37.6]	-2.4086	-0.1814
(-37.6, -31.2]	-0.6015	-0.1134
(-31.2, -24.7]	0.4028	-0.0821
(-24.7, -18.2]	0.8673	-0.0706
(-18.2, 40]	1.1752	-0.0650

TABLE VIII
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 6-LINE SEGMENTS
APPROXIMATION OF $16 \cdot q_1(V_d) = A_7 \cdot V_d + B_7$

Region of V_d (mV)	A_7	B_7
[-70, -57.1]	0.0031	0.0002
(-57.1, -37.6]	0.0351	0.0021
(-37.6, -5.3]	0.1099	0.0049
(-5.3, 14.1]	0.1940	0.0053
(14.1, 27.1]	0.3305	0.0034
(27.1, 40]	0.5104	-0.0015

TABLE IX
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 6-LINE SEGMENTS
APPROXIMATION OF $q_2(V_d) = A_8 \cdot V_d + B_8$

Region of V_d (mV)	A_8	B_8
[-70, -45.6]	0	0.9999
(-45.6, -21.1]	-0.0027	0.9998
(-21.1, 3.3]	-0.0075	0.9997
(3.3, 15.6]	-0.0140	0.9997
(15.6, 27.8]	-0.0211	0.9998
(27.8, 40]	-0.0321	1.0001

TABLE X
THE PARAMETER VALUES OF INDIVIDUAL LINES FOR THE 1-LINE SEGMENT
APPROXIMATION OF $I_{KS2}(V_d) = A_9 \cdot V_d + B_9$

Region of V_d (mV)	A_9	B_9
[-70, 40]	7	0.6296

response of individual function are showed in Fig. 8, 9, 10, and 11, respectively. As mentioned earlier, we consider 32 bits for registers (1 bit for sign, 8 bits for integer part and 23 bits for fractional part). Therefore, inputs and the corresponding outputs of our multipliers, adders and subtractions are 32 bits.

We used FPGA for hardware implementation. By providing an array of logic components that can be easily configured in a desired mode, FPGAs are reconfigurable devices for implementation of digital systems, [8]. Recent studies investigated that FPGA provide an appropriate platform for designing neuromorphic systems [29]. The digital circuit was simulated and synthesized using VIVADO, resulting a maximum clock frequency of 318.173 MHz. The resource utilization of the FPGA implementations is summarized in Table XI.

C. Error calculation

Error between the proposed models and the original P-R model is defined as Root Mean Square Error (RMSE), and calculated by (49). Furthermore, we calculate normalized RMSE (NRMSE) as (50). In (50), Y_{real} defines the original

Fig. 8. Scheduling diagram for proposed model. (a) Somatic compartment voltage. (b) Dendrite compartment voltage.

Fig. 9. Scheduling diagram for proposed model. (a) $h[n]$. (b) Sodium current for somatic compartment $I_{Na}[n]$.

Fig. 10. Scheduling diagram for proposed model. (a) $n[n]$. (b) Potassium current of the somatic compartment, $I_K[n]$.

values of the function, and Y_{approx} is the corresponding approximated value. Considering Table XII, which lists the RMSE and NRMSE for piecewise linear approximations of different functions, the dynamic performance of the system is preserved.

$$RMSE(Y_{real}, Y_{approx}) = \sqrt{\frac{\sum_{i=1}^n (Y_{real} - Y_{approx})^2}{n}} \quad (49)$$

Fig. 11. Scheduling diagram for proposed model. (a) The persistent sodium current for dendrite compartment, $I_{NaP}[n]$. (b) $q[n]$. (c) The slow potassium current for dendrite compartment, $I_{KS}[n]$.

TABLE XI
LOW-LEVEL DEVICE UTILIZATION SUMMARY

Slice logic utilization	Used	Available
Number of slice LUTs	23,293	53,200
Number of bonded IOBs	9	200
Number of slice registers	172	106,400

TABLE XII
THE RMSE AND NRMSE FOR ORIGINAL AND PROPOSED FUNCTIONS

Function	RMSE	NRMSE
h_1	5.1418e-05	0.0070
h_2	2.6482e-04	0.0092
I_{Na2}	0.2079	0.0082
n_1	7.5419e-05	0.0031
n_2	2.0424e-04	0.0102
I_{NaP}	5.1601e-04	0.0055
q_1	8.6683e-06	0.0073
q_2	9.5981e-06	0.0088
I_{KS2}	1.3311e-04	1.7314e-04

$$NRMSE(Y_{real}, Y_{approx}) = \frac{RMSE}{Y_{max} - Y_{min}} \quad (50)$$

IV. SIMULATION RESULTS

In this section, software and hardware implementation results are discussed. Indeed, the primary objective is to examine the feasibility of FPGA implementation of the model and to show that hardware can reproduce the model responses. By adjusting P , I_{soma} and g_c , different patterns of simplified Pinsky-Rinzel model can be obtained. In the first set of simulations, bursting activities is examined. In simulation shown in Fig. 12, $g_c=1$, $p=0.15$ and for (a),(b),(c) $I_{soma}=3$ and for (d),(e),(f) $I_{soma}=19.7$.

Fig. 12. The time response of the neuron membrane potential of soma (V_s) and dendrite (V_d) components in mV. (a), (d) MATLAB simulations of the original simplified P-R model. (b), (e) MATLAB simulations of the proposed piece-wise linear model. (c), (f) VIVADO simulation of the designed digital circuit. In these simulations $(g_c, p) = (1, 0.15)$ and in (a), (b), (c) $I_{soma} = 3$ and in (d), (e), (f) $I_{soma} = 19.7$. The first and the third panels show the bursting activities of somatic voltage. The second and the forth panels show the bursting activities of dendrite voltage.

Fig. 13. The time response of the neuron membrane potential of soma (V_s) and dendrite (V_d) components in mV. (a), (d) MATLAB simulations of the original simplified P-R model. (b), (e) MATLAB simulations of the proposed piecewise linear model. (c), (f) VIVADO simulation of the designed digital circuit. In these simulations $p = 0.15$ and in (a), (d) $(g_c, I_{soma}) = (1, 23)$, in (b), (c) $(g_c, I_{soma}) = (1, 34)$ and in (d), (e), (f) $(g_c, I_{soma}) = (5, 3)$. The first and the third panels show the bursting activities of somatic voltage. The second and the forth panels show the bursting activities of dendrite voltage.

Fig. 14. The time response of the neuron membrane potential of soma (V_s) and dendrite (V_d) components in mV. (a), (d) MATLAB simulations of the original simplified P-R model. (b), (e) MATLAB simulations of the proposed piecewise linear model. (c), (f) VIVADO simulation of the designed digital circuit. In these simulations $(g_c, p) = (0.1, 0.15)$ and in (a), (b), (c) $I_{soma}=7$ and in (d), (e), (f) $I_{soma}=20$. The first and the third panels show spiking and bursting activities of somatic voltage. The second and the forth panels show spiking and bursting activities of dendrite voltage.

Fig. 15. Phase plane of original and proposed model. (a), (b), (c) show the phase plane of original simplified P-R model. (d), (e), (f) show the phase plane of the proposed digital circuit. The dynamics of network is preserved in the proposed circuit.

TABLE XIII
THE RMSES AND NRMSES BETWEEN OBTAINED RESULTS IN THE ORIGINAL AND PROPOSED MODELS IN MATLAB SIMULATIONS

	$(g_c, p, I_{soma}) = (1, 0.15, 3)$		$(g_c, p, I_{soma}) = (1, 0.15, 19.7)$		$(g_c, p, I_{soma}) = (1, 0.15, 23)$	
	RMSE	NRMSE	RMSE	NRMSE	RMSE	NRMSE
V_s	0.0138	0.1333	0.0179	0.1814	0.0195	0.1988
V_d	0.0123	0.2208	0.0130	0.2575	0.0104	0.2075
	$(g_c, p, I_{soma}) = (5, 0.15, 3)$		$(g_c, p, I_{soma}) = (0.1, 0.15, 7)$		$(g_c, p, I_{soma}) = (0.1, 0.15, 20)$	
	RMSE	NRMSE	RMSE	NRMSE	RMSE	NRMSE
V_s	0.0095	0.1175	0.0270	0.2162	0.0428	0.3515
V_d	0.0093	0.1237	0.0091	0.2344	0.0110	0.2963

TABLE XIV
THE RMSE AND NRMSE BETWEEN OBTAINED RESULTS OF THE PROPOSED DIGITAL CIRCUIT IN VERILOG WITH THOSE IN MATLAB SIMULATIONS

	$(g_c, p, I_{soma}) = (1, 0.15, 3)$		$(g_c, p, I_{soma}) = (1, 0.15, 19.7)$		$(g_c, p, I_{soma}) = (1, 0.15, 23)$	
	RMSE	NRMSE	RMSE	NRMSE	RMSE	NRMSE
V_s	0.0066	0.0633	0.0121	0.1234	0.0171	0.1846
V_d	0.0040	0.0715	0.0058	0.1150	0.0081	0.1646
	$(g_c, p, I_{soma}) = (5, 0.15, 3)$		$(g_c, p, I_{soma}) = (0.1, 0.15, 7)$		$(g_c, p, I_{soma}) = (0.1, 0.15, 20)$	
	RMSE	NRMSE	RMSE	NRMSE	RMSE	NRMSE
V_s	0.0060	0.0745	0.0233	0.1873	0.0318	0.2607
V_d	0.0058	0.0764	0.0027	0.0719	0.0018	0.0478

In Fig. 12, the first and the third panels show the bursting activities of somatic compartment (blue color) and the second and the fourth panels display the bursting activities of dendrite compartment (red color). Fig. 12(a), (d) illustrates the MATLAB simulations of the membrane potential of original simplified P-R model, and (b), (e) are the MATLAB simulations of the proposed piece-wise linear model, and (c), (f) are the VIVADO simulation of the designed digital circuit. Comparing different panels, it is evident that both piecewise linear model and digital circuit produce similar responses to original biophysical P-R model. Additionally, by increasing the injection current to soma (i.e., $I_{soma}=19.7$), the firing frequency is also increased and tonic bursting activities can be appeared in digital circuit.

Next, we examine the spiking activities in the digital circuit of the 2-compartmental model. Fig. 13 shows the spiking activities of the simplified P-R neuron model. For simulation illustrated in Fig. 13, $p = 0.15$ and for (a) $g_c = 1$, $I_{soma} = 23$, for (b), (c) $g_c = 1$, $I_{soma} = 34$, for (d), (e), (f) $g_c = 5$, $I_{soma} = 3$. In Fig. 13, the first and the third panels show the bursting/spiking behavior of somatic compartment (blue color) and the second and the forth panels display the bursting/spiking activities of dendrite compartment (red color). As can be seen, the results of MATLAB simulation of the membrane potential of original simplified P-R model which is shown in Fig.13 (a), (d) not only have good agreement with the MATLAB simulations of the proposed piecewise linear model illustrated in (b), (e) but also with the VIVADO simulation of the digital circuit shown in (c), (f).

In the third set of simulations, we investigate the effect of variation of g_c , which couple the somatic and dendritic components. When somatic and dendritic compartments are

strongly coupled (e.g. $g_c = 5$), the spikes will be generate as shown in Fig. 13 (d), (e), (f). On the other hand, when g_c is weak (e.g. $g_c=0.1$), the burst entirely change and the dendritic component cannot follow somatic part, showing a small hump during a burst, as shown in the second panel of Fig. 14 (a), (b), (c). In Fig. 14 (d), (e), (f), a larger current is injected to the soma, thus the bursting patterns disappears and spiking activities are modulated in frequency.

The phase plane, (V_s-h), (V_s-n), (V_d-q), of the simplified P-R neuron model (a), (b), (c) and for the proposed digital circuit (d), (e), (f) are shown in Fig. 15. As can be observed, although there are also some quantitative differences, the general shape and the qualitative behavior of trajectories are similar. Regarding the obtained results, one can conclude that the dynamical characteristics of the original system are preserved by the designed circuit.

Computing RMSE and NRMSE, the reliability of the proposed model is supported. Table XIII shows the RMSE and NRMSE between biophysical P-R and proposed piecewise linear models in MATLAB, and Table XIV presents the RMSE and NRMSE between simulations of digital circuit in VIVADO with the original model in MATLAB.

Considering Fig. 12, 13, 14, and 15, the digital circuit maintains the essential properties of its biological counterpart in different conditions. Considering the important criteria from the hardware point of view such as scaling up the designed circuit, reducing the digital implementation cost and keeping low power operation while obtaining results similar to the biophysical model of P-R model, the proposed circuit produces acceptable results.

V. Conclusion

Design and effective implementation of spiking neural network in hardware is now an active domain of research and several procedures have been proposed for designing and construction of brain like processing systems for conventional computing. The focus of this paper was to propose a piecewise linear model and a digital circuit for the simplified Pinsky-Rinzel neuron model to verify that both proposed linearized and digital circuit can produce similar responses as the biophysical model does. This work can be considered as a basic step towards creating adaptable and bio-plausible hardware-based neural network using simplified Pinsky-Rinzel neuron model. The results of the FPGA implementation are in agreement with those Matlab and Vivado simulations. Future works aim at extending this model and creating a neural population model and constructs a spiking neural network in digital domain. These issues will be addressed in future study.

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