Dissipative Solitons with Extreme Spikes

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Abstract—We have found dissipative solitons with extreme spikes in the laser model described by the complex cubic-quintic Ginzburg-Landau equation. They exist in both normal and anomalous dispersion cases.

Keywords—component; dissipative soliton; rogue waves

I. INTRODUCTION

Dissipative solitons (DS) in laser systems are generated due to the intricate balance of nonlinearity, dispersion, gain and loss [1,2]. The complexity of this balance leads to a large variety of soliton profiles [3]. These include common bell-shaped solitons, flat top solitons, creeping solitons and many other nontrivial forms. Moreover, DS can evolve on propagation changing shape periodically, chaotically or lead to the formation of rogue waves.

One of the most unusual DS dynamics found recently is the appearance of sharp peaks on top of a more stable wider soliton [4], that serves as a background. These sharp peaks are localized in both the transversal and the longitudinal dimensions. Sharp peaks may appear regularly or chaotically. In the latter case, they are known as spiny solitons [5]. Remarkably, these types of solitons are not something exceptional. First discovered in an isolated region of parameters [4,5] they have been also found in another region [6] which is not related to the original one. In both regions, the cavity dispersion was normal. In this work, we make a further extension of the parameter region where dissipative solitons with extreme spikes can be observed. Namely, we have continuously varied the dispersion parameter to reach the area of anomalous dispersion while keeping intact the rest of the equation parameters.

II. RESULTS

As in previous works [1-3], the specific model we are dealing with is the cubic-quintic complex Ginzburg-Landau equation with the same parameters as in [3]. Extreme pulsations, observed earlier in the normal dispersion regime, are characterized by narrow spikes developed on top of a roughly stationary soliton that serves as pedestal supporting these peaks. In order to characterise these solutions, we use as main pulse parameter the energy, $Q(z)$.

Fig.1 shows the dependence of the calculated maximum (dashed lines) and minimum (dotted lines) of energy, Q , on the cavity dispersion. Parameters of the CGLE are taken to be:

$\varepsilon=1$, $\delta=-0.1$, $\nu=0.1$, $\beta=0.3$, while for the gain saturation, μ , we took three different values -0.001, -0.0025 and -0.003. These three values provide different bifurcation patterns.

Figure 1. The maximal (dashed curves) and minimal (dotted curves) soliton energy vs the cavity dispersion, D , when D changes from normal to anomalous. The red curves are for $\mu=-0.001$, the blue ones for $\mu=-0.0025$ and the green for $\mu=-0.003$. The values of other equation parameters are shown inside the figure. The red curves have two intervals of discontinuity in this range of D -values dividing the family of pulsating solutions into three different sets separated by regions with no localized solutions. The blue curves show a small region of bistability with two coexisting types of pulsations. The three curves visibly overlap when D is positive (anomalous dispersion) but differ significantly when D becomes increasingly negative (normal dispersion).

When $\mu = -0.001$ (red curves), there are three different families of pulsating solutions separated by smaller regions where no localized solutions exist. The amplitudes of the energy oscillations $Q(z)$ for these three families differ considerably. The maximum energy is above 100 for the left hand side curve (when D is close to -1 while it is below 20 for the right hand side curve (when $D > -0.5$). When $\mu = -0.0025$ there are only two families of solutions. Moreover, their regions of existence overlap. There is a small region of D -values around $D \sim -0.6$ with bistability. Finally, when $\mu = -0.003$, there is only one continuous curve for each of the values Q_m and Q_M . This means that only one branch of solitons exists in the whole interval of D -values of interest. For D greater than -0.4, the $Q_{M,m}$ -curves for all three values of μ nearly coincide. Moreover, these curves hardly change with D . They also exhibit a much smaller energy variation from minimum to maximum giving the false impression that the pulsations are not as extreme as for the cases around $D = -1$.

The extremity of pulsations is best visible in the evolution plots along the z -axis. The periodic evolution of the energy, Q , is shown in Fig.2. For this demonstration, we have chosen $\mu = -$

0.001 and two values of D at the opposite ends of the dispersion interval, $D=1$ (red solid curves) and $D=-1$ (blue dotted curves). We can see, from Fig.2 that the period of oscillations is long in the case of normal dispersion. The solution remains in the form of a low amplitude pedestal soliton for most of the period with the spike occupying a small part of the period. On the contrary, the period becomes shorter in the case of anomalous dispersion so that the duration of the spikes is now comparable to the period. The energy, Q , oscillates with stronger magnitude when $D=-1$. These oscillations are almost ten times smaller when $D=+1$. However, the oscillations of the peak amplitude in the two cases are comparable.

Figure 2. Evolution of the energy, Q , for a few periods of extreme pulse oscillations when $\mu=-0.001$, and D is either 1 or -1. The red curve is for the anomalous dispersion case (positive D), and the blue curve is for the normal dispersion one (negative D). The green region mark the evolution shown in Fig.3.

3D-plots of the pulse evolution around an extreme peak are shown in Fig.3 for the case of the anomalous dispersion. This plot shows the evolution within the z -interval highlighted by the green shaded rectangle in Fig.2. The panel (a) is the amplitude profile of the pulse in the (t,z) -plane while the panels (b) and (c) show false-color plot of the pulse amplitude and phase respectively on the same (t,z) -plane. This figure provides more detailed features of the pulse evolution. For $D=1$ (anomalous dispersion case), the extreme peak is much narrower and significantly shorter than in the case of normal dispersion.

III. CONCLUSIONS

The appearance of extreme spikes on top of wider dissipative solitons is a striking phenomenon that deserves further studies both theoretical and experimental. By far, we have shown that extreme spikes can appear regularly or chaotically. In the latter case, they can be considered as rogue waves [5]. Extreme spikes can appear for a wide range of the system parameters. Previous studies revealed at least two sets of parameters that admit extreme spikes. The present study widened the range of parameters to include the continuous change of the dispersion parameter of the cavity from normal to anomalous. Remarkably, the anomalous dispersion case considered here provides considerably sharper spikes than the normal dispersion one. When building lasers generating ultra-short pulses, this can be considered as an advantage. The variable dispersion has been chosen for the sake of its convenience in changing in experiments. Clearly, changing other parameters would also provide a continuous range of

existence of extreme spikes. We have demonstrated this possibility changing the gain saturation parameter μ from -0.003 to -0.001. Investigating the whole range of parameters and finding new regions admitting extreme spikes could be done in future works on this subject.

Figure 3. a) Extreme pulse evolution for the anomalous dispersion case. This plot is for the green region shown in Fig.2. The lower panels b) and c) are the false color plots of the amplitude and the phase of the same data as on the upper panel.

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