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Physics Letters B 640 (2006) 117–120
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Will black holes eventually engulf the Universe?

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Received 13 June 2006; accepted 16 July 2006

Available online 10 August 2006

Editor: N. Glover

Abstract

The Babichev–Dokuchaev–Eroshenko model for the accretion of dark energy onto black holes has been extended to deal with black holes with non-static metrics. The possibility that for an asymptotic observer a black hole with large mass will rapidly increase and eventually engulf the Universe at a finite time in the future has been studied by using reasonable values for astronomical parameters. It is concluded that such a phenomenon is forbidden for all black holes in quintessential cosmological models.

PACS: 98.80.-k; 04.70.-s

Keywords: Accretion; Black holes; Dark energy

The discovery of the current acceleration of the Universe has driven a plethora of theoretical models to account for the dark stuff which has been invoked to justify observations of distant supernovae Ia, cosmic microwave background anisotropy, microlensing and the statistics of quasars and clusters [1–3], all aiming at determining the equation-of-state parameter w of the Universe, $p = w\rho$. It seems still possible and even quite feasible that the value of w be less than -1, even though this would induce serious problems for the consistency and stability of cosmic dark energy [4–9]. However, even if $w > -1$, for which most of the above problems are no longer present, the most popular dark energy model, i.e. the quintessence scenario, may pose a serious potential difficulty. That difficulty appears when we consider the accretion of a quintessence scalar field with constant parameter $w > -1$ onto black holes [10–12]. In fact, in the simplest case first discovered by Babichev et al. [13,14], the rate of Schwarzschild black hole mass is given by

$$\dot{M} \equiv \frac{dM}{dt} = 4\pi A M^2\left[p(\rho_\infty) + \rho_\infty\right], \tag{1}$$

where $p(\rho_\infty)$ and ρ_∞ respectively are the pressure and energy density of the Universe at the asymptotic limit $r \to \infty$ and, in this case, $A = 4$ [13]. One can then see from the onset that for $p+\rho > 0$ in a quintessence model with equation of state $p(t) = w\rho(t) = w\rho_0(a(t)/a_0)^{-3(1+w)}$, constant $-1 < w < -1/3$ and corresponding scale factor

$$a(t) = a_0\left[1 + \frac{3}{2}(1+w)\left(\frac{8\pi\rho_0}{3}\right)^{1/2}(t-t_0)\right]^{\frac{2}{3(1+w)}}, \tag{2}$$

integrating then Eq. (1), we obtain for the mass of the black hole

$$M(t) = M_0\left[1 - \frac{4\pi A\rho_0 M_0(1+w)(t-t_0)}{1+(6\pi\rho_0)^{1/2}(1+w)(t-t_0)}\right]^{-1}. \tag{3}$$

It follows that the increase of black hole mass may be so quick as to yield a black hole mass corresponding to a size that would eventually exceed the size of the Universe itself, at a finite time in the future $t = t_{bs}$, with

$$t_{bs} = t_0 + \frac{1}{[4\pi A\rho_0 M_0 - (6\pi\rho_0)^{1/2}](1+w)}. \tag{4}$$

It can be checked that such a rather weird behaviour would also occur in the case of Kerr black holes [15].

Nevertheless, the whole extreme black hole swelling phenomenon could simply be thought to be an artifact resulting

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0370-2693
doi:10.1016/j.physletb.2006.07.067

from the use of static metrics which do not induce any non-zero Θ_0^r component of the black hole momentum–energy tensor, and hence implying no internal energy flow. In this case, the accretion procedure used to derive the rate is in principle only valid for small accretion rates, not for large rates and even less for the extreme rates leading to a blow-up of the black hole size. Our first task therefore will be checking under what conditions, if any, the above catastrophic phenomenon may take place. The simplest non-static metric that would still contain a time-dependence enough to induce internal non-zero energy-flow component Θ_0^r to overcome the above approximation on the rate is one in which the black-hole mass is allowed to depend generically on time, that is for a Schwarzschild metric, (we use natural units so that $G = c = 1$):

$$\mathrm{d}s^2 = \left(1 - \frac{2M(t)}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{2M(t)}{r}\right)^{-1}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right). \tag{5}$$

One can generalize the mechanism advanced in Refs. [13,14] to this non-static case. So, in first place, we integrate in r the time component of the conservation law $T^{\nu}_{\mu;\nu} = 0$ which in this case no longer leads to a relativistic Bernoulli equation. So we obtain

$$(p+\rho)\left(1 - \frac{2M}{r} + u^2\right)^{1/2} ur^2 M^{-2} e^{\int_\infty^r f(r,t)\,\mathrm{d}r} = C_1(t), \tag{6}$$

where $C_1(t)$ is a time dependent function which has the physical dimensions of an energy density, $u = \mathrm{d}r/\mathrm{d}s$ is the r component of four-velocity and

$$f(r,t) = \frac{\partial_0 T_0^0}{T_0^r} - \frac{4\pi r}{1 - 2M/r}\left(T_0^0 - T_r^r\right). \tag{7}$$

Secondly we integrate the projection on the four-velocity of the energy–momentum conservation law, $u_\mu T^{\mu\nu}_{;\nu} = 0$; so we have

$$ur^2 e^{\int_{\rho_\infty}^{\rho} \frac{\mathrm{d}\rho}{p(\rho)+\rho}} e^{\int_\infty^r g(r,t)\,\mathrm{d}r} = -A(t), \tag{8}$$

where

$$g(r,t) = \frac{(1 - 2M/r + u^2)^{1/2}}{(1 - 2M/r)u}\frac{\partial_0 \rho}{p+\rho} + \frac{\dot{M}(1 - 2M/r + 2u^2)}{ur(1 - 2M/r)^2(1 - 2M/r + u^2)^{1/2}} + \frac{\partial_0 u}{(1 - 2M/r)(1 - 2M/r + u^2)^{1/2}}, \tag{9}$$

with $\partial_0 \equiv \partial/\partial_t$ and $A(t)$ is a positive time-dependent function (note that $u < 0$). In obtaining Eqs. (6)–(9) we have used for dark energy an energy–momentum tensor T^{μ}_{ν} corresponding to a perfect fluid. In order to evaluate the function $A(t)$ we can use the property that $A(t) = \lim_{r\to\infty} ur^2$, so that $A(t)$ is not an explicit function of t, because r does not depend on t for the simplest non-static metric (5), and u depends only on t through M. Then, as the physical dimensions of $A(t)$ must be $(\text{meters})^2$, we must have $A(t) = M^2 A'$, with A' a constant. At low rate of change with time we must recover the expressions of Ref. [13], so A' must be equal $A = 4$ as one will see clearerly below. From Eqs. (6) and (8), it then follows:

$$(p+\rho)\left(1 - \frac{2M}{r} + u^2\right)^{1/2} e^{-\int_{\rho_\infty}^{\rho} \frac{\mathrm{d}\rho}{p(\rho)+\rho}} e^{-\int_\infty^r \mathrm{d}r[g(r,t) - f(r,t)]} = B(t), \tag{10}$$

where $B(t) = -C_1(t)/A' = p(\rho_\infty) + \rho_\infty$.

Relative to the energy flow induced in the quintessential fluid, the black hole mass rate can be expressed as $\dot{M} = -\int T_0^r\,\mathrm{d}S$, in which $\mathrm{d}S = r^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$, $0 \leqslant \theta \leqslant \pi$ and $0 \leqslant \phi \leqslant 2\pi$. Then the above equations lead finally to

$$\dot{M} = 4\pi A' M^2 (p+\rho) e^{-\int_\infty^r f(r,t)\,\mathrm{d}r}. \tag{11}$$

One can see that for the relevant physical case of an asymptotic observer; i.e. $r \to \infty$, one recovers Eq. (1) also for the non-static black hole metric given by Eq. (5), such as it occurs in the case of wormholes [16]. We note furthermore that this asymptotic result is also valid for the Kerr–Newman black hole as it has been shown [15] that the effects due to the presence of angular momentum and electric charge on the accretion of dark energy all vanish as $r \to \infty$. It follows that, at least asymptotically, the big swelling of black holes leading to a swallowing of the Universe is not an artifact coming from metric staticity, although for finite values of the radial coordinate, we see that the rate equation is changed in such a way as to not guarantee the occurrence of an extreme black hole swelling.

However, once we have shown the asymptotic consistency and accuracy of Eq. (1), we must check how current observational data fit the extent of this phenomenon, or even ultimately prevent it from occurring at all. We consider again a quintessence model described by Eq. (2) and energy density $\rho(t) = \rho_0 (a(t)/a_0)^{-3(1+w)}$, integrating expression (11) for an asymptotic observer. We then have

$$M = \frac{M_0}{1 - F(t)}, \tag{12}$$

in which

$$F(t) = \frac{4\pi A' \rho_0 M_0 (1+w)(t - t_0)}{1 + (6\pi\rho_0)^{1/2}(1+w)(t - t_0)}. \tag{13}$$

One can note then that $F(t) > 0$ when $t > t_0$ and $\dot{F}(t) \equiv \mathrm{d}F(t)/\mathrm{d}t > 0$; therefore $F(t)$ is an increasing function and the limit of $F(t)$ when t goes to infinity is finite and equal to $(8\pi\rho_0/3)^{1/2} A' M_0$. Then $F(t)$ has a asymptote. When we consider a universe model which contains only dark energy (which is assumed to be a sufficiently good approximation for the future times where the big swelling of the black holes could be thought to occur), ρ_0, the energy density at the coincidence time, is given by

$$\rho_0 = \rho_n (a_n/a_0)^{3(1+w)} = \rho_n (1+z)^{3(1+w)}, \tag{14}$$

in which the subindex n indicates present value. On the other hand, since the spatial curvature of the Universe k is thought to be nearly zero, the current density will be equal to the current critical density, that is

$$\rho_n = \frac{3H_n^2}{8\pi}. \tag{15}$$

Now, from the WMAP data, we can obtain that $(8\pi\rho_n/3)^{1/2} = H_n \sim 10^{-26}$ (meters)$^{-1}$. One can express the initial black hole mass as $M_0 = X_0 M_\odot \sim X_0 \times 10^3$ (meters), so that the asymptotic value of $F(t)$ is $\sim X_0 \times 10^{-23} \ll 1$, generally, and so $M \sim M_0$ even at infinitely large times. In obtaining this estimate we have taken into account that the value of constant A' should be the same as that for A in Eq. (1). Thus, the main conclusion of the present Letter is that for current observational data and for solar-mass or even supermassive black holes with masses even larger than $10^{10} M_\odot$ the accretion of dark energy with $w > -1$ onto black holes leads to a very smooth increase of the hole size and the phenomenon of an engulfing of the Universe by the black hole is largely prevented. In fact, it could never occur. We note that it is only for extremely massive hypothetical black holes with masses on the range of $M \sim 10^{23} M_\odot$ or larger, that such a phenomenon could eventually take place. It could then be possible that if w kept constant and accretion of ordinary or dark matter would continue enlarging black holes in the future, then these black holes might finally increase larger than the Universe itself due to dark energy accretion.

Clearly, present observational indications seem to imply that w is not constant and takes on values less than -1, so that in principle, the occurrence of the considered catastrophic phenomenon becomes actually quite unlikely at any time in the far future. However, if $\dot{w} > 0$ and dark energy would therefore cross the dividing barrier at $w = -1$ in the future, then the following speculative reasoning might be in order.

If the Universe will expand forever induced by dark energy with $w > -1$, then it would commonly be believed than in about hundred thousand billion years the last stars will die out, some of them leaving a black hole behind. These black holes would evaporate by the Hawking process while the remaining matter very slowly decayed. Thus both the cooled stars and the dilute gas, and later the black holes formed at the end of the star lifetime and those supermassive ones that stood at the center of galaxies, will disappear from the Universe in its remote future, leaving rare electrons and positrons spread over huge distances from each other [17]. An unsolved question is however, how big can a black hole grow? By the holographic bound we know that black holes are the most entropic objects and by the laws of black hole mechanics that coalescence of black holes implies a neat increase of entropy. It appears then quite plausible, in principle, that superimposed to the above process leading to the thermal death of the Universe, before evaporating, black hole would continue swallowing ordinary and dark matter or finally eating each other so that the total balance of entropy increase be optimized to a maximum and actually the evaporation process will effectively start with immensely huge black holes. However, if we assume that the Universe is expanding in size at the speed of light, then its radius would be 13.7 billion light years, and its diameter would be 27.4 billion light years. Converting this to meters and cubing gives a volume of 2×10^{30} kg. This gives a total of 10^{23} stars. Thus, roughly speaking, for a black hole to undergo the big swelling leading to engulf the entire Universe we would need to start with an initial black hole mass which equals the mass of the entire Universe. This is nevertheless impossible because of the holographic bound for entropy, $S < S_{bh}$, where S_{bh} would be the entropy of a black hole with the same energy as the whole Universe, and the associated feature that receding black holes at large distance can never coalerse. It follows that in quintessence models black holes can never engulf the Universe.

Actually, a question similar to what has been discussed in the present Letter was already posed for primordial black holes in the early universe by Zeldovich and Novikov who claimed [18] that these primordial black holes could have grown as fast as the horizon mass by ordinary matter accretion. Also like for the case of large black holes accreating dark energy in the late Universe studied in the present Letter, it was later shown by Carr and Hawking [19] that there was no substantial growth of the primordial black holes due to accretion over the age of the Universe.

Acknowledgements

The authors thank Salvador Robles and Alessandro Mantelli for useful discussions, technical help and encouragement. The authors acknowledge John Barrow for calling our attention to Refs. [18,19]. This work was supported by MEC under Research Project No. FIS2005-01181.

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