Multiscale and multifractional spacetimes in quantum gravity: status report

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Main message:

- Although there are many independent quantum gravities, they all share some universal features: **(1)** dimensional flow, **(2)** entanglement entropy, **(3)** complex dimensions.

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- Their cosmology is poorly known but extremely interesting.

Outline

1. A landscape in quantum gravity

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2. Universal properties
   - Dimensional flow
   - Entanglement entropy

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02/42– Motivations

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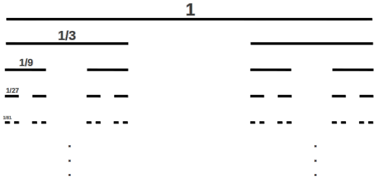
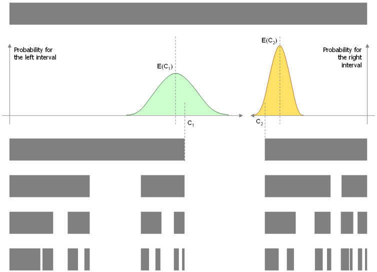
- Dim. flow and UV finiteness? Renormalizabile gravity?
- Theory and phenomenology (from particle physics to cosmology) of fractal spacetimes? Very preliminary results in the 1980s [Svozil 1986; Eynk 1989a,b; Müller, Schäfer 1986a,b].

03/42– Example of fractal: Cantor set

Deterministic

Random





04/42– ABC of multiscale spacetimes

G.C. EPJC **76** (2016) 181 [arXiv:1602.01470]

A. Dimensional flow occurs: [A1] At least two of the dimensions $d_{\rm H}$, $d_{\rm S}$, and $d_{\rm W}$ vary. [A2] Flow is continuous from the IR to a UV cut-off. [A3] Flow occurs locally (prevents false positive).

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B. Integer dimension observed at a finite number of points (e.g., UV and IR extrema).
C. Weakly multifractal spacetime: if $d_{\rm W}=2d_{\rm H}/d_{\rm S}$ and $d_{\rm S}\leq d_{\rm H}$ at all scales.

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C. Weakly multifractal spacetime: if $d_{\rm W} = 2d_{\rm H}/d_{\rm S}$ and $d_{\rm S} \leq d_{\rm H}$ at all scales.

D. Strongly multifractal spacetime: if, in addition of satisfying A–C, it is nowhere differentiable.

05/42– Landscape of multiscale theories

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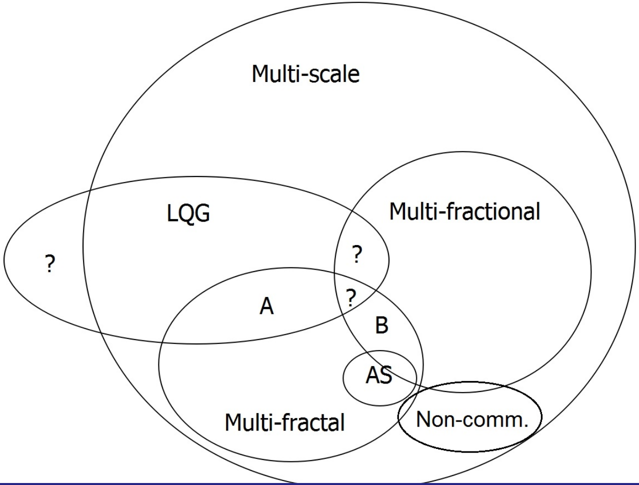
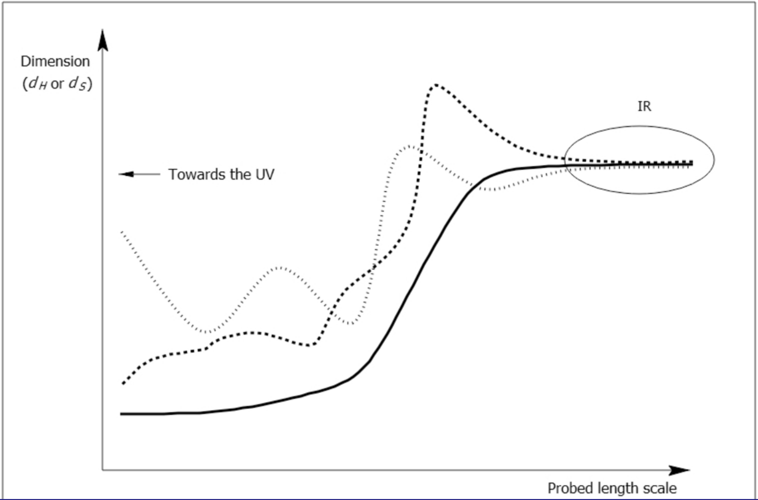


Outline

06/42– Slow flow



07/42– Dimensions near the IR

$$d(\ell) \simeq D + b\left(\frac{\ell_*}{\ell}\right)^c + \text{(log oscillations)}, \qquad d = d_\mathrm{H}, d_\mathrm{S}$$

		D	b_H	c_H	b_S	c_S
Asymptotic safety		4	0	—	< 0	> 0
CDT		4	0	—	< 0	2
Near black holes		D	0	—	$\frac{D+1}{2}$	2
Nonlocal gravity and string field theory		D	0	—	< 0	2
Fuzzy spacetimes		D	0	—	$-D$	2
Gravity with quantum particles		3	0	—	$-\frac{21}{16}$	2
κ-Minkowski	bicovariant ∇^2, AN(3)	4	0	—	-2	2
κ-Minkowski	bicovariant ∇^2, AN(2)	3	0	—	$-\frac{3}{2}$	2
κ-Minkowski	bicrossproduct ∇^2	4	0	—	1	2
κ-Minkowski	cyclic invariance (o.s.)	D	< 0	1	?	?
Hořava–Lifshitz gravity		D	0	—	< 0	> 0
GFT, spin foams, LQG (o.s.)		$D(= 4)$	< 0	2	> 0	2

08/42– Flow-equation theorem

G.C., PRD **95** (2017) 064057 [arXiv:1609.02776]

Varying $d_{\rm H}$

If the Hausdorff dimension of spacetime changes with the probed scale, and if it does so slowly at large scales, then the D-volume is uniquely determined as

$$\mathcal{V}(\ell) \simeq \ell^D + \left|\frac{\ell}{\ell_*}\right|^{D\alpha} F_\omega(\ell), \qquad F_\omega(\ell) = 1 + \sum_{n>0} F_n(\ell)$$

$$F_n(\ell) = A_n \cos\left(n\omega \ln \frac{\ell}{\ell_\infty}\right) + \sum_n B_n \sin\left(n\omega \ln \frac{\ell}{\ell_\infty}\right)$$

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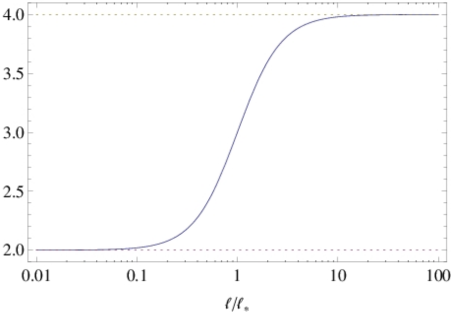
Similar statement for the spectral dimension $d_{\rm S}$.

09/42– Multiscaling

Scaling property: $\mathcal{V}(\lambda\ell) \sim \lambda^{D\alpha}\mathcal{V}(\ell) \qquad \Rightarrow \qquad d_{\rm H} = D\alpha$



10/42– Discreteness

G.C., JHEP **01** (2012) 065; G.C., arXiv:1705.01619

Exactly the same measure found in fractal geometry [Ren et al. 1996–2003; Nigmatullin & Le Méhauté 2003].

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Oscillatory part of $\mathcal{V}(\ell) = \ell^D + \ell^{D\alpha} F_\omega(\ell)$ invariant under a DSI:

$$F_\omega(\lambda_\omega^n \ell) = F_\omega(\ell)\,, \qquad \lambda_\omega = \exp(-2\pi/\omega)\,, \qquad n = 0, 1, 2, \dots$$

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DSIs appear in fractals and in complex and critical systems (earthquakes, financial crashes,. . .) [Sornette 1998].

$\omega = d_{\mathrm{H}\mathbb{C}}$ complex dimension!

Can we observe complex dimensions?

11/42– Entanglement entropy in quantum gravity

- Entanglement entropy is an important actor in the quantum gravity show: signals the emergence of classical spacetime. (a) Einstein eqs. recovered from thermodynamics if the e.e. is finite [Jacobson 1995]. (b) It prescribes how two spacetime regions can connect or are torn away classically depending on whether the d.o.f. within these regions entangle and disentangle at the quantum level [Van Raamsdonk 2010].
- **What are the conditions on a spacetime geometry leading to a finite entanglement entropy *density* ρ_d?**

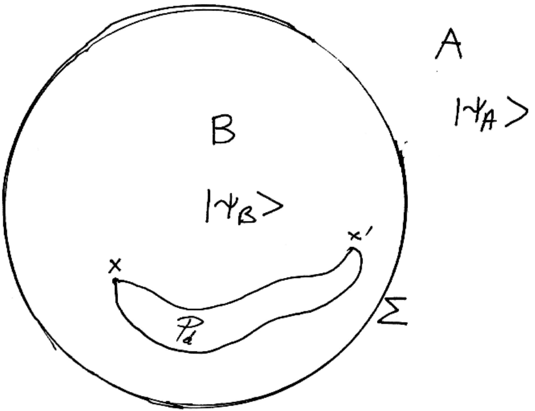
12/42– Entanglement (von Neumann) entropy: quantum-mechanics

Measuring entanglement $=$ counting the number of degrees of freedom on the boundary.

$$S_A = -\mathrm{tr}_A(\rho_A \ln \rho_A)\,, \qquad \rho_A = \mathrm{tr}_B \left|\Psi\right\rangle \left\langle\Psi\right|\,.$$

No entanglement: $\left|\Psi\right\rangle = \left|\Psi_A\right\rangle \otimes \left|\Psi_B\right\rangle$

12/42– Entanglement (von Neumann) entropy: quantum-mechanics



13/42– Entanglement entropy: QFT

Trace over QFT d.o.f. [Casini, Huerta 2009; Nesterov, Solodukhin 2010,2011; Solodukhin 2011]:

$$S \sim \mathcal{A}_d(\Sigma)\,\rho_d := \mathcal{A}_d(\Sigma)\int_{\epsilon^2}^{\infty}\frac{\mathrm{d}\sigma}{\sigma}\mathcal{P}_d(\sigma)\,,\qquad d = D-2$$

UV finiteness $\Rightarrow$ finite ρ_d? No:

$$\lim_{x\to y} G(x,y) \propto \int_0^{\infty}\mathrm{d}\sigma\,\mathcal{P}_4(\sigma) \quad \text{but} \quad \mathcal{P}_4(\sigma) \propto \int \mathrm{d}^4 p\, \mathrm{e}^{-\sigma C(p)}$$

Rules out all QFTs on noncompact momentum space for any $C(p)$ [Nesterov, Solodukhin 2010].

14/42– Spectral zeta function

Spectral theory, $\Box\psi=\lambda\psi$: $\frac{1}{\lambda^s}=\frac{1}{\Gamma(s)}\int_0^{+\infty}\mathrm{d}\sigma\,\sigma^{s-1}\mathrm{e}^{-\sigma\lambda}$.
Mellin transform of return probability:

$$\zeta_d(s):=\frac{1}{\Gamma(s)}\int_0^{+\infty}\mathrm{d}\sigma\,\sigma^{s-1}\mathcal{P}_d(\sigma)\sim\sum_n\frac{1}{\lambda_n^s}\,.$$

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$$\boxed{\rho_d = \lim_{s\to 0}\Gamma(s)\,\zeta_d(s)}$$

15/42– Constant spectral dimension

Popular definition $\mathcal{P}_D \sim \sigma^{-d_S/2}$ (QG):

$$d_S = -2 \lim_{\sigma \to 0} \frac{\ln \mathcal{P}_D(\sigma)}{\ln \sigma} \to -2 \frac{\mathrm{d} \ln \mathcal{P}_D(\sigma)}{\mathrm{d} \ln \sigma} .$$

Alternative definition [Bessis et al. 1983,1984]: pole of ζ_D at $\sigma \ll 1$ (fractal geometry):

$$\mathcal{P}_D(\sigma) = \frac{1}{2\pi \mathrm{i}} \int_{\epsilon - \mathrm{i}\infty}^{\epsilon + \mathrm{i}\infty} \mathrm{d}s\, \zeta_D(s) \Gamma(s)\, \sigma^{-s} .$$

Can we extend it to geometries with variable d_S?

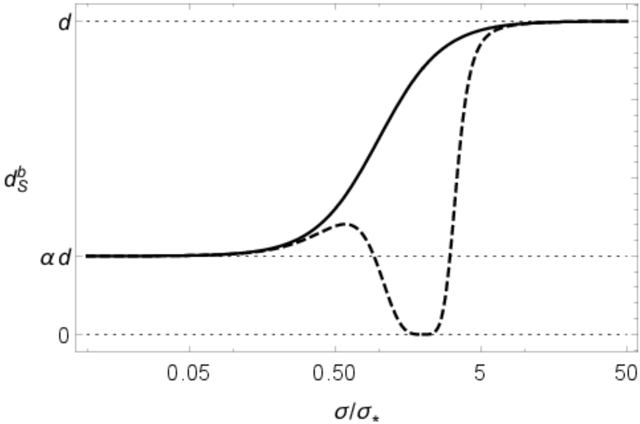
16/42– General result

Arzano, G.C., arXiv:1704.01141

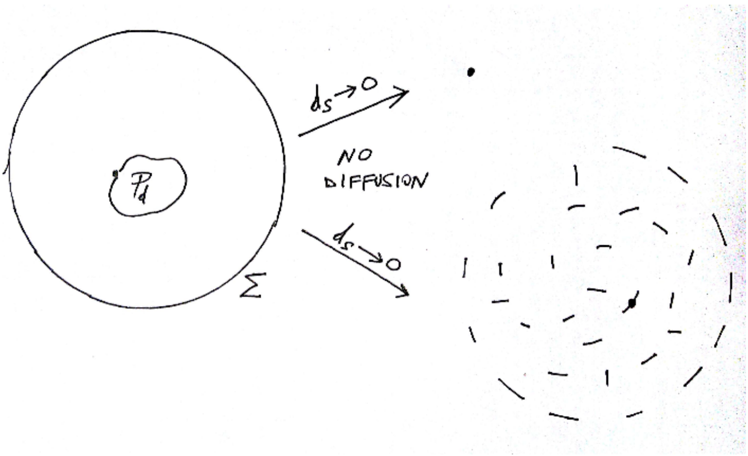
Necessary condition for a finite entropy density

If the entanglement entropy density ρ_d is finite and nonzero, then the spectral dimension d_S^b of the spatial boundary never vanishes at any scale.

17/42– Dimensional flows with $0 < \rho_d < \infty$



18/42– Interpretation: degenerate or rough boundary



19/42– Examples

Arzano, G.C., arXiv:1704.01141

$\rho_d = \infty$ or $\rho_d = 0$:

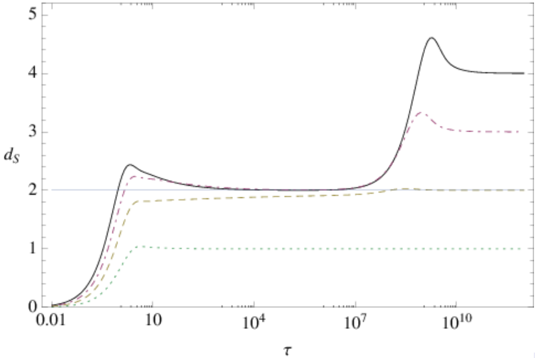
- $C(p) = |p|^{2\gamma}$: $d_S = \text{const}$.
- κ-Minkowski (UV finite): $d_S^{UV} = 0$ [Arzano et al. 2015,2016].
- GFT/LQG/spin foams: $d_S^{UV} = 0$ [G.C., Oriti, Thürigen 2014,2015].
- Nonlocal quantum gravity (UV finite): $d_S^{UV} = 0$ [G.C., Modesto 2014].

$0 < \rho_d < \infty$:

- String theory: $d_S^{UV} = 2$ [G.C., Modesto 2014].
- CDT: only if $d_S^{UV} \neq 2$.
- Multifractional spacetimes: $d_S^{UV} = D\alpha \neq 0$.

20/42– LQG, spin foams, GFT

General states $|\psi\rangle = \sum_{j,\mathcal{C}} a_{j,\mathcal{C}} |j,\mathcal{C}\rangle$. Discreteness combinatorial effects drive d_S to zero [G.C., Oriti, Thürigen 2014, 2015], hence $\rho_d = \infty$ (or 0).



Outline

20/42– Multifractional theories

G.C., JHEP **03** (2017) 138 [arXiv:1612.05632]

$$S[\phi^i] = \int d\varrho(x)\, \mathcal{L}[\mathcal{D}_x, \phi^i]\,, \qquad d\varrho(x) = \prod_\mu dq^\mu(x^\mu)$$

Same measure, different choices of Lagrangian symmetries ($v = \partial_x q$):

1. Weighted derivatives: $\mathcal{D}_x = v^{-1/2}\partial_x(v^{1/2}\,\cdot\,)$.
2. q-derivatives (multifractal): $\partial_q = v^{-1}\partial_x$.
3. Fractional derivatives (multifractal): $\mathbb{D}_x \sim \partial_x^\alpha$.

21/42– Status: 2012–2017

	$\mathcal{D}^2$	$\Box_q$	$\mathbb{D}^2$
Foundations	✓	✓	?
Relativistic mechanics	✓	✓	?
QFT and Standard Model	✓	✓	✓?
Perturbative renormalizability	✗	✗	✓?
Gravity and cosmology	✓	✓	?
Phenomenology: particles	✓	✓	?
Phenomenology: astrophysics	✓	✓	?
Phenomenology: inflation	?	✓	?
Phenomenology: dark energy	?	?	?

23/42– UV divergence, $\rho_d < \infty$: multifractional theory with q-derivatives

$$\zeta_D(s) = \zeta_{D,\alpha}(s) = \frac{\sigma_*^s}{(1-\alpha)(4\pi\sigma_*)^{\frac{D}{2}}} \frac{\Gamma\left[\frac{D-2s}{2(1-\alpha)}\right]\Gamma\left[\frac{2s-D\alpha}{2(1-\alpha)}\right]}{\Gamma\left(\frac{D}{2}\right)\Gamma(s)} \, .$$

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$$d_{\rm S}^{\rm IR} = D\,, \qquad d_{\rm S}^{\rm UV} = D\alpha$$

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$$d_S^{\rm IR} = D\,, \qquad d_S^{\rm UV} = D\alpha$$

$$\rho_d = \frac{\Gamma\left[\frac{d}{2(1-\alpha)}\right]\Gamma\left[-\frac{d\alpha}{2(1-\alpha)}\right]}{(1-\alpha)(4\pi\sigma_*)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)}, \qquad \alpha \neq \frac{2}{D}, \frac{4}{2+D}, \frac{6}{4+D}, \dots$$

24/42– Observational constraints

WEIGHTED DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
$\alpha_{\rm QED}$ quasars	—	—	—	G.C., Magueijo, Rodríguez, PRD 2014
CMB black body	—	—	—	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
Lamb shift	$< \mathbf{10^{-23}}$	$< 10^{-14}$	$> 10^{7}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
$\alpha_{\rm QED}$ measurements	$< \mathbf{10^{-26}}$	$< 10^{-17}$	$> 10^{10}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
GWs and GRBs	—	—	—	G.C., EPJC 2017

q-DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
primordial CMB (!)	weak	weak	weak	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
CMB black body	—	—	—	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
Muon lifetime	$< \mathbf{10^{-13}}$	$< 10^{-5}$	$> 10^{-3}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
Lamb shift	$< 10^{-23}$	$< 10^{-15}$	$> \mathbf{10^{7}}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
$\alpha_{\rm QED}$ measurements	—	—	—	G.C., Nardelli, Rodríguez, PRD 2016(b)
GWs	$< 10^{-22}$	$< 10^{-14}$	$> \mathbf{10^{7}}$	G.C., EPJC 2017
GRBs $\sim$	$< 10^{-32}$	$< 10^{-24}$	$> \mathbf{10^{26}}$	G.C., EPJC 2017

25/42– q-derivatives: gravity

G.C., JCAP **12** (2013) 041 [arXiv:1307.6382]

$$ {}^q\Gamma^\rho_{\mu\nu} := \tfrac{1}{2} g^{\rho\sigma}\left(\frac{1}{v_\mu}\partial_\mu g_{\nu\sigma} + \frac{1}{v_\nu}\partial_\nu g_{\mu\sigma} - \frac{1}{v_\sigma}\partial_\sigma g_{\mu\nu}\right) , $$

$$ {}^qR^\rho{}_{\mu\sigma\nu} := \frac{1}{v_\sigma}\partial_\sigma {}^q\Gamma^\rho_{\mu\nu} - \frac{1}{v_\nu}\partial_\nu {}^q\Gamma^\rho_{\mu\sigma} + {}^q\Gamma^\tau_{\mu\nu}\,{}^q\Gamma^\rho_{\sigma\tau} - {}^q\Gamma^\tau_{\mu\sigma}\,{}^q\Gamma^\rho_{\nu\tau} . $$

Action:

$$ S = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\, v\,\sqrt{-g}\,({}^qR - 2\Lambda) + S_\mathrm{m} . $$

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Action:

$$ S = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\, v\,\sqrt{-g}\,({}^qR - 2\Lambda) + S_{\rm m}\,. $$

Einstein equations:

$$ {}^qR_{\mu\nu} - \frac{1}{2} g_{\mu\nu}({}^qR - 2\Lambda) = \kappa^2\,{}^qT_{\mu\nu}\,. $$

26/42– q-derivatives: local inertial frames

- LIF: centered on the observer, locally isomorphic to multifractional Minkowski spacetime, q-Poincaré transformations

$$q'^{\mu}(x^{\mu}) = \Lambda_{\nu}^{\ \mu} q^{\nu}(x^{\nu}) + a^{\mu}$$

Each and every LIF has its own anomalous geometry $q(x)$.

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- Relational measurements. Choosing measurement units is a convention. Measuring a size or a velocity by an observer in a multifractal world is not different from measuring them in a normal non-anomalous world: $v_x = \Delta x/\Delta t$ vs. $v_q = \Delta q(x)/\Delta q(t)$.

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- Relational measurements. Choosing measurement units is a convention. Measuring a size or a velocity by an observer in a multifractal world is not different from measuring them in a normal non-anomalous world: $v_x = \Delta x/\Delta t$ vs. $v_q = \Delta q(x)/\Delta q(t)$. Measurements of **dimensionless** observables do discriminate! $v_x(O_1)/v_x(O_2)$ vs. $v_q(O_1)/v_q(O_2)$ where $scale(O_1) \ll scale(O_2)$.

27/42– FRW cosmology

$$\frac{H^2}{v^2} = \frac{\kappa^2}{3}\rho + \frac{\Lambda}{3} - \frac{\mathsf{K}}{a^2}\,, \qquad \dot\rho + 3H(\rho + P) = 0$$

$$\rho = \frac{\dot\phi^2}{2v^2} + V(\phi)$$

Ordinary slow-roll approximation unnecessary.

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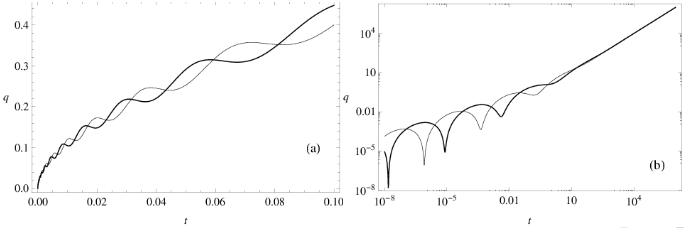
$$\rho = \frac{\dot\phi^2}{2v^2} + V(\phi)$$

Ordinary slow-roll approximation unnecessary. **Cyclic** universe



28/42– Inflationary spectra

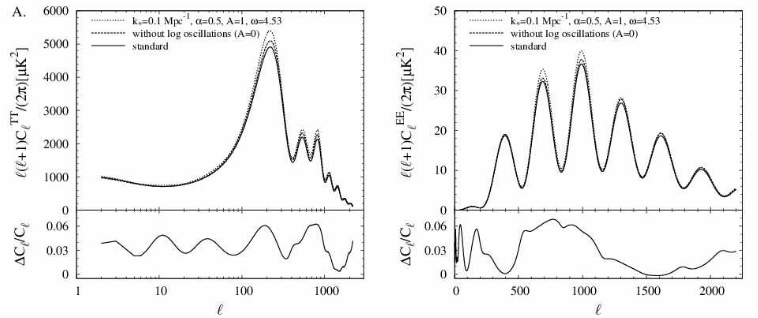
General behaviour (from $p(k)=1/q(1/k)$):

$$P_{\rm s}=\mathcal{A}_{\rm s}\tilde{k}^{n_{\rm s}-1}\sim\mathcal{A}_{\rm s}\left(\frac{k}{k_*}\right)^{\alpha(n_{\rm s}-1)}[F_\omega(\ln k)]^{1-n_{\rm s}}\,.$$

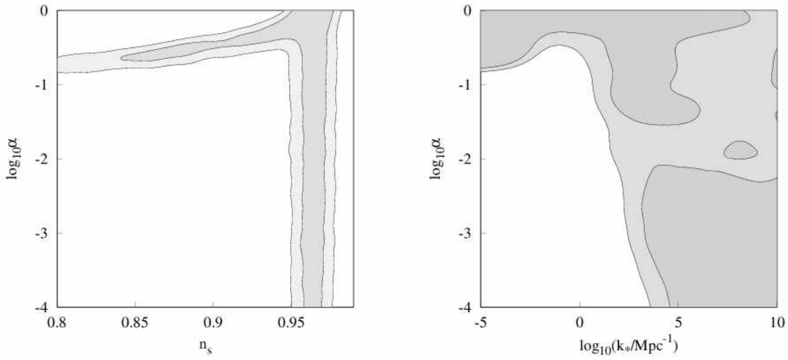
Scale invariance without strong slow-roll approximation and a log-oscillating pattern.
$\rightarrow$ Spacetime discrete at scales $\sim\ell_\infty$ (totally disconnected?).
Visible effect of this geometry: not "holes" in the fabric of spacetime but a logarithmic modulation of the power spectrum of primordial fluctuations!

29/42– CMB spectra

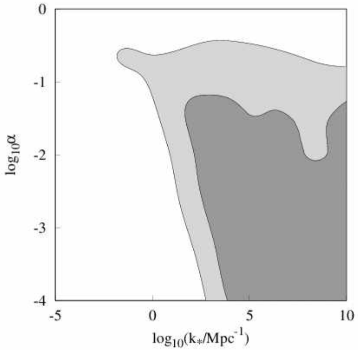
G.C., Kuroyanagi, Tsujikawa, JCAP **08** (2016) 039



30/42– 2D contours without log-oscillations



31/42– 2D contours with log-oscillations



E.g. $N = 4$: **Upper bound** $\alpha < 0.1$. In general, $\alpha \lesssim 0.1 - 0.6$.

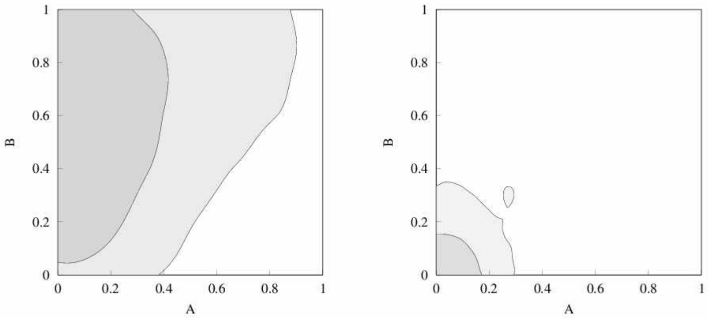
32/42– Consequences for spacetime dimension

The Hausdorff dimension of space $d_{\rm H}^{\rm space}=3\alpha$ in the UV cannot exceed

$N=2:$	$d_{\rm H}^{\rm space}\lesssim 0.3$	(UV)
$N=3:$	$d_{\rm H}^{\rm space}\lesssim 1.9$	(UV)
$N=4:$	$d_{\rm H}^{\rm space}\lesssim 1.7$	(UV)

Counter-intuitive!

33/42– 2D contours with log-oscillations



Example: $N = 2$, $\alpha = 0.1, 0.5$

34/42– Upper bounds on A_1, B_1

Amplitudes of log oscillations of geometry cannot exceed

$$\begin{aligned} N=2: &\qquad A<0.3,\ B<0.4\\ N=3: &\qquad A<0.3,\ B<0.2\\ N=4: &\qquad A<0.4,\ B<1.0 \end{aligned}$$

First constraints of this kind.

35/42– Beyond first harmonic

G.C., arXiv:1705.01619; G.C., Ronco, arXiv:1706.02159

Parametrization inspired by critical systems [Gluzman, Sornette 2002]:

$$A_n = 2\xi \frac{\mathrm{e}^{-\gamma n}}{n^u} \cos(\psi_n + \beta_n), \quad B_n = -2\xi \frac{\mathrm{e}^{-\gamma n}}{n^u} \sin(\psi_n + \beta_n), \quad \beta_n := n\omega \frac{\ell_\infty}{\ell_*}.$$

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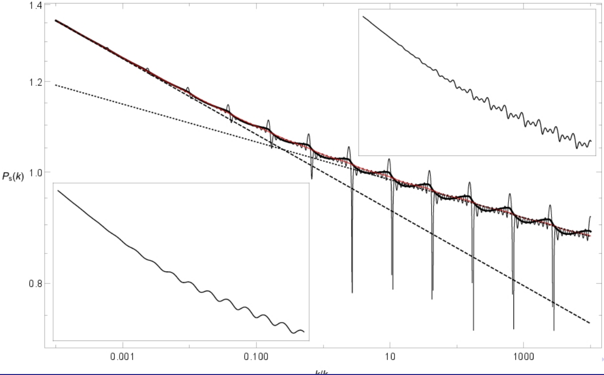
Nowhere differentiable measure (stochastic spacetime): fast-varying (ergodic mixing) phases

$$\psi_n = \Omega n \ln(\Omega n)\,, \qquad \psi_n = \Omega n^2\,, \qquad \psi_n = \Omega e^{n/\Omega}\,,$$

36/42– Beyond first harmonic

G.C., arXiv:1705.01619; G.C., Shafieloo, *in progress*

Discrete Scale Invariance:
imprint of complex dimensions in the CMB



37/42– Summary: future lines

Physical origin of a finite entropy density:

- Main agent is dimensional flow. But not sufficient (compact-momentum-space and nonlocal examples).
- Role of discreteness is less clear, but it might turn out to be a liability rather than an asset (LQG/GFT example).
- String theory and the multifractional example show that discreteness is not necessary.

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Multifractional theories:

- Full study of the theory with fractional derivatives.

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Multifractional theories:

- Full study of the theory with fractional derivatives.

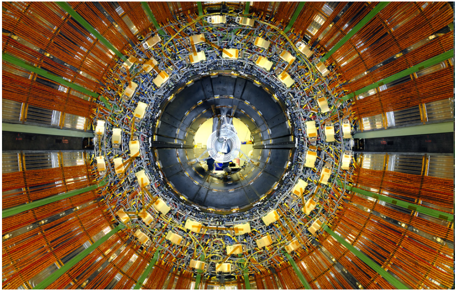
Cosmology with q-derivatives:

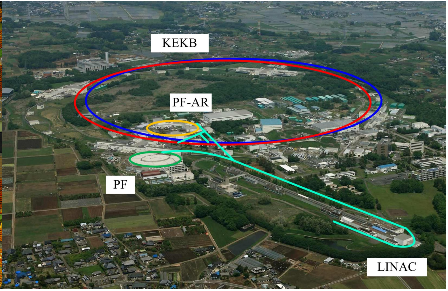
- Theory and phenomenology fully studied: only dark energy is missing!
- Geometry cannot sustain inflation but it could act as dark energy.

Outline

38/42– **Accelerators**

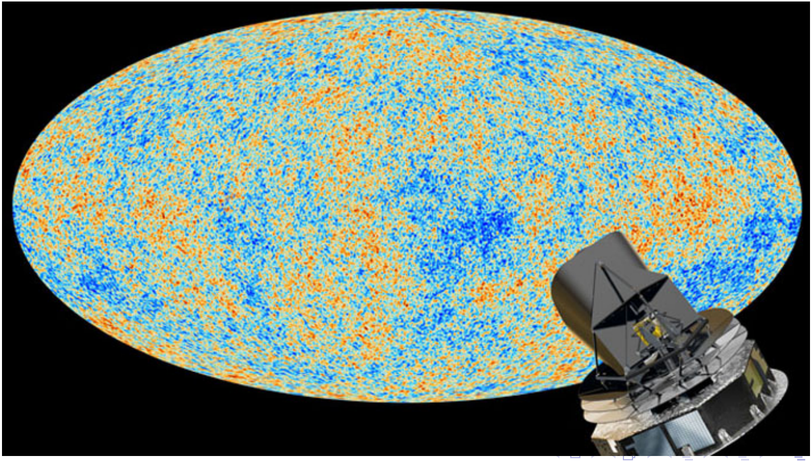
LHC (ongoing), KEK (ongoing)





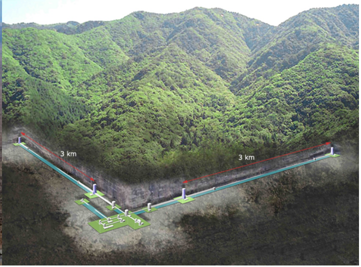
39/42– CMB and polarization

PLANCK (finished), LiteBIRD (2022)



40/42– Gravitational waves

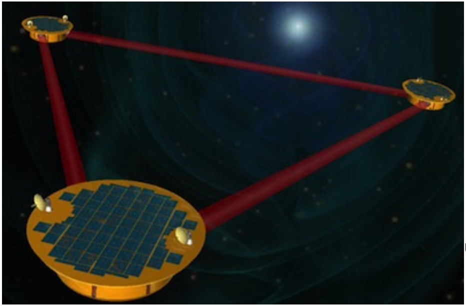
aLIGO (ongoing), KAGRA (2018) (ground-based)

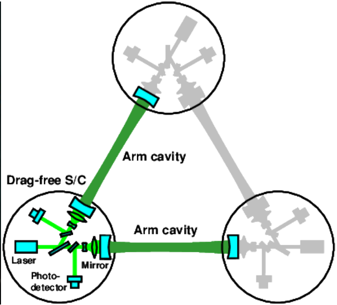


41/42–

Gravitational waves

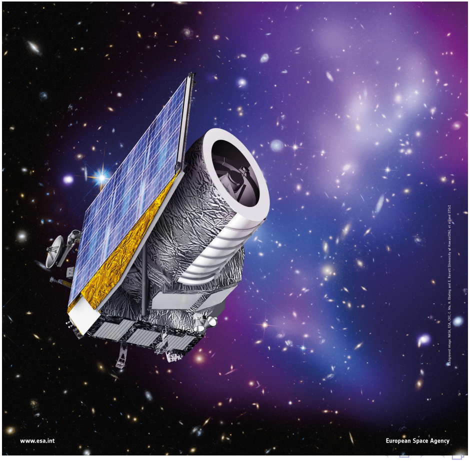
eLISA (2034), DECIGO (proposed) (space-borne)





42/42– Dark energy

Euclid (2020)



どうもありがとうございました！

Thank you!

¡Muchas gracias!

Grazie!

Muito obrigado!

Danke schön!