

Detecting quantum gravity in the sky

Status of multifractional theories

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01/38– Universal features in QG

Many theories of quantum gravity: string theory, perturbative QG, asymptotic safety, CDT, HL gravity, noncommutative spacetimes, nonlocal gravity, LQG, spin foams, GFT, All different, but with **something in common**:

01/38– Universal features in QG

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- Many of them predict a **fuzzy spacetime**: intrinsic uncertainty in measurements of times and distances.

01/38– Universal features in QG

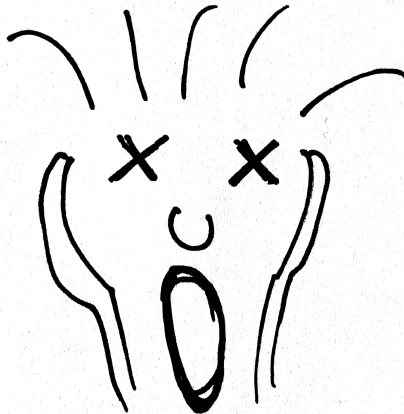
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- Many of them predict a **fuzzy spacetime**: intrinsic uncertainty in measurements of times and distances.
- **All QGs are multiscale**. **Dimensional flow**: Changing behaviour of correlation functions, spacetime with scale-dependent dimension d . $d < 4$ in the UV. **Universal** feature in QG [’t Hooft 1993; Carlip 2009; G.C. PRL 2010].

01/38– Universal features in QG

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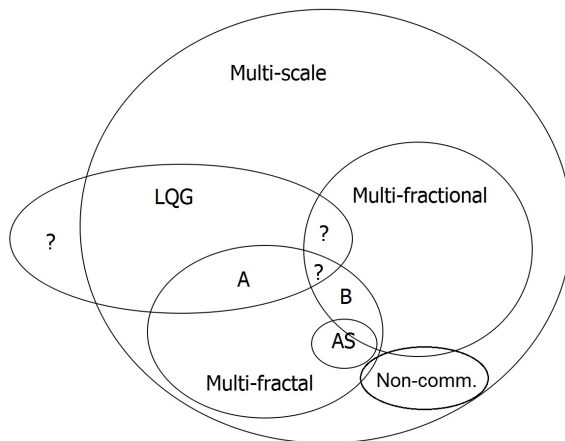


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02/38– Landscape of quantum gravities

G.C. EPJC **76** (2016) 181 [arXiv:1602.01470]





04/38— Two questions

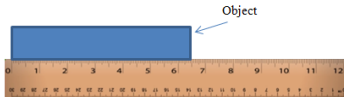
- Is it possible to study all QGs simultaneously and extract useful information from their universal features?

04/38— Two questions

- Is it possible to study all QGs simultaneously and extract useful information from their universal features?
- Are dimensional flow and fuzziness **observable**?

05/38– Measuring distances and times

Newton: comparing an object with another one and the duration of an event with that of another one.

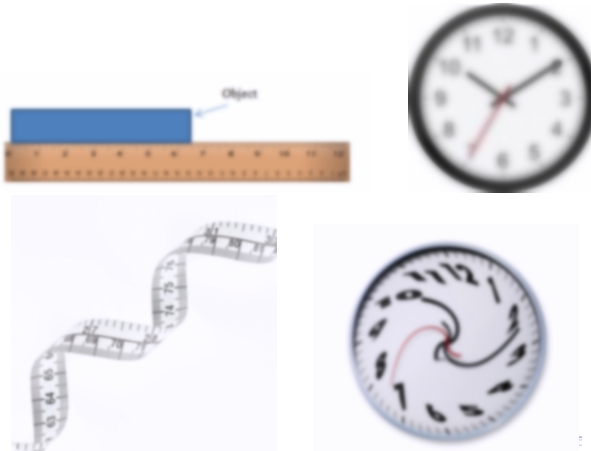


05/38– Measuring distances and times

Einstein: there are no absolute rulers and clocks.



06/38– Measuring distances and times with uncertainties: blur effect



07/38– Is this it?

We do **not** see blurred contours. Information on the **scales** at which spacetime is fuzzy still missing.



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08/38– Fuzziness in QG

- Spacetimes with noncommutative geometry (coordinates).
- Asymptotic safety (minimal resolution) [Reuter & Schwindt 2006,2007].
- LQG (discreteness of distance/area/volume operator spectra).
- Phenomenological models [Modesto & Nicolini 2010].

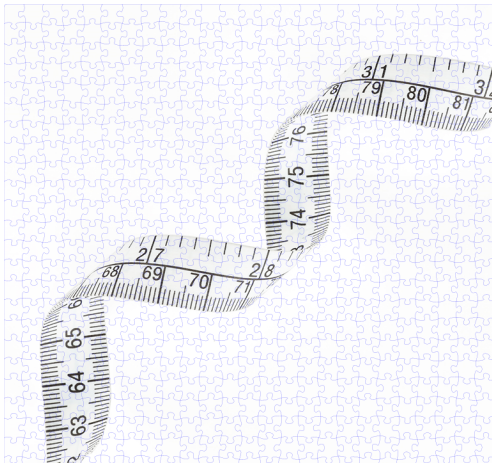
09/38– What is the origin of fuzziness?

Quanta of geometry always fluctuating.



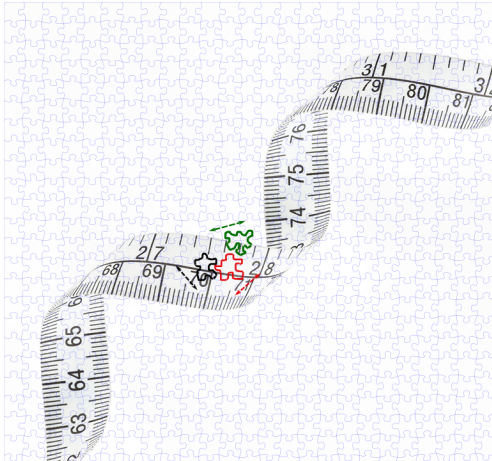
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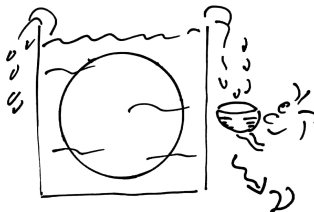
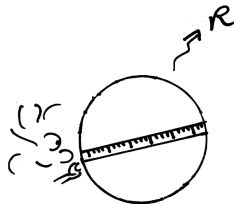
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10/38– Hausdorff dimension d_H

Scaling of the volume \mathcal{V} of a ball w.r.t. its radius: $\mathcal{V}(R) \sim R^{d_H}$.

① Measure the radius.

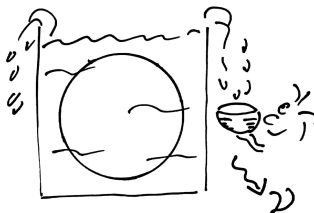
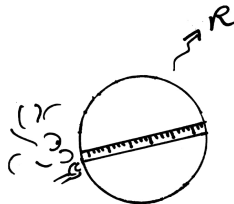


$$R^{d_H} \sim \mathcal{V}$$

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- ① Measure the radius.
- ② Measure the volume.

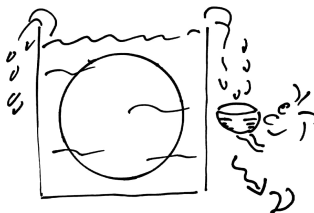
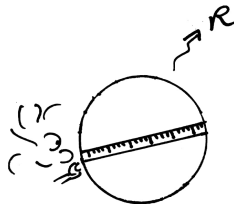


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10/38– Hausdorff dimension d_H

Scaling of the volume \mathcal{V} of a ball w.r.t. its radius: $\mathcal{V}(R) \sim R^{d_H}$.

- 1 Measure the radius.
- 2 Measure the volume.
- 3 Calculate d_H .



$$R^{d_H} \sim \mathcal{V}$$

11/38– Hausdorff and spectral dimension

Hausdorff dimension d_H Scaling of the volume \mathcal{V} of a ball (cube)

$$d_H := \frac{d \ln \mathcal{V}(\ell)}{d \ln \ell}, \quad \mathcal{V} \sim \ell^{d_H}$$

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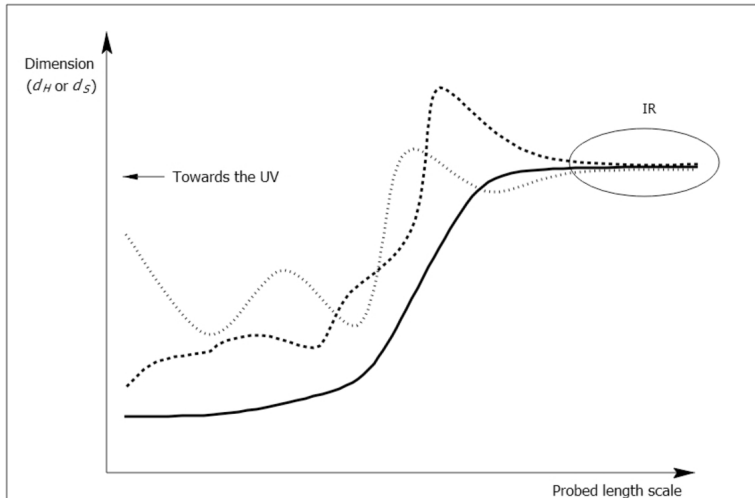
Spectral dimension d_S

Probing spacetime with resolution $1/\ell$: $(\partial_{\ell^2} - \square)K(x, x'; \ell) = 0$,
 $\mathcal{P}(\ell) = \int d^D x K(x, x; \ell)$.

Scaling of the return probability \mathcal{P}

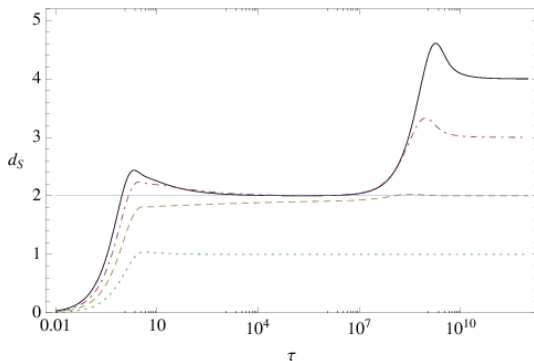
$$d_S := -\frac{d \ln \mathcal{P}(\ell)}{d \ln \ell}, \quad \mathcal{P} \sim \ell^{-d_S}$$

12/38– Dimensional flow



13/38– LQG, spin foams, GFT

General states $|\psi\rangle = \sum_{j,c} a_{j,c} |j, c\rangle$. Discreteness combinatorial effects drive d_S to zero [G.C., Oriti, Thürigen 2014, 2015], hence $\rho_d = \infty$ (or 0).



14/38– ABC of multiscale spacetimes

G.C. EPJC 2016 [arXiv:1602.01470]

- A. **Dimensional flow** occurs: [A1] At least two of the dimensions d_H , d_S , and d_W vary. [A2] Flow is continuous from the IR to a UV cut-off. [A3] Flow occurs locally (prevents false positive).

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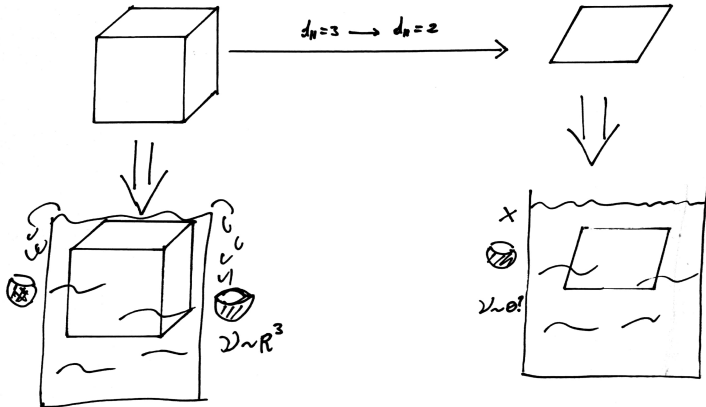
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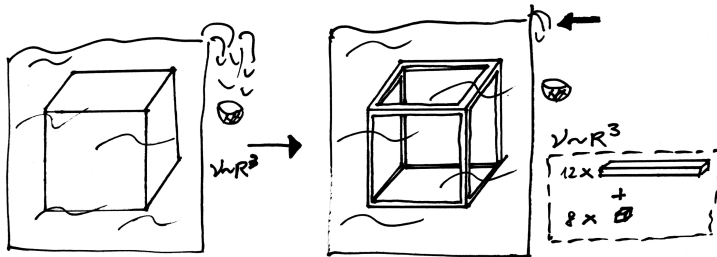
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- D. **Strongly multifractal spacetime**: if, in addition of satisfying A–C, it is nowhere differentiable.

15/38– Dimensional flow: pressing cubes?

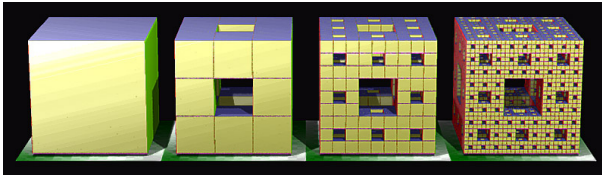


16/38– Dimensional flow: carving out cubes?



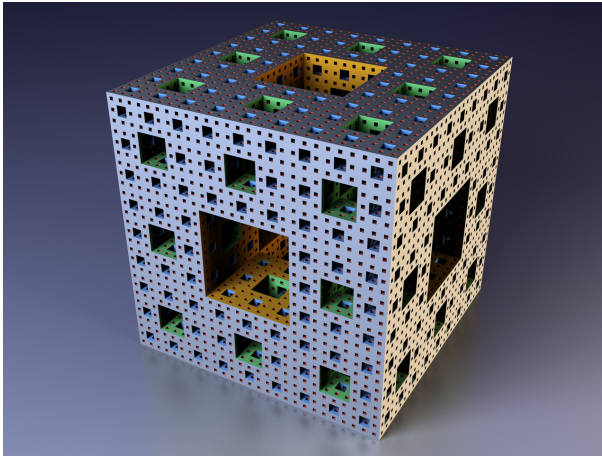
17/38– Dimensional flow: fractals?

Menger sponge:

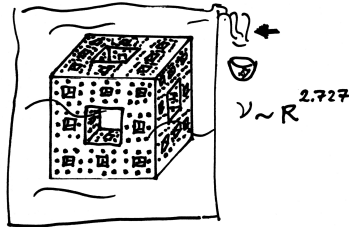


17/38– Dimensional flow: fractals?

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18/38– Dimensional flow: fractals?



19/38– Is this it?

- We do **not** measure fractal cubes. Information on the **scales** at which spacetime is fractal still missing.

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- We do **not** measure fractal cubes. Information on the **scales** at which spacetime is fractal still missing.
- **Multi-fractals**: fractal geometry with a given dimension at certain scales, but “smooth” at large scales.

20/38– Dimensions near the IR

$$d(\ell) \simeq D + b \left(\frac{\ell_*}{\ell} \right)^c + (\log \text{ oscillations}), \quad d = d_H, d_S$$

	D	b_H	c_H	b_S	c_S
Asymptotic safety	4	0	—	< 0	> 0
CDT	4	0	—	< 0	2
Near black holes	D	0	—	$\frac{D+1}{2}$	2
Nonlocal gravity and string field theory	D	0	—	< 0	2
Fuzzy spacetimes	D	0	—	$-D$	2
Gravity with quantum particles	3	0	—	$-\frac{21}{16}$	2
κ -Minkowski bicovariant ∇^2 , AN(3)	4	0	—	-2	2
κ -Minkowski bicovariant ∇^2 , AN(2)	3	0	—	$-\frac{3}{2}$	2
κ -Minkowski bicrossproduct ∇^2	4	0	—	1	2
κ -Minkowski cyclic invariance (o.s.)	D	< 0	1	?	?
Hořava–Lifshitz gravity	D	0	—	< 0	> 0
GFT, spin foams, LQG (o.s.)	$D(= 4)$	< 0	2	> 0	2

21/38– Flow-equation theorem

G.C., PRD **95** (2017) 064057 [arXiv:1609.02776]

Varying d_H

If the Hausdorff dimension of spacetime changes with the probed scale, and if it does so slowly at large scales, then the D -volume is uniquely parametrized as

$$\mathcal{V}(\ell) \simeq \ell^D + \left| \frac{\ell}{\ell_*} \right|^{D\alpha} F_\omega(\ell), \quad F_\omega(\ell) = 1 + \sum_{n>0} F_n(\ell)$$

$$F_n(\ell) = A_n \cos \left(n\omega \ln \frac{\ell}{\ell_\infty} \right) + B_n \sin \left(n\omega \ln \frac{\ell}{\ell_\infty} \right)$$

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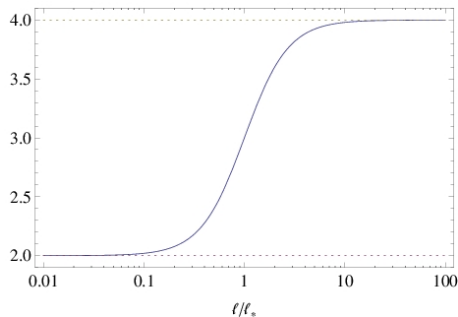
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Parameters $\alpha, \omega, \ell_*, \ell_\infty, A_n, B_n$ determined by the **dynamics**.
Similar statement for the spectral dimension d_S .

22/38– Property 1: multiscaling

Scaling property: $\mathcal{V}(\lambda\ell) \sim \lambda^{D\alpha} \mathcal{V}(\ell) \Rightarrow d_H = D\alpha$



23/38– Property 2: discreteness

G.C., JHEP **01** (2012) 065 [arXiv:1107.5041]; PRD **96** (2017) 046001
[arXiv:1705.01619]

Oscillatory part of $\mathcal{V}(\ell) = \ell^D + \ell^{D\alpha} F_\omega(\ell)$ invariant under a **DSI**:

$$F_\omega(\lambda_\omega^n \ell) = F_\omega(\ell), \quad \lambda_\omega = \exp(-2\pi/\omega), \quad n = 0, 1, 2, \dots$$

∞ **many scales** $\lambda_\omega^{\pm n} \ell_\infty$, departure from usual UV/IR dichotomy.

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DSIs appear in fractals and in **complex** and **critical** systems
(earthquakes, financial crashes, ...) [Sornette 1998].

$$\omega = d_{\mathbb{H}\mathbb{C}} \quad \text{complex dimension!}$$

Can we observe complex dimensions?

24/38– What is the origin of spacetime “fractality”?

- **Good question!** It seems it is generated by the same mechanism of fuzziness [Amelino-Camelia, G.C. & Ronco, PLB 2017, arXiv:1705.04876; G.C. & Ronco, NPB 2017, arXiv:1706.02159].

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- Conjecture: they are two ways of describing the same phenomenon.

25/38– Multifractional theories

G.C., JHEP **03** (2017) 138 [arXiv:1612.05632]

$$S[\phi^i] = \int d\varrho(x) \mathcal{L}[\mathcal{D}_x, \phi^i], \quad d\varrho(x) = \prod_{\mu} dq^{\mu}(x^{\mu})$$

Same measure, different choices of Lagrangian symmetries

($v = \partial_x q$):

- ① Weighted derivatives: $\mathcal{D}_x = v^{-1/2} \partial_x (v^{1/2} \cdot)$.
- ② q -derivatives (multifractal): $\partial_q = v^{-1} \partial_x$.
- ③ Fractional derivatives (multifractal): $\mathbb{D}_x \sim \partial_x^{\alpha}$.

26/38– Status: 2012–2017

	\mathcal{D}^2	∂_q^2	\mathbb{D}^2
Foundations	✓	✓	?
Relativistic mechanics	✓	✓	?
QFT and Standard Model	✓	✓	✓?
Perturbative renormalizability	✗	✗	✓?
Black holes	✓	✓	?
Gravity and cosmology	✓	✓	?
Phenomenology: particles	✓	✓	?
Phenomenology: astrophysics	✓	✓	?
Phenomenology: inflation	?	✓	?
Phenomenology: dark energy	?	?	?

27/38– Observational constraints ($\alpha \ll 1$)

WEIGHTED DERIVATIVES	t_* (s)	ℓ_* (m)	E_* (GeV)	
Muon lifetime	—	—	—	G.C., Nardelli, Rodríguez, PRD 2016(b)
Lamb shift	$< 10^{-23}$	$< 10^{-14}$	$> 10^{-2}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
α_{QED} measurements	$< 10^{-26}$	$< 10^{-18}$	$> 10^1$	G.C., Nardelli, Rodríguez, PRD 2016(b)
$\Delta\alpha_{\text{QED}}/\alpha_{\text{QED}}$ quasars	$< 10^{11}$	$< 10^{20}$	$> 10^{-37}$	G.C., Magueijo, Rodríguez, PRD 2014
GWs, GRBs	—	—	—	G.C., EPJC 2017
Cherenkov radiation	—	—	—	G.C., JHEP 2017
CMB black-body and primordial spectra	—	—	—	—
q -DERIVATIVES	t_* (s)	ℓ_* (m)	E_* (GeV)	
Muon lifetime	$< 10^{-11}$	$< 10^{-3}$	$> 10^{-13}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
Lamb shift	$< 10^{-21}$	$< 10^{-13}$	$> 10^{-4}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
α_{QED} measurements	—	—	—	G.C., Nardelli, Rodríguez, PRD 2016(b)
$\Delta\alpha_{\text{QED}}/\alpha_{\text{QED}}$ quasars	—	—	—	G.C., Magueijo, Rodríguez, PRD 2014
GWs*	$< 10^{-22}$	$< 10^{-14}$	$> 10^{-2}$	G.C., EPJC 2017, JHEP 2017
GRBs*	$< 10^{-39}$	$< 10^{-30}$	$> 10^{14}$	G.C., EPJC 2017, JHEP 2017
Cherenkov radiation*	$< 10^{-57}$	$< 10^{-49}$	$> 10^{33}$	G.C., JHEP 2017
CMB black-body spectrum	—	—	—	—
primordial CMB (!)	weak	weak	weak	G.C., Kuroyanagi, Tsujikawa, JCAP 2016

* Strongest bounds are avoidable in the “stochastic” version of the theory.

28/38– Theory with q -derivatives: gravity

G.C., JCAP **12** (2013) 041 [arXiv:1307.6382]

$${}^q\Gamma_{\mu\nu}^{\rho} := \frac{1}{2}g^{\rho\sigma} \left(\frac{1}{v_{\mu}}\partial_{\mu}g_{\nu\sigma} + \frac{1}{v_{\nu}}\partial_{\nu}g_{\mu\sigma} - \frac{1}{v_{\sigma}}\partial_{\sigma}g_{\mu\nu} \right) ,$$

$${}^qR^{\rho}_{\mu\sigma\nu} := \frac{1}{v_{\sigma}}\partial_{\sigma}{}^q\Gamma_{\mu\nu}^{\rho} - \frac{1}{v_{\nu}}\partial_{\nu}{}^q\Gamma_{\mu\sigma}^{\rho} + {}^q\Gamma_{\mu\nu}^{\tau}{}^q\Gamma_{\sigma\tau}^{\rho} - {}^q\Gamma_{\mu\sigma}^{\tau}{}^q\Gamma_{\nu\tau}^{\rho} .$$

Action:

$$S = \frac{1}{2\kappa^2} \int d^Dx \, v \sqrt{-g} ({}^qR - 2\Lambda) + S_m .$$

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Einstein equations:

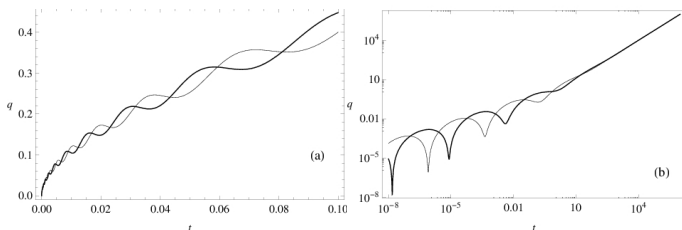
$${}^qR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}({}^qR - 2\Lambda) = \kappa^2 {}^qT_{\mu\nu} .$$

29/38– FRW cosmology

$$\frac{H^2}{v^2} = \frac{\kappa^2}{3} \rho, \quad \dot{\rho} + 3H(\rho + P) = 0$$

$$\rho = \frac{\dot{\phi}^2}{2v^2} + V(\phi)$$

Ordinary **slow-roll** approximation **relaxed**. **Cyclic** universe



30/38– Inflationary spectra

General behaviour (from $p(k) = 1/q(1/k)$):

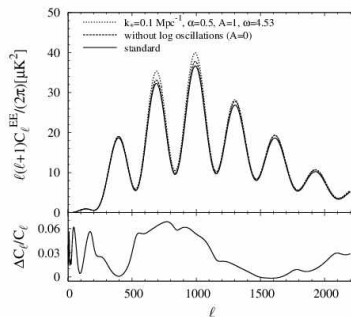
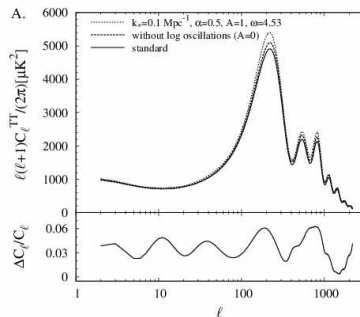
$$P_s = \mathcal{A}_s \tilde{k}^{n_s-1} \sim \mathcal{A}_s \left(\frac{k}{k_*} \right)^{\alpha(n_s-1)} [F_\omega(\ln k)]^{1-n_s}.$$

Scale invariance without strong slow-roll approximation and a log-oscillating pattern.

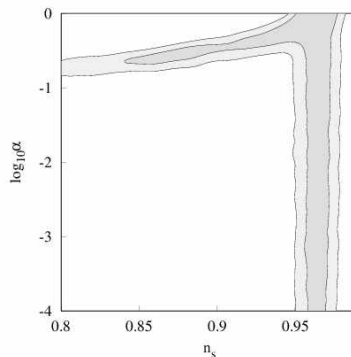
→ Spacetime discrete at scales $\sim \ell_\infty$ (totally disconnected?).

Visible effect of this geometry: not “holes” in the fabric of spacetime but a logarithmic modulation of the power spectrum of primordial fluctuations.

31/38– CMB spectra

G.C., Kuroyanagi, Tsujikawa, JCAP **08** (2016) 039 [arXiv:1606.08449]

32/38– Likelihood contours

G.C., Kuroyanagi, Tsujikawa, JCAP **08** (2016) 039 [arXiv:1606.08449]

33/38– Estimates on d^{UV} and bounds on amplitudes

$$\omega = \frac{2\pi\alpha}{\ln N}, \quad N = 2, 3, 4, \dots$$

The Hausdorff dimension of space $d_{\text{H}}^{\text{space}} = 3\alpha$ in the UV cannot exceed

$$\begin{aligned} N = 2 : \quad d_{\text{H}}^{\text{space}} &\lesssim 0.3 & (\text{UV}) \\ N = 3 : \quad d_{\text{H}}^{\text{space}} &\lesssim 1.9 & (\text{UV}) \\ N = 4 : \quad d_{\text{H}}^{\text{space}} &\lesssim 1.7 & (\text{UV}) \end{aligned}$$

Amplitudes of **log oscillations of geometry** cannot exceed

$$\begin{aligned} N = 2 : \quad A_1 &< 0.3, B_1 < 0.4 \\ N = 3 : \quad A_1 &< 0.3, B_1 < 0.2 \\ N = 4 : \quad A_1 &< 0.4, B_1 < 1.0 \end{aligned}$$

34/38– Beyond first harmonic

G.C., G.C., PRD **96** (2017) 046001 [arXiv:1705.01619]; G.C., Ronco, NPB **923** (2017) 144 [arXiv:1706.02159]

Parametrization inspired by critical systems [Gluzman, Sornette 2002]:

$$A_n = 2\xi \frac{e^{-\gamma n}}{n^u} \cos(\psi_n + \beta_n), \quad B_n = -2\xi \frac{e^{-\gamma n}}{n^u} \sin(\psi_n + \beta_n), \quad \beta_n := n\omega \frac{\ell_\infty}{\ell_*}.$$

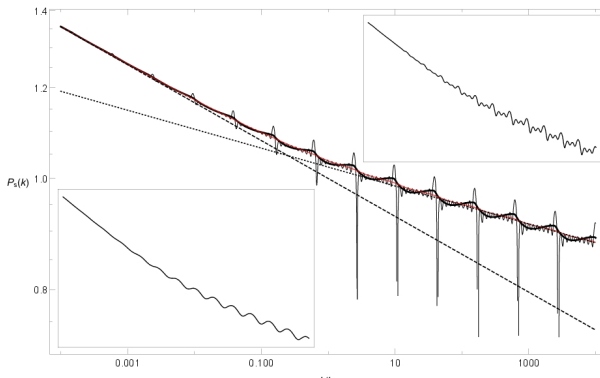
Nowhere differentiable measure (stochastic spacetime):
fast-varying (ergodic mixing) phases

$$\psi_n = \Omega n \ln(\Omega n), \quad \Omega n^2, \quad \Omega e^{n/\Omega}$$

35/38– Beyond first harmonic

G.C., PRD **96** (2017) 046001 [arXiv:1705.01619]; G.C., Shafieloo, *in progress*

Discrete Scale Invariance:
imprint of **complex dimensions** in the CMB



36/38– Late-time acceleration from geometry?

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- q -derivatives:

$$\frac{H^2}{v^2} = \frac{\kappa^2}{3} \rho, \quad \frac{\ddot{a}}{a} = -\frac{\kappa^2}{6} v^2 (\rho + 3P) + H \frac{\dot{v}}{v}.$$

$v \simeq 1 + |t/t_*|^{\alpha-1}$, $H\dot{v}/v \propto (\alpha - 1)|t/t_*|^{\alpha-2} < 0$, $\ddot{a} = 0$ for $w = P/\rho < -1/3$. But measure factors $1/v^2 < 1$ in kinetic terms, potentials can dominate in not-so-strong SR.

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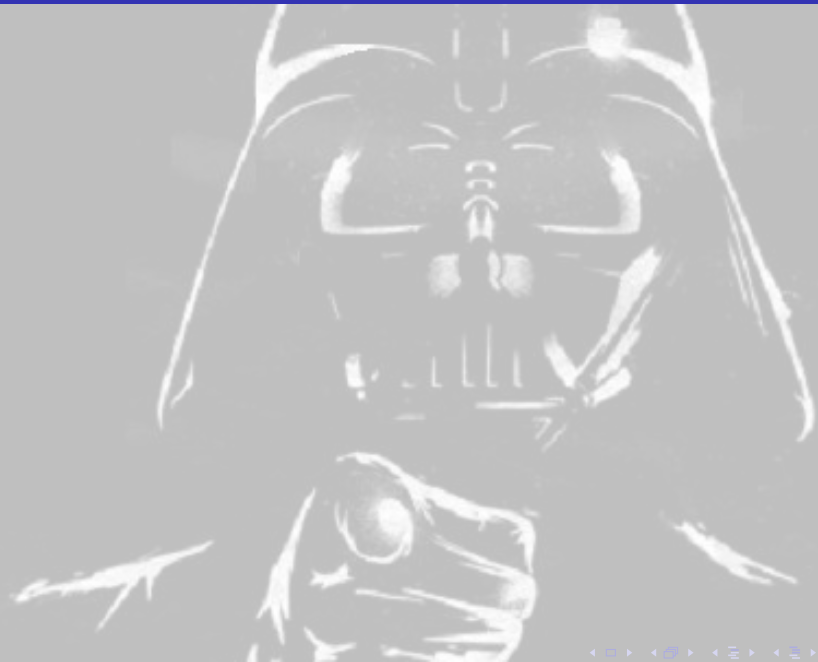
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- Weighted derivatives:**

$$H^2 = \frac{\kappa^2}{3} \rho + \frac{\Omega}{2} \frac{\dot{v}^2}{v^2} + \frac{U(v)}{6v}, \quad \frac{\ddot{a}}{a} = -\frac{\kappa^2}{6} (\rho + 3P) + \frac{U(v)}{6v}.$$

$\Omega = -3/2 + f(v) < 0$, $U(v) > 0$ “potential” for v determined by geometry and dynamics. Term $\propto \Omega(\dot{v}/v)^2 < 0$ like a phantom, term $\propto U/v > 0$ like a time-varying Λ .



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- Familiarity: cosmological equations (not phenomenology!) very similar to **ST theories**.
- Phenomenology: rich (also particle physics and astrophysics) and unusual.
- Falsifiability: no fine tuning allowed, very easy to falsify (easier than **ST theories**).

38/38—

Conclusions

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38/38—

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38/38– Conclusions

- Different theories of quantum gravity have common characteristics: fuzziness and dimensional flow.
- There are general results describing dimensional flow and linking this with fuzziness.
- Multifractional theories are the ideal testing ground to study these theoretical features and link them with **observations**.
- They display effects of anomalous geometry at all scales an in all fields of physics, some of which are yet to be explored. **Easily falsifiable!**

どうもありがとうございました！

Merci beaucoup!