Multi-photon scattering tomography with coherent states

Tomás Ramos and Juan José García-Ripoll
Instituto de Física Fundamental IFF-CSIC, Calle Serrano 113b, Madrid 28006, Spain

The achievement of strong coupling between quantum impurities and propagating photons at low dimensions [1–3] has spun a renewed interest in the theory of single- and multi-photon scattering. From earlier developments using wavefunction theory, the field has evolved into sophisticated calculations of one- and two-photon scattering matrices using input-output theory [4], path integral formalism [5], and has been even extended into the ultrastrong coupling regime [6]. Experiments, however, have been mostly constrained to estimates of the single-photon transmission and reflection coefficients [1] or cross-Kerr phases between coherent beams [7].

In this work we develop an experimental framework for determining the multi-photon scattering matrices of an arbitrary and possibly unknown point-like quantum impurity. To this end, we combine tools from scattering theory and homodyne detection in quantum optics, giving rise to an experimentally feasible method which does not require single-photon sources nor single-photon detectors.

In particular, we place the scatterer in a one-dimensional waveguide or channel (see Fig. 1), and we probe it with coherent state inputs of the form,

$$|\Psi_{\rm in}\rangle = |\alpha\rangle_{k_1,\ldots,k_m} = e^{-\frac{|\alpha|^2}{2}} e^{\sum_{\sigma=1}^{m}\alpha_\sigma A^\dagger_{k_\sigma}}|0\rangle, \qquad (1)$$

where $A^\dagger_{k_\sigma}$ are bosonic creation operators of a photon wave-packet centered at different momenta k_σ, with $\sigma = 1,\ldots,m$. In addition, α_σ are the coherent displacement coefficients for each momentum mode, with $|\alpha|^2 = \sum_{\sigma=1}^{m}|\alpha_\sigma|^2$, and $|0\rangle$ denotes the vacuum state. After the interaction with the scatterer, the photonic output state reads, $|\Psi_{\rm out}\rangle = S|\Psi_{\rm in}\rangle$, where the components of the scattering matrix read,

$$S_{p_1\cdots p_n k_1\cdots k_m} = \langle 0|A_{p_1}\cdots A_{p_n} S A^\dagger_{k_1}\cdots A^\dagger_{k_m}|0\rangle. \qquad (2)$$

They describe the probability amplitudes for transitions from m photons with momenta $k_1,\ldots,k_m$ to n photons with momenta $p_1,\ldots,p_n$. Notice that we do not impose any condition on the scattering matrix (besides unitarity $SS^\dagger = S^\dagger S = 1$ and $S|0\rangle = |0\rangle$ which always hold), allowing us to describe any multi-photon process in one-dimension, including the case when the number of photons is not conserved. To perform the detection of photons with different momenta $p_1,\ldots,p_n$, we pass the forwardly scattered photons through a multi-port beam splitter operation $U_{\rm BS}$ (see Fig. 1), which involves A_{p_1} and $n-1$ additional channels $A^2_{p_2},\ldots,A^n_{p_n}$, and thus allows us to measure quadratures of the form,

$$\langle A_{p_1}A^2_{p_2}\ldots A^n_{p_n}\rangle = \langle\Psi_{\rm out}|U^\dagger_{\rm BS}A_{p_1}A^2_{p_2}\ldots A^n_{p_n}U_{\rm BS}|\Psi_{\rm out}\rangle. \qquad (3)$$

Our main result is a relation between the output quadratures moments in Eq. (3) and the general scattering matrix elements in Eq. (2), with an error scaling as $\sim|\alpha|^2$ in the case of highly attenuated coherent state inputs, $|\alpha| \ll 1$, and thus can be made negligibly small.

This allows for the realization of a tomographic protocol to determine the multi-photon scattering matrix elements of an unknown quantum scatterer as shown in detail in our work. An additional feature of the scheme is that the measurement of the quadratures in Eq. (3) is insensitive to vacuum or amplification noise as it involves bosonic fields at independent channels. This is particularly useful for implementations with superconducting circuits as all noise from the amplifiers is canceled out without previous calibration. We close this work with a practical study of the protocol in the case of two- and three-level system scatterers, showing its feasibility under realistic experimental conditions such as losses and dephasing.

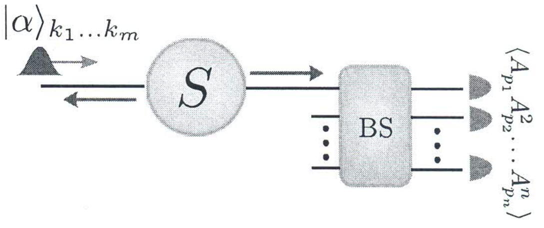

FIG. 1. A one-dimensional waveguide transports incoming photons that are transformed by the quantum impurity into right and left outgoing states, through a unitary transformation S, the scattering matrix. We show that by probing the system with attenuated coherent state wave-packets, $|\alpha| \ll 1$, in combination with homodyne detection at the output, it is possible to reconstruct the multi-photon components of S with negligible errors $\sim|\alpha|^2 \ll 1$.

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