The Relativistic Quantum Information North (RQIN 2017)

July 4th–7th, 2017
Yukawa Institute for Theoretical Physics, Kyoto University

On time and position in quantum theory

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Causal order is under quantum examination

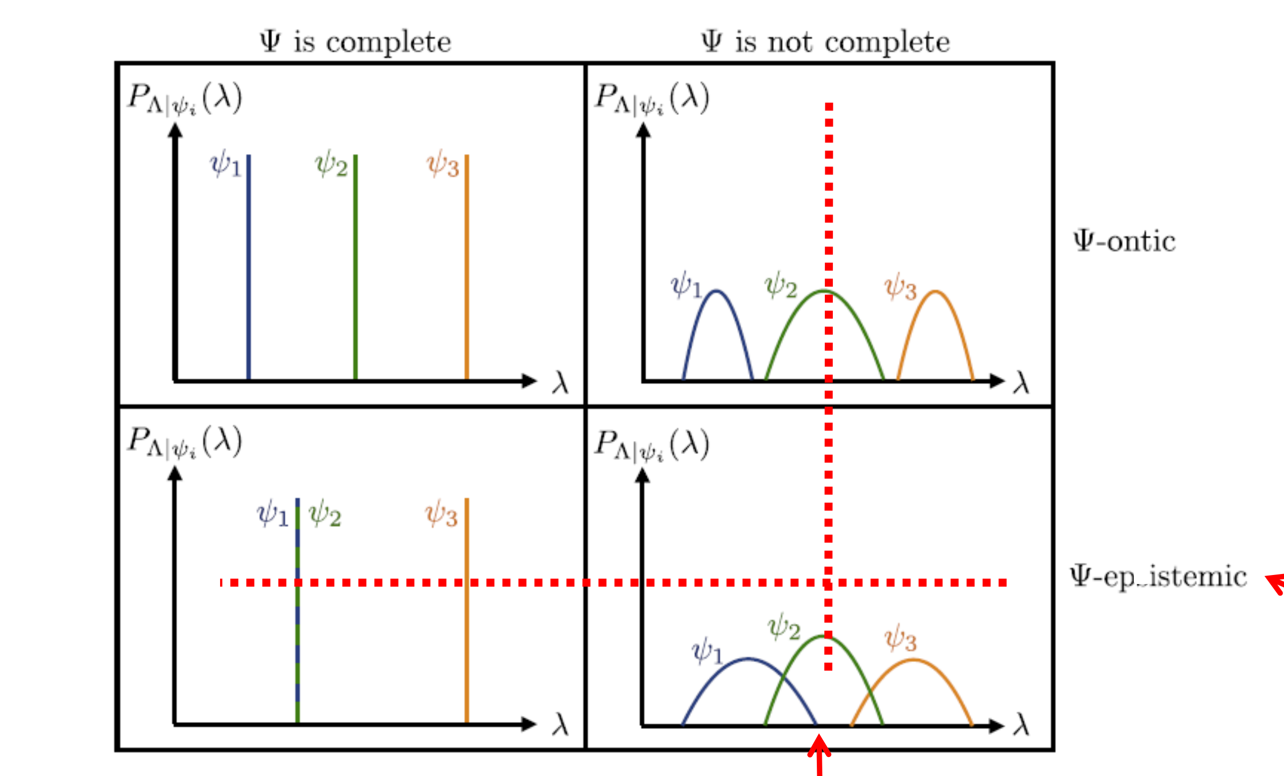
A problem with time

Quantum time

Measuring in space time local and non local operators

Different states for the same event

Quantum time again



Ψ is ontic and complete

Roger Colbeck and Renato Renner
New J. Phys. 19 (2017) 013016

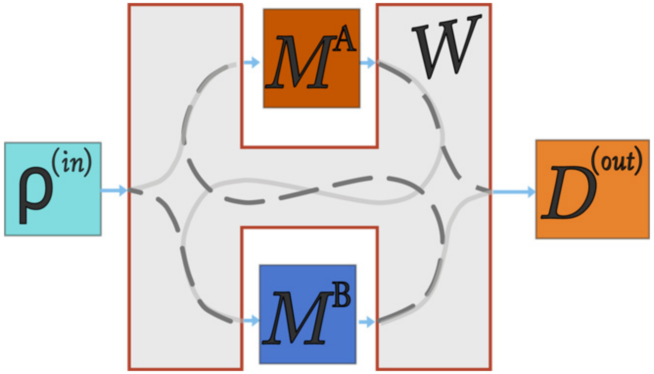
Roger Colbeck and Renato Renner
Nat. Commun. 2 411 (2011)

etc

$H \longrightarrow H_A \otimes H_B$

measurements → *projectors*

any experiment takes place in spacetime and therefore has a causal order



A process with indefinite causal order

by almost 7 SDs

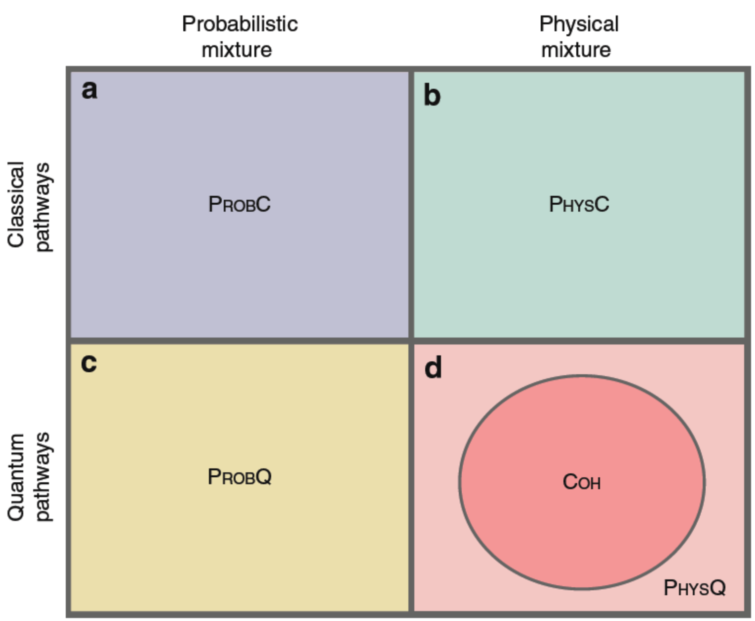
$$|w_{\text{SWITCH}}\rangle = \frac{1}{\sqrt{2}}(|w^{A\to B}\rangle|0\rangle^C + |w^{B\to A}\rangle|1\rangle^C)$$

$$W_{\text{SWITCH}} = \text{Tr}_{\mathcal{H}^{(\text{out})}}(|w_{\text{SWITCH}}\rangle\langle w_{\text{SWITCH}}|)$$

$$\text{Tr}(SW^{\text{n-sep}}) < 0$$

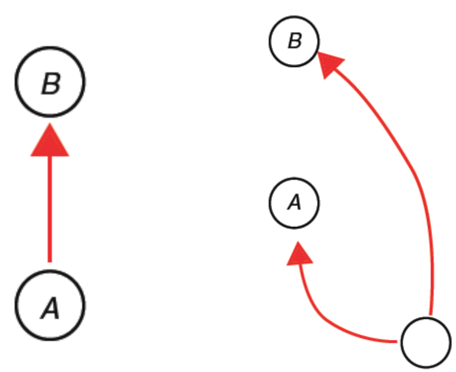
Giulia Rubino et al. Sci Adv 2017;3:e1602589

S decomposes in terms of operations made in the laboratory $\rho_z^{(\text{in})} \otimes M_{a,x}^A \otimes M_{b,y}^B \otimes D_d^{(\text{out})}$



J. W. MacLean et al, Nat. Comm. 8, 15149 (2017)

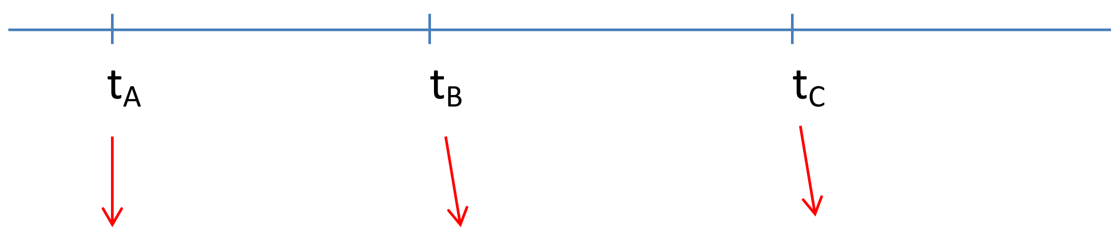
the causal map COH is intrinsically quantum in both the common-cause and cause-effect pathways and it exhibits a quantum Berkson effect.



Causal order:

1. If A happens before B on the same machine, then $A < B$
2. If A the sending of a message and B the reception of the same message, $A < B$
3. If $A < B$ and $B < C$ then $A < C$
4. For all A, $A \nless A$ … (or use $\leq$)

Quantum Mechanics has problems with this



Can there be $|\varphi>_{t_A}$ $|\varphi>_{t_b}$ $|\varphi>_{t_C}$ in $\mathcal{H}$?

$$|\Psi> \in \mathrm{P}(t_A, t_B) \Rightarrow \quad \mathrm{P}(t_A, t_B)|\Psi> = |\Psi>, \quad U_T|\Psi> \in \mathrm{P}(t_A + T, t_B + T)$$

$U_T = e^{itH}$ with $\sigma(H)$ bounded, define $f(t) =< U_t\Psi, PU_t\Psi >$

Analiticity: either $f(t) \neq 0$ on a dense open set, or $f(t)$=0 $\forall\ t \in \mathbb{R}$

Hegerfeldt, Phys. Rev. Lett. **72**, 596 (1994)

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$$f(t) =< U_t\Psi, P(a,b)U_t\Psi > =< \Psi, P(a+t, b+t)\Psi >, t \in \mathbb{R}$$

Consider $|\Psi > \in P(a,b)$ and $t > |b-a|$

$$f(t) =< P(a,b)\Psi, P(a+t, b+t)\Psi > = 0 \Longrightarrow f(t)=0, \ \forall\ t \in \mathbb{R}$$

Then, $0 = f(0) =< \Psi, P(a,b)\Psi > = < \Psi, \Psi >$ $\Rightarrow |\Psi > = 0$

Halvorson https://www.princeton.edu/~hhalvors/papers/

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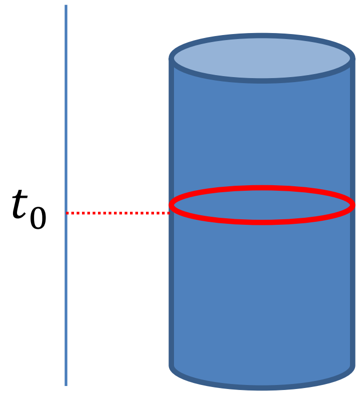
Way outs

i. $P(a+t, b+t)$ not orthogonal to $P(a,b)$ PVM to POVMs

ii. $\sigma(H)$ unbounded

clock *system*

$[\hat{T},\hat{\Omega}] = i$ $[\hat{Q},\hat{P}] = i\hbar$



Unleashing time

the state of the system given that the clock shows t

time is what is shown in a clock

$$\mathbb{J}|\Psi\rangle\rangle = 0$$

$$\mathcal{H} = \mathcal{H}_T \otimes \mathcal{H}_S, \qquad \mathbb{J} = \hbar\Omega \otimes \mathbb{I}_S + \mathbb{I}_T \otimes H_S\,, \qquad \sigma(\Omega, T, \mathbb{J}) = \mathbb{R}$$

$$|\Psi\rangle\rangle = \int_{-\infty}^{+\infty} dt\, |t\rangle|\psi(t)\rangle = \int_{-\infty}^{+\infty} d\omega\, |\omega\rangle|\tilde{\psi}(\omega)\rangle$$

$$\langle t|\mathbb{J}|\Psi\rangle\rangle = 0 \implies \left(-i\hbar\frac{\partial}{\partial t} + H_S\right)|\psi(t)\rangle = 0, \quad \langle\omega|\mathbb{J}|\Psi\rangle\rangle = 0 \implies (\hbar\omega + H_S)|\tilde{\psi}(\omega)\rangle = 0$$

Pauli problem: as $\mathbb{J}|\Psi\rangle\rangle = 0$ on physical states $\implies \langle\langle\Phi|[T,\mathbb{J}]|\Psi\rangle\rangle = 0$

Pauli problem $\mathbb{J}|\Psi\rangle\rangle = 0$ on physical states $\Longrightarrow \langle\langle\Phi|[T,\mathbb{J}]|\Psi\rangle\rangle = 0$

$\langle\langle\Phi|[T,\mathbb{J}]|\Psi\rangle\rangle = \langle\langle\Phi|\hbar[T,\Omega] + [T,H]|\Psi\rangle\rangle$

$[T,\Omega] = i \Longrightarrow \langle\langle\Phi|[T,H]|\Psi\rangle\rangle = -i\hbar,$

$\sigma(H) = \sigma(T) = \mathbb{R}$ on physical states!

NO, $\mathbb{J}|\Psi\rangle\rangle = 0 \nRightarrow T\mathbb{J}|\Psi\rangle\rangle = 0$

Weyl sequence $|\Psi_n\rangle\rangle = \left(\frac{2}{\pi n}\right)^{1/4} \int dt\, e^{t^2/n}|t\rangle|\psi(t)\rangle$

$\lim_{n\to\infty} |(\mathbb{J} - \lambda)|\Psi_n\rangle\rangle|^2 \longrightarrow 0$ for $\lambda = 0$ (essential eigenvalue)

$|\mathbb{J}|\Psi_n\rangle\rangle|^2 = \frac{1}{n} \longrightarrow 0$, $|\mathbb{J}|\Psi_n\rangle\rangle|^2 = \frac{3}{4} \Longrightarrow$ \ $\langle\langle\Psi_n|T\mathbb{J}|\Psi_n\rangle\rangle = \frac{i}{2}$

Then $\langle\langle\Psi_n|[T,\mathbb{J}]|\Psi_n\rangle\rangle = i$ and $[T,\Omega] = i \Longrightarrow \langle\langle\Phi|[T,H]|\Psi\rangle\rangle = 0$

No Pauli problem

Beware of promoting T to an operator in $\mathcal{H}_S$

Simplest mishap: time of arrival operator

$$t \ \sim \ \frac{mq}{p} \longrightarrow \widehat{T} \ \sim \frac{m\hat{Q}}{\hat{P}}$$ (with appropriate ordering)

Measurement of time of arrival in QM

a) Prepare a particle state and detect its arrival (results in $\Delta t \sim 1/E_K$)

Or

b) Measure $\hat{T}$ for the set up … results of b) do not correspond to those of a)…ETC

Aharonov, Oppenheim, Popescu, Reznik, Unruh PRD (1998)

Measurement and reduction in QFT

covariant state reduction

Hellwig, Krauss PRD 1970

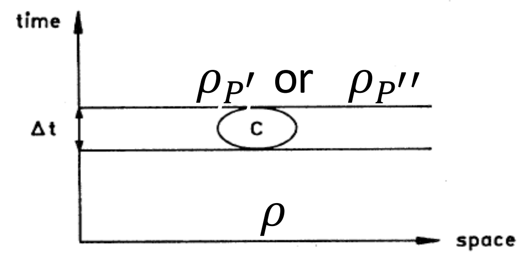
System in $\mathcal{H}$ density ρ $\text{tr}\rho = 1$

Measurement of A at t_0, $\langle A \rangle = \text{tr}\rho A$

Quantum Mechanics

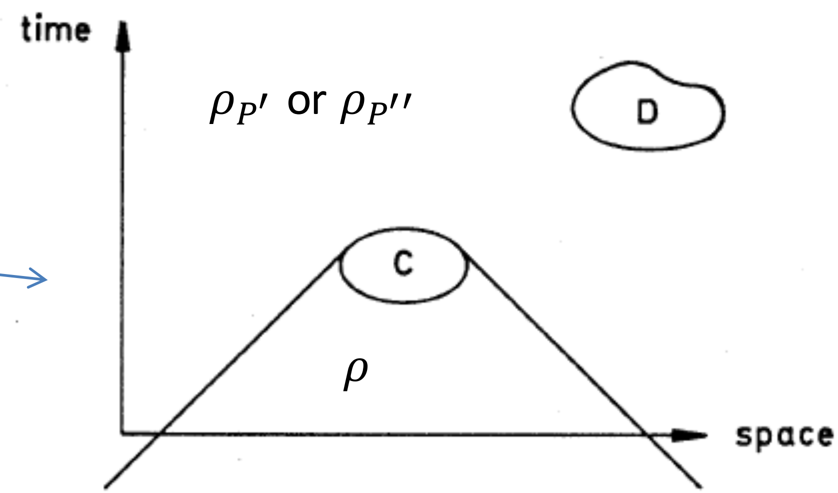
$$\rho \longrightarrow \rho_{P'} = P\rho\, P + (1-P)\rho(1-P)$$

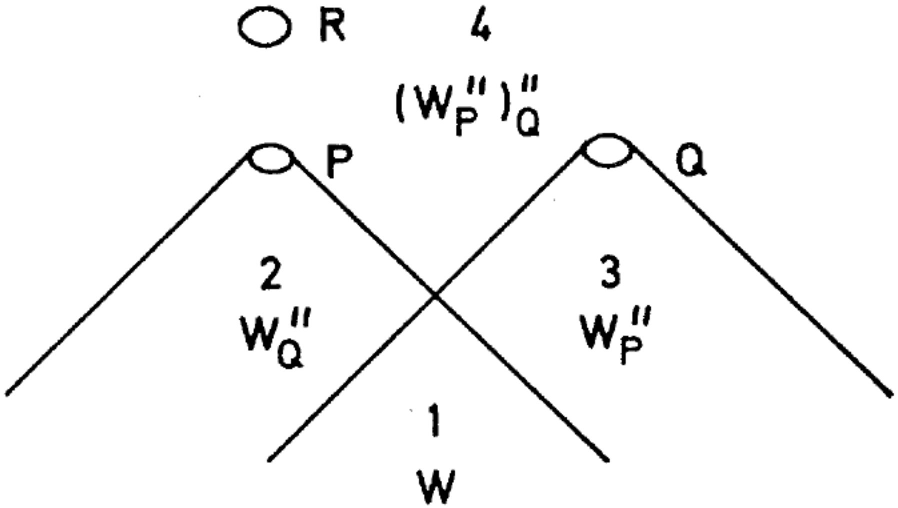
$$\rho \longrightarrow \rho_{P''} = P\rho\, P / tr\rho\, P$$



Hellwig, Krauss: in QFT

state reduction in $V_+ \cap V'$





P and Q selective measurements on W

P: $W \longrightarrow W_{P''}$ in 3 , 4

$W \longrightarrow W$ in 1 , 2

Q: $W \longrightarrow W$ in 1

$W_{P''} \longrightarrow W_{P''}$ in 3

$W \longrightarrow W_{Q''}$ in 2

$W_{P''} \longrightarrow = \frac{QW_{P''}Q}{tr\,(QW_{P''})} = \frac{QPWPQ}{tr(QPW)}$ in 4

P Q spatially separated , R in the future

R measure on state $(W_{P''})_{Q''} = \frac{QW_{P''}Q}{tr\,(QW_{P''})} = \frac{QPWPQ}{tr(QPW)} = \frac{PQWQP}{tr(PQW)} = \left(W_{Q''}\right)_{P''}$

Independent of $t_P \prec t_Q$ or $t_P \succcurlyeq t_Q$

Proposal grounded on strict locality in QFT

Measurement and reduction in QFT

Frame dependent states

Aharonov, Albert PRD 1984

Warning : Histories may be frame dependent

Prepare a particle in state

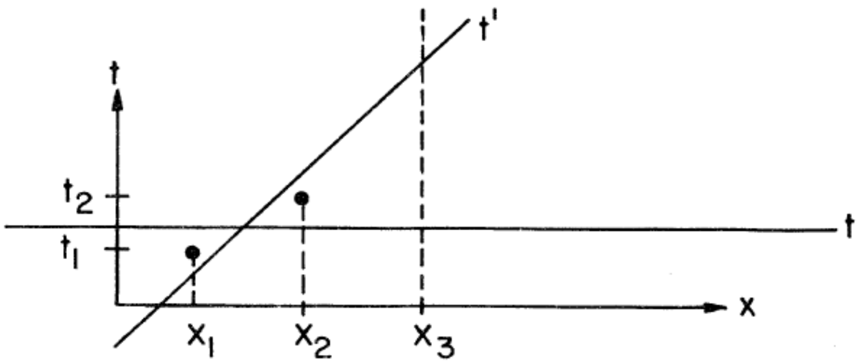
$$|\alpha\rangle = |x_1\rangle + |x_2\rangle + |x_3\rangle, \quad t < t_1$$

Measure at $t = t_1$ ... particle not in x_1

$$|\beta\rangle = |x_2\rangle + |x_3\rangle, \quad t_1 < t < t_2$$

Measure at $t = t_1$ ... particle not in x_1

$$|x_3\rangle \qquad t_2 < t$$



In frame K' where (x'_2, t'_2) precedes (x'_1, t'_1)

$$|\alpha'\rangle \qquad t < t_2$$

$$|\gamma'\rangle = |x'_1\rangle + |x'_3\rangle, \quad t'_2 < t' < t'_1$$

$$|x'_3\rangle \qquad t'_1 < t'$$

Observer K $\quad |\beta\rangle \quad t_1 < t < t_2$

Observer K' $\quad |\gamma'\rangle \quad t'_2 < t' < t'_1$

A non local system evolves undisturbed from $t = -\infty$ to $t = -\infty$

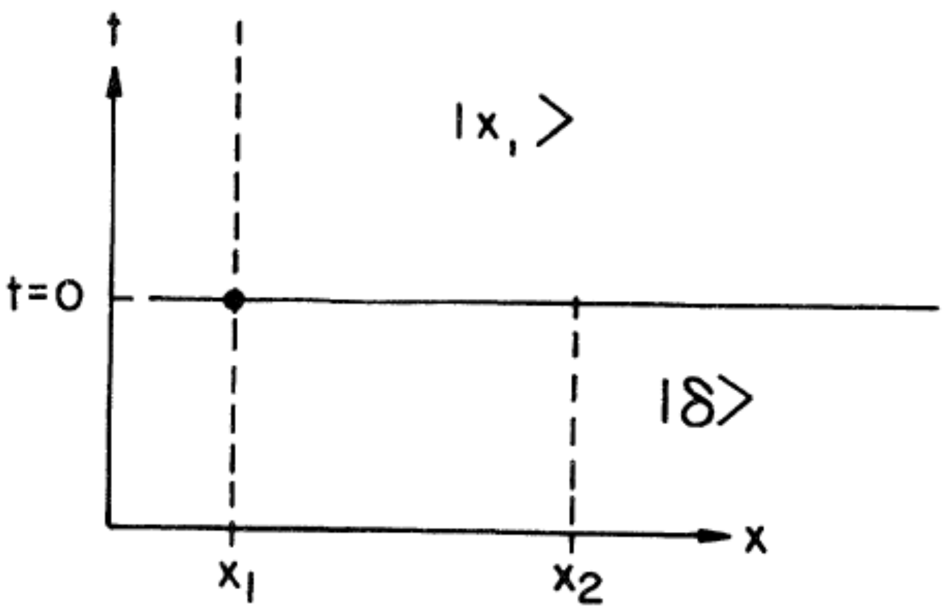
At $t = -\infty$ prepare a particle in $|\delta\rangle = |x_1\rangle + |x_2\rangle$, $D|\delta\rangle = \delta\,|\delta\rangle$ const. of motion

Check that particle is in $|\delta\rangle$ at an instant t using $H = H_1 + H_2$, $\quad H_i = g_i(t) q_i \sigma_z^{(x_i)}$

If prepared in $|\delta\rangle$ the procedure gives δ and leaves the state as it was

Now, suppose a local detector at $x_1 = 0$ founds the particle at $x_1 = 0$ at $t = 0$

From then on $t \in (0, \infty)$, the particle remains in the box

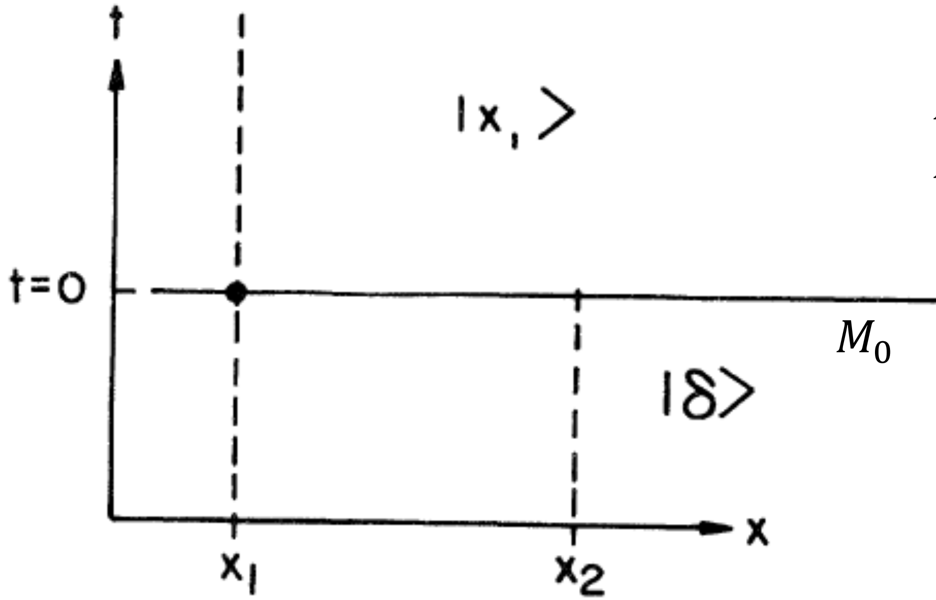


measurements at $t = 0$ and $t = -\infty$
do not commute:

measure ment of x gives $x = x_1 = 0$
Measurent of D gives D= δ

M_∞

$[M_-, M_0] \neq 0$



M_- position $\longrightarrow x_1 = 0 \quad (t = 0)$

M_- D $\longrightarrow D = \delta \quad (t = -\infty)$

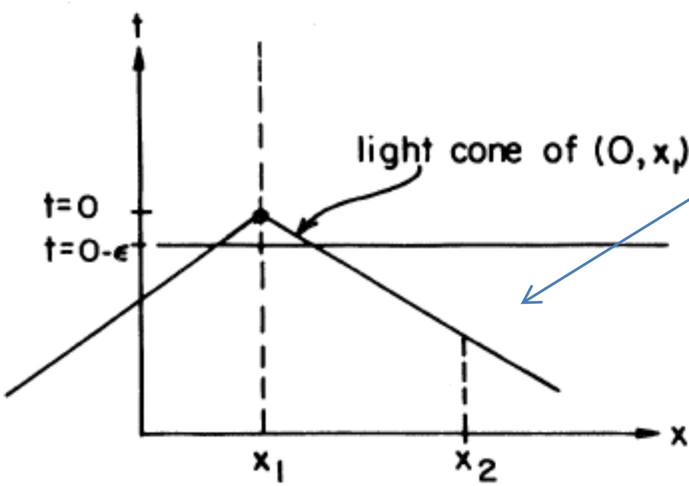
M_0

History 1. collapse at $t = -\infty$, later at $x_1 = 0$

History 2. at $t = 0$ $|\delta\rangle \longrightarrow |x_1\rangle$

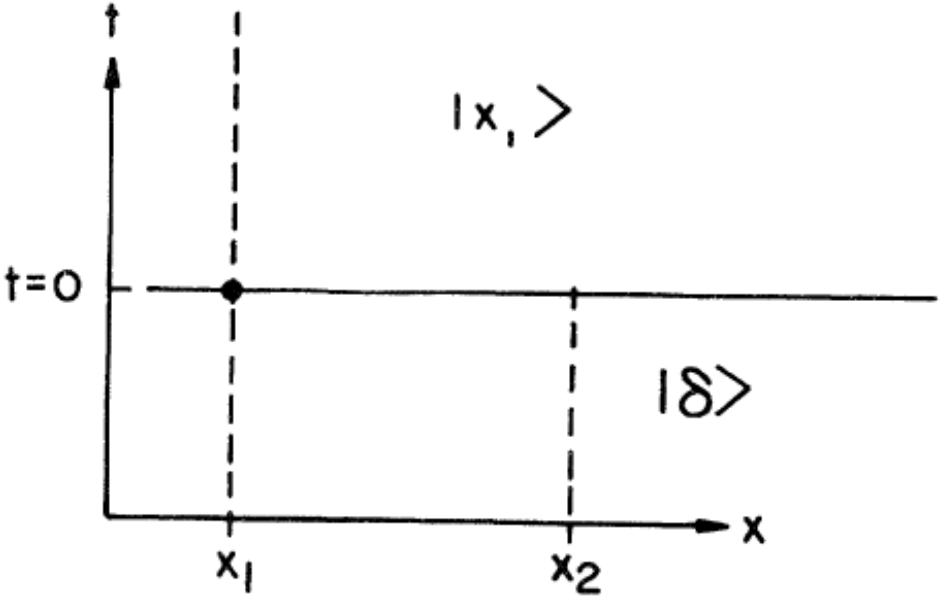
frame independent but $t = 0, t' \neq 0$

$M_{-\infty}$



reduction not along V- (not Hellwig-Kraus)

Instantáneous reduction



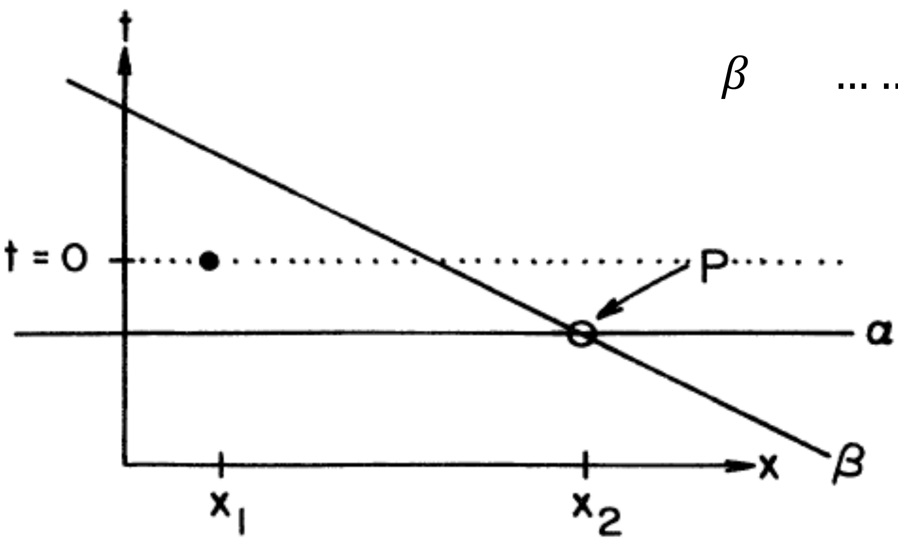
A and B using instantaneous reduction in their frames

conflicting accounts

not a single history in space-time

α monitors state history in K → reduction at $t = 0$

β … … … … … … … K' → reduction at $t' = 0$



$$\alpha\text{: } |\delta\rangle \quad \Psi(\mathrm{P}) = \frac{1}{\sqrt{2}}$$

$$\beta : |x_1\rangle \quad \Psi(\mathrm{P}) = 0$$

$\Psi(\mathrm{x}, \mathrm{t})$ is not enough

Needed a functional on spacelike hypersurfaces most basic level: spacelike hypersurfaces characterized by their relative velocities

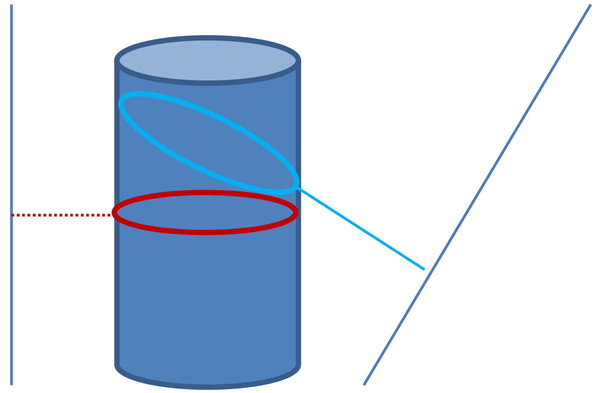
Clocks evolve with their proper times characterized by their relative velocities

The Hamiltonian constraint serve to fix a time on the physical subspace of $\mathcal{H}_T \otimes \mathcal{H}_S$

Consider a set of clocks α, $\mathcal{H}_T \longrightarrow \mathcal{H}_{T_\alpha}, \mathbb{J} \longrightarrow \mathbb{J}_\alpha$ For each α a physical subspace with states given by $\mathbb{J}_\alpha|\Psi_\alpha\rangle\rangle$

clock $[\hat{T}, \hat{\Omega}] = i$

system $[\hat{Q}, \hat{P}] = i\hbar$



Is something like this what we are looking for?

Thanks for your attention

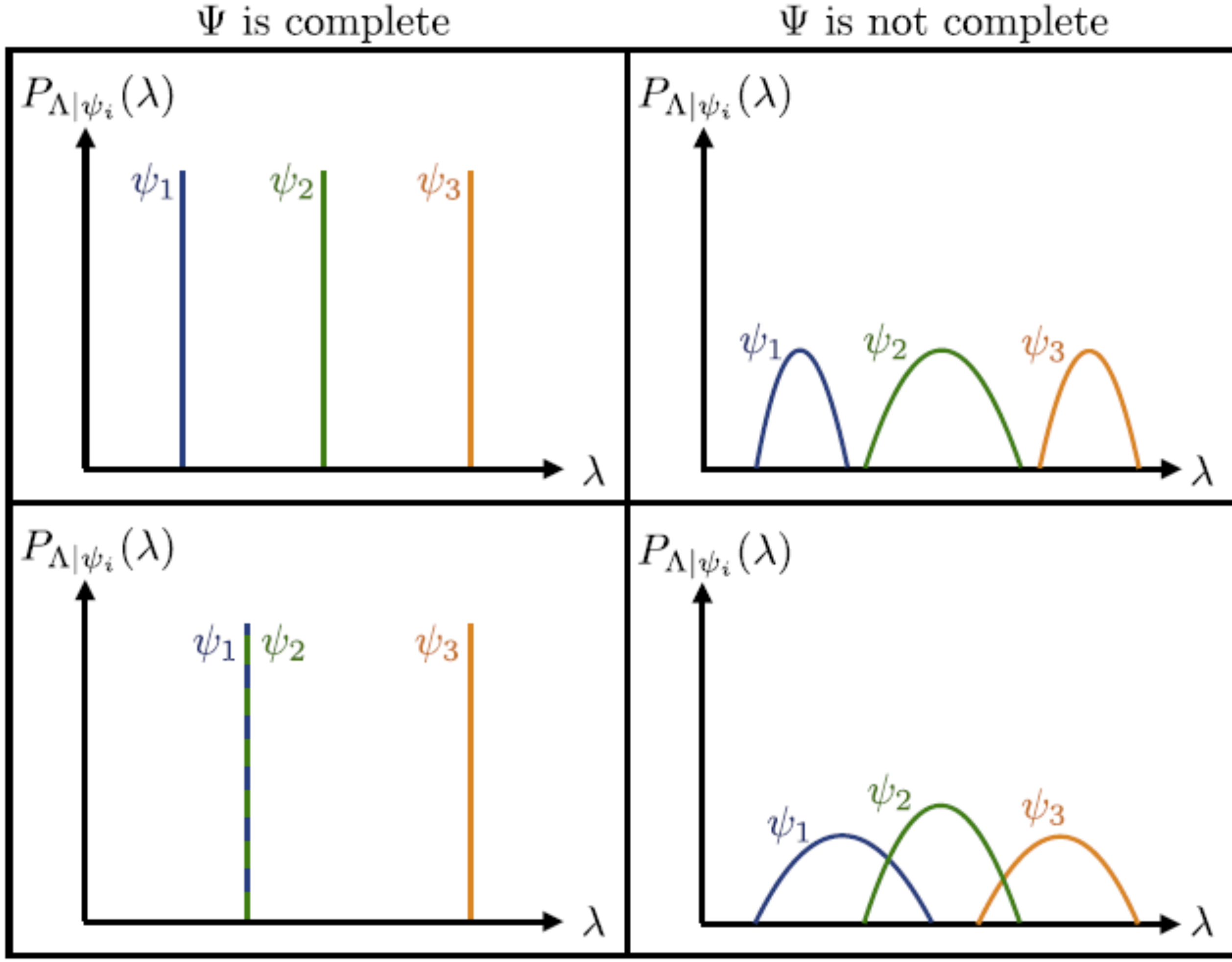


Figure 1. The different possible roles of the wave function Ψ. A model that uses a variable Λ to de
either Ψ-ontic or Ψ-epistemic, depending on whether or not the wave function Ψ is uniquely det
denoted by λ). Conversely, the relevant parts of Λ may be determined by Ψ, in which case Ψ is co
respect to an appropriate causal order), [17] rules out the right column, [16] rules out the bottom
well as [14], based on different assumptions) rules out the bottom row.