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FITTING CRITERIA FOR PRODUCTION MODELS

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INTRODUCTION

The dynamic production model presented last year by Lleonart **et al.** (1985) is applied to a new set of data in the present paper. The basis of the model has not been modified, but fitting methods are discussed and a new, more appropriate criterion to fit the data and to estimate parameters is introduced.

DESCRIPTION OF THE MODEL

Traditional production models are based on the steady-state hypothesis and provide a formula to calculate the theoretical catch in equilibrium for year i (Y_{ei}). The two production models used in ICSEAF are Schaefer's:

$$Y_{ei} = af_i + bf_i^2 \qquad (1)$$

and Fox's:

$$Y_{ei} = af_i \exp(bf_i) \qquad (2)$$

where f_i = fishing effort in year i.

Gulland's method consists of modifying the above equations as follows:

$$Y_{ei} = af_i + bf_i\bar{f}_i, \qquad (1')$$

and,

$$Y_{ei} = af_i \exp(b\bar{f}_i), \qquad (2')$$

where $\bar{f}_i$ = average fishing effort over the last few years (usually three).

This modification is a step towards a dynamic approach to production model stock assessment.

In the traditional models, total allowable catch (TAC) is calculated assuming cpue to be constant, using the equation:

$$Y_{ci} = \frac{Y_{i-1}}{f_{i-1}} f_i \qquad (3)$$

where:

Y_{i-1} = catch for the previous year
f_{i-1} = effort for the previous year
f_i = effort for the current year.

In this equation catch is calculated under the assumption that cpue remains constant for two years, so that the resulting prediction of Y represents one extreme, where $Y=Y_{ci}$. Equations (1) or (2) (depending on the model chosen) predict catch under the other extreme situation, where $Y=Y_{ei}$.

The Lleonart **et al.** (1985) model introduces a new parameter, g, called **inertia**, which weights these two extreme cases,

Y_{ei} and Y_{ci}, in the following manner:

$$\hat{Y}_i = gY_{ci} + (1 - g)Y_{ei}, \qquad (4)$$

where $\hat{Y}_i$ = estimated catch in year i.

Further details on g and the geometrical interpretation thereof are given in Lleonart **et al.** (1985).

The model expressed in equation (4) is in fact dynamic, since a one-year memory is contained in the term Y_{ci}. MSY, f_{MSY} and $f_{0,1}$ can be calculated as usual.

TOTAL ALLOWABLE CATCH

The traditional operating method is to fit the model as if the population were in a steady state (equivalent to "0" inertia, in terms of the dynamic model described above), but to calculate the TAC's as if cpue were constant, (i.e., assuming the value for the current year to be equal to that of the preceding year), thus assuming inertia = 1 in this step of the procedure.

In comparison, in Lleonart's **et al.** (1985) dynamic model, the curve fitting and TAC calculations are based on a single inertia criterion, using the same value in both cases. Equation (4) is used to calculate TAC's under the said model, Y_{ei} being derived from equation (1) for Schaefer's model and from equation (2) for the Fox model.

FITTING THE DATA

Fitting criterion

In order to estimate the parameters of a model it is necesary to have a fitting criterion. In the traditional production models, functional regression between cpue and f is used. For Schaefer's model the following function is minimized:

$$ss = \Sigma \left| \frac{Y_i}{f_i} - \frac{\hat{Y}_i}{f_i} \right| \left| \bar{f}_i - \hat{\bar{f}}_i \right| .$$

For the Fox model, the function to be minimized is:

$$ss = \Sigma \left| \ln \frac{Y_i}{f_i} - \widehat{(\ln \frac{Y_i}{f_i})} \right| \left| \bar{f}_i - \hat{\bar{f}}_i \right|$$

(f_i and $\bar{f}_i$ are defined as in Gulland's method.)

The authors do not defend any hypothesis for the use of such minimization criteria, usually employed simply because the parameters involved can be easily estimated by linear regression.

A minimization criterion should be based on an hypothesis regarding the distribution pattern of the parameters studied, which hypothesis is to be tested by analysis of residuals (Draper and Smith 1981). Such an analysis was attempted by the authors, but, unfortunately, the historical series of catch and effort data is not long enough to reach significant conclusions. Nevertheless, Kirkwood (1981), in a statistical study on whales, considers that catches might follow a Poisson distribution, in which case the function to be minimized is:

$$ss = \Sigma (\sqrt{Y_i} - \sqrt{\hat{Y}_i})^2$$

because the square root transformation of a Poisson-distributed variable ensures asymptotic normality.

Analogous reasoning may be applied to hakes, although bearing in mind that while whales are scarce, Cape hakes are abundant, so that catches may be assumed to follow a normal, rather than a Poisson, distribution. These considerations led the authors to choose minimization of the following equation as a criterion:

$$ss = \Sigma(Y_i - \hat{Y}_i)^2 \qquad (5)$$

In a previous paper (Lleonart **et al.** 1985) the Kirkwood hypothesis was followed to be able to compare the results obtained with those of Butterworth (1984), calculated in accordance with the said criterion.

Traditional Schaefer model

In the traditional Schaefer model, as modified by Gulland, parameters are easily estimated following the normal distribution hypothesis from the substitutions given below:

$$C = \Sigma Y_i f_i; \quad D = \Sigma Y_i f_i \bar{f}_i$$

$$E = \Sigma f_i^2; \quad F = \Sigma f_i^2 \bar{f}_i;$$

$$G = \Sigma f_i^2 \bar{f}_i^2.$$

Therefore:

$$a = \frac{C - bF}{E} \; ; \quad b = \frac{DE - CF}{GE - F^2} \; .$$

Traditional Fox model

For the traditional Fox model, as modified by Gulland, the equations become:

$$a = \frac{\Sigma\, f_i Y_i \, \exp(b\bar{f}_i)}{\Sigma\, f_i^2 \, \exp(2b\bar{f}_i)}$$

For b, the following equation can be solved by approximation (using the Newton-Raphson method, for example), from:

$$\Sigma\, Y_i f_i \bar{f}_i \, \exp(b\bar{f}_i) - a \, \Sigma\, f_i^2 \bar{f}_i \, \exp(2b\bar{f}_i) = = 0.$$

Dynamic Schaefer model

In the dynamic Schaefer model, minimization can be effected by analytical methods. In order to simplify the expression, the following substitutions are performed:

$D = \Sigma\, X_i Y_i f_i$; $E = \Sigma\, Y_i f_i$;
$F = \Sigma\, Y_i f_i^2$; $H = \Sigma\, X_i^2 f_i^2$;
$P = \Sigma\, X_i f_i^2$ $Q = \Sigma\, X_i f_i^3$;
$R = \Sigma\, f_i^2$; $S = \Sigma\, f_i^3$;
$T = \Sigma\, f_i^4$;

where:

$$X_i = \frac{Y_{i-1}}{f_{i-1}} ,$$

and all the summations are extended from i=2 to n (where n is the total number of years).

From (1), (3), (4) and (5) the following equations are obtained by standard minimization methods:

$$b = \frac{FR - ES - g(QR - PS)}{(1 - g)\,(TR - S^2)} ,$$

$$a = \frac{E - gP - b(1-g)S}{(1 - g)R} , \text{ and}$$

$$-D+a(E+P-2gP-a(1-g)R)+b(F+Q-2gQ-b(1-g)T)- -2ab(1-g)S+gH = 0,$$

which can be solved for g by approximation using the Newton-Raphson or other similar method.

Dynamic Fox model

No analytical solution was found for the dynamic Fox model, so a numerical non-linear fitting algorithm was used instead.

RESULTS AND DISCUSSION

Each set of data was run six times: for each of the Schaefer and Fox models, three criteria were applied, as follows:

- traditional fitting of the traditional model,
- new fitting of the traditional model,
- new fitting of the dynamic model.

In all cases, the following parameters were estimated: a, b, g (for dynamic approaches only), ss/(n-1), MSY, f_{MSY}, TAC_{MSY}, and $TAC_{0,1}$. The parameter ss/(n-1) is more suitable than ss, since it allows for comparison among the various fittings.

The data used were taken from ICSEAF (1985), with an addendum for 1984 provided by the Secretariat. Cpue estimations for the first half of 1985 were not available, so 1984 data were used to estimate curve parameters and calculate TAC's. Two different series of cpue, from columns v and xv of Table 2 in ICSEAF (1985), were used for Divisions 1.3+1.4, while the series under columns f and k of Table 3 of the said paper were used for Division 1.5. The input data are reproduced in Tables 1 through 3, and the results in Tables 4 through 8.

In the traditional models, three-year running averages have been used since Gulland's modification, i.e., two degrees of freedom are lost, corresponding to the data points for the first and second years. In the present dynamic versions, however, just one degree of freedom is lost, corresponding to the data point for the first year, because the dynamic versions have a one-year memory. Thus, only in cases where the data for the second year of the series contribute significantly to the variance, the value of ss/(n-1) obtained using the traditional models may be lower than that for the dynamic approach, since only the latter includes the data point for the second year. This situation is reflected in Table 6, for the dynamic Schaefer model; otherwise, the dynamic approach results in an improved value of ss/(n-1).

The inertia values estimated by the dynamic model under discussion were over 0,7 for Divisions 1.3+1.4, around 0,7 for Division

1.6, and under 0,47 for Division 1.5.

The Schaefer models are better fitted than the Fox models for Divisions 1.3+1.4 and 1.5, while the opposite is true for Division 1.6. In all cases, however, the differences are small.

The improvement achieved by application of the dynamic models was greatest in the case of Division 1.6, where the decrease in the values of ss/(n-1) was most significant.

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TABLE 1. Cape hakes in Divisions 1.3+1.4: catch and effort, 1965-1984 (input data used in assessments)

Year	Catch ('000 t)	Fishing effort[1]	Fishing effort[2]
1965	93,5	0,220 519	52,528 091
1966	212,4	0,680 769	162,137 405
1967	195,0	0,898 618	214,285 706
1968	382,7	1,671 179	398,645 844
1969	320,5	1,526 191	364,204 559
1970	402,5	1,880 841	447,222 229
1971	365,6	1,766 184	420,229 889
1972	606,1	3,544 444	841,805 481
1973	377,6	2,776 471	662,456 177
1974	313,8	2,932 710	697,333 313
1975	309,4	3,094 000	736,666 687
1976	369,9	3,698 000	880,476 196
1977	277,5	3,189 655	565,173 096
1978	258,1	3,147 561	590,617 859
1979	172,3	2,426 761	423,341 522
1980	90,5	1,028 409	201,111 115
1981	92,1	0,959 375	165,945 938
1982	176,4	1,618 697	327,343 262
1983	215,8	1,488 276	367,632 019
1984	228,5	1,718 045	357,031 250

1 - effort index, calculated from ICSEAF 1985, Table 2, column xv
2 - effort in '000 h, calculated from ICSEAF 1985, Table 2, column v

TABLE 2. Cape hakes in Division 1.5: catch and effort, 1965-1984 (input data used in assessments)

Year	Catch ('000 t)	Fishing effort[1]	Fishing effort[2]
1965	99,7	0,445 045	49,351 486
1966	122,2	0,466 347	50,488 842
1967	199,4	1,356 551	146,627 197
1968	247,7	1,794 783	208,134 445
1969	206,2	1,793 278	198,295 212
1970	224,7	2,043 009	224,731 003
1971	229,7	1,594 847	167,633 575
1972	214,0	2,140 260	237,806 671
1973	290,3	2,903 230	299,302 063
1974	195,7	2,796 029	301,110 779
1975	178,7	2,179 378	232,089 615
1976	211,9	3,653 448	407,500 000
1977	154,5	2,239 594	241,456 253
1978	125,1	2,233 536	240,534 637
1979	140,1	1,892 824	202,998 550
1980	74,6	1,050 521	111,323 875
1981	120,6	1,419 341	147,126 831
1982	130,1	1,548 321	154,832 153
1983	123,3	1,369 500	136,949 997
1984	141,8	1,540 804	157,504 440

1 - effort index, calculated from ICSEAF 1985, Table 3, column k
2 - effort in '000 h, calculated from ICSEAF 1985, Table 3, column f

TABLE 3. Cape hakes in Division 1.6: catch and effort, 1955-1984 (input data used in assessments

Year	Catch ('000 t)	Fishing effort[1]
1955	115,4	6 666,667
1956	118,2	7 557,544
1957	126,4	7 674,560
1958	130,7	8 038,130
1959	146,0	8 979,090
1960	159,9	9 237,435
1961	148,7	12 299,420
1962	147,6	10 409,027
1963	169,5	12 133,142
1964	162,3	11 116,438
1965	203,0	18 726,936
1966	195,0	18 344,309
1967	176,7	17 652,348
1968	143,6	14 345,654
1969	165,1	19 153,133
1970	142,5	19 709,543
1971	202,0	28 490,831
1972	243,9	49 782,242
1973	157,7	31 734,407
1974	123,0	26 451,612
1975	89,6	19 231,115
1976	143,9	26 896,074
1977	102,3	17 521,917
1978	101,1	17 142,372
1979	92,7	15 123,002
1980	101,5	18 528,831
1981	100,7	17 328,400
1982	86,0	14 645,657
1983	73,7	11 352,389
1984	82,7	12 408,246

1 - effort in days, calculated from ICSEAF 1985, Table 4

TABLE 4. Cape hakes in Divisions 1.3+1.4: output data for Schaefer and Fox models, using cpue data from ICSEAF (1985), Table 2, column v

Parameters estimated	SCHAEFER			FOX		
	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model
a	1,080 14	1,027 68	0,836 95	1,211 70	1,116 72	0,986 91
b	-9,774 46 E-4	-7,692 99 E-4	-6,230 73 E-4	-1,520 26 E-3	-1,162 45 E-3	-0,130 47 E-2
g			0,717 16			0,710 98
ss/(n-1)	5 652,63	3 791,16	965,564	5 430,27	4 085,30	1 016,29
MSY ('000 t)	298,405	343,211	281,060	293,212	353,408	278,276
f(MSY) ('000 h)	552,531	667,933	671,630	657,783	860,253	766,465
TAC(MSY) ('000 t)	353,620	427,477	387,762	420,981	550,562	429,190
TAC (0,1) ('000 t)	318,258	384,730	356,140	329,005	430,264	350,768

TABLE 5. Cape hakes in Divisions 1.3+1.4: output data for Schaefer and Fox models, using cpue from ICSEAF (1985), Table 2, column xv

Parameters estimated	SCHAEFER			FOX		
	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model
a	270,194	251,833	196,507	328,937	286,186	255,990
b	-61,736 1	-48,184 3	-39,796 3	-0,439 024	-0,325 453	-0,405 59
g			0,755 91			0,749 02
ss/(n-1)	7 187,14	4 793,31	1 029,74	7 086,33	5 233,31	1 074,20
MSY ('000 t)	295,632	329,048	242,578	275,632	323,494	232,187
f(MSY) (index)	2,188 30	2,613 25	2,468 90	2,277 78	3,072 64	2,465 56
TAC(MSY) ('000 t)	291,043	347,559	307,424	302,944	408,662	303,893
TAC (0,1) ('000 t)	261,939	312,803	282,011	236,757	319,369	248,618

TABLE 6. Cape hakes in Division 1.5: output data for Schaefer and Fox models, using cpue from ICSEAF (1985), Table 3, column f

Parameters estimated	SCHAEFER			FOX		
	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model
a	1, 660 37	1,553 08	1,362 93	2,056 91	1,804 19	1, 711 4
b	-3,804 87 E-3	-3,151 97 E-3	-2,456 19 E-3	-4,328 82 E-3	-3,503 83 E-3	-0,344 08 E-2
g			0,419 78			0,411 78
ss/(n-1)	1 201,43	983,825	999,606	1 260,85	1 097,54	1 076,04
MSY ('000 t)	181,138	191,312	189,071	174,804	189,428	182,974
f(MSY) ('000 h)	218,190	246,366	277,447	231,010	285,402	290,633
TAC(MSY) ('000 t)	196,371	221,729	214,523	207,909	256,862	215,338
TAC (0,1) ('000 t)	176,734	199,556	202,944	162,485	200,737	188,830

TABLE 7. Cape hakes in Division 1.5: output data for Schaefer and Fox models, using cpue from ICSEAF (1985), Table 3, column k

Parameters estimated	SCHAEFER			FOX		
	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model
a	182,242	165,672	143,420	226,024	104,020	178,33
b	-46,209 0	-36,037 2	-27,803 6	-0,480 563	-0,378 482	-0,364 33
g			0,466 55			0,459 63
ss/(n-1)	1 380,41	1 067,71	914,999	1 336,46	1 138,98	978,531
MSY ('000 t)	179,684	190,408	184,952	173,025	188,586	180,068
f(MSY) (Index)	1,971 93	2,298 62	2,579 16	2,080 89	2,642 14	2,744 75
TAC(MSY) ('000 t)	181,418	211,473	209,367	191,442	243,077	213,368
TAC (0,1) ('000 t)	163,276	190,326	197,301	149,616	189,964	185,319

TABLE 8. Cape hakes in Division 1.6: output data for Schaefer and Fox models, using cpue from ICSEAF (1985), Table 4

Parameters estimated	SCHAEFER			FOX		
	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model	Trad. fit, trad. model	New fit, trad. model	New fit, dynam. model
a	18,676 4	12,986 5	12,178 6	22,563 3	18,613 0	21,073
b	-0,540 477	-0,253 221	-0,272 375	-5,682 02 E-2	-4,468 44 E-2	-5,949 9 E-2
g			0,747 81			0,681 44
ss/(n-1)	4 603,67	1 428,49	256,865	1 221,52	1 088,66	260,103
MSY ('000 t)	161,343	166,503	136,135	146,085	153,237	130,291
f(MSY) ('000 d)	17,277 7	25,642 5	22,356 4	17,599 4	22,379 2	16,807 0
TAC(MSY) ('000 t)	115,243	171,036	145,843	117,388	149,269	117,897
TAC (0,1) ('000 t)	103,718	153,932	134,348	91,740 9	116,654	100,059