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Measurement and Modeling of the Effective Thermal Conductivity of Sintered Silver Pastes

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ABSTRACT

The effective thermal conductivity of sintered porous pastes of silver is modelled through two theoretical methods and measured by means of three experimental techniques. The first model is based on the differential effective medium theory and provides a simple analytical description considering the air pores like ellipsoidal voids of different sizes, while the second one arises from the analysis of the scanning-electron-microscope images of the paste cross sections through the finite element method. The predictions of both approaches are consistent with each other and show that the reduction of the thermal conductivity of porous pastes can be minimized with spherical pores and maximized with pancake-shaped ones, which are the most efficient to block the thermal conducting pathways. A thermal conductivity of 151.6 W/m·K is numerically determined for a sintered silver sample with 22% of porosity. This thermal conductivity agrees quite well with the one measured by the Lateral Thermal Interface Material Analysis for a suspended sample and matches, within an experimental uncertainty smaller than 16%, with the values obtained by means of Raman thermometry and the 3ω technique for two samples buried in a silicon chip. The consistence between our theoretical and experimental results demonstrates the good predictive performance of our theoretical models to describe the thermal behavior of porous thermal interface materials and to guide their engineering with a desired thermal conductivity.

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I. INTRODUCTION

Thermal interface materials (TIMs) are critical for optimizing the thermal contacts between two surfaces and improving the heat dissipation across them. This is particularly important in electronic devices, whose performance, power, lifetime, and miniaturization rely on the effective dissipation of heat.[1–9] TIMs provide mechanical strength to the junction between a heat sink and a heat source, and their thermal conductivity represents the bottleneck for the heat flow through the mating surfaces, which are usually not smooth neither at the same temperature. Among the wide variety of TIMs (e.g. adhesives, solders, gels, pads, and pastes), those based on a metallic paste have become of great interest due to their relatively high thermal conductivity.[4, 5, 10, 11] A thermal paste consists of a matrix that is filled with a thermally conductive solid in the form of particles. The particle size is usually in the order of tens of nanometers to fill up the microscale valleys in the surfaces topography. As the number of conductive particles increases, the thermal conductivity of a thermal paste increases, but its conformability decreases, and therefore the solid content should not be excessive.[12]

Thermal pastes made up of carbon nanoparticles (with a size of about 30 nm) in the form of carbon black, have been proposed as TIMs with great potential applications, due to the compressibility and low electrical conductivity of these particles with respect to metals.[13–17] The nanostructure and squishability of carbon black contribute to the spreadability of the paste, while the low electrical conduction is desirable for avoiding possible short-circuiting. Recently, Lin and Chung [18] showed that a thermal paste with nanoparticles of fumed metal oxides, such as zinc oxide, aluminum oxide, titanium dioxide, and silicon dioxide exhibit negligible electrical conductivity while keeping their thermal performance as effective as the ones with carbon black. Even though the thermal pastes based on carbon black and metal oxides exhibit better conformability and spreadability than those with pure metallic nanoparticles (e.g. silver or copper), they are not as thermally conductive as these latter ones. Carbon nanotubes have been also proposed as conductive filler of thermal pastes, due to their high axial thermal conductivity and relatively small diameter. Xu et al.[19] reported that a thermal paste with a volume fraction of 0.6% of single-walled carbon nanotubes exhibits a thermal conductance of $2 \cdot 10^5 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$, which is comparable to the one of a solder, but smaller than the corresponding one for carbon black, which is equal to

$3 \cdot 10^5 \text{ W}\cdot\text{m}^{-2} \cdot \text{K}^{-1}$. [20–22] In general, materials exhibiting high thermal conductivity, such as metals, carbons, and composites based on these two materials can be efficient thermal interface materials, however, the usual presence of air voids (pores) during the sintering of these thermally conductive pastes diminishes their thermal conductivity and therefore their performance. This reduction depends on the geometry and concentration of pores, and it is not well quantified to date. [23–25] An accurate measurement and detailed modeling of their thermal properties through analytical models and computational simulations is therefore required.

The objective of this work is to measure and model the effective thermal conductivity of sintered porous silver pastes. This will allow us to understand the effects of the pore shape, size, and concentration on the thermal performance of these pastes, and therefore to optimize their design and manufacture for applications as a thermal glue of materials.

II. THEORETICAL MODEL

In this section, a methodology for modelling the thermal conductivity of porous materials by means of analytical and numerical calculations is presented.

A. Analytical Approach

This model describes the thermal paste as a porous medium with ellipsoidal pores of different sizes and random orientations, as shown in Fig. 1. The thermal conductivity of this medium can be determined by applying the differential effective medium theory, [26–28] which is based on a process of incremental homogenization. This procedure consists in building up the porous medium by increasing the volume fraction (porosity) p of pores through infinitesimal fractions, such that in each step, the resulting medium can be considered as a homogeneous one. [27] Assuming that the thermal conductivity of the air pores is negligible in comparison to the one k_m of the matrix, the effective thermal conductivity k of the porous medium is given by [27]

$$\int_{k_m}^k \frac{dx}{xA(x)} = -\ln(1 - p), \quad (1)$$

FIG. 1: Schematics of (a) a porous paste made up of (b) ellipsoidal pores of arbitrary size and fixed shape ($b/a = \text{constant}$) embedded in a matrix of thermal conductivity k_m .

where the parameter A is defined by the behaviour of k at low porosities ($p \ll 1$), as follows $k = k_m [1 + A(k_m)p]$. Equation (1) is valid for low and high porosities and for porous media with a wide distribution of pore size and shape,[26, 27] which is the case of most practical interest.

A suitable model to describe k at low porosities was reported by Nan et al.,[29] who generalized Maxwell's previous result.[30] For ellipsoidal air pores with random orientation and negligible thermal conductivity with respect to the one of the matrix, Nan et al.[29] results reduce to

$$\frac{k}{k_m} = \frac{1 - p}{1 + (n - 1)p}, \quad (2a)$$

$$n = \frac{3L + 1}{6L(1 - L)}, \quad (2b)$$

where the geometrical factor L of the ellipsoidal pores depends only on their aspect ratio $\rho = b/a$ (Fig. 1(b)) and is given by

$$L = \frac{\rho^2}{2(\rho^2 - 1)} \begin{cases} 1 - \frac{\text{arccosh}(\rho)}{\rho\sqrt{\rho^2 - 1}}, & \rho > 1, \\ 1 + \frac{\arccos(\rho)}{\rho\sqrt{1 - \rho^2}}, & \rho < 1. \end{cases} \quad (3)$$

The Taylor series expansion of Eq. (2a) yields the coefficient $A = -n$. After inserting this result into Eq. (1) and carrying out the required integration, the following expression

of the effective thermal conductivity k of the porous paste shown in Fig. 1(a) is obtained

$$k = k_m(1 - p)^n, \quad (4)$$

which is a simple power law. Note that under a first-order approximation on the porosity p , Eq. (4) reduces to Eq. (2a), as expected. Equations (3) and (4) establish that k is driven by the pore shape, is lower than the thermal conductivity k_m of the matrix, and increases as the exponent n decreases. Table 1 summarizes the values of L and n for regular pore shapes and shows that porous media with spherical pores are expected to exhibit higher thermal conductivities than those with cylindrical and flat ones.

TABLE I: Geometrical parameters of pores

Pore shape	Geometrical factor L	Exponent n
Spherical	1/3	3/2
Cylindrical	1/2	5/3
Flat	0	∞

Taking into account that the porous pastes usually exhibit a wide distribution of the pore shapes, a better estimation of the exponent n can be obtained by averaging over all possible values of the geometrical factor L , as follows

$$n = \int_0^{1/2} \frac{3L + 1}{6L(1 - L)} P(L) dL, \quad (5)$$

where $P(L)$ is the probability distribution of L , and hence it satisfies the following normalization condition

$$\int_0^{1/2} P(L) dL = 1. \quad (6)$$

After determining the probability distribution $P(L)$ of the pore shapes, for a particular porous medium, an average estimation of n can be calculated by means of Eq. (5).

B. Numerical Approach

This approach is based on the Finite Element Method (FEM) analysis of Scanning Electron Microscope (SEM) images of cross sections of a silver paste sample that are thresholded in a black and white figure. Each pixel is converted into a 2D finite element to which the

FIG. 2: Schematic illustration of the finite element analysis procedure based on SEM microstructural images.

properties of either Ag or air (pore) are assigned. The thermal conductivities used in this analysis for Ag and air are 419 W/m·K and 0 W/m·K, respectively. A schematic overview of this procedure is shown in Fig. 2.

To calculate the thermal conductivity of the sintered paste material by means of our numerical analysis, the computational homogenization approach is used.[31, 32] This method considers a microstructural representative volume element (RVE) of the material (selected window in Fig.2) supporting the propagation of heat under steady state conditions. The size of this unit cell should be big enough to become statistically representative, but small enough not to increase the computational time too much. The heat flux vector $\vec{q}_m$ within this RVE is then determined by the steady-state heat balance

$$\nabla_m \cdot \vec{q}_m = 0, \quad (7)$$

where ∇_m denotes the gradient operator with respect to the local RVE coordinates. The constitutive relations between the heat flux and the temperature gradient are assumed to be known for each microstructural constituent and they are described by the isotropic linear Fourier's law, with temperature independent thermal conductivities. For each analysis, the macroscopic reference temperature θ_M and the macroscopic temperature gradient $\nabla_M \theta_M$ are prescribed. The temperature field θ_m within the RVE (microscopic temperature) can then be expressed as

$$\theta_m(\vec{x}) = \theta_M + \nabla_M \theta_M \cdot (\vec{x} - \vec{x}_1) + \theta_f(\vec{x}), \quad (8)$$

where $\vec{x}$ is the position vector of a point within the RVE and $\vec{x}_1$ is the position vector of a reference point, arbitrarily selected to be node 1 shown in Fig. 2; θ_f is a microfluctuation temperature field, which represents the microscale local deviations, due to the heterogeneous microstructure, from the linear temperature distribution dictated by the prescribed macroscopic temperature gradient. The macroscopic temperature gradient $\nabla_M \theta_M$ is set as the volume average of the microscopic one $\nabla_m \theta_m$

$$\nabla_M \theta_M = \frac{1}{V} \int_V \nabla_m \theta_m dV, \quad (9)$$

V being the RVE volume. Substitution of Eq. (8) into Eq. (9) and the application of the divergence theorem lead to the following constraint on the microfluctuation field

$$\int_{\Gamma} \theta_f \hat{n} d\Gamma = \vec{0}, \quad (10)$$

where Γ stands for the RVE boundary and $\hat{n}$ is the outward unit vector normal to the boundary. To satisfy Eq. (10), periodic boundary conditions are applied requiring the equality of the microfluctuations on the opposite Right-Left and Top-Bottom RVE boundaries: $\theta_f^R = \theta_f^L$ and $\theta_f^T = \theta_f^B$. Making use of Eq. (8), these constraints can be rewritten in terms of the microscale temperature θ_m as

$$\theta_m^R = \theta_m^L + \nabla_M \theta_M \cdot (\vec{x}^R - \vec{x}^L), \quad (11a)$$

$$\theta_m^T = \theta_m^B + \nabla_M \theta_M \cdot (\vec{x}^T - \vec{x}^B). \quad (11b)$$

The microscopic boundary value problem given by Eq. (7), with the boundary conditions in Eqs. (11a) and (11b), is solved by the Finite Element Method through the MarcMentat software of the MSC corporation.[33] The result of the simulations is the microscale heat flux field $\vec{q}_m$ resulting from a prescribed macroscopic temperature gradient, as shown in Fig. 2. The macroscopic effective heat flux $\vec{q}_M$ can then be computed as the volume average of its microscale counterpart

$$\vec{q}_M = \frac{1}{V} \int_V \vec{q}_m dV. \quad (12)$$

The macroscopic second-order tensor of thermal conductivity K_M , which in general may be anisotropic, is then determined by

$$\vec{q}_M = K_M \cdot \nabla_M \theta_M. \quad (13)$$

In this work, the temperature gradient is imposed along the vertical direction of the micrograph pictures (as this is the direction of the interconnect), and therefore the effective thermal conductivity along this direction will only be computed.

III. EXPERIMENTAL PROCEDURES

A. Sample sintering

Silver pastes provided by the Heraeus company were dried at 75 °C for two hours in air atmosphere. They were then sintered at 200 °C with a pressure of 5 MPa sets for 60 minutes by a presser of the Lauffer company. The obtained strip samples of porous silver and a their typical SEM image are shown in Figs. 3(a) and 3(b), respectively. The white and black colors stand for the silver and air pores, respectively.

FIG. 3: (a) Strip samples of sintered porous silver and (b) their typical SEM image.

B. Thermal characterization

To validate the theoretically predicted values of the thermal conductivity of our sintered silver (SAG) samples, their thermal conductivity was measured by means of the Lateral Thermal Interface Material Analysis (LaTIMATM), Raman thermometry, and the 3ω technique, whose particular experimental setups used in this work, are briefly described below.

1. Lateral Thermal Interface Material Analysis

This novel technique consists in setting a constant thermal current along a free standing thin film and measuring the heat flux and the temperature gradient of the sample through thermocouples and an IR camera, respectively, as shown in Fig. 4. The thermal conductivity is then determined by using a simple formula,[34] which is analogous to the ASME standard. The experimental setup was designed to minimize possible artifacts related to the sample materials and their geometry, and to ensure a measurement accuracy better than $\pm 5\%$. The dimensions of the oblong SAG samples analyzed by means of the LaTIMA technique were $5 \cdot 20 \text{ mm}^2$ with thicknesses of $150 \text{ }\mu\text{m}$ (SAG1) and $50 \text{ }\mu\text{m}$ (SAG2).

FIG. 4: Measuring principle of the LaTIMATM experimental setup.

2. Raman thermometry: a contactless approach

This technique is based on illuminating a sample surface with a laser beam and measuring its temperature rise as a function of the absorbed power. The laser incident energy should be higher than the sample bandgap to generate a heat current through the electron-phonon interactions. This is achieved by the inelastic scattering (Raman effect) and absorption of the laser beam incident photons by the sample electrons in the valence band. These "hot" electrons subsequently decay emitting optical and acoustic phonons and therefore rising the local temperature. The scattered light is collected by an optical system and analyzed with a spectrometer, as described elsewhere.[35–37]

To measure the thermal conductivity of our sintered silver samples by means of this contactless technique, they were used as TIMs to bond a $20 \text{ }\mu\text{m}$ -thick Si chip to a copper

FIG. 5: Scheme of the experimental measurement system for Raman thermometry and the 3ω technique. The Au resistor is used as the 3ω heater and sensor.

heat sink, as shown in Fig. 5. The incident laser beam was focused onto the Si surface and the temperature rise (ΔT) at the illuminated Si surface was recorded as a function of the absorbed power (P_{abs}). The effective thermal conductivity k_{eff} of the Si-TIM-Cu sub-system was then determined by considering it like an isotropic semi-infinite medium,[36] for which $k_{eff} = 2P_{abs}/(\pi d \Delta T)$, with $d = 700$ nm being the diameter of the laser spot. Raman thermometry hence does not allow differentiating the contribution of each layer to the effective thermal conductivity of this sub-system, however its simplicity makes it ideal for a fast initial measurement. Given that the thermal dissipation in the Si chip is driven by the overall thermal conductivity of the Si-TIM-Cu sub-system, from an applied perspective, this quantity is usually of primary interest.

3. 3ω technique: a depth resolved study

The popular and accurate 3ω technique is depth sensitive through its frequency response to the voltage third harmonic of the injected current.[38] For the layered system shown in Fig.5, this frequency dependence is monitored via the Au resistor deposited on the top of the native Al_2O_3 layer, whose presence is intrinsic. Among the several approaches to apply this technique to a multilayered system, [39, 40] we used the one that allows us taking measurements over a relatively wide range of frequencies without any restriction on the number or thickness of the individual layers.[40] The search algorithm associated with the model was implemented in Matlab and is able to straightforwardly extract the thermal

conductivity of our TIM samples.

Two TIM samples of sintered silver were fabricated with thicknesses of $13\text{ }\mu\text{m}$ and $48\text{ }\mu\text{m}$. The frequency response of these samples was observed to be between 10 Hz and 650 Hz. The thermal conductivity of both samples was then extracted from the frequency dependence of the voltage third harmonic.

IV. RESULTS AND DISCUSSIONS

In this section, we present and analyze the predictions of the proposed analytical and numerical models in comparison with our experimental results for the thermal conductivity of sintered porous samples.

A. Thermal conductivity measurements

FIG. 6: Effective thermal conductivity measured by the LaTIMA technique, in comparison with (a) its values reported in the literature for different materials and (b) our numerical result for sintered silver samples.

Figure 6(a) shows the comparison of the thermal conductivity measured by means of the LaTIMA technique for different materials, with their corresponding well-known values reported in the literature. The measurements were performed for two samples of each material. The rather good agreement between the experimental and the literature values for the

FIG. 7: Temperature rise measured by Raman thermometry on the illuminated Si surface, as a function of the absorbed power by two TIM samples of sintered silver paste. The dashed and dotted lines represent the best least squares fittings to the experimental data.

eight materials, demonstrates the high accuracy of the LaTIMA technique and establishes a benchmark for measuring the thermal conductivity of our porous SAG samples, which is shown in Fig. 6(b). Note that the thermal conductivity of the two samples depends on their thicknesses, which is commonly observed in sintered die attach materials treated even with identical processing conditions. As one can see, the experimental results validate our numerical predictions shown in Fig. 13(b), with a deviation smaller than 10%, for the relevant thinner SAG2 sample.

The temperature rise measured by Raman thermometry on the illuminated Si surface is displayed in Fig. 7, as a function of the absorbed power. The increase of the temperature with the absorbed power is nearly driven by a straight line, whose slope, according to the discussion in the above sub-section III.B.2, determines the effective thermal conductivities $k_{13} = 115 \text{ W/m}\cdot\text{K}$ and $k_{48} = 140 \text{ W/m}\cdot\text{K}$ of the Si-TIM-Cu sub-systems with TIM thicknesses of $13 \text{ }\mu\text{m}$ and $48 \text{ }\mu\text{m}$, respectively. Note that k_{48} is comparable to but smaller than the ones determined numerically (Fig. 13(b)) and by the LaTIMA technique (Fig. 8(b)), for a single TIM (SAG2). This slight difference indicates that the thermal boundary resistances in the Si-TIM-Cu sub-system are low and that the sintered silver TIM provides a rather good thermal performance to conduct heat from the Si chip towards the copper heat sink. On the other hand, the difference between k_{13} and k_{48} arises from the semi-infinite medium approximation applied to determine them, which reduces the experimental accuracy

FIG. 8: Absorbed-power dependence of the temperature rise measured by the 3ω method on the illuminated Si surface, for two TIM samples of sintered silver paste. The continuous lines stand for the finite element (FE) simulations performed for different sample thermal conductivities.

of Raman thermometry, especially for the thinner probed system.

Figures 8(a) and 8(b) show the dependence on the absorbed power of the temperature rise measured with the 3ω method on the illuminated Si surface, for two TIM samples. The comparison of the experimental data with the numerical results obtained through finite element (FE) simulations yielded the thermal conductivities $k_{s13} = (150 \pm 10)$ W/m·K and $k_{s48} = (130 \pm 10)$ W/m·K for the sintered silver TIMs with thicknesses of $13 \mu\text{m}$ and $48 \mu\text{m}$, respectively. Therefore, within an experimental uncertainty smaller than 16%, both samples have the same thermal conductivity, which is practically independent of the samples thicknesses. This relatively high uncertainty arises from the multiple layers of the probed system (Fig. 5), which increases the number of input parameters and hence the systematic errors to determine the thermal conductivity of the TIMs. It is thus clear that this experimental uncertainty represents the tradeoff to measure the thermal conductivity of a thin film at which one does not have direct access (buried layer). More importantly, the TIM thermal conductivity measured with the 3ω method agrees quite well with the one obtained through the LaTIMA technique (Fig. 6(b)).

B. Pore shape and orientation

We study here the influence of microstructural morphological features on the effective thermal conductivity of porous media. Several void shapes with varying number of corners, curvature, and orientation with respect to the applied temperature gradient are considered. To this purpose, various 2D unit cells with fixed porosity have been generated, each containing a single void of a particular shape in the centre. The numerically computed effective thermal conductivity normalized by the thermal conductivity of pure silver is shown in Figs. 9(a) and 9(b), where the symbols indicate the pore shape. The thermal conductivity clearly varies for unit cells with different pore shapes, showing that the geometry of pores plays an important role on the resulting thermal performance of the material, which cannot be determined by the porosity only. Note that rounded pores and the ones with more corners yield higher thermal conductivities. The curvature of the pore edges has also a clear impact, with convex pore shapes providing a higher thermal conductivity than concave ones. The orientation of the larger pore side along the direction of the temperature gradient has also a positive effect to avoid the reduction of the effective thermal conductivity. These results are reasonable and determined by the material channels available for the heat transport. For a fixed porosity and concave pore shapes and pores oriented perpendicularly to the temperature gradient, these channels are narrower than those for convex and parallel oriented pores. Examples of the computed heat flux fields for pores with four corners and varying edge curvature are shown in Figs. 10.

The effect of the shape of pores with random orientation is now studied by means of the analytical model, which considers convex shapes only. Figure 11 shows that the exponent n takes its minimal value for spherical pores ($\rho = 1$) and increases for other ellipsoidal ones. For cigar-shaped pores ($\rho > 1$), n increases with ρ until reaching its asymptotic value of $5/3$, for perfect cylindrical pores. On the other hand, for pancake-shaped pores, n increases without limit as ρ decreases. It is therefore clear that the pancake-shaped pores have a stronger effect than the cigar-shaped ones on the thermal conductivity of porous pastes. In addition, for $\rho > 0.3825$, there are two aspect ratios, one larger than and another smaller than unity, for which the exponent n takes the same value. This indicates that porous materials made up of pores with different shapes may have the same thermal conductivity.

The normalized thermal conductivity of the porous material, predicted by the analytical

FIG. 9: Relative thermal conductivity computed for unit cells with pores of different (a) number of corners and curvatures, and (b) shapes and orientations. The symbols indicate the pore shapes and orientations.

FIG. 10: Computed heat flux field for the unit cells with (a) concave, (b) straight, and (c) convex curvatures of pore edges.

model is shown in Figs. 12(a) and 12(b) as a function of the pore aspect ratio and porosity, respectively. Note that, irrespective of the porosity, the maximum thermal conductivity of the porous paste occurs for spherical pores, which is consistent with the numerical results shown in Figs. 9. Pastes with pancake-shaped pores exhibit a sizeable reduction of their thermal conductivity, such that flatter pores yield lower thermal conductivities. The difference between the thermal conductivities for spherical and cylindrical pores is relatively small and it reduces as the porosity decreases.

FIG. 11: Exponent n as a function of the aspect ratio $\rho = b/a$ of ellipsoidal pores with random orientation.

FIG. 12: Normalized thermal conductivity of a porous material as a function of the (a) pore aspect ratio and (b) porosity.

C. Theoretical predictions for Porous Silver

The thermal conductivity of the porous silver sample shown in the micrograph of Fig. 13(a) is here computed by means of the FEM analysis. The porosity of this material is approximately 0.22. In order to include sufficient statistics while keeping the FEM model size manageable, several microstructural models have been obtained by cutting image windows of different sizes. The computed effective thermal conductivity as a function of the window size is shown in Fig. 13(b), with the mean value and standard deviation indicated for several

FIG. 13: (a) Window cuts on a SEM image of a porous silver sample (porosity $p \approx 0.22$) and (b) its computed effective thermal conductivity as a function of the window size.

windows of the same size. The uncertainty on the retrieved thermal conductivity, due to the stochasticity of the microstructure is smaller than 7% for all window sizes. The average values of the thermal conductivity for each window size do not deviate significantly and vary between 148.9 $\text{W/m}\cdot\text{K}$ and 156.3 $\text{W/m}\cdot\text{K}$. This indicates that the overall average value of 151.6 $\text{W/m}\cdot\text{K}$ is a good representation of the effective thermal conductivity of the sample of the porous silver material. Note that this range of values is not predicted by the simple rule of mixtures (326.8 $\text{W/m}\cdot\text{K}$), which once more emphasizes the significant role of the pore geometrical effects. According to Fig. 12(b), the predictions of the analytical and FEM methods are in good agreement for spherical pores. This confirms that the thermal conductivity of materials with pores of regular shapes is described by the simple formula in Eq. (4). On the other hand, for real porous samples, which exhibit a wide distribution on the pore sizes and shapes, as shown in Fig. 10(a), the numerical prediction for the thermal conductivity displayed in Fig. 13(b) agrees with the analytical one for pancake-shaped pores of aspect ratio $b/a = 14.2$ (Fig. 12(b)). This fitted value is quite reasonable due to the significant presence of flat pores in the sample. Furthermore, these theoretical predictions are confirmed and validated by the experimental data shown in Figs. (6) and 8. This consistence between our theoretical and experimental results demonstrates the good predictive performance of our analytical and numerical models to fully describe the thermal

conductivity of porous TIMs, as a function of their porosity and pores size and shape.

V. CONCLUSIONS

The effective thermal conductivity of sintered porous pastes of silver has been theoretically and experimentally studied. The theoretical modeling is based on both the differential effective medium theory and the finite element method, whose consistent predictions show that the pore geometry and orientation play a significant role on the effective thermal conductivity of porous pastes. Both models have shown that the reduction of the thermal conductivity can be minimized with spherical pores and maximized with pancake-shaped ones. A thermal conductivity of 151.6 W/m·K has been numerically determined for a sintered silver sample with 22% of porosity. This agrees quite well with the thermal conductivity measured by the lateral thermal interface material analysis for a suspended sample and matches, within an experimental uncertainty smaller than 16%, with the values obtained by means of Raman thermometry and the 3ω technique for two silver samples buried in a silicon chip. The good agreement between our theoretical and experimental results have demonstrated the reliability of our methodology to perform the thermal characterization of porous thermal interface materials and to guide their engineering with a desired thermal conductivity.

ACKNOWLEDGEMENTS

This work was supported by the NANOTHERM project co-funded by the European Commission under the "Information and Communication Technologies", Seven Framework Program, and the Grant Agreement N° 318117.

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