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Identification of Above-Zone Pressure Perturbations Caused by Leakage from Those Induced by Deformation

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Abstract

Pressure changes in the above zone, i.e., the overlying aquifer of an injection zone separated by a sealing caprock, are usually attributed to leakage through wells. However, pressure changes can be induced geomechanically due to rock deformation without any hydraulic connection between the injection zone and the above zone where the pressure change is observed. To account for these two causes of pressure change in the above zone, we develop an analytical solution to evaluate the deformation-induced pressure changes and we derive an asymptotic analytical solution for pressure perturbations caused by leaking wells. The analytical models compare well with available numerical/analytical solutions. Using the analytical solutions for the deformation- and leakage- induced pressure changes, we propose a graphical diagnostic plot to determine the cause of pressure change. Considering that the pressure change is caused by leakage, we then use the asymptotic solution to develop an easy-to-use fully graphical methodology to characterize leaking wells. This methodology improves a previous analysis methodology that was based on an inverse modeling algorithm that can be highly instable and

computationally expensive. Based on the graphical method presented here, the slopes and intercepts of the proposed line-fitted graphs are used to determine the leak location and transmissibility. We apply the graphical method to an example problem to illustrate its application procedure and effectiveness in differentiating deformation-induced pressure changes from leaking wells. Overall, the diagnostic plot proposed here proves to be useful to determine the cause of the above-zone pressure change.

Keywords: fluid injection; leakage; geomechanics; diagnostic plot; analytical solutions

1. Introduction

Fluid injection in subsurface permeable saline formations is increasing significantly due to the growing interest in various applications, including wastewater disposal, gas storage, CO₂ sequestration, geothermal energy and enhanced oil/gas recovery. Containment of the injected fluids is needed to meet the regulatory requirements and/or to ensure efficiency of the intended processes. The injected fluids can leak to overlying formations in presence of leakage pathways. Improperly plugged and abandoned (P&A) wells are considered as high potential leakage pathways (Gasda et al., 2004; Nicot, 2009). In presence of leakage pathways, the injection zone (IZ) pressurization caused by fluid injection may lead to mobilisation of the IZ's native fluid and/or the injected fluids toward the surface. A leakage-induced pressure change can travel quickly and be sensed over broad areas. In addition, today's pressure transducer technology allow measuring pressure at high frequency, resolution and accuracy at relatively

low cost. These characteristics make the pressure monitoring a viable option for detecting leakage in the subsurface.

To detect leakage, the pressure can be measured both in the IZ and in an above-zone (AZ), i.e., the overlying aquifer separated from the IZ by a confining layer. AZ's pressure change should be more sensitive to leakage because it should only correspond to leakage events (Meckel et al., 2013). Leakage-induced pressure changes in the IZ and AZ have been studied for CO₂ storage in saline aquifers using both semi-analytical and numerical models (Birkholzer et al., 2009; Kang et al., 2014; Mathias et al., 2011; Mathias et al., 2009; Nordbotten et al., 2005; Pruess, 2011). CO₂ leakage is preceded by brine leakage and the pressure change corresponding to this brine leakage can be evaluated analytically using single-phase models (Cihan et al., 2011; Nordbotten et al., 2004; Zeidouni, 2012, 2014; Zeidouni et al., 2011). Vilarrasa et al. (2010a) presented a methodology to calculate an average CO₂ density that can be used to estimate the brine-equivalent of the injected CO₂ volume that may be used in single-phase models. Rapid brine leakage detection is crucial to the avoidance of potential CO₂ leakage from the reservoir and to minimize the amount of leaking brine.

Several researchers studied the inversion of the AZ's pressure response to determine the leak characteristics (Chabora and Benson, 2009; Jung et al., 2013; Sun et al., 2013; Wang and Small, 2014; Zeidouni and Pooladi-Darvish, 2012a, b). Inverse algorithms are applied to pressure data generated using analytical/numerical models to infer leakage volume and location. While inverse modeling can be useful, it requires robust and computationally expensive inversion techniques that may not be easy to implement in practice. Furthermore, inverse models can be very instable when the unknown parameters are not fully independent, which has been shown

to be the case for the leakage problem (Zeidouni and Pooladi-Darvish, 2012a). Thus, developing graphical approaches, such as those used in conventional pressure transient analysis (e.g. Spivey and Lee, 2013), may be very convenient for analyzing the pressured data for leaking well characterization. Here, we introduce a technique to obtain all the leaking well parameters graphically.

Apart from leakage, pressure changes in the AZ may be induced geomechanically without any hydraulic communication between the IZ and AZ. When a fluid is injected into IZ, the IZ expands, causing deformation-induced pressure changes into the adjacent caprock and the AZ. As a result of this deformation, fluid pressure drops in the caprock in response to injection into the IZ (Hsieh, 1996; Kim and Parizek, 1997; Rodrigues, 1983; Vilarrasa et al., 2010b; Vilarrasa et al., 2013). This phenomenon is known as reverse-water level fluctuations or Noordbergum effect (Verruijt, 1969). Furthermore, deformation-induced pressure changes also affect the AZ (Hsieh, 1996). Though a small volume of the lower portion of the AZ close to the injection well has the same sign of the pressure change induced in the IZ at early times of injection, the rest of the AZ presents, like the caprock, a reverse-water level fluctuation (Hsieh, 1996). Thus, it is important to determine this deformation-induced pressure perturbation and how it differs from leakage-induced pressure change.

In the following, we derive an analytical solution to determine the characteristics of AZ pressure changes induced by deformation. Next, we derive an asymptotic analytical solution for pressure changes associated with well leakage. We propose a diagnostic plot based on the analytical solutions to determine whether pressure change is caused by leakage or deformation. In addition, we develop a graphical method to analyze the pressure data for leaking well

characterization. In an example problem, we compare and illustrate the agreement of our analytical solutions with available numerical/analytical solutions. We apply the diagnostic method to illustrate how the pressure changes corresponding to deformation and leakage can be differentiated. Finally, we apply our proposed graphical method to analyze the pressure data for characterizing the leaking well.

2. Analytical models

To develop our analytical models, we use the physical configuration (shown in Fig. 1) considered in several analytical/numerical studies on well leakage systems (Avci, 1994; Cihan et al., 2011; Javandel et al., 1988; Nordbotten et al., 2004). The system consists of two permeable formations linked by a leaking well which are otherwise separated by a sealing caprock (configuration and variables used in equations are shown in Fig. 1). Fluid is injected at a constant flow rate q in the IZ and pressure is measured in an observation well both in the IZ and AZ. Properties of the injected fluid are identical to those of the original fluids of the IZ and AZ. The IZ and AZ are assumed sufficiently large so that they act infinitely throughout the pressure observation period. The zones are also considered isotropic and homogeneous with constant properties (e.g., permeability, porosity, and compressibility). Leakage can occur in the vertical direction through a leaking well of radius r_l and permeability k_l . The leaking well is at a distance R from the injection well and distance r from the observation well. A single-phase, 1-D, radial-flow system is considered in IZ and AZ.

In the following, the analytical solution for deformation-induced (in absence of leakage) pressure change is first derived followed by the derivation of the analytical solution corresponding to a leaking well.

2.1. Pressure change due to deformation

To analytically calculate the deformation-induced pressure change that occurs in the AZ, we assume that the overpressure is due, exclusively, to the deformation induced by fluid injection. Therefore, we assume that there is no vertical flux from the IZ to the AZ through the caprock. This assumption of no-flow through the caprock is reasonable for short times of injection, when overpressure has not had enough time to propagate into the caprock. However, for long injection times, i.e., several years of injection, overpressure will propagate into the caprock (Chang et al., 2013; Vilarrasa and Carrera, 2015) and eventually the resident brine will flow vertically towards the AZ.

The vertical displacement due to expansion of the IZ due to injection, assuming no horizontal displacement is (Vilarrasa et al., 2013)

$$u = \phi c_t h \Delta p' , \quad (1)$$

where ϕ is the IZ porosity, $c_t = (c_w + c_r)$ is the total compressibility (considered identical for IZ and AZ), c_w is water compressibility, c_r is rock compressibility, h is the IZ thickness and $\Delta p'$ is the overpressure induced in the IZ, which can be closely approximated by (Cooper and Jacob, 1946)

$$\Delta p' = \frac{q\mu}{4\pi kh} \ln \left(\frac{2.25kt}{\mu\phi c_t r^2} \right), \quad (2)$$

where q is the flow rate, μ is fluid viscosity, k is the IZ permeability, t is time and r is the radial distance from the injection well.

If we assume that the caprock has a negligible permeability, i.e., lower than 10^{-19} m^2 , there will be no flow across it and therefore, no overpressure due to fluid movement will occur. Thus, the overlying layers will not expand similarly to the aquifer due to fluid injection (Rutqvist et al., 2010). However, there will be a pressure change due to injection-induced deformation. The expansion of the IZ deforms the overlying layers, causing both bending and compression of the AZ. Bending expands the pore volume of the AZ (Vilarrasa et al., 2013), which will cause a pressure drop, and compression contracts the pore volume, which will cause a pressure buildup. Here, we only consider the compression of the AZ caused by the IZ expansion. The initial pressure drop caused by bending may be larger than the overpressure caused by compaction at very short injection times, which would cause a pressure decrease. Since this effect can easily be distinguished from the pressure perturbation caused by leakage because they have opposite signs, we do not include bending effects in this analytical solution. Apart from bending, assuming that there is no vertical flow, the vertical expansion that takes place within the IZ due to overpressure will progressively dissipate towards the surface causing compression of the adjacent layers. This compression will be proportional to the IZ expansion and could be calculated, for instance, with the solution of Okada (1992). Taking into account this progressive attenuation of the vertical deformation, the volumetric deformation of the AZ can be expressed as

$$\varepsilon_v = \Delta\phi_m = \varepsilon_z = -\frac{Cu}{h_m}, \quad (3)$$

where ε_z is the vertical deformation, ϕ_m is the AZ porosity, C is the attenuation constant of the vertical displacement within the AZ and h_m is the thickness of the AZ. The volumetric deformation equals the change in porosity of the AZ. This reduction in porosity (note the negative sign in Equation (3)) will induce an overpressure because the resident fluid will have to accommodate into a smaller pore volume.

Mass conservation of water in the AZ can be expressed as (Bear, 1972)

$$\frac{\partial(\rho\phi_m)}{\partial t} = -\nabla \cdot (\rho Q) , \quad (4)$$

where ρ is fluid density and Q is the volumetric flux, which can be expressed using Darcy's law (assuming horizontal flow)

$$Q = -\frac{k_m}{\mu} \nabla p_m , \quad (5)$$

where p_m is pressure in the AZ and k_m is the AZ permeability.

Neglecting the fluid compressibility and using Equation (5), the mass conservation of the AZ (Equation (4)) yields

$$\frac{\partial\phi_m}{\partial t} = \frac{k_m}{\mu} \nabla^2 p_m . \quad (6)$$

Applying the temporal derivative to Equation (3) and using Equations (1) and (2) and assuming that the IZ porosity remains constant gives

$$\frac{\partial\phi_m}{\partial t} = -\frac{q\mu\phi_c C}{4\pi k h_m t} . \quad (7)$$

Combining Equations (6) and (7) and taking into account that the Laplacian operator in radial coordinates equals

$$\nabla^2(\bullet) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial(\bullet)}{\partial r} \right), \quad (8)$$

the mass conservation becomes

$$-\frac{q\mu^2\phi c_t C}{4\pi k k_m h_m t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Delta p_m}{\partial r} \right) \quad \text{where } \Delta p_m = p_m - p_{mi}, \quad (9)$$

where p_{mi} is the initial pressure in the AZ.

Integrating Equation (9) and considering the pressure symmetry at the injection well ($d\Delta p_m/dr = 0$ at $r \rightarrow 0$) and that the overpressure is null at the radius of influence, give the following expression for the deformation-induced pressure change in the AZ

$$\Delta p_m = \frac{q\mu^2\phi c_t C}{16\pi k k_m h_m t} (r_i^2 - r^2), \quad (10)$$

where r_i is the radius of influence given by (see e.g. Sabet, 1991, Chapter 1)

$$r_i = \sqrt{4\eta t}, \quad (11)$$

where $\eta = k / (\phi\mu c_t)$ is the IZ hydraulic diffusivity.

Replacing Equation (11) into (10), the deformation-induced pressure change in the AZ results in

$$\Delta p_m = \frac{q\mu C}{4\pi k_m h_m} \left(1 - \frac{r^2}{4\eta t} \right). \quad (12)$$

To generalize the analytical solution, the deformation-induced pressure change in the AZ can be expressed in dimensionless variables. For a monitoring well located at a distance L from the injection well, Equation (12) becomes

$$p_{mD} = \frac{C}{2T_D} \left(1 - \frac{L_D^2}{4t_D} \right), \quad (13)$$

where $L_D = L/r_w$ is the dimensionless injector-monitor distance, r_w is the well radius, $t_D = \eta t / r_w^2$ is the dimensionless time, the dimensionless pressure is $p_{mD} = (2\pi k h \Delta p_m) / (q\mu)$, and the dimensionless transmissivity is $T_D = k_m h_m / kh$.

The derivative of the dimensionless pressure change in the AZ with respect to the logarithm of the dimensionless time reads

$$\frac{dp_{mD}}{d \ln t_D} = \frac{C}{8T_D} \frac{L_D^2}{t_D}. \quad (14)$$

Hereafter, we use the tradition of referring to the pressure derivative with respect to logarithm of time as pressure derivative (Bourdet, 2002).

The solution for deformation-induced pressure changes in the AZ implies the following:

(i) The solution has the similarity property with similarity variable r^2/t . Pressure measurements from different locations when plotted versus the similarity variable should overlap.

(ii) Log-log plot of dimensionless pressure derivative ($dp_{mD}/d \ln t_D$) versus dimensionless time provides a linear behavior with a negative unit-slope and intercept of $\log \left(\frac{C}{8T_D} L_D^2 \right)$. Such a

plot can serve as a diagnostic plot to distinguish the deformation-induced pressure change.

The same plot for a leaking well is of interest which will be discussed in the next section.

- (iii) The dimensionless pressure change reaches a constant $p_{mD} = C/2T_D$ at late time. If $t_D/L_D^2 \geq 100$, using the constant p_{mD} , predicts the pressure change within 1% error. In other words, after $t_D = 100L_D^2$ the deformation-induced pressure change in the AZ may be negligible. After this time any pressure change can be interpreted as caused by another source, e.g., well leakage. This is important as it implies that the early time ($t_D < 100L_D^2$) pressure measurements may be highly influenced by deformation and may not be interpreted for leakage. To characterize leakage, the pressure data may be acquired over a sufficiently long time period so that $t_D \gg 100L_D^2$.

2.2. Pressure change due to well leakage

Avci (1994) derived a Laplace domain analytical solution for the two-layer system shown in Fig. 1 considering leakage through a well. Based on Avci (1994), the dimensionless pressure in AZ at the observation well (located at distance L from the injection well and distance l from the leaking well) is given by Equation (15) and the dimensionless leakage rate is given by Equation (16).

$$\bar{p}_{mD} = \frac{\bar{q}_{ID}(s)}{T_D} K_0 \left(\sqrt{\frac{s}{\eta_D}} l_D \right), \quad (15)$$

where

$$\bar{q}_{lD} = \frac{\frac{1}{s} K_0(\sqrt{s} R_D)}{K_0(\sqrt{s} r_{lD}) + \frac{1}{T_D} K_0\left(\sqrt{\frac{s}{\eta_D}} r_{lD}\right) + \frac{1}{\alpha}} \quad (16)$$

and

$$R_D = \frac{R}{r_w}, \quad t_D = \frac{t}{r_w}, \quad r_{lD} = \frac{r_l}{r_w}, \quad \alpha = \frac{r_l^2 k_l}{2kh h_l}, \quad \eta_D = \frac{\eta_m}{\eta}, \quad q_{lD} = \frac{q_l}{q}. \quad (17)$$

In the above equations K_0 is the modified Bessel functions of the second kind of order zero, s is the Laplace transform dummy variable and η_m is the hydraulic diffusivity of the AZ. q_l is the leakage rate. Considering late-time approximation and using series expansion and truncation, we obtain a real-time asymptotic solution. The derivation of the asymptotic solution from the Laplace-domain solution is given in the Appendix. Based on this solution, the dimensionless pressure at the observation well in AZ is given by Equation (18) and the dimensionless leakage rate is given by Equation (19),

$$p_{mD} = \frac{1}{(1+T_D)} \left(-\frac{\gamma}{2} - \frac{1}{2} \ln \left(\frac{R_D^2}{4t_D} \right) \right) - \frac{q_{lD}}{2T_D} \left(\kappa + \ln \left(\frac{t_D^2}{\eta_D} \right) \right), \quad (18)$$

$$q_{lD} = \frac{1}{(1+1/T_D)} \left(1 - C(t_D) \left(\kappa + \ln(R_D^2) \right) \right), \quad (19)$$

where

$$C(t_D) = \frac{1}{(\kappa - 2\gamma + \ln(4t_D))} - \frac{\gamma}{(\kappa - 2\gamma + \ln(4t_D))^2} + \frac{\gamma^2 - \pi^2/6}{(\kappa - 2\gamma + \ln(4t_D))^3}, \quad (20)$$

$$\kappa = \frac{2T_D}{\alpha(T_D+1)} + \ln \left(\frac{\eta_D^{\frac{1}{(T_D+1)}}}{r_{lD}^2} \right) \quad (21)$$

A number of observations can be made from the solution above:

(i) The term $\left(-\frac{\gamma}{2} - \frac{1}{2} \ln \left(-\frac{R_D^2}{4t_D} \right) \right)$ on the right-hand side (RHS) of Equation (18) is the late-time

approximation of $\frac{-1}{2} Ei \left(-\frac{R_D^2}{4t_D} \right)$, which represents pressure change due to injection in the

IZ at the leaking well. The two terms on the RHS of Equation (18) are close in magnitude and are of opposite sign. We have found that the use of the ln-form of the solution (as in Equation (18)) leads to a smooth and monotonic behavior of Equation (18) at early times.

(ii) In the limit as $t_D \rightarrow \infty$, Equation (19) approaches zero, meaning that a steady-state leakage rate can be obtained that equals $T_D/(T_D+1)$. Therefore, if the transmissivity ratio is large, the steady-state leakage rate approaches the injection rate ($q_{lD} \approx 1$). When T_D is small, $q_{lD} \approx T_D$.

Based on Equations (19) and (20), the leakage rate reaches steady-state if $\frac{\kappa + \ln(R_D^2)}{\kappa - 2\gamma + \ln(4t_D)}$

is negligibly small. Beyond such a time, the leak can be treated as a constant rate sink/source.

(iii) For large values of α , Equation (21) becomes independent of α , suggesting that pressure and the leakage rate become independent of α when α is large. α is the main dimensionless parameter affecting the leakage rate and corresponds to leaking well transmissibility (permeability $\times$ cross-sectional area) which we designate as the *leakage coefficient*.

(iv) Taking the derivative of Equation (18) and using only the first term in Equation (20) gives:

$$\frac{dp_{mD}}{d \ln t_D} = \frac{1}{2(1+T_D)} \left(1 - \frac{\left(\kappa + \ln \left(\frac{t_D^2}{\eta_D} \right) \right) \left(\kappa + \ln (R_D^2) \right)}{\left(\kappa - 2\gamma + 2 \ln (2) + \ln (t_D) \right)^2} \right). \quad (22)$$

Unlike the deformation-induced derivative where derivative decreases with increasing t_D , the leakage-induced derivative increases with increasing t_D . Moreover, based on Eq. (22) the log-log plot of the derivative versus t_D for the leaking well is always nonlinear and concave down. This provides sufficient information for distinction between the leakage-induced and deformation-induced pressure changes. Log-log plot of the derivative versus time for the pressure change induced by deformation is linear and decreases with time (based on Eq. (14)) and that for leaking well is non-linear, concave downward and increases with time (based on Eq. (22)).

3. Leaking well parameterization

Equation (18) gives the dimensionless pressure in the AZ (p_{mD}) at a given time and location. The following parameters of the leaking well are required to determine the pressure change: R_D , t_D , r_{lD} , and α . As discussed in Zeidouni and Pooladi-Darvish (2012a), for practical purposes, the leaking well radius (represented by r_{lD}) does not affect the pressure independently as long as the leakage coefficient, α , is constant. Therefore, the leaking well was characterized by three parameters: R_D , t_D , and α . r_{lD} is assumed known and can be considered equal to 1. L_D is the

dimensionless distance between the injection and observation wells which is known. In short, the leaking well can be characterized by: R_D , λ_D , and α .

As discussed in the next section, pressure observation at more than one location is required to enable characterization of a single leaking well. When multiple monitoring wells are involved, multiple λ_D values will be unknown. Working with the location coordinates (instead of distances) will reduce the number of unknowns. Therefore, it is more convenient to work with the location coordinates when measurements at multiple observation wells are available. We establish the convention of using (0, 0) as the location of the injector. Then, the leaking well is parameterized by its coordinates (x_l, y_l) and the leakage transmissibility. In dimensionless form, the leaking well parameters are x_{lD} (x_l/r_w), y_{lD} (y_l/r_w) and α . The relationship between the leaking well coordinates and distance R is

$$R_D = \sqrt{x_{lD}^2 + y_{lD}^2}. \quad (23)$$

The location of the monitoring well (x_m, y_m) is related to the distances λ and L by

$$\lambda_D = \sqrt{(x_{lD} - x_{mD})^2 + (y_{lD} - y_{mD})^2}, \quad (24)$$

$$L_D = \sqrt{x_{mD}^2 + y_{mD}^2}, \quad (25)$$

where $x_{mD} = x_m/r_w$ and $y_{mD} = y_m/r_w$.

In the following we use the above solution to arrive at useful forms that can be used to graphically evaluate the unknowns of the leaking well parameters properly.

4. Graphical method development

4.1. Graphical determination of leakage coefficient, α

Leakage coefficient, α , represents the leaking well transmissibility. The graphical method for estimating α is presented in Zeidouni and Pooladi-Darvish (2012b) based on the fact that the

plot of $\frac{1}{\sqrt{1-2(1+T_D)\frac{dP_{mD}}{d\ln t_D}}}$ versus $\ln(t_D)$ is linear at late-time. The slope (m) and intercept (b)

of the line-fitted data will give:

$$\kappa = \frac{b}{m} + 2\gamma - 2\ln(2), \quad \text{where} \quad \kappa = \frac{2T_D}{\alpha(T_D + 1)} + \ln\left(\frac{\eta_D^{\frac{1}{(T_D+1)}}}{r_{lD}^2}\right), \quad (26)$$

$$\left(\kappa + \ln(R_D^2)\right)\left(\kappa + \ln\left(\frac{r_D^2}{\eta_D}\right)\right) = \frac{1}{m^2}. \quad (27)$$

α can be calculated using eq. (26).

4.2. Graphical determination of leaking well location (x_{lD} , y_{lD})

Determination of the location parameters can be difficult. Using inverse modelling techniques, Zeidouni and Pooladi-Darvish (2012a) arrived at the following conclusions: (1) Pressure inversion is non-unique: exactly the same pressure response can be obtained for different sets of leaking well parameters, (2) leak parameters are highly interdependent: any error in estimation of one parameter translates into errors in the other parameters, (3) pressure is

much more sensitive to α than location parameters: any error in α translates to much larger errors in the location parameters. The first conclusion implies that at least two observation wells are required to enable fully characterization of a leaking well. The observation wells must be located so that one of them is off the line connecting the other to the injection well. The second and third conclusions suggest that the pressure data should be interpreted in a way to minimize the effect of α estimation from location parameters determination. Also, to obtain the location parameters pressure observation at several points may be required. This can be made by mounting several hydraulically separated pressure gauges along the path of a single directional observation well.

One would expect that location parameters can be determined based on Equation (27) using data from at least two observation wells. Considering typical ranges for the system properties provides that $\kappa \in [-4.6, 1.2 \times 10^9]$ while $\ln(R_D^2) \in [4.6, 18.4]$ and $\ln(l_D^2/\eta_D) \in [0.0, 23.0]$. κ varies on a much wider range and can be orders of magnitudes larger than $\ln(R_D^2)$ and $\ln(l_D^2/\eta_D)$. Therefore, any error in estimating α can cause a significant error in estimating the location parameters when using Equation (27). Thus, information derived from eq. (27) on the location parameters can be of little value. Here, we develop an alternative method to minimize the impact of α on the location parameters estimation. The method requires at least 3 observation points.

Consider 3 observation points 1, 2 and 3 located at (x_{mD1}, y_{mD1}) , (x_{mD2}, y_{mD2}) , and (x_{mD3}, y_{mD3}) .

The leak-monitor distances l_{D1} , l_{D2} , and l_{D3} are given by

$$z_{Di} = \sqrt{(x_{iD} - x_{mDi})^2 + (y_{iD} - y_{mDi})^2} \quad \text{where } i = 1, 2, 3. \quad (28)$$

Rewriting Equation (18) for the two observation wells 1 and 2, followed by adding and subtracting the resulting equations gives

$$\frac{\ln\left(\frac{z_{D2}}{z_{D1}}\right)}{\kappa + \ln\left(\frac{z_{D1}z_{D2}}{\eta_D}\right)} = \frac{\left(\frac{dp_{mD1}}{d \ln t_D} - \frac{dp_{mD2}}{d \ln t_D}\right)}{\frac{1}{1+T_D} - \left(\frac{dp_{mD1}}{d \ln t_D} + \frac{dp_{mD2}}{d \ln t_D}\right)}. \quad (29)$$

For observation wells 2 and 3 we get

$$\frac{\ln\left(\frac{z_{D3}}{z_{D2}}\right)}{\kappa + \ln\left(\frac{z_{D2}z_{D3}}{\eta_D}\right)} = \frac{\left(\frac{dp_{mD2}}{d \ln t_D} - \frac{dp_{mD3}}{d \ln t_D}\right)}{\frac{1}{1+T_D} - \left(\frac{dp_{mD2}}{d \ln t_D} + \frac{dp_{mD3}}{d \ln t_D}\right)}. \quad (30)$$

The above equations suggest that a plot of Δ_1 and Δ_2 (given by Equation (31)) as a function of time will give a zero-slope line which intersects the y-axis at g_1 and g_2 (given by Equation (32)), respectively. Evaluation of g_1 and g_2 provides a system of two equations with two unknowns that can be readily solved for x_{iD} and y_{iD} .

$$\Delta_i = \frac{\left(\frac{dp_{mD,i}}{d \ln t_D} - \frac{dp_{mD,i+1}}{d \ln t_D}\right)}{\frac{1}{1+T_D} - \left(\frac{dp_{mD,i}}{d \ln t_D} + \frac{dp_{mD,i+1}}{d \ln t_D}\right)} \quad \text{where } i = 1, 2, \quad (31)$$

$$g_i = \frac{\ln\left(\frac{b_{i+1}}{b_i}\right)}{\kappa + \ln\left(\frac{b_i b_{i+1}}{\eta_D}\right)} \quad \text{where } i = 1, 2. \quad (32)$$

5. Example problem

The domain geometry and fluid properties of the example problem are given in Table 1. Water is injected at a rate of 864 m³/day into a 30-m thick IZ which is separated from a 10-m thick AZ by a 4-m thick confining layer. Both zones contain water with the same properties as the injected water. The porosity and total compressibility is identical for both zones and equal to 0.1 and 10⁻⁹/Pa, respectively. The permeability is 10⁻¹³ m² for both IZ and AZ. The zones are initially at hydrostatic equilibrium. The pressure is observed in the AZ at a well located at a distance of 75 m from the injection well. Well radius is 0.1 m for both wells.

To obtain leakage-induced pressure changes we append a leaking well to the above configuration. The leaking well is located at $(x_l, y_l) = (46\text{m}, 19.6\text{m})$ which is 50 m from the injection well located at the origin of the coordinate system. Leaking well permeability is 10⁻¹⁰ m² and its radius is identical to the injection well radius (0.1 m).

5.1. Deformation-induced pressure comparison

The deformation-induced pressure changes for this example are obtained both with the proposed analytical solution (recall Section 2.1) and a hydro-mechanical numerical solution

solved with the finite element code CODE_BRIGHT (Olivella et al., 1994; Olivella et al., 1996). The model is identical to that solved for the analytical solution, but including the mechanical equations. The top of the injection zone is placed at 2 km depth. To avoid boundary effects, the model extends radially 20 km and we model the overburden above the AZ up to the surface and the basement below the IZ down to 4 km deep. The mechanical boundary conditions are set to no displacement perpendicular to all boundaries, except at the upper boundary, where the atmospheric pressure is imposed. The caprock permeability is set to a very small value ($k_{cap}=10^{-23}$ m²) to ensure that there is no flow across the caprock and thus, pressure changes in the AZ are only due to deformation. The Young's modulus of the caprock is set to 5 GPa and its Poisson ratio to 0.3.

The AZ pressure changes calculated using our analytical solution (considering $C=1$) are compared to the numerical results in Figure 2. Though the deformation-induced pressure changes predicted by the analytical solution do not show a good fitting with the numerical results, the long-term pressure evolution is well captured. One reason for this early mismatch may be that the analytical solution accounts only for vertical deformation, but the IZ-caprock-AZ system deforms in a more complex way. In particular, radial and bending deformation are probably controlling the volumetric strain evolution at early times of injection (Hsieh, 1996; Vilarrasa et al., 2013). This discrepancy suggests that this analytical solution cannot be used to estimate the initial pressure drop in the AZ induced by the expansion of the IZ. However, for the purpose of discriminating pressure perturbations in the AZ caused by leakage from those induced by deformation, this analytical solution satisfactorily captures the long-term deformation-induced pressure changes in the AZ (see Figure 3).

Figure 3 displays the pressure derivative in the AZ as a function of time (both in dimensional and dimensionless forms). The fitting between the numerical and analytical solutions is very good for the long-term tendency. Though the trends are different at the beginning of injection, the analytical and numerical solutions have the same slope for long injection times. The difference at the beginning of injection between the numerical and analytical derivatives (up to $L_D^2/t_D \cong 0.02$) is the cause of the mismatch in pressure evolution (recall Figure 2). On the other hand, the coincidence in the slope of the pressure derivative between the numerical and analytical solutions suggests that the long-term deformation-induced pressure changes in the AZ are mainly caused by the vertical compression of the AZ induced by the expansion of the IZ. Furthermore, since the logarithm of the derivative linearly decreases with the logarithm of time, the pressure changes induced by deformation occur only for a limited amount of time. Thus, if pressure perturbations are measured in the AZ after some days of injection, they will be most likely due to leakage.

Overall, this analytical solution cannot predict the initial pressure drop in the AZ that is induced by the expansion of the IZ. However, this is not a limitation for the methodology that we are proposing because the initial pressure drop has the opposite sign that a pressure perturbation that would be caused by leakage and thus, they are easily distinguishable. At later times, the deformation-induced pressure change tends to increase with time, so it has the same sign as the pressure perturbation caused by leakage. Since the analytical solution captures well the temporal evolution of the deformation-induced pressure changes in the AZ in the long-term, the analytical solution can be used to discriminate between leakage and deformation-induced pressure changes.

5.2. Leakage-induced pressure comparison

The leakage-induced pressure change calculated by our asymptotic solution is compared to that calculated by Avci's (1994) Laplace-domain solution. Note that these solutions provide the pressure changes caused merely by leakage. The solutions are compared in Figures 4 and 5 in terms of pressure change and pressure derivative both in dimensional and dimensionless forms. The solutions are in very good agreement in terms of pressure change. The derivatives are not in full agreement initially but reach agreement after 1 day of injection.

5.3. Leakage versus deformation

The log-log diagnostic plot of derivative versus time is given in Figure 6 for the pressure changes induced by well leakage (using Laplace domain solution) and deformation (using numerical and analytical solutions). The derivative corresponding to leakage increases with time and is concave downward. The derivative for deformation exhibit decreasing linear trend at late-time. The analytically derived derivative for deformation shows that the trend is indeed following a negative unit-slope with intercept of 5.32 ($=\log\left(\frac{1}{8T_d}L_d^2\right)=\log\left(\frac{3}{8}750^2\right)$). Therefore, the diagnostic plot proves to be useful in identification of pressure changes induced by deformation versus leakage.

It should be noted that given the linearity of the governing diffusivity equation we solved here, we can use the same solutions to determine the pressure response to a combination of leakage

and deformation. However, we limit the scope of this paper to study the pressure for the leakage and deformation separately.

5.4. Graphical leaking well characterization

5.4.1. Leakage Coefficient, α

We apply the graphical method introduced in this paper to determine the leaking well properties. The method presented in Zeidouni and Pooladi-Darvish (2012b) showed that leaking well leakage coefficient, α , can be calculated using line-fitted plot of

$$\frac{1}{\sqrt{1-2(1+T_D)\frac{dP_{mD}}{d\ln t_D}}} \text{ (referred to by LHS in Figure 7) versus } \ln(t_D). \text{ The plot is given in Figure 7}$$

based on which the intercept and slope are $b=0.864$ and $m=0.012$, respectively. Using Equation (26) we obtain $\kappa \approx 71.77$ and $\alpha = 6.97 \times 10^{-3}$. The graphically calculated α is in close agreement with that used to generate the data which was $\alpha = 6.94 \times 10^{-3}$.

5.4.2. Leaking well location

In this section we determine the location parameters based on the plot of Equation (31) as a function of time. Two additional monitoring wells are required (making a total of 3 monitoring points in the AZ) as discussed in Sec. 4.2. The additional monitoring points are located at $(x_{m2}, y_{m2}) = (75 \text{ m}, 20 \text{ m})$ and $(x_{m3}, y_{m3}) = (0, 20 \text{ m})$ as indicated in Table 2. In dimensionless terms, the location of the monitoring points are (x_{mD}, y_{mD}) of $(750, 0)$, $(750, 200)$ and $(0, 200)$. Each pair of wells provide one graph leading to 3 graphs overall. However, two graphs are sufficient for

determining the two location parameters. The two graphs are shown in Figure 8. g_1 and g_2 , given by Equation (32), are the late-time steady values of Δ_1 and Δ_2 . Therefore, from Figure 8, we obtain $g_1 = -0.00225$ and $g_2 = 0.00550$. Using these values of g_1 and g_2 along with Equation (32) and solving for the location parameters of the leak we obtain: $x_{ID} = 460$ and $y_{ID} = 196$ which are exactly identical to the correct values of the leak location parameters.

6. Summary and Conclusions

Monitoring pressure perturbations in an AZ separated from the IZ by a sealing caprock can be useful to evaluate containment of the injected fluid. However, AZ pressure changes are not necessarily caused by leakage and can be induced by geomechanical deformation. In this paper, we developed two analytical solutions to estimate the pressure changes in the AZ induced by deformation and well leakage. The analytical model for deformation-induced pressure changes is obtained by translating the vertical displacements into temporal variations in porosity and combining them with the diffusivity equation. Analytical solution for pressure change caused by leaking well was introduced by Avci (1994) which was obtained in Laplace domain. We obtain real-time domain asymptotic solution by series expansions and truncations of Avci's solution terms to make the analytical Laplace inversion possible. The deformation-induced pressure change is found to stabilize at late-time and its derivative (with respect to logarithm of time) follows a linear behavior when plotted versus time on a log-log graph.

Comparing the deformation-based analytical solution to numerical results for an example problem shows that the solution closely captures the late-time behavior of the pressure and its

derivative, which permits discriminating pressure perturbations caused by leakage from those induced by deformation. The asymptotic solution for leakage-induced pressure change shows excellent agreement with Avci's solution. Based on the analytical solution for deformation- and leakage- induced pressure changes, the log-log plot of dimensionless derivative versus dimensionless time can serve as a diagnostic plot to determine the cause of pressure changes that are observed. On this log-log plot the deformation-induced derivative follows a linear trend with negative unit slope while the leakage-induced derivative increases non-linearly and concaves downward.

The leakage-induced analytical solution is further utilized to develop a graphical technique to characterize leaking wells if the pressure change is found to be due to well leakage. As a result, the leaking well can be characterized graphically without the need for complex inverse algorithms. Applying to an example problem, the graphical method is shown to be able to obtain leaking well parameters very precisely.

Appendix: Derivation of Asymptotic Solution

For late time period: $s \rightarrow 0$, and we can make the following approximations (Abramowitz and Stegun, 1970)

$$K_0(x) \approx -\gamma - \ln(x/2) \quad (\text{A.1})$$

on the basis of which, Equation (16) simplifies to

$$\bar{q}_{ID} = \frac{-\frac{\gamma + \ln(\sqrt{s}R_D/2)}{s}}{-\frac{\gamma + \ln(\sqrt{s}r_{ID}/2)}{1} - \frac{\gamma + \ln\left(\sqrt{\frac{s}{\eta_D}}r_{ID}/2\right)}{T_D} + \frac{1}{\alpha}} \quad (\text{A.2})$$

Further rearrangement to make the Laplace variable appear only in the denominator yields

$$\bar{q}_{ID} = \frac{T_D}{s(T_D + 1)} \left(1 + \frac{\ln\left(\frac{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} R_D^2}{r_{ID}^2}\right)}{\ln\left(\frac{e^{2\gamma} r_{ID}^2}{4} \frac{1}{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} s}\right)} \right), \quad (\text{A.3})$$

or

$$\bar{q}_{ID} = \frac{1}{(1+1/T_D)} \left(\frac{1}{s} - \ln \left(\frac{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} R_D^2}{r_{ID}^2} \right) \right) \left(\frac{-1}{s \ln \left(\frac{e^{2\gamma} r_{ID}^2}{4} \frac{1}{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} s} \right)} \right). \quad (A.4)$$

The inverse of Equation (A.4) is

$$q_{ID} = \frac{1}{(1+1/T_D)} \left(1 - \ln \left(\frac{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} R_D^2}{r_{ID}^2} \right) \right) \mathcal{L}^{-1} \left(\frac{-1}{s \ln \left(\frac{e^{2\gamma} r_{ID}^2}{4} \frac{1}{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} s} \right)} \right). \quad (A.5)$$

The last term in the RHS of Equation (A.5) should be evaluated in the real-time domain. Yeh and Wang (2007) present a late-time approximation of the Laplace inverse of $1/(s \ln (s/\kappa))$ in calculating well-bore flux for the constant pressure well-test problem. The inverse is based on truncating four terms of the expansion series provided by Ritchie and Sakakura (1956) for the Laplace inverse of $1/(s \ln (s/\kappa))$. Three-term and two-term approximations of the inversion were presented by Jaeger (1943) and Carslaw and Jaeger (1959), respectively. The following is obtained for the last term in the RHS of Equation (A.5), considering a three-term truncated series

$$\mathcal{L}^{-1} \left\{ \frac{-1}{s \ln \left(\frac{e^{2\gamma} r_{ID}^2}{4} \frac{1}{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} s} \right)} \right\} = \frac{1}{\ln \left(\frac{4\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}}}{e^{2\gamma} r_{ID}^2} t_D \right)} - \frac{\gamma}{\left(\ln \left(\frac{4\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}}}{e^{2\gamma} r_{ID}^2} t_D \right) \right)^2} + \frac{\gamma^2 - \frac{\pi^2}{6}}{\left(\ln \left(\frac{4\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}}}{e^{2\gamma} r_{ID}^2} t_D \right) \right)^3} \quad (\text{A.6})$$

Using Equation (15) and Equations (A.1) and (A.4), we obtain

$$p_{mD} = \frac{1}{2(1+T_D)} \left(\ln(t_D) - \gamma - \ln \left(\frac{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} R_D^2 \rho_D^2}{4\eta_D r_{ID}^2} \right) + \ln \left(\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} \frac{R_D^2}{r_{ID}^2} \right) \right. \\ \left. \ln \left(\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} \frac{\rho_D^2}{\eta_D r_{ID}^2} \right) \mathcal{L}^{-1} \left[\frac{-1}{s \ln \left(\frac{e^{2\gamma} r_{ID}^2}{4} \frac{1}{\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} s} \right)} \right] \right). \quad (\text{A.7})$$

Equation (A.7) can also be written as

$$p_{mD} = \frac{1}{(1+T_D)} \left(\frac{-\gamma}{2} - \frac{1}{2} \ln \left(\frac{R_D^2}{4t_D} \right) \right) - \frac{q_{ID}}{2T_D} \ln \left(\eta_D^{\frac{1}{(T_D+1)}} e^{\frac{2T_D}{\alpha(T_D+1)}} \frac{\rho_D^2}{\eta_D r_{ID}^2} \right). \quad (\text{A.8})$$

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Table 1. Values of the parameters considered in the example problem

<i>Parameter</i>	<i>value</i>
k_l , leak permeability (m ²)	10 ⁻¹⁰
R , leak-injector distance (m)	50
h_l , leakage interval (m)	24
r_l , leaking well radius (m)	0.1
μ , water viscosity (Pa.s)	10 ⁻³
AZ compressibility (1/Pa)	10 ⁻⁹
AZ porosity (fraction)	0.1
k_m , AZ permeability (m ²)	10 ⁻¹³
h_m , AZ thickness (m)	10
IZ compressibility (1/Pa)	10 ⁻⁹
IZ porosity (fraction)	0.1
k , IZ permeability (m ²)	10 ⁻¹³
h , IZ thickness (m)	30
q , injection rate into the storage aquifer (m ³ /s)	0.01
r_w , injection well radius (m)	0.1

Table 2. Location Coordinates for the Base Case

<i>Parameter</i>	<i>Value</i>
x , abscissa of the injection well location (m)	0
y , ordinate of the injection well location (m)	0
x_l , abscissa of the leak location (m)	46
y_l , ordinate of the leak location (m)	19.6
x_{m1} , abscissa of the monitoring well location (m)	75
y_{m1} , ordinate of the monitoring well location (m)	0
x_{m2} , abscissa of the monitoring well location (m)	75
y_{m2} , ordinate of the monitoring well location (m)	20
x_{m3} , abscissa of the monitoring well location (m)	0
y_{m3} , ordinate of the monitoring well location (m)	20

Fig. 1. Schematic view of the leaking well connecting IZ to AZ. h_l is leakage interval which is the vertical distance from the middle of IZ to middle of AZ ($h_l = h_c + (h + h_m)/2$).

Figure 2. Evolution of the deformation-induced pressure change in the AZ using numerical and analytical solutions.

Figure 3. Evolution of the deformation-induced pressure derivative in the AZ given by the numerical solution and the analytical solution. The plot includes both axes with units and their equivalent in dimensionless form.

Figure 4. The pressure change (in dimensionless and dimensional forms) in the AZ using Avci (1994) and the asymptotic solution provided here.

Figure 5. The derivative (in dimensionless and dimensional forms) in the AZ using Avci (1994) and the asymptotic solution provided here.

Figure 6. Diagnostic plot for distinction between leakage- and deformation-induced pressure changes for the example problem.

Figure 7. Graphical calculation of the leakage coefficient.

Figure 8. Plot of Δ_1 and Δ_2 in Equation (31) for two pairs of monitoring points: 1-2 and 2-3.