QRPA calculations of stellar weak-interaction rates

P. Sarriguren

Instituto de Estructura de la Materia

CSIC, Madrid, Spain

Zakopane Conference on Nuclear Physics:
Extremes of Nuclear Landscape.
August 28 - September 4 (2016)

Weak-decay rates

Problem :

- Weak-decay rates determine late stages of stellar evolution.
- Experimental extrapolations or theoretical predictions:
Reproduce exp information on $T_{1/2}$ and BGT under terrestrial conditions.

Theoretical approach :

- Deformed HF+BCS+QRPA formalism with Skyrme forces and residual interactions in both ph and pp channels.

Results : Weak-decay rates at various (ρ, T) in stellar scenarios

- pf-shell nuclei. Main constituents of stellar core in presupernova formations: Sc, Ti, V, Mn, Fe, Co, Ni, Zn isotopes.
BGT measured in laboratory (Charge exchange reactions) and compared with benchmark Shell Model calculations.
- Neutron rich Zr-Mo isotopes: r process.
- Neutron-deficient waiting-point isotopes (Ni-Sn) at (ρ, T) typical of rp-process.

Weak-decay rates

$$\lambda = \ln 2 (T_{1/2})^{-1} = \frac{\ln 2}{D} \sum_{if} P_i(T) B_{if} \Phi_{if}(\rho, T)$$

Initial states thermally populated

$$P_i(T) = \frac{2J_i + 1}{G} e^{-E_i/(kT)}, \quad G = \sum_i (2J_i + 1) e^{-E_i/(kT)} \quad P_{i=g.s.}(T=0) = 1$$

Nuclear structure

$$B_{if}(GT) = \left(\frac{g_A}{g_V} \right)_{\text{eff}}^2 \left\langle f \left\| \sum_k \sigma^k t_{\pm}^k \right\| i \right\rangle^2$$

Phase space factors :

$$\beta^+, \text{ EC} : \Phi_{if} = \Phi_{if}^{EC} + \Phi_{if}^{\beta^+}$$

$$\beta^- : \Phi_{if} = \Phi_{if}^{\beta^-}$$

$\lambda(\rho, T)$ are different from laboratory ($P_i, \text{ cEC}$)

Phase space factors

$$\Phi_{if}^{\beta^-} = \int_1^{Q_{if}} \omega \sqrt{\omega^2 - 1} (Q_{if} - \omega)^2 F(Z+1, \omega) [1 - S_e(\omega)] [1 - S_v(Q_{if} - \omega)] d\omega$$

$$\Phi_{if}^{\beta^+} = \int_1^{Q_{if}} \omega \sqrt{\omega^2 - 1} (Q_{if} - \omega)^2 F(-Z+1, \omega) [1 - S_p(\omega)] [1 - S_v(Q_{if} - \omega)] d\omega$$

$$\Phi_{if}^{cEC} = \int_{\omega_\ell}^{\infty} \omega \sqrt{\omega^2 - 1} (Q_{if} + \omega)^2 F(Z, \omega) S_e(\omega) [1 - S_v(Q_{if} + \omega)] d\omega$$

$$\Phi^{oEC} = \frac{\pi}{2} \left[q_K^2 g_K^2 B_K + q_{L_1}^2 g_{L_1}^2 B_{L_1} + \dots \right]$$

q Neutrino energy

g Radial components of the e-wf at $r=0$

B Exchange and overlap corrections

Distribution functions

$S_p = S_v = 0$

S_e : Fermi-Dirac distribution

$$S_e = \frac{1}{\exp[(\omega - \mu_e)/(kT)] + 1}$$

$$Q_{if} = \frac{1}{m_e c^2} (M_p - M_d + E_i - E_f)$$

Nuclear Structure

Skyrme HF + def + BCS + QRPA (ph,pp)

Skyrme HF

Fragmentation

Smooth occupations

New excitations

Redistribution and reduction of the strength

pnQRPA with separable forces

Transition amplitudes

$$\langle \omega_K | \sigma_K t^- | 0 \rangle = \sum_{\pi\nu} \left(q_{\pi\nu} X_{\pi\nu}^{\omega_K} + \tilde{q}_{\pi\nu} Y_{\pi\nu}^{\omega_K} \right)$$

$$\tilde{q}_{\pi\nu} = u_\nu v_\pi \Sigma_K^{\pi\nu}, \quad q_{\pi\nu} = v_\nu u_\pi \Sigma_K^{\pi\nu}, \quad \Sigma_K^{\pi\nu} = \langle \nu | \sigma_K | \pi \rangle$$

Neutron-rich isotopes: Ge - Pd

80-94Ge, 86-100Se, 90-104Kr, 94-108Sr,
100-114Zr, 104-118Mo, 110-124Ru, 114-128Pd

r-process: supernovae, n-star mergers
Nucleosynthesis of heavy elements

Zr-Mo B(GT)

Zr-Mo Half-lives

Impact of deformation

$$r = \log_{10} \left[\frac{T_{1/2}(\text{calc})}{T_{1/2}(\text{exp})} \right]$$

Average
position

$$M_r = \frac{1}{n} \sum_{i=1}^n r_i$$

Mean square
fluctuation

$$\sigma_r = \left[\frac{1}{n} \sum_{i=1}^n (r_i - M_r)^2 \right]^{1/2}$$

		10^{Mr}	10^{σ_r}
Global	Sph	1.105	10.213
	def	0.937	3.088
$0.25 < \beta < 4$	sph	0.260	3.285
	def	1.062	1.518

P.S. PRC 91, 044304 (2015)

Weak decay rates in pf-shell nuclei

Main constituents of stellar core in presupernovae. Comparison with :

- exp. (n,p), (p,n)
- SM calculations

GT properties: Test of QRPA

SM: NPA 653, 439 (1999)
QRPA: NPA 716, 230 (2003)

Weak decay rates in pf-shell nuclei

Calculations for several isotopes:

Sc, Ti, V, Fe, Mn, Ni, Co, Zn

exp: charge exchange reactions
(n,p)

$$B_{if}(GT) \propto \langle f | \sigma t | i \rangle^2$$

Accumulated GT strength

A.L. Cole PRC 86, 015809 (2012)

P.S. PRC87, 045801 (2013), PRC93, 054309 (2016)

Weak decay rates in pf-shell nuclei

$$\lambda(\rho, T) \propto \sum_{if} P_i(T) \mathbf{B}_{if} \Phi_{if}(\rho, T)$$

Electron Capture from e^- plasma

$$\Phi_{if}^{cEC} = \int_{\omega_\ell}^{\infty} \omega \sqrt{\omega^2 - 1} (Q_{if} + \omega)^2 F(Z, \omega) \times S_e(\omega) [1 - S_\nu(Q_{if} + \omega)] d\omega$$

$$S_e = \frac{1}{\exp[(\omega - \mu_e)/(kT)] + 1}$$

$$Q_{if} = \frac{1}{m_e c^2} (M_p - M_d + E_i - E_f)$$

Si burning

Ia Supernova

Medium-mass neutron-deficient isotopes

X-ray bursts: Source of intense X-ray emissions generated by thermonuclear runaway in the H-rich environment of an accreting n-star, fed from a binary red giant companion.

Nucleosynthesis mechanism: rp capture process

rp-process: Proton capture reaction rates are orders of magnitude faster than the competing β^+ -decays.

Waiting point nuclei: When the dominant p-capture is inhibited, the reaction flow waits for a slow beta-decay to proceed further.

Time scale

Isotope abundances

Light curve profile

Medium-mass neutron-deficient isotopes

Waiting point nuclei in rp-processes

Shape coexistence

Beyond full Shell Model

Ni, Zn, Ge, Se, Kr, Sr, Zr, Mo, Ru, Pd, Cd, Sn
(N=Z and neighbors)

Constrained HF+BCS

Gamow-Teller strength: Theory and Experiment

ISOLDE: Total absorption spectroscopy

Exp: Poirier et al. PRC69, 034307 (2004)

Exp: Nacher et al. PRL92, 232501 (2004)

β^+/EC half-lives: Theory and Experiment

$$T_{1/2}^{-1} = \frac{(g_A / g_V)_{\text{eff}}^2}{6200} \sum_f \Phi^{\beta^+/\text{EC}} \left| \langle f \| \beta^+ \| i \rangle \right|^2$$

$$(g_A / g_V)_{\text{eff}} = 0.74 (g_A / g_V)_{\text{bare}}$$

$$\Phi_{if}^{\beta^+} = \int_1^{Q_{if}} \omega \sqrt{\omega^2 - 1} (Q_{if} - \omega)^2 F(Z, \omega) d\omega$$

$$\Phi^{\text{EC}} = \frac{\pi}{2} \left[q_K^2 g_K^2 B_K + q_{L_1}^2 g_{L_1}^2 B_{L_1} + \dots \right]$$

Good agreement with
experiment:
Reliable extrapolations
to high ρ and T

Weak decay rates in rp-process

^{74}Sr

Deformed pn-QRPA

B(GT) and $T_{1/2}$: Good agreement with experiment

Competition β^+/EC

P.S. PRC83, 025801 (2011)

^{78}Sr

Weak decay rates in rp-process

^{74}Sr

^{76}Sr (WP)

P.S. PRC83, 025801 (2011)

^{78}Sr

Conclusions

Nuclear structure model (**deformed Skyrme HF+BCS+QRPA**)

- Study of decay properties in different mass regions.
Astrophysical applications.
- Reproduce half-lives and main features of GT strength distributions extracted from beta-decay and/or charge exchange reactions.
- Weak-decay rates at (ρ, T) typical of astrophysical scenarios:
 - pf-shell nuclei (presupernova): EC rates from QRPA comparable quality to benchmark SM calculations.
 - Neutron-rich Zr-Mo isotopes (r process).
 - Neutron-deficient WP nuclei (rp-process):
EC/ β^+ competition

Residual interactions

Particle-hole residual interaction consistent with the HF mean field

$$V_{ph} = \frac{1}{16} \sum_{sts't'} \left[1 + (-1)^{s-s'} \underline{\vec{\sigma}_1 \cdot \vec{\sigma}_2} \right] \left[1 + (-1)^{t-t'} \underline{\vec{\tau}_1 \cdot \vec{\tau}_2} \right] \frac{\delta^2 E}{\delta \rho_{st}(\vec{r}_1) \delta \rho_{s't'}(\vec{r}_2)} \delta(\vec{r}_1 - \vec{r}_2)$$

$$\begin{aligned} V_{ph}^{\sigma\tau} &= \frac{1}{16} \sum_{sts't'} (-1)^{s-s'} (-1)^{t-t'} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\delta^2 E}{\delta \rho_{st}(\vec{r}_1) \delta \rho_{s't'}(\vec{r}_2)} \delta(\vec{r}_1 - \vec{r}_2) \\ &= \frac{1}{16} \left[-4t_0 - 2t_1 k_F^2 + 2t_2 k_F^2 - \frac{2}{3} t_3 \rho^\alpha \right] \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 \delta(\vec{r}_1 - \vec{r}_2) \end{aligned}$$

Average over nuclear volume

→

Separable forces

$$\left\{ \begin{aligned} V_{GT}^{ph} &= 2\chi_{GT}^{ph} \sum_{K=0,\pm 1} (-1)^K \beta_K^+ \beta_{-K}^-, \quad \beta_K^+ = \sigma_K t^+ = \sum_{\pi\nu} \langle \nu | \sigma_K | \pi \rangle a_\nu^+ a_\pi \\ \chi_{GT}^{ph} &= -\frac{3}{8\pi R^3} \left\{ t_0 + \frac{1}{2} k_F^2 (t_1 - t_2) + \frac{1}{6} t_3 \rho^\alpha \right\} \\ V_{GT}^{pp} &= -2\kappa_{GT}^{pp} \sum_{K=0,\pm 1} (-1)^K P_K^+ P_K, \quad P_K^+ = \sum_{\pi\nu} \langle \nu | (\sigma_K)^+ | \pi \rangle a_\nu^+ a_\pi^+ \end{aligned} \right.$$

pnQRPA with separable forces

Phonon operator

$$\Gamma_{\omega_K}^+ = \sum_{\gamma_K} \left[X_{\gamma_K}^{\omega_K} A_{\gamma_K}^+ - Y_{\gamma_K}^{\omega_K} A_{\bar{\gamma}_K} \right], \quad A_{\gamma_K}^+ = \alpha_n^+ \alpha_{\bar{p}}^+$$

$$\Gamma_{\omega_K} |0\rangle = 0, \quad \Gamma_{\omega_K}^+ |0\rangle = |\omega_K\rangle$$

pnQRPA equations

Transition amplitudes

$$\langle \omega_K | \sigma_K t^\pm | 0 \rangle = \mp M_{\pm}^{\omega_K}$$

$$M_-^{\omega_K} = \sum_{\pi\nu} \left(q_{\pi\nu} X_{\pi\nu}^{\omega_K} + \tilde{q}_{\pi\nu} Y_{\pi\nu}^{\omega_K} \right)$$

$$M_+^{\omega_K} = \sum_{\pi\nu} \left(\tilde{q}_{\pi\nu} X_{\pi\nu}^{\omega_K} + q_{\pi\nu} Y_{\pi\nu}^{\omega_K} \right)$$

$$\tilde{q}_{\pi\nu} = u_\nu v_\pi \sum_K^{\pi\nu}, \quad q_{\pi\nu} = v_\nu u_\pi \sum_K^{\pi\nu}, \quad \sum_K^{\pi\nu} = \langle \nu | \sigma_K | \pi \rangle$$

Single-particle states from deformed Skyrme Hartree-Fock
 u, v: Occupation amplitudes from BCS pairing

B(GT) in the lab. system

$$I_i K_i (0^+ 0) \rightarrow I_f K_f (1^+ K)$$

$$B_\omega(GT^\pm) = \delta(\omega_0 - \omega) \langle \omega_0 | \sigma_0 t^\pm | 0 \rangle^2 + 2\delta(\omega_1 - \omega) \langle \omega_1 | \sigma_1 t^\pm | 0 \rangle^2$$

Isotopic evolution (β , r_c): Ge-Pd n-rich isotopes

Shape dependence of GT distributions in neutron-deficient: Pb isotopes

B(GT) strength distributions

- Not very sensitive to : Skyrme force and pairing treatment
- Sensitive to : Nuclear shape

Signatures of deformation

PRC 72, 054317 (2005), PRC 73, 054302 (2006)

β -decay patterns: Theory vs. Experiment

^{192}Pb

^{190}Pb

Preliminary experimental data (unpublished)

TAS (ISOLDE/CERN) A. Algora et al.

Weak decay rates in pf-shell nuclei

($Q < 0$) large: sensitive low-lying exc.

P.S. PRC87, 045801 (2013)

Weak decay rates in pf-shell nuclei

Q small

Odd-A nuclei. Excited states

Odd-A nuclei: Contributions of excited states