

UNIVERSITAT POLITÈCNICA DE CATALUNYA

MASTER THESIS

**Non-linear Model Predictive Control
Applied to PEM Fuel Cells: Anode Pressure
and Humidity Regulation**

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Abstract

In this thesis, a nonlinear model predictive control (NMPC) strategy is proposed to regulate the humidity and pressure in a Proton Exchange Membrane Fuel Cell (PEMFC) anode. The proposed control strategy uses two controllers in cascade to control the humidity and pressure in the anode, separately. This approach is used in order to overcome the difficulties caused by two dynamics with time-constants orders of magnitude apart. The inner loop, with the fastest dynamics, regulates the pressure in the anode with the set-point provided by the outer loop. The outer loop regulates the relative humidity in the anode using the temperature in the anode humidifier and also the reference pressure in the anode. The controllers developed in this thesis are based on the explicit non-linear equations describing the mass balances in the fuel cell. With this strategy, safety and performance constraints for pressure and humidity can be guaranteed and external disturbances, as changes in stack current demand, are rejected. Simulation results are presented to show the capabilities of the proposed controller under different settings and control laws. The results obtained show satisfactory regulation of the humidity and pressure with promising performance regulating the humidity with the pressure constrained to a single value. The approach followed can be used to extend this design to the anode and cathode of similar PEMFC systems with similar characteristics.



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Contents

Abstract	iii
Acknowledgements	v
List of Figures	ix
List of Tables	xi
Abbreviations	xiii
1 Introduction	1
1.1 Motivation	1
1.2 Objective	2
1.3 Thesis Outline	3
2 Background and State of the Art	5
2.1 Fuel Cells	5
2.1.1 Fuel Cell Fundamentals	5
2.1.2 Auxiliary Components	7
2.2 PEMFC Control-Oriented Modeling	9
2.3 Nonlinear Model Predictive Control	10
2.3.1 General Description	10
2.3.2 NMPC Formulation	12
3 Problem Statement	15
4 Mathematical Model	19
4.1 Variables description	20
4.2 Control-Oriented Models	21
4.2.1 Pressure Control-Oriented Model	22
4.3 Humidity Control-Oriented Model	24
4.4 Complete Model	25
5 Controller Design	27
5.1 Pressure Controller (Inner Loop)	28

5.2	RH Controller (Outer Loop)	31
6	Results	35
6.1	Simulation Scenario	35
6.2	Key Performance Indicators	35
6.2.1	Overshoot	36
6.2.2	Settling time	36
6.2.3	Smoothness	36
6.2.4	Steady State Error	37
6.3	Main Results	37
7	Conclusion	43
A	Appendix A	45
	Bibliography	47



List of Figures

2.1	Scheme of a PEMFC	6
2.2	Scheme of the PEMFC design under study. Based on Kunusch et al. [2013a]	7
3.1	Block diagram of the control system	18
4.1	System architecture with controllers and their corresponding internal model.	22
5.1	Comparison between full model and reduced model	30
5.2	Comparison between different H_p length	31
6.1	Illustration of overshoot and settling time.	37
6.2	Response of the internal loop with a disturbance at 5 seconds mark the stack current demand is doubled.	38
6.3	Response of the external loop without perturbations.	39
6.4	Response of the external loop with a step change in the RH objective with the freedom to set the pressure in the anode.	40
6.5	Response of the external loop with a step change in the RH objective and a step change in the current drawn, doubling the initial demand.	42



List of Tables

- 6.1 KPI for different values in the weight matrices. 40
- 6.2 KPI for different values in the weight matrices with P_{an} constrained to a single value. 41

- A.1 Values of the constants and coefficients of the mathematical model 45



Abbreviations

FC	F uel C ell
PEMFC	P roton E xchange M embrane F uel C ell
MEA	M embrane E lectrode A ssembly
MPC	M odel P redictive C ontrol
NMPC	N on-linear M odel P redictive C ontrol
KPI	K ey P erformance I ndicator



Chapter 1

Introduction

1.1 Motivation

Hydrogen fuel cells are one of the most promising technologies regarding energy production thanks to their high efficiency and due to the fact hydrogen is a clean source of energy. Particularly, proton exchange membrane fuel cells (PEMFC) provide high power density making them viable for portable and vehicular power applications, as well as for stationary plants. A typical PEMFC power system is composed of several auxiliary interconnected components, as presented in [Pukrushpan et al. \[2004\]](#). The energy is produced in the cell stack subsystem where the hydrogen, supplied from the anode, and the oxygen, supplied from the cathode, react. The energetic efficiency of this reaction depends on several factors such as the concentrations of the reactants, the degradation of the membrane, the temperature in the cell stack, the pressure of the gases and the humidity across the membrane. It is therefore necessary a control system to maintain optimal conditions in order to avoid a degradation in the membrane while maximizing the closed-loop performance. The control problem is complex due to the numerous variables that affect the process and the interconnections among them. Short life of the membrane is a barrier for its massive commercialization so extending its lifespan is one of the main interests in this field.

The relative humidity (RH) in both anode and cathode channels has a capital importance both in the preservation of the membrane and the energetic performance of the PEMFC. The importance

of RH lies in the need of high humidity in the anode for high proton conductivity without saturating the ambient that could cause flooding in the membrane, blocking the channels and pores of the gas diffusion layers. The flooding of the membrane results in a poor performance and it also leads to corrosion. As the water is produced in the cathode, the flooding is a phenomenon appearing more frequently in the cathode than in the anode. In this work, a control system is presented to achieve the suitable regulation of both the RH and the pressure in the anode while rejecting the disturbances produced by the electrochemical reaction. Ideally, partial pressure of hydrogen in the anode must be high enough to avoid starvation in the PEMFC, a phenomenon produced by the lack of the required reactant reducing the lifetime of the fuel cell and its general performance. The excess of hydrogen pressure first implies an excess of mechanical stress in the membrane electrode assembly (MEA).

Model Predictive Control (MPC) is a control technique widely used in the industry since 1980s. It is characterized by the ability of solving the optimal control problem taking into account current state of plant and also the future state of it. This anticipation capability is the major advantage in front of other techniques. The extension of the classical MPC to the domain of nonlinear systems, nonlinear model predictive control (NMPC), allows the inclusion of explicit models of the process. The use of the explicit plant, including its nonlinearities, provides more accurate predictions compared to the use of a approximate linearization.

NMPC paradigm provides useful tools to deal with complex systems such as a fuel cell. In the case under study in this thesis the impossibility of obtaining an accurate approximation of the process leads to the use of a nonlinear system. The need of constraint managing and the inherent nature of constraint management of the NMPC result in the ideal control strategy for the objective of this thesis.

1.2 Objective

The general objective of this thesis to design a controller for the regulation of the pressure and relative humidity in the anode of a PEMFC. Controlling these two variables in a fuel cell presents problems due to the different dynamics of the variables. The objective is to design a



cascade-loop control architecture with the adaptation of a lumped parameter model to each of the controllers, to overcome these problems.

The controllers designed in this project are set to regulate humidity and pressure to a given point to provide optimal performance and extend the lifespan of the modelled fuel cell. The study of the optimal set points of these variables is out of the scope of this thesis, instead suitable set points in a reasonable range are used. The general objective may be divided among this specific objectives:

- Analyse the main features of fuel cell model.
- Design a suitable control architecture for pressure and humidity regulation.
- Develop and adapt internal models for NMPC controllers.
- Achieve suitable pressure in the anode.
- Compare different settings of the designed NMPC controller in simulation.

1.3 Thesis Outline

This thesis is organized as follows. In Chapter 2 the state of the art regarding PEMFC and NMPC is reviewed and the basic background in MPC is introduced. In Chapter 3 the case study of the current thesis is presented with its features and control objectives. The study of the mathematical model and the description of the variables in the system, and their physics, are described in Chapter 4. The design of the controllers in the cascade-loop architecture is discussed in Chapter 5. The results extracted from the simulations are presented and analysed in Chapter 6. Finally, in Chapter 7 conclusions of this thesis are presented along some suggested lines of future work.

Results from this thesis have been submitted as regular paper to the 5th IFAC Conference on Nonlinear Model Predictive Control 2015 (NMPC'15), which will take place in Seville, Spain on September 17 - 20, 2015.



Chapter 2

Background and State of the Art

This chapter reviews the basic literature regarding Fuel Cells and Model Predictive Control. Fuel cells and MPC are briefly introduced and special emphasis is put in studies focused on NMPC and anode control. It provides the necessary concepts in order to achieve the main objective of this work, which is to develop a control-oriented model and an NMPC controller for the anode pressure and humidity regulations.

2.1 Fuel Cells

2.1.1 Fuel Cell Fundamentals

Fuel cells are electrochemical devices that convert the chemical energy into electricity through a chemical reaction. In the reaction, two agents are involved: the fuel and the oxidant. The products of the reaction are water, heat, and, as previously mentioned, electricity. A fuel cell is composed by a cathode, an anode and an electrolyte that allows charges to move between the two sides of the fuel cell. The electrolyte can be an acid, which is a fluid with free H^+ ions, or certain polymers that can contain mobile free H^+ . Current is generated when electrons go from the anode to the cathode across an external circuit as presented in Figure 2.1. Fuel cells can be classified by the type of electrolyte they use [Barbir, 2005], for example:

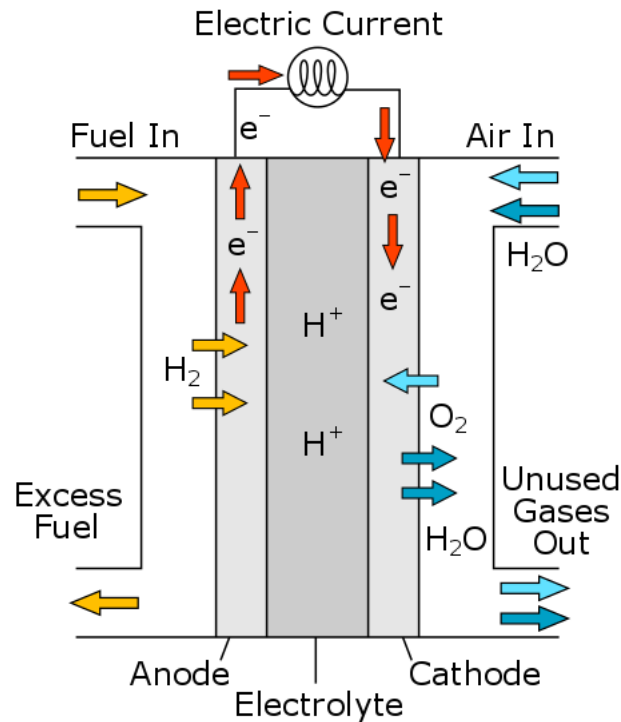


FIGURE 2.1: Scheme of a PEMFC

- **AFC:** Alkaline fuel cells use concentrated KOH as electrolyte and they can operate at temperatures ranging from 50°C to 250°C.
- **PEMFC:** Proton exchange membrane fuel cells use a thin proton conductive polymer membrane and they operates at 30-100°C.
- **PAFC:** Phosphoric acid fuel cells use concentrated phosphoric acid as the electrolyte and they operate at 150-220°C.
- **MCFC:** Molten carbonate fuel cells have the electrolyte composed of a combination of alkali (Li, Na, K) carbonates, they operate at 600-700°C.
- **SOFC:** Solid oxid fuel cells use a solid, nonporous metal oxide as electrolyte. These cells operate at 800-1000°C.

This thesis is developed using a mathematical model of a PEMFC and its auxiliary systems reported in [Kunusch et al., 2011]. The voltage of a cell fuel is quite small when drawing useful



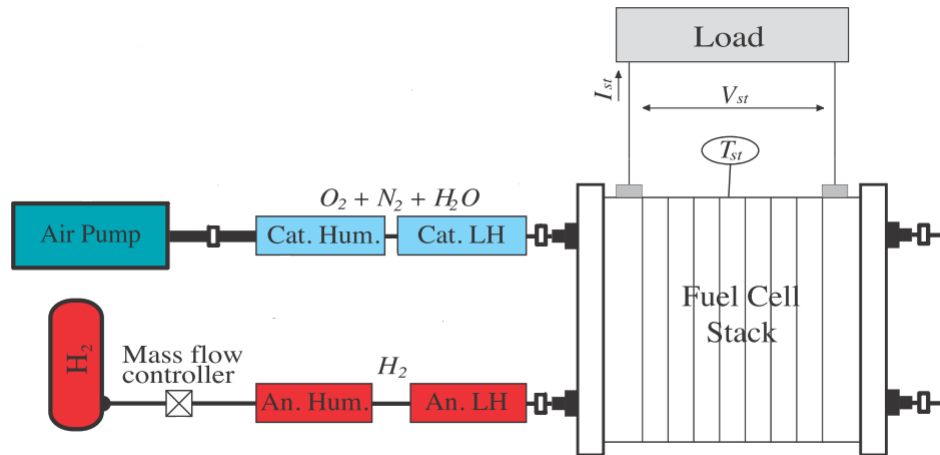


FIGURE 2.2: Scheme of the PEMFC design under study. Based on [Kunusch et al. \[2013a\]](#)

current. Many fuel cells are connected in series in order to produce useful voltage. The group of cell fuels is called stack [[Larminie and Dicks, 2003](#)].

2.1.2 Auxiliary Components

The hydrogen and oxygen must be properly conditioned to provide satisfactory performance and preserve the lifespan of the fuel cell. The auxiliary components that provide the adequate conditioning are: the reactant supplies, the humidifiers and the line heaters. Humidifiers and line heaters are similar for both anode and cathode. The focus of this auxiliary devices is on the devices that condition the hydrogen as this thesis is focused on the anode. The auxiliary devices are set in a configuration as in Figure 2.2.

Reactant Supply

The high purity hydrogen is required for PEMFC limits the supply methods that can be used. These methods are basically: compressed gas, solid metallic hydrides [[Chen et al., 2003](#)] and cryogenic liquid. The PEMFC under study is supplied by a compressed tank of hydrogen of high purity. The oxygen content in the air is enough for powering fuel cells. Air is supplied by means of either a compressor or a fan in most of the cases. The fan is used in open-cathode designs, in which the system works at ambient pressure. Also some laboratory designs can be

found using compressed air tanks. The supply of oxygen is a capital issue for the performance and the preservation of PEMFC [Gerard et al., 2010], [Taniguchi et al., 2004].

Humidifier

Humidification is one of the most important aspects of the fuel cell. The membrane needs high humidity, close to 100%, for high proton conductivity but saturation must be avoided. A vapour-saturated ambient causes flooding in the membrane, liquid water blocks the channels and pores in the gas diffusion layers. The water is produced in the cathode but the water can travel across the membrane so the humidity must be regulated in both anode and cathode. Under certain operating conditions the moisture produced in the cathode would be sufficient but under normal operation anode and cathode need a humidification systems.

There are different methodologies to humidify a gas but here it is presented the water exchange through a permeable material. Hydrogen or air to be humidified are supplied through a permeable membrane, respectively. There appears a humidity gradient that provides diffused vapour to the gas to be humidified. The degree of humidification is regulated by adjusting the water temperature within the humidifier [Kunusch et al., 2011].

The requirements of humidity and stoichiometric conditions to avoid early degradation and to extend the lifetime of fuel cells are presented in Schmittinger and Vahidi [2008], remarking the importance of water management where humidity regulation is an important issue. Kunusch et al. [2013a] present an important work for this current thesis since it tackles the observability problem of the water transport across the membrane. This issue is closely related to the humidity regulation. They consider a series of observers for the water transport across the membrane that are essential in order to estimate the RH at the anode.

Line heater

Line heaters increase the temperature of the gases before entering the stack. The objective of heating the gases is to prevent condensation of the vapour contained in the humidified gases and avoid the flooding, previously mentioned. With a higher gas temperature, the same amount of vapour enters the stack but the higher temperature implies a higher saturation pressure. Another



feature provided by the higher temperature is the capacity to regulate the relative humidity without changing the operation temperature of the stack thus changing the temperature of the water in the humidifier will only affect the moisture in the gas.

2.2 PEMFC Control-Oriented Modeling

The use of a real plant to test different operating conditions is not a suitable possibility due to the availability of it and the risk of damaging it with the experimental operating conditions. A simulation model is used to test new control strategies, different temperature or pressures setups among other operating conditions. PEMFC modelling is a vast field of research with a wide variety of model topologies depending on the objective of the study (performance, durability, etc.).

The domain of the model can vary from a lumped parameter model, which simplifies the description of the behaviour of spatial distributed entities to approximate discrete entities under some assumptions, to 3D models with distributed parameter, where the entity behaviour is described with its spatial distribution taken into account. Some examples of the variety of the domain of PEMFC models are: lumped parameter the model proposed by [Kunusch et al. \[2011\]](#), this model is used in this thesis; a 1+1D model proposed by [Mangold et al. \[2010\]](#), where spatially distributed one-dimensional volumes are modelled; and finally a 3D model of the gas diffusion layers (GDL) proposed by [Thiedmann et al. \[2008\]](#).

The dynamics of fluids, like the gases in a PEMFC, are described by PDEs in an infinite-dimension state space. Lumped parameter models represent physical systems with time-dependent ODEs obtained by approximating the dynamics of the fluid. Spatial distributed models are based on spatial discretizations of the PDEs in order to obtain more detailed description of the fluid dynamics. Distributed models can provide important information regarding the chemical reaction and water formation. This detailed information comes at expense of higher computational costs.

[Kunusch et al. \[2011\]](#) present an analytical model designed for non-linear control and observation purposes. This model has been validated experimentally in laboratory PEMFC test-bench. The

approach followed in the design of the model is a combination between a theoretical, shown by [Pukrushpan et al. \[2004\]](#), and empirical based on experimental data.

2.3 Nonlinear Model Predictive Control

2.3.1 General Description

Model predictive control (MPC) is a control technique in which the control action is obtained solving an open-loop optimal control problem over a finite-time horizon. The control problem is solved on-line at each sampling time and the first element of the control action sequence, corresponding to current time-sample, is applied to the plant [[Maciejowski, 2002](#)]. The current state of the plant is used as initial conditions for the open-loop constrained optimal control problem for the next sampling instant.

The main advantage of MPC is the ability to obtain an optimal solution regarding the cost function and a set of constraints. The constraints may involve both inputs and states of the plant. Plants usually present a set of constraints and bounds due to physical and safety limitations. These constraints can be equality or inequality constraints in reference of any of the states and inputs of the system. Inherent multi-variable control and constraint management are the major advantages of MPC approach over other techniques [[Mayne et al., 2000](#)]. Additionally, the cost function can include terms relating to energy consumption, plant degradation or economy of the process as control objectives. Due to its flexibility, MPC approach is widely used in the industry. The major drawback of this technique is the dependence of a numerical solver fast enough to solve the control problem in the limited time between sampling-times. Also, the performance of a MPC controller is limited by the accuracy of the model, the accuracy of the predictions and the control actions taken accordingly is directly related to the accuracy of the model.

The Nonlinear Model Predictive Control (NMPC) is an extension of MPC including nonlinear systems. The NMPC approach considers plants and constraints that can be nonlinear [[Grüne and Pannek, 2011](#)]. The extension of NMPC takes advantage of explicit nonlinear models of the plant to obtain a control sequence more accurate in relation to the real plant. The extension of NMPC arises from the difficulty of obtaining a suitable linear model of the real plant. The need



to use nonlinear model is usually related to the impossibility to obtain a suitable linearization of the real plant. Other cases could be related to the use of nonlinear constraints even the system can be linearized with suitable accuracy. The major disadvantages of the NMPC are higher computational burden and loss of convexity of the optimization problem.

In the field of fuel cells, MPC and NMPC techniques have been previously applied as in the work of [Gruber et al. \[2012\]](#). This work presents an NMPC design for the airflow regulation in a PEMFC in order to guarantee the oxygen excess in the cathode and ensure performance and safety conditions. The work from [Vahidi et al. \[2004\]](#) tackles the issue of oxygen starvation in the cathode by using a linear MPC with an auxiliary power source, showing the capabilities of anticipating the possible energy shortages produced by oxygen starvation. In order to reduce the computational burden, [Panos et al. \[2012\]](#) present an explicit/multi-parametric MPC, in which they avoid the need for repetitive online optimization. The optimization problem of the MPC is solved off-line by parametric optimization to obtain the optimal solution as an optimal mapping of the current state, output measurements and reference trajectory instead of demanding online optimization. Additionally, [Danzer et al. \[2009\]](#) proposes an MPC design to prevent starvation, where the goal is achieved with a scheme that incorporates actuator limitations and state constraints in the control design. In [Luna et al. \[2015\]](#), an NMPC strategy is proposed to regulate the concentrations of the different gas species inside a PEMFC anode gas channel. The purpose of the regulation relies on the rejection of the unmeasurable perturbations that affect the system: the hydrogen reaction and water transport terms. A distributed parameter model is used, taking into account spatial variations along the channel.

Other modern control techniques applied to PEMFC are found in the literature. [Shao et al. \[2014\]](#) introduces a fault diagnosis system based on an ANN (artificial neural network) ensemble method that improves the stability and reliability of the PEMFC systems. Sliding mode control paradigm is used in [Kunusch et al. \[2012\]](#) and [Kunusch et al. \[2013b\]](#), where a robust control solution is proposed to solve the air supply control problem in autonomous PEMFC-based systems. A Super Twisting controller is designed using a nonlinear model of a laboratory fuel cell test station and in [Kunusch et al. \[2013b\]](#) the proposed control strategy is successfully implemented in the laboratory test bench. Regarding PEMFC observers, [Arcak et al. \[2004\]](#) present a nonlinear adaptive observer design to estimate the partial pressure of hydrogen in the anode channel of a fuel cell. A precise knowledge of this pressure is of importance to ensure reliable and efficient

operation of the fuel cell power system. Their design makes use of a monotonic nonlinear growth property of the voltage output on hydrogen partial pressures at the inlet and at the exit of the channel.

2.3.2 NMPC Formulation

The explicit nonlinear model used for the optimal control problem is based on a discrete-time nonlinear plant presented as:

$$x(k+1) = f(x(k), u(k), v(k)), \quad (2.1a)$$

$$y(k) = g(x(k), u(k), w(k)), \quad (2.1b)$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$, $v \in \mathbb{R}^{n_x}$, and $w \in \mathbb{R}^{n_y}$ are the control action, state, measured output, perturbation and noise vectors, respectively, at time instant k . Mapping functions $f(x(k), u(k), v(k))$ and $g(x(k), u(k), w(k))$ might not be linear. This system is subject to a set of constraints defined by:

$$G(x, u, y) \leq 0, \quad (2.2a)$$

$$H(x, u, y) = 0. \quad (2.2b)$$

Then the optimal control problem is formulated:

$$\min_{\mathbf{u} \in \mathbb{R}^{n_u H_p}} J(\mathbf{u}, \mathbf{x}, \mathbf{y}) \quad (2.3)$$

where J is the cost function to minimize, subject to the constraints (2.2a) and (2.2b), and H_p is the prediction horizon. Moreover, \mathbf{u} , \mathbf{x} , and \mathbf{y} are the sequences defined as:

$$\mathbf{u} \triangleq \{u(k | k), u(k+1 | k), u(k+2 | k), \dots, u(k+H_p-1 | k), u(k+H_p | k)\},$$

$$\mathbf{x} \triangleq \{x(k+1 | k), x(k+2 | k), \dots, x(k+H_p-1 | k), x(k+H_p | k), x(k+H_p+1 | k)\},$$

$$\mathbf{y} \triangleq \{y(k+1 | k), y(k+2 | k), \dots, y(k+H_p-1 | k), y(k+H_p | k), y(k+H_p+1 | k)\},$$



where $u(k + i | k)$ denotes the prediction of the control action at time $k + i$ performed at time k . The same nomenclature applies to the states and outputs sequences. The predictions are obtained as follows:

$$\begin{aligned}x(k + 1 + i | k) &= f(x(k + i | k), u(k + i | k), v(k + i | k)), \\y(k + i | k) &= g(x(k + i | k), u(k + i | k), w(k + i | k)),\end{aligned}$$

with i ranging from 0 to H_p and $x(k|k) = x_0$, being x_0 the initial conditions. Disturbances and noise (v, w) may be modelled or unmodelled depending on the control problem but they will be always bounded. Given the problem is feasible, there will be an optimal sequence of control inputs \mathbf{u}^* . The first element of this sequence, $u^*(k | k)$, is applied to the system as a control action. After the control action is applied, the outputs are measured and the state of the plant updated. The updated states of the plant are used as initial conditions and the optimal control problem is solved again. This procedure is repeated iteratively along a simulation scenario. If the states could not be fully observed, an observer system would be required to recover the states of the plant.



Chapter 3

Problem Statement

The goal is to develop a control system for a PEMFC anode subsystem that regulates the RH and pressure in the anode channels. The pressure will be regulated to reach an optimal value according to performance and preservation parameters of the PEMFC. The study of this parameters is out of the scope of this work. The humidity will be regulated to reach high humidity levels in the anode but always avoiding the saturation of the vapour. It has been previously presented the flooding phenomenon and the controller will avoid it to preserve the fuel cell. The controller obtained will be tested under a simulation scenario with a simulation model.

The characteristics of the fuel cell system under study has several measured variables that provide information about the system. In the anode part, the measured variables are: the pressure in the anode channels (P_{an}) and the pressure in the anode humidifier $P_{hum,an}$. In the fuel cell stack, the temperature (T_{st}), the current (I_{st}) and voltage (V_{st}) are also available. Also the input hydrogen flow (W_{H2}) and the temperature in the anode humidifier $T_{hum,an}$ are controlled and measured. Additionally the measure of the RH in the anode (RH) is also available.

To achieve the desired RH in the anode at steady state, two inputs are used: the temperature of the humidifier and the hydrogen inflow in the system. The pressure in the anode can also be set externally thus the regulation of RH could be only achieved by changes in the temperature in the humidifier.

The regulation of humidity and pressure faces two principal perturbations: the water transport across the membrane and the changes in the current in the stack. In this thesis, the former is considered observed even though the dynamics of this are unknown. The later, the stack current, is a perturbation that is measured and provides information about hydrogen consumption. In the simulations, the controller will face changes of current demand to test the capabilities of handling a change on the hydrogen consumption while achieving the control objectives.

The control problem will be formulated as an non-linear constrained optimization problem. The form to proceed is to establish an objective function, that is a function of the inputs and outputs, to be minimized. The constrains of this optimization problem will be the maximum and minimum bounds of the different variables that will be given by physical and safety reasons. There is an additional constraint regarding the temperature in the anode humidifier that is worth particular attention, the humidifier only has a heating system but it does not have a cooling system. This means, it can be actively increased by providing energy to the heating system but it only decreases passively by dissipating the heat.

The analytical model of the anode channels, obtained from [Kunusch et al. \[2013a\]](#) and [Kunusch et al. \[2011\]](#), can be described as follows:

$$\begin{aligned}\dot{m}_{H_2}(t) &= f_1(W_{H_2}(t), m_{H_2}(t), P_{an}(t)), \\ \dot{P}_{an}(t) &= f_2(m_{H_2}(t), P_{amb}(t), I_{st}(t), W_{v,mem}(t), RH(t)), \\ \dot{RH}(t) &= f_3(m_{H_2}(t), P_{an}(t), W_{v,mem}(t), RH(t), T_{hum,an}(t), \Pi_{hum,an}(t)),\end{aligned}$$

where P_{amb} is the ambient pressure, Π is the power supplied to the humidifier and m_{H_2} is the mass of hydrogen in the humidifier. This last variable is closely related to the pressure in the humidifier ($P_{hum,an}$), as it will be shown later. The analytical model is composed by continuous-time equations and it needs to be discretized in order to design an NMPC controller in discrete time. Assuming the time between samples (Δt) small enough, the discrete model will keep the properties of the continuous model. The discretization of the system will be carried out using



the Euler method. In the general case a dynamic system can be discretized as follows:

$$\dot{x} \triangleq \frac{dx}{dt} = f(x),$$

with a time sample small enough we can approximate:

$$\frac{\Delta x}{\Delta P} = \frac{x(t_k + \Delta t) - x(t_k)}{\Delta t} \approx f(x)$$

$$x(t_k + \Delta t) = f(x)\Delta t + x(t_k).$$

For simplicity, the temporal dependence will be expressed as multiples of the sampling time T_s . A function evaluated at sample-time k is equivalent to being evaluated at time t_k , where $t_k = T_s k$:

$$x(k+1) = f(x)T_s + x(k).$$

The discrete-time system will have the following form:

$$m_{H2}(k+1) = m_{H2}(k) + f_1(k)T_s$$

$$P_{an}(k+1) = P_{an}(k) + f_2(k)T_s$$

$$RH(k+1) = RH(k) + f_3(k)T_s.$$

The dynamic nature of the two inputs, W_{H2} and $\Pi_{hum,an}$, is quite different and the response time of the system to a change of the input hydrogen flow is orders of magnitude faster than the response time to a change in the temperature of the humidifier set-point.

There is strong interaction interaction between both controlled variables. Variations in $T_{hum,an}$, produced by $\Pi_{hum,an}$, will cause a variation in $P_{hum,an}$ that it will change the inflow to the anode thus changing P_{an} . The same chain effect can be seen when a variation in W_{H2} : this variation in W_{H2} changes $W_{v,inj}$ causing a variation of the RH .

The solution proposed is a cascade loop architecture with a sampling time accordant to the dynamics of the humidity and pressure separately, as seen in Figure 3.1. With this design the inner loop manages the fastest dynamics of the whole plant without losing accuracy and the outer loop manages the slowest. The inner loop controls the pressure of the anode by using the

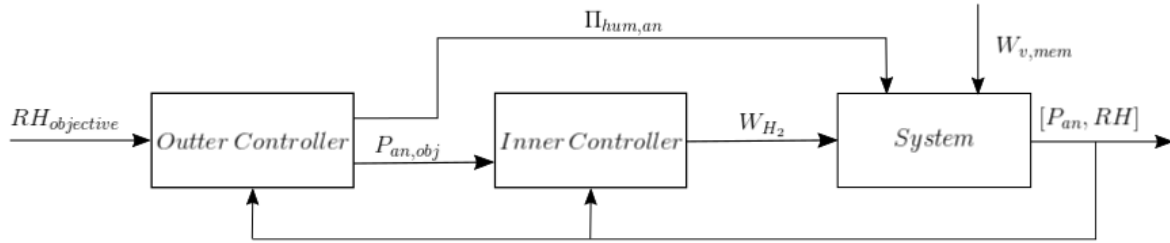


FIGURE 3.1: Block diagram of the control system

hydrogen inflow as control action while the humidity controller uses the power supplied to the heating system in the humidifier. The humidity controller does not have to take into account the dynamics of the pressure inner loop because the inner loop regulates the pressure fast enough for the outer loop to assume the change is instantaneous. The outer loop modifies two variables that affect the inner loop: the set-point of the anode pressure and the temperature of the humidifier. The set-point of anode pressure is the control objective of the inner controller. The temperature of the humidifier is a measured perturbation for the inner controller. The temperature of the humidifier produces a variation in the pressure in the humidifier. Although it is a measured perturbation and the outer controller has a model of the variation of the temperature, the model of the humidifier temperature is not used in the inner loop because the difference of time constant between the two loops. During the prediction horizon, the temperature is assumed to remain constant, even though there is a small variation.



Chapter 4

Mathematical Model

The following natural step towards designing the controller is to describe analytically the model used as a baseline of this thesis. The anode of a PEMFC can be modelled following different approaches but, in this case, the model used is focused on the auxiliary systems around it: the humidifiers, the manifolds and line heaters. The dynamics of the electrochemical reaction are simplified. This was modelled in the previous work of [Kunusch et al. \[2011\]](#) and [Kunusch et al. \[2013a\]](#). The model reported includes many variables and parameters that would make the control problem too difficult. So a simplified version of it with the focus on the anode and anode humidifier, is used. The mass balance in the anode part is modelled as:

$$\dot{m}_{H_2}(t) = W_{H_2}(t) - W_{H_2,an,in}(t) \quad (4.1a)$$

$$\dot{P}_{an}(t) = W_{H_2,an,in}(t) - W_{H_2,an,out}(t) - W_{H_2,react}(t) \quad (4.1b)$$

$$RH(t) = (-W_{v,an,out}(t) - W_{v,mem}(t) + W_{v,inj}(t)). \quad (4.1c)$$

Before making an exhaustive analytical description of both internal and external control-oriented models, a qualitative description of the main variables involved in the model is presented in the following section. These variables appear in both control-oriented models.

All pressures, fluxes, temperature and relative humidity, denoted by P , W , T and RH , respectively, are dependent on time. The time dependence notation is dropped for clarity in some

equations but all of these values, with their respective subindices, will remain dependent on time with the exception of the ones clearly stated as constant.

4.1 Variables description

The main function of an PEMFC has been previously described but some details were left to closer examination. The first focus will be in the variables involved in the humidifier dynamics.

The humidifier adds vapour to the input flow of hydrogen leaving it with a relative humidity close to 100% and this is given by the saturation pressure that is determined by the temperature of the humidifier. The higher the temperature, the higher the partial vapour pressure with the same relative humidity. This temperature is going to be controlled but the system is designed in such a way that only positive increments of temperature can be made so the only way to reduce the temperature is to let the system dissipate the excess of heat. This temperature has a maximum allowed value given by the stack temperature. Stack temperature is assumed to be set to an optimum point for the PEMFC performance, if the temperature of the gasses entering the fuel cell are below the desired temperature they are heated to the required point in line heaters for the anode and cathode. Taking this into consideration the temperature in the humidifier will remain between the ambient temperature and stack temperature. In the real operation of this system it will always be much closer to the stack temperature than to the room temperature. Humidifier pressure is an available measurement of the system and it can be directly related to the hydrogen mass in the humidifier. The characteristics of the fuel cell system impose some physical constraints that need to be addressed. The minimum pressure in the humidifier must be higher than the pressure of the anode so that the hydrogen can flow to the anode and the maximum pressure will be set to a safe value to protect the equipment and respect some model limitations. Equation (4.1a) describes the dynamical behaviour of the hydrogen mass in the humidifier (m_{H_2}). The inlet hydrogen flow in the humidifier, W_{H_2} , is used as control action for the anode pressure regulation. This flow is provided by a pressurized tank with high purity hydrogen. The flow of hydrogen that leaves the humidifier is heated in the anode line heater and finally enters the anode ($W_{H_2,an,in}$).



The second state P_{an} , as it has been said before, is the total pressure in the anode. The term $W_{H_2,an,in}$ that appears in (4.1b) is the same described previously, the flow of hydrogen leaving the humidifier. $W_{an,out}$ is the gas flow leaving the anode without reacting or being transported through the polymer membrane.

Anode temperature will be regulated by the line heater mentioned before. This temperature is assumed to be perfectly controlled to the optimum point. The pressure of the anode must be lower than the pressure into the humidifier but higher than the ambient pressure because a positive differential pressure is needed so that not external air enters the anode.

The last term that appears in the dynamics of the state P_{an} is $W_{H_2,react}$, this term can be seen as a perturbation because it only depends of the configuration of the fuel cell and the intensity drawn from it.

The current across the stack (I_{st}) is an observed perturbation given by the operation mode of the fuel cell and directly related to the power it supplies.

The last variable interfering with the system is a perturbation, the water transport across the membrane ($W_{v,mem}$). This variable has a complex dynamics that are not modelled in the dynamic model and is considered measured perturbation. The water transport across the membrane is a consequence of the electro-chemical reaction and the different relative humidities in both the cathode and anode.

The third and last state is more complex than the previous ones although this complexity is not evident at the beginning. The first two terms of the equation $W_{v,an,in}$ and $W_{v,an,out}$ are similar to the previously seen. $W_{v,an,in}$ is the vapour flow that enters to the anode, it is obtained as a proportion of the vapour injected to the hydrogen flow in the humidifier and the relation of the partial pressures of the vapour and hydrogen in the line heater.

4.2 Control-Oriented Models

In Chapter 3 the architecture of the controller set-up has been presented, in this section the different models used for each controller and for simulations are presented. In Figure 4.1 the

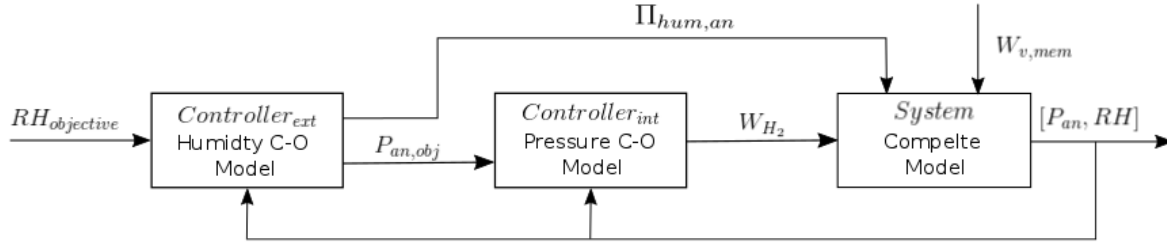


FIGURE 4.1: System architecture with controllers and their corresponding internal model.

system is represented with a block diagram with both controllers and their corresponding internal model.

4.2.1 Pressure Control-Oriented Model

This control-oriented model describes two phenomena: how the supplied hydrogen is humidified and what happens in the anode of the fuel cell with this humidified hydrogen. These phenomena are described from the point of view of mass balances taking into account the conservation mass principle and the ideal gases law.

For the objective of this work, it is important to know the mass and behaviour of m_{H_2} , whose change is represented by the dynamic process:

$$\dot{m}_{H_2} = f_1 = W_{H_2} - W_{H_2,an,in}. \quad (4.2)$$

The change of the hydrogen mass it depends on the hydrogen supplied to the humidifier W_{H_2} and the flow of hydrogen going to the anode $W_{H_2,an,in}$. The behaviour of W_{H_2} is set externally and will be used as a control action for the system. Besides, $W_{H_2,an,in}$ is a variable obtained after the linearisation of the nozzle equation. It can be described as a bivariate function parametrized by $T_{hum,an}$. The approximation is the polynomial:

$$W_{H_2,an,in} = C_0 + C_1(P_{hum} - P_{an}), \quad (4.3)$$

where C_0 and C_1 are values determined experimentally [Kunusch et al., 2011] and they are shown in Table A.1. The values of P_{hum} and P_{an} are measured from the system. Moreover, P_{hum} is



directly related with the m_{H_2} by the ideal gas law and will be described as

$$P_{hum} = K_1 m_{H_2}$$

$$K_1 = \frac{G_h T_{hum}}{V_{hum}},$$

where K_1 is the factor obtained by ideal gases law that relates mass and pressure, V_{hum} is the volume of the humidifier and G_h is the molar mass of the hydrogen.

The dynamical behaviour of the anode is more complex, here there is the influence of hydrogen, as before, the dynamics of the diffused water vapour and some phenomena from the stack and the cathode. The focus of interest is the pressure dynamics in the anode expressed as a function of the different inputs, outputs and stack current. This is given by

$$\dot{P}_{an} = f_2 = ((W_{H_2,an,in} - W_{H_2,react} - W_{h_2,out})R_h + (W_{v,inj} - W_{v,out} - W_{v,mem})R_v) \frac{T_{st}}{V_{an}}. \quad (4.4)$$

The term $W_{v,inj}$ is the amount of vapour added, dependent on the hydrogen flux, temperature and pressure in the anode:

$$W_{v,inj} = \frac{G_v RH_{hum} P_{sat}(T_{hum})}{G_h P_{hum}} W_{H_2,an,in}. \quad (4.5)$$

The terms RH_{hum} and $P_{sat}(T_{hum})$ are the RH and saturation pressure in the humidifier respectively. The RH_{hum} is very close 100% when the humidifier under nominal operation. $P_{sat}(T_{hum})$ is expressed as follows:

$$P_{sat,hum}(T_{hum}) = 10^{3+\gamma(T_{hum})}, \quad (4.6)$$

where

$$\gamma(T_{hum}) = \alpha_0 + \alpha_1 T_{hum} + \alpha_2 T_{hum}^2 + \alpha_3 T_{hum}^3 + \alpha_4 T_{hum}^4.$$

The coefficients of this polynomial are found in Table A.1.

The hydrogen consumed in the electrochemical reaction, $W_{H_2,react}$, only depends on I_{st} and constant parameters, i.e.,

$$W_{H_2,react} = I_{st} \frac{G_h n}{2F},$$

being n the number of cells and F Faraday's constant. The outflow in anode, W_{out} , is dependent on a nozzle constant and the differential pressure between P_{an} and P_{amb} and

$$W_{out} = K_{an,n}(P_{an} - P_{amb}).$$

With the measure of RH , the proportion of vapour in W_{out} can be known as follows:

$$\begin{aligned} W_{v,out} &= (1 - \omega)W_{out}, \\ W_{H2,out} &= \omega W_{out}, \\ \omega &= \frac{1}{\frac{G_v R_v}{G_h R_h} \frac{m_{v,an}}{m_{H2,an}} + 1}, \\ m_{v,an} &= \frac{P_{sat}(T_{st})V_{an}RH}{T_{st}}, \\ m_{H2,an} &= \frac{(P_{an} - P_{sat}(T_{st})RH)V_{an}}{R_h T_{st}}, \end{aligned}$$

where $m_{v,an}$ and $m_{H2,an}$ are the mass of vapour and hydrogen in the anode, respectively.

The last variable concerning the anode mass balance is the water transport in the membrane ($W_{v,mem}$) that has unmodeled dynamics in the control-oriented model due its complexity but it can be also observed. The parameter ω indicates the mass relation of hydrogen and vapour in the anode.

The remaining of terms in equation (4.4) are: R_h , hydrogen specific constant; R_v , vapour specific constant; T_{st} , PEMFC stack temperature and V_{an} , anode volume. The values of this constants can be found in Table A.1. Finally is worth pointing out that the the internal model is discretized using a sampling time of 0.1s.

4.3 Humidity Control-Oriented Model

This model describes the changes of humidity in the anode in relation to the temperature in the humidifier. The dynamics of the temperature model is orders of magnitude slower than the dynamics of the humidifier and anode masses, thus will be considered instantaneous changes seen as observable perturbations. The same basic ideas are used in the pressure control oriented model but assuming that the pressures are instantaneously self-regulated. The heating model is



assumed to be a first order system where the input $\Pi_{hum,an}$ is the power supplied to the heating resistor. The discrete-model of the heating system is

$$T_{hum}(k + 1) = -\Omega T_{hum}(k) + \Pi_{hum,an} \Delta t, \quad (4.7)$$

where Ω is the heat dissipation rate. The humidity system can be described as

$$\dot{RH} = f_3 = (-W_{v,an,out} - W_{v,mem} + W_{v,inj}),$$

where all the terms are previously described.

4.4 Complete Model

The complete model has 8 states. It models the whole system including all the cathode dynamics and a model of $W_{v,mem}$ so it is a quite accurate reference to apply the control. The original model has as inputs the voltage of the air compressor that relates to the input air flux and the hydrogen input flux, this model assumes the temperature of the anode humidifier remains constant so some slight modifications are performed to adapt the model. This modifications are regarding the implementation of the model but the theoretical approach of it is exactly the same. The description of this model is found in [Kunusch et al. \[2011\]](#). The model presented was validated experimentally and provides a useful information about how the controller would perform on a physical setup. This model is used for simulations purposes with the sampling time equal to the pressure control-oriented model. The cathode part is not studied in this work but is configured to provide satisfactory conditions for the purposes of this project.

The complete model is not the addition of the control-oriented models (COM). The complete model is slightly different than the COM because no assumptions of are made regarding the constant values of the perturbations or instant changes in the pressure. The simulation model is used to close the control loop. The COM are used to compute the optimal inputs and once they are obtained they are applied to the simulation model. The outputs obtained from it are used as initial conditions for the optimization problem in the following iteration.



Chapter 5

Controller Design

In this chapter, two controllers needed are designed separately and individually tested in different scenarios to demonstrate their effectiveness regulating anode pressure and humidity. The two loops have different time constants and can be seen separately. The inner loop, in charge of pressure regulation, will assume constant values for RH_{hum} , RH and T_{hum} . In the outer loop, P_{hum} and P_{an} will be assumed to change instantaneously. The inner loop is able to reach the set point of the pressures fast enough to ignore their transient behaviour in the outer loop. This values, considered constant in the optimization process, are updated at each time step even though the dynamics of the change are ignored. In MPC approach the control horizon can differ from the prediction horizon but it is important to note that in this project are the same noted as H_p .

5.1 Pressure Controller (Inner Loop)

The objective is to find the optimal value of W_{H2} supplied to the system to obtain the desired value of P_{an} . The model is described as a discrete-time non-linear state space as follows:

$$m_{H2}(k+1) = f_1(k)T_s + m_{H2}(k),$$

$$P_{an}(k+1) = f_2(k)T_s + P_{an}(k),$$

$$y_1(k) = K_1 m_{H2}(k),$$

$$y_2(k) = P_{an}(k).$$

It is necessary to define the prediction horizon (H_p) for the NMPC controller. If the control-oriented model, referred also as reduced model, is compared with the full model, it shows that both have a similar behavior until the time mark of 2 seconds approximately (Figure 5.1). Taking into account that both models are discretized with a sampling time (T_s) of 0.1 s, it is safe to use controllers with a prediction horizon up to 20.

The optimization problem is expressed as follows:

$$\min_{W_{H2} \in \mathbb{R}^{H_p}} \sum_{k=0}^{H_p} J(P_{an}(k), W_{H2}(k))$$



subject to:

$$P_{amb} \leq P_{hum} \leq P_{hum,max}, \quad (5.1a)$$

$$P_{hum} \leq P_{an} \leq P_{an,max}, \quad (5.1b)$$

$$P_{an} \leq P_{hum}, \quad (5.1c)$$

$$\Delta W_{H2,min} \leq \Delta W_{H2} \leq \Delta W_{H2,max}, \quad (5.1d)$$

$$0 \leq W_{H2} \leq W_{H2,max}, \quad (5.1e)$$

$$m_{H2}(k+1) = f_1(k)T_s + m_{H2}(k), \quad (5.1f)$$

$$P_{an}(k+1) = f_2(k)T_s + P_{an}(k), \quad (5.1g)$$

with

$$J(P_{an}(k), W_{H2}(k)) = (P_{an}(k) - P_{an,ref}(k))^2 w_{P_{an}} + \Delta W_{H2}(k)^2 w_{\Delta W_{H2}},$$

and $w_{P_{an}}$, $w_{\Delta W_{H2}}$ are the weight matrices. The matrix ΔW_{H2} is the increment of the control action in relation to the last applied control action. This is a frequent way to penalize the control action and avoid steady state error. A penalty in the change causes a smoother control action signal. A sharp control action in real systems should be avoided because it could damage the components of the plant. The way of finding the optimal value for these matrices (controller tuning) is out of the scope of this thesis. For the purposes of this thesis, suitable values have been found from simulation results via trial-and-error procedures. The first two constraints of this optimization problem have been set as safety constraints of the pressures in the anode and in the humidifier (5.1a)-(5.1b). Additionally, in order to represent a unidirectional valve from humidifier to the anode, a restriction forces the pressure in the humidifier to be higher or equal than that in the anode (5.1c). Then, two constraints regarding W_{H2} are set: one defines the maximum and minimum inlet flow W_{H2} (5.1e) and the other constraint bounds the W_{H2} change rate between two consecutive control actions (5.1d). The change rate of W_{H2} is defined as: $\Delta W_{H2}(k) \triangleq W_{H2}(k) - W_{H2}(k-1)$. Finally, the last constraints are the restrictions imposed by the dynamical system (5.1f)-(5.1g).

Upon closer examination of the dynamic equations of the system, one can notice that the humidifier acts as a buffer between the input (W_{H2}) and the actual output of the system (P_{an}). This coupled with a short prediction horizon produces a big control action that increases the

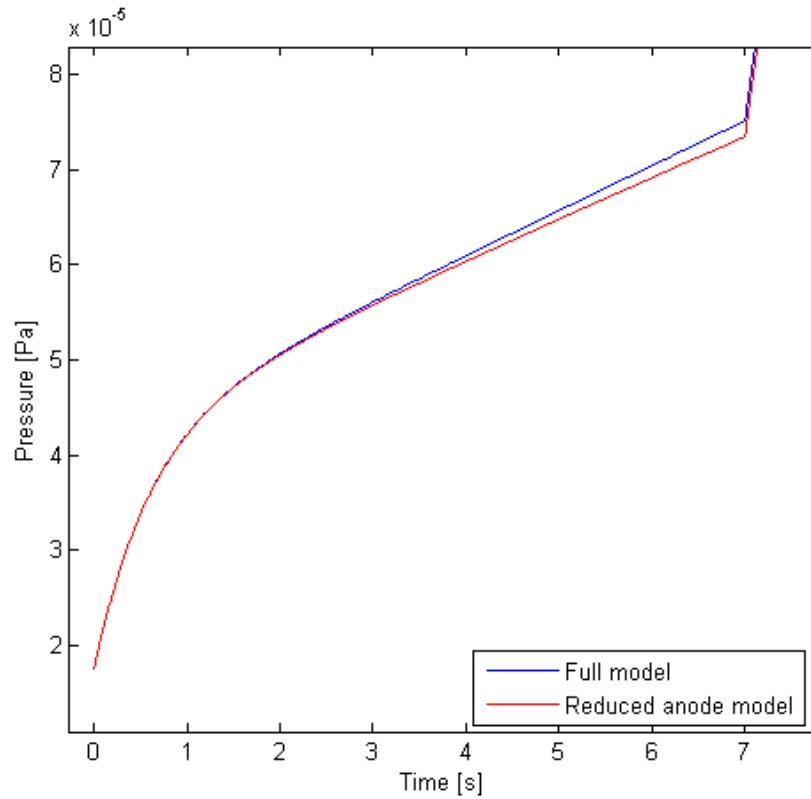
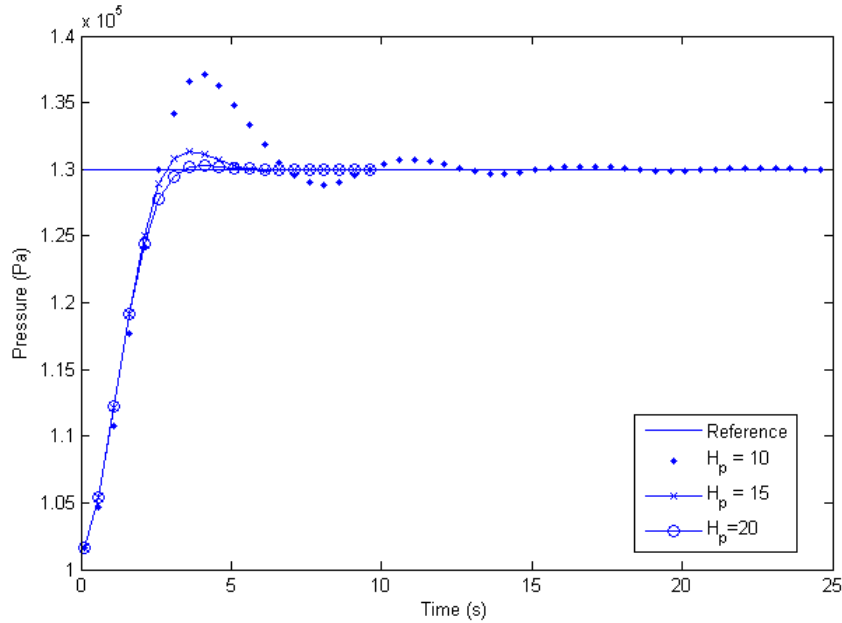


FIGURE 5.1: Comparison between full model and reduced model

pressure in the humidifier putting the system close to the constraints. This extra pressure in the humidifier will cause an increase in the inflow to the anode making the system difficult or even impossible to control. The buffer effect could be avoided providing also a set point for the humidifier but this would make the system slower if the set point is constant. In order to provide a dynamic set point another level of optimizer would be required increasing the complexity of the system prohibitively. A softer response could be achieved with a penalty on the control action but this would include a steady state error also undesired. As a result, the better option is to choose a large H_p without compromising the performance of the system. By simulation it is found that $H_p = 15$ provides a satisfactory results and performance. In Figure 5.2 it is shown the response of the system with different H_p settings.



FIGURE 5.2: Comparison between different H_p length

5.2 RH Controller (Outer Loop)

This external controller provides two signals to the pressure controller: $\Pi_{hum,an}$ and $P_{an,ref}$. In the previous section, a full detailed model was used as internal model. In the outer loop, the dynamics of P_{hum} and P_{an} are ignored, and P_{an} is assumed to follow exactly $P_{an,ref}$. The objective to regulate the RH is achieved mostly by the change of the temperature of the humidifier but as mentioned in Section 3, the temperature can only be decreased passively, the control action $\Pi_{hum,an}$ only provides positive increments of temperature, so the extra manipulable $P_{an,ref}$ can help achieving the desired RH . Between the two inputs is desirable to use the temperature primarily and avoid the excess expenditure of H_2 that would result as an excessive $P_{an,ref}$. Hence,

the optimization problem related to this controller is expressed as follows:

$$\begin{aligned} \min_{[\Pi_{hum,an}, P_{an,ref}]' \in \mathbb{R}^{2H_{p,ext}}} & \sum_{k=0}^{H_p} J(RH(k), \Pi_{hum,an}(k), P_{an,ref}(k)) \\ \text{s.t. :} & \\ & RH_{min} \leq RH \leq RH_{max} \\ & P_{an,min} \leq P_{an} \leq P_{an,max} \\ & \Pi_{hum,an,min} \leq \Pi_{hum,an} \leq \Pi_{hum,an,max} \\ & \Delta \Pi_{hum,an,min} \leq \Delta \Pi_{hum,an} \leq \Delta \Pi_{hum,an,max} \\ & T_{hum,an,min} \leq T_{hum,an} \leq T_{hum,an,max} \\ & RH(k+1) = RH(k) + f_3(k)T_s \end{aligned}$$

with

$$\begin{aligned} J(RH(k), \Pi_{hum,an}(k), P_{an,ref}(k)) &= (P_{an,ref}(k) - P_{an,optim}(k))^2 w_{P_{an,ref}} \\ &+ \Delta \Pi_{hum,an}(k)^2 w_{\Delta \Pi_{hum,an}} + (RH(k) - RH_{ref}(k))^2 w_{RH}. \end{aligned}$$

Matrices $w_{P_{an,ref}}$, $w_{\Delta \Pi_{hum,an}}$ and w_{RH} are the weight matrices. $\Delta \Pi_{hum,an}$ is the increment of the control action in relation to the last control action. The constraint in $P_{an,ref}$ could be bounded to a single value set externally and then the controller would just adjust the humidity in the anode via the temperature of the anode.

In order to determine $H_{p,ext}$ for this controller, it is necessary to take into account two factors: the settling time of the pressure subsystem and the time constant of the humidifier temperature. The sampling time is 20s, five times the time the inner loop takes to reach steady state. With the sampling time in mind, a balance must be found between a horizon that allows to make predictions long enough, in time units, to make significant predictions without being too expensive computationally. The balance is found with $H_{p,ext} = 15$, allowing predictions of events 5 minutes ahead of the current sampling time. It is important to achieve RH without increasing excessively the temperature because it is hard to decrease it, this implies avoiding overshooting at expense of a slower system. A rule of thumb to set the weight matrices in such a way is to



penalize heavily the state representing the pressure in the anode and the slew rate of the power applied to the humidifier.

Given this external controller has two degrees of freedom a fixed anode pressure could be set and control the RH via $T_{hum,an}$. This could be a valid approach when the economy of H_2 is a primary objective, setting a very low anode pressure would reduce the input of the hydrogen in the system. This option will be studied in Chapter 6.



Chapter 6

Results

6.1 Simulation Scenario

Simulations have been carried out using `fmincon` routine in MATLAB® 2010b 64-bits running on an Intel® Core™2 Duo CPU E8600 @ 3.33 GHz with 8GB of RAM. The simulation conditions are set to a fixed set-point of RH to provide optimal performance for the PEMFC. The process assumes observability of the variables $W_{v,mem}$ and RH , the observability of the first is solved by [Kunusch et al. \[2013a\]](#) and the second can be measured with a humidity sensor in the outlet flow. The system will be simulated first with two degrees of freedom for the controller and the second with a fixed low pressure so the controller only regulates the temperature of the humidifier. Both simulations will face a perturbation in the form of a change of I_{st} demand, the demand in I_{st} will be doubled.

6.2 Key Performance Indicators

In order to evaluate the performance of the controller, 4 key performance indicators (KPI) are going to be proposed. These indicators will provide a quantitative measure of the performance of the controller in the simulation scenario. The evaluation of the different settings will take place in a time interval beginning at the sample k_i when the system reached steady state then a change

in the reference signal and a perturbation will be introduced to the system. The evaluation will end up at k_f when the simulation finishes.

6.2.1 Overshoot

It was previously mentioned that the system cannot cool down the humidifier, so a high overshoot in the RH should be avoided. A high overshoot could flood the membrane with the problems it entails. This measure will be the maximum RH value minus the set-point value divided by the set-point, i.e.,

$$KPI_{ov} = \frac{RH_{max} - RH_{ref}}{RH_{ref}}. \quad (6.1)$$

In Figure 6.1 this measure is illustrated.

6.2.2 Settling time

Settling time KPI, KPI_t measures the time it takes for the system output to enter and remain within a specified error band. In this case a band of 5% will be used. Notice this percentage is over the change of the signal, not an absolute 5% of the RH measure. In Figure 6.1 it is shown graphically.

6.2.3 Smoothness

The smoothness performance indicator measures the changes in the control action. Abrupt changes may be harmful for the actuators in the real system and produce extra stress for the system in general. The smoothness KPI is defined as:

$$KPI_{\Delta u} = \frac{1}{(k_f - k_i)} \sum_{k=k_i}^{k_f} (\Delta u(k))^2. \quad (6.2)$$

The values k_i and k_f are the initial and final time samples, respectively, of the evaluation. The change in the control action $\Delta u(k)$ is defined as $\Delta u(k) = u(k) - u(k - 1)$.



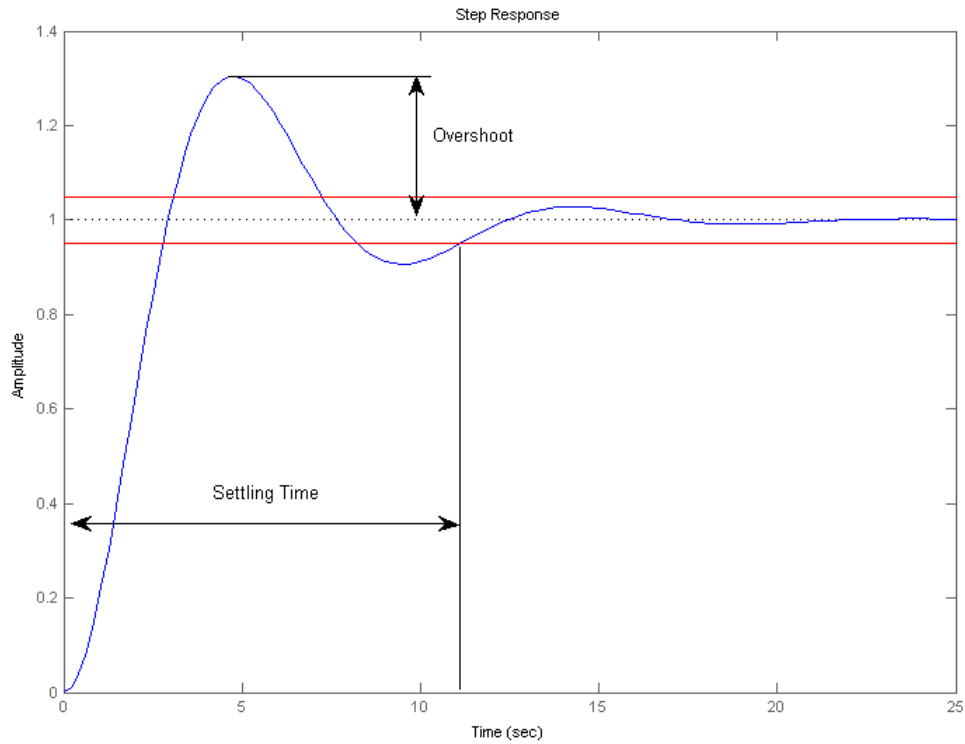


FIGURE 6.1: Illustration of overshoot and settling time.

6.2.4 Steady State Error

This indicator measures the deviation of the humidity with respect to the set-point in steady state. This accounts for small oscillations once the value of the error is under the 5% previously mentioned. This will be quantified using the mean absolute error (MEA):

$$KPI_e = \frac{1}{(k_f - k_i)} \sum_{k=k_i}^{k_f} |RH(k) - RH_{ref}|. \quad (6.3)$$

6.3 Main Results

The main challenge the pressure controller faces is the changes in the $P_{an,obj}$ and the disturbance introduced by a change in I_{st} . There will be a step change in the current drawn and as well as in the $P_{an,obj}$ and both perturbations are rejected with no steady state error with a suitable transient behaviour, no overshoot and fast response. As stated before, this response allows

the external controller to assume the values change instantaneously, the settling time of this subsystem is orders of magnitude smaller than the sampling time of the external controller. The change of I_{st} it has a small impact over the pressure subsystem, it can be seen in Figure 6.2 at 5 seconds in the simulation. There is a slight change in the pressure but the effects are rejected quite fast. The change in the pressure is due to the change in H_2 demand when stack current increases.

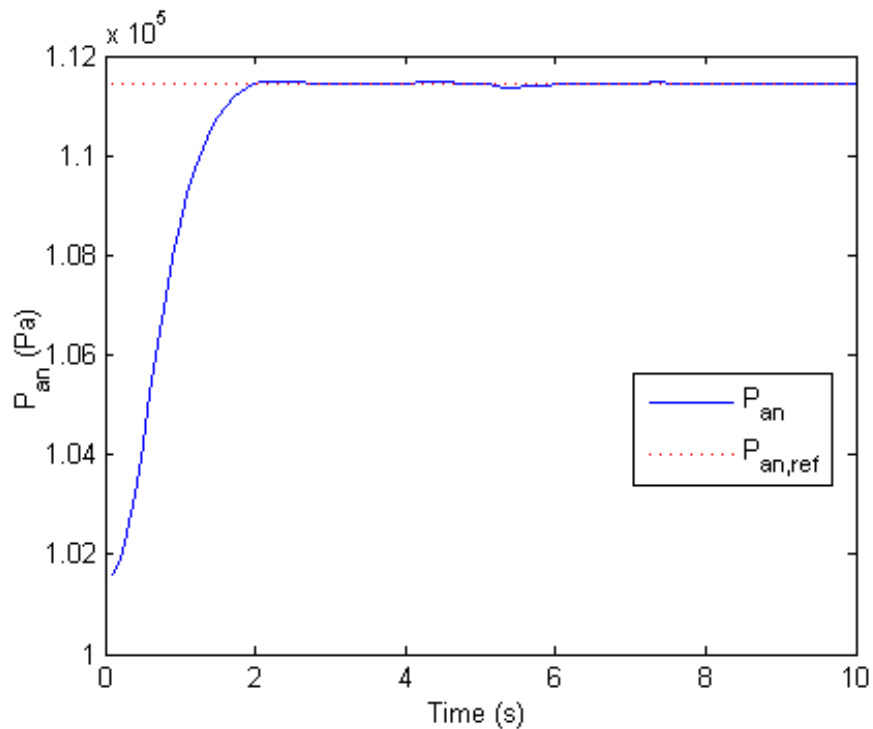


FIGURE 6.2: Response of the internal loop with a disturbance at 5 seconds mark the stack current demand is doubled.

It is interesting to check the external controller assuming no perturbations from the inner loop. Basically the changes in pressure and the demand of hydrogen produce changes in the water transport through the membrane that are not modelled. In Figure 6.3, it can be seen how RH is regulated with an appropriate stationary response. In this simulation the set-point of the pressure in the anode ($P_{an,obj}$) is not set by the controller but set externally so this response is obtained by using only the temperature. It is important to note that this is just the partial simulation of the external controller uncoupled. Later this possibility is analysed more extensively.



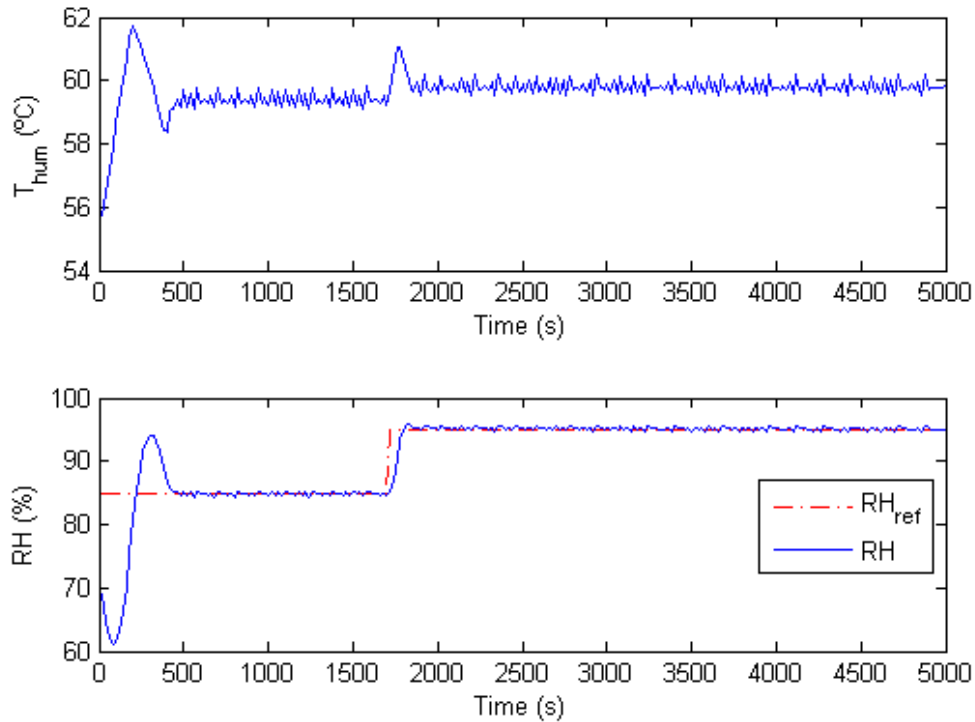


FIGURE 6.3: Response of the external loop without perturbations.

So far the results presented are both systems isolated but the interesting point is to show the full capabilities of the whole system, pressure and temperature loops, tested in the model of the FC. A degradation in the performance compared to the isolated systems is to be expected because the number of perturbations both systems transmit each other. The effect of the external disturbance, the change in the current drawn from the FC, is completely rejected. A comparison between different configurations can be seen in Table 6.1. The KPI obtained by the simulations show that with a high penalty on the control action and low penalty on regulation error, the system enters an oscillatory state where it is not able to settle within a satisfactory error margin. On the other hand, an opposite configuration with low penalty on control action and high penalty on regulation error reaches a steady state quite fast but it produces a small oscillation due to the aggressive behaviour of the controller. A compromise between smoothness, settling time and overshoot must be found to achieve a satisfactory response for the system for a general case. Particular cases might consider more aggressive or passive controllers but, for a general case, a satisfactory behaviour has been found with: $w_{\Delta\Pi} = 0.2$ and $w_{RH} = 25$. The results produced are shown graphically in Figure 6.4 where the simulation is carried out with a change of RH_{ref}

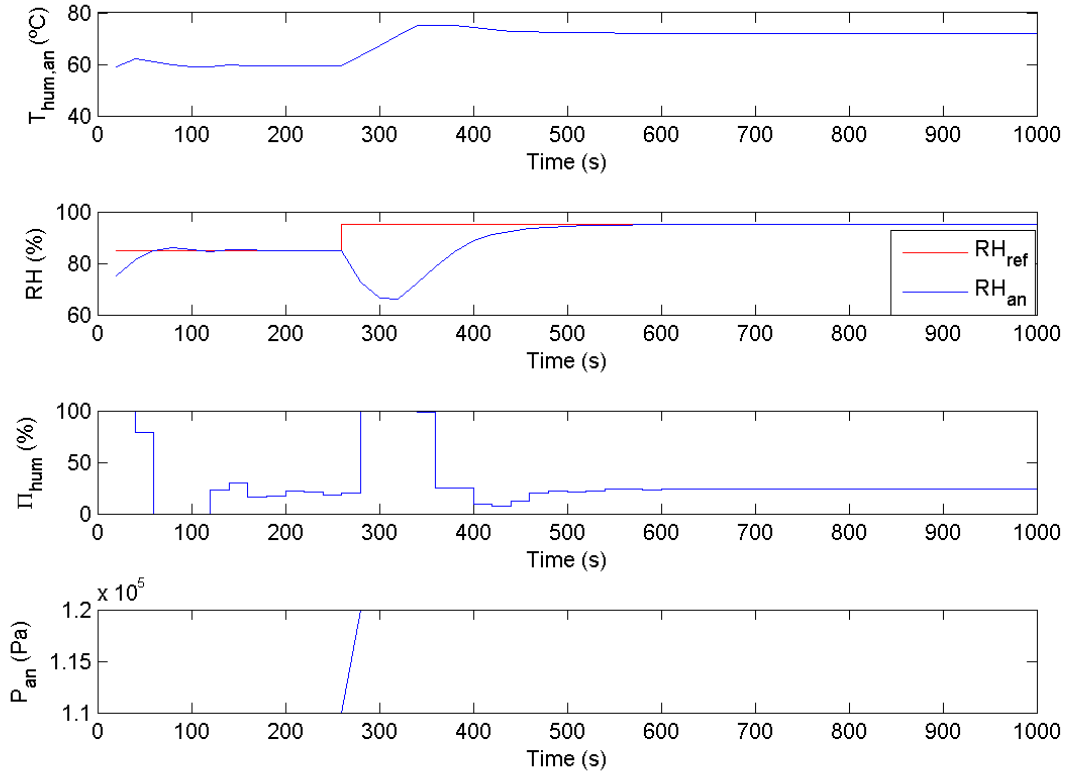


FIGURE 6.4: Response of the external loop with a step change in the RH objective with the freedom to set the pressure in the anode.

TABLE 6.1: KPI for different values in the weight matrices.

$w_{\Delta\Pi}$	w_{RH}	KPI_{ov}	KPI_t	$KPI_{\Delta u}$	KPI_e
0.5	5	16%	-	55.78	-
0.25	10	10.6%	-	51.07	-
0.2	25	0	260s	47.22	3.95%
0.1	50	0.4%	180s	58.93	4.65%
0.05	100	0.3%	160s	52.69	4.19%
0.01	200	0.3%	140s	54.72	4.28%

in order to show the dynamic response of the system.

When the RH objective is increased there is an increase also in P_{an} . This increase in the pressure causes a temporal drop in the RH purging the anode. This drop in RH can be seen in Figure 6.4. This makes the settling time longer but provides useful extra pressure to avoid



TABLE 6.2: KPI for different values in the weight matrices with P_{an} constrained to a single value.

$w_{\Delta\Pi}$	w_{RH}	KPI_{ov}	KPI_t	$KPI_{\Delta u}$	KPI_e
0.25	10	8.55%	-	25.66	-
0.2	25	8.5%	-	53.24	-
0.1	50	0.4%	40s	45.5	2.27%
0.05	100	0.3%	40s	45.44	2.12%
0.01	200	0.3%	40s	45.51	2.11%

constraint violation regarding the maximum RH allowed.

The controller without the possibility to set the reference of the pressure in the anode is an interesting option in case economizing the hydrogen was the priority. This configuration has a quite similar behaviour to the general configuration, with the freedom to set the objective of the pressure in the anode. The way to proceed is to constrain the values of P_{an} to a single value. The tuning of the controller is also easier, because there is one less degree of freedom, but when the working operations is close enough to the saturated anode, the system will not have the possibility to purge the system with excess of hydrogen. This action can decrease fast the quantity of vapour in the anode. Different conditions have been tested in simulation and evaluated using the KPI previously described. Simulations results of this system can be seen in Figure 6.5 with: $w_{\Delta\Pi} = 0.05$ and $w_{RH} = 100$. The results show little variation once the controller is able to produce a satisfactory response. When $P_{an,ref}$ is not changed, the dynamics of the system are simplified and the external controller is able to predict better future outcomes.

The computation of the optimal solution in average took 15.64 seconds per iteration, for the internal loop and 0.91 seconds per iteration for the outer one. On one hand, the computational time in the outer loop is satisfactory because it stays below the response time of the system and it could be applied in a real system. On the other hand, the computational time for the inner loop must be drastically decreased in order to be applied in real time, the computation time is too high compared to the sampling time of the pressure.

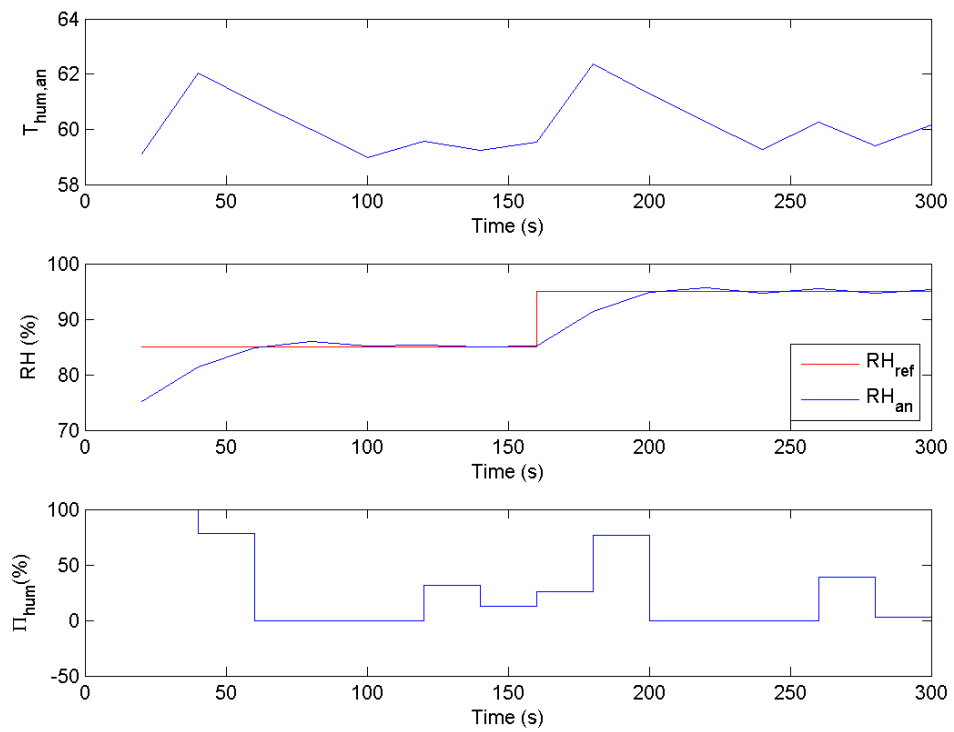


FIGURE 6.5: Response of the external loop with a step change in the RH objective and a step change in the current drawn, doubling the initial demand.



Chapter 7

Conclusion

The NMPC controller has been designed and applied satisfactorily in the PEMFC anode subsystem, allowing the control of critical variables for the lifespan of the membrane. The controller met the control requirements set: follow specific humidity and pressure set-points and reject the perturbation caused by a variation of current demand. Promising results have been obtained in the simulation scenario. The results showed a better performance of the controller with a fixed pressure in the anode, a condition that makes sense physically and economically. Indeed, this approach is an interesting option regarding future works where the control of both anode and cathode humidity will be performed. The economy of the hydrogen has not been studied in this project but it is an important key to take into account when designing the weight matrices of the controller. The general approach followed provides an easy way to design the NMPC controller that considers the value of the membrane, the cost of the hydrogen and the electrical cost of the heating system in the humidifier. The flexibility of the NMPC presented provides a wide spectrum of possible controllers considering different control objectives. A remarkable issue of this approach is the computational burden that this kind of optimization problem carries. The time it takes the numerical solver to find the optimal solutions in each time step is quite high for the inner loop thus making difficult to apply to a real PEMFC. Even though the time spent in the optimization for the inner loop makes it not ready to be implemented in a real setting. It is important to keep in mind that these are the first approach to design this kind of control laws for this application. The objective of this thesis is accomplished by the results obtained by simulation. The external loop controller, on the other hand, is capable of finding the optimal

solution in time between time samples. This means the external controller could be applied to PEMFC in a laboratory setting.

A reimplementaion of the model with the performance as prime objective is an interesting approach for future works. With this change the time spend solving the optimization problem would be considerably reduced. The model could also be improved by adding a more details of the humidifier dynamics. The current model presents a series of limitations regarding how the vapour injected to the hydrogen flow is represented. The current humidifier model is adequate to simulate nominal working conditions but a more detailed one could be used to simulate startup, high demand, no demand, and shut-down situations. Finally a last suggestion for future works is to implement a similar controller architecture in order to control anode and cathode humidity. This would be a quite interesting line of work regarding the water management in PEMFC systems.



Appendix A

Appendix A

TABLE A.1: Values of the constants and coefficients of the mathematical model

Parameter	Value	Units
C_0	1.0836×10^{-5}	-
C_1	3.3510×10^{-9}	-
α_0	-1.69×10^{-10}	-
α_1	3.85×10^{-7}	-
α_2	3.39×10^{-4}	-
α_3	0.143×10^{-9}	-
α_4	20.92	-
Fuel cell stack temperature (T_{st})	65	°C
Fuel cell line heater temperature (T_{lh})	65	°C
Humidifier relative humidity (RH_{hum})	0.95	
Hydrogen gas constant (R_h)	4.124×10^3	Nm/kg/°K
Vapour gas constant (R_v)	461.5×10^3	Nm/kg/°K
Faraday constant (F)	96485 C/mol	
Hydrogen molar mass (G_v)	2.01×10^{-3}	kg/mol
Vapour molar mass (G_h)	18.02×10^{-3}	kg/mol
Volume anode (V_{an})	0.02	l
Volume humidifier (V_{hum})	0.002	l
Anode nozzle restriction (K_{an})	3.28×10^{-10}	kg/s/bar
Number of cells (n)	7	-
Ambient pressure (P_{amb})	1.013×10^5	Pa



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