Wavelet Filtering to Extract Coherent Vortices from Altimetric Data

ANTONIO TURIEL, JORDI ISERN-FONTANET, AND EMILIO GARCÍA-LADONA

Institut de Ciències del Mar (CSIC), Barcelona, Spain

(Manuscript received 1 July 2005, in final form 5 March 2007)

ABSTRACT

Because of the optimal features of wavelet processing, the use of wavelets for describing and analyzing signals in 2D turbulence has been generalized since a decade ago. In spite of the close analogy between 2D turbulence and geophysical fluid dynamics, few works have tried to generalize the rich framework of wavelet techniques to the study of experimental signals in oceanography. In this paper, the authors extend a prominent wavelet technique designed for the study of direct numerical simulations (DNSs) on 2D turbulence, the coherent vortex simulation, and analyze with it ocean velocity fields obtained from sea surface height maps derived from satellite altimetry. The authors demonstrate the pertinence of this technique to describe altimetry data, resulting in a description of oceanic flows with a reduced number of degrees of freedom. In particular, it is shown that the western Mediterranean circulation is well approximated by a field of extracted coherent vortices when an appropriate wavelet basis is employed as a filter; however, about one-third of the energy is lost in this description, evidencing important differences between results obtained on the frame of 2D turbulence and oceanic data.

1. Introduction

Remotely sensed products from satellites are excellent for assessing fundamental properties of the ocean dynamics because of their rate of acquisition and their large-area synopticity. Satellite data, complemented by the effort made during many campaigns of in situ measurements and several international research programs developed during the last 30 yr, have substantially improved our understanding of the oceans at the mesoscale level (Robinson 1983; Kamenkovich et al. 1986). Salient mesoscale structures, such as ocean eddies and fronts, are regularly observed (frequently by visual inspection of infrared images), thus allowing one to study their properties and dynamics in detail (Nihoul and Jamart 1989; Hooker et al. 1995, 1997). The ubiquitous presence of nearly horizontal coherent vortices seems to be a fundamental ingredient in ocean dynamics. Coherent vortices are known to play a key role in World Ocean dynamics due to their effectiveness in moving energy and matter through the ocean and their impact

on mixing (see, e.g., Armi et al. 1989; Olson 1991). A detailed characterization of their spatial distribution and temporal evolution is essential as vortices modify the statistics of ocean flows and need to be taken into account when modeling dispersion and diffusion in oceans (Davis 1987; Provenzale 1999; Bracco et al. 2000b; Pasquero et al. 2001).

The role of coherent vortices in flow dynamics has been linked to the research on turbulence since the beginning. To decompose the motion of flows under a high Reynolds number into their main dynamic modes, classical techniques such as Fourier modal decomposition or empirical orthogonal functions (see, e.g., Holmes et al. 1996) have been employed; however, these methods are nonlocal and so the spatial localization of mesoscale features becomes uncertain. An alternative approach is to regard the flow as an ensemble of localized vortices that dominate the motion. This conceptual model has been particularly useful for two-dimensional turbulence, where coherent structures emerge spontaneously due to the development of the enstrophy–energy cascades (McWilliams 1984; Jiménez 1996). The idea of regarding structures of concentrated vorticity as the fundamental building blocks in turbulent flows has been largely exploited in the past to make numerical simulations of the flow and also as a conceptual model to evaluate statistical and mixing properties,

Corresponding author address: Antonio Turiel, Physical Oceanography Group, Institut de Ciències del Mar, CMIMA (CSIC), Passeig Marítim de la Barceloneta, 37–49 E-08003 Barcelona, Spain.
E-mail: turriel@cmima.csic.es

scaling laws, etc. (Benzi et al. 1988; Babiano et al. 1987; McWilliams 1990; Weiss and McWilliams 1993).

At the core of such an approach is the proper definition of a coherent vortex, which in the case of analyzing flow fields (normally pressure, velocity, or vorticity fields) from both numerical models and experiments is far from being trivial. Finding objective ways of selecting and defining coherent vortices has been an active area of research in the last decade (Chong et al. 1990; Jeong and Hussain 1995; Haller 2005). Most of them are based on a combination of geometrical and dynamical properties that must be satisfied in the flow regions occupied by coherent vortices. In general, geophysical flows in the atmosphere and the ocean are not purely 2D, but fortunately at midlatitudes, the dynamics of geophysical flows at the mesoscale fits well in the quasigeostrophic (QG) approximation, so it can be reasonably described by 2D turbulence (Rhines 1979). Under these conditions, a useful criterion based on the Okubo–Weiss parameter (Okubo 1970; Weiss 1991) has allowed the partitioning of fluid flow in two components—the vortex-dominated regions and the background field; it is therefore an appropriate criterion to deduce vortex structures (McWilliams 1984; Elmhahidi et al. 1993). The Okubo–Weiss criterion has been successfully applied to extract, track, and analyze the statistical properties of marine eddies (Isern-Fontanet et al. 2003; Morrow et al. 2004; Isern-Fontanet et al. 2006a). Furthermore, the observed non-Gaussianity in the probability density functions (PDFs) of velocity fields evaluated from ocean measurements is thought to be the result of the presence of coherent structures, in a similar way to what is obtained from numerical simulations of 2D turbulence (see, e.g., Llewellyn Smith and Gille 1998; Gille and Llewellyn Smith 2000; Bracco et al. 2000a, 2004; Schorghofer and Gille 2002; LaCasce 2005; Isern-Fontanet et al. 2006b).

A rather new approach to the flow separation problem, still on the frame of 2D turbulence, has been introduced through wavelet analysis of the vorticity field (Siegel and Weiss 1997; Farge et al. 1999; Farge and Schneider 2001). Flow is decomposed as a coherent, inhomogeneous, non-Gaussian component formed by a set of vortices at different scales and an incoherent homogeneous Gaussian component characteristic of the background flow (Farge et al. 1999). When applied to numerical simulations, this method has shown good performance for vortex extraction and efficient filtering of the unsolved modes. The method is known as coherent vortices simulation (Farge et al. 1999); in this paper we will refer to this technique by the more descriptive name of coherent vortex separation (CVS). Very recently Ruppert-Felsot et al. (2005) have applied the

same concept to explore data from laboratory experiments on a rotating turbulent flow with successful results in terms of flow separation between coherent and incoherent modes of the vorticity field. Although the results have only been applied on the frame of pure turbulence studies in simple geometries (periodic and cylindric boundaries), it is reasonable to think that they can be applied to oceans, as previous studies suggest that an important part of oceanic dynamics could be characterized by a precise knowledge of coherent eddies. Hence, a technique such as CVS would be undoubtedly of interest in this context. This is precisely the scope of this paper: to present an application of CVS for altimetric maps obtained in the western Mediterranean basin and to demonstrate that CVS is a technique that allows for drastically reducing the number of degrees of freedom necessary to reproduce the main statistical properties of the basin dynamics. We will optimize CVS by performing a search on the best adapted wavelet basis and assessing the quality using several parameters.

The paper is organized as follows. The next section is devoted to the description of the data that will be used for our implementation of CVS. In section 3 we will present the basic theoretical aspects in CVS, while section 4 is devoted to showing the experimental results. We will discuss those results in section 5, and the overall conclusions of our work are presented in section 6.

2. Description of the data

Given the direct relation between altimetric data and sea-state dynamics, in this study we have used sea level anomaly (SLA) maps produced by Collecte Localisation Satellites (CLSs) in Toulouse, France, which combine the signal of the European Remote Sensing Satellite (ERS) and Ocean Topography Experiment (TOPEX)/Poseidon (T/P) altimeters. These maps are processed with the usual corrections (sea-state bias, tides, inverse barometer, etc.) and with improved ERS orbits using TOPEX/Poseidon as a reference (AVISO 1996; LeTraon and Ogor 1998). SLAs are regularly produced by subtracting a 4-yr mean value (1993–96), and prior to the analysis, data are low-pass filtered using a 35-km median filter and a Lanczos filter with a cutoff wavelength of 42 km in order to reduce altimetric noise (Larnicol et al. 1995). The repeat periods for the *ERS-1* and T/P are 35 and 10 days, respectively, and the cross-track distances are 0.7° and 2.8° . SLA maps are built for 10-day periods, using an improved space/time objective analysis method (which takes into account long wavelength errors) on a regular grid of $0.2^\circ \times 0.2^\circ$ (LeTraon et al. 1998; Ayoub et al. 1998). The use of altimetric

data limits the phenomena that can be studied. On the one hand, the sampling periodicity of altimeters aliases frequencies below 20 and 70 days in T/P and *ERS-I* data, respectively. On the other hand, satellite track separation and time sampling limit the capability of altimetric maps to observe fast propagating structures or short-lived ones. Furthermore, recently it has been demonstrated that a two-satellite scenario may underestimate an important portion of the signal variance and may deform the shape of some eddies (Pascual et al. 2006). However, previous studies have demonstrated that sampling characteristics of altimetric maps are good enough to capture and track the largest and most energetic vortices observed in the western Mediterranean Sea (e.g., Millot 1999; Pascual et al. 2002; Larnicol et al. 2002; Puillat et al. 2002; Font et al. 2004).

Velocities are estimated from the sea level anomaly h' , as usual, assuming a geostrophic relationship

$$u = -\frac{g}{f} \frac{\partial h'}{\partial y}, \quad v = \frac{g}{f} \frac{\partial h'}{\partial x},$$

where g is gravity and f is the Coriolis parameter. We will call the resulting velocity field $\mathbf{v} = (u, v)$ the geostrophic velocity field.

For this study we will consider a subset of geostrophic velocity fields for an area of the western Mediterranean Sea and different times. The area under study covers 35° – 45° N latitude by 2° W– 10° E longitude. The time periods correspond to summer months in 1996 (from 15 July to 31 August), 1997 (1 August to 15 September), and 1998 (1 May to 15 June); so, each one of these periods gives rise to 5 complete altimetric maps and hence 5 realizations of the geostrophic velocity field, and 15 velocity fields intotal.

3. Coherent vortex separation: Theoretical background

a. Introduction

Initially developed to analyze seismic signals, wavelets have become a tool for a wider range of applications including characterization of statistical properties using maxima lines of the wavelet transform (Muzy et al. 1991), scaling properties in general (Davis et al. 1994), determination of cloud structure (Roux et al. 2000), detection and statistical characterization of precipitation (Gupta and Waymire 1993; Turiel et al. 2005a), assessment of streamlines in sea surface temperature satellite images (Turiel et al. 2005b; Isern-Fontanet et al. 2007), and many other applications. It has become a powerful tool to analyze dynamical properties of turbulent flows from both laboratory and nu-

merical origins (Farge 1992). The essence of CVS lies in the efficient use of the wavelet analysis. Wavelet bases allow one to represent complex signals without fixing a privileged observation scale, because wavelet bases are scale-invariant. For that reason, wavelet bases are very appropriate in dealing with signals that lack a characteristic scale of interaction or that possess many different scales at which the relevant physical processes take place (Mallat 1999). With the use of wavelet bases we do not predetermine the adequate size for a coherent vortex; this will result from the analysis.

The velocity field and the vertical component of the vorticity can be directly retrieved one from the other in 2D flows (by means of the Biot–Savart law), so we can work with either field without losing information about the motion. As our interest is to extract the so-called coherent vortices, we will focus on the vorticity, trying to design an appropriate filter to separate the relevant coherent modes.

Let us express this construction in a more mathematical, formal way. Let $\mathbf{v}(\mathbf{x})$ be the vector field of velocities at each point $\mathbf{x}$ of the sea surface. The vorticity ω associated with the velocity field is then

$$\omega = \nabla \times \mathbf{v}.$$

In the case of a 2D fluid, only the z component of the vorticity is nonvanishing, so we will work with this component, which we will denote as $\omega \equiv \omega_z = \partial_x v - \partial_y u$. In this case, Biot–Savart’s law gives a way to retrieve the velocity from vorticity, namely,

$$\mathbf{v} = \nabla \times (\Delta^{-1} \omega),$$

and so we can pass from velocity to vorticity and vice versa.

b. Overview of wavelet theory

Wavelet analysis was first introduced to perform local Fourier analysis, that is, to characterize the local oscillatory properties of a signal (period, wavelength, phase, and amplitude) by just taking into account a small neighborhood of the instant under study. Such an instantaneous characterization allows one to follow the time evolution of the wave characteristics and at the same time gives access to the precise location at which a given oscillation is taking place. The concept of “wavelet” (elementary wave) implies that the basic oscillatory feature is no longer a plane wave of infinite extent but a fast decaying oscillation that extinguishes after a few full periods; a classical example of such a “fast-decaying” wave is given by Gabor functions $G(x, \sigma, \lambda)$, which can be defined as

$$G(x, \sigma, \lambda) \equiv \sin(x/\lambda)e^{-(1/2)(x^2/\sigma^2)}.$$

In Gabor functions, the package extent is controlled by the scale scope parameter σ , while the frequency is governed by the wavelength parameter λ . For a given signal $f(x)$, the local amplitudes associated with local oscillations are thus characterized by projections $\alpha^G(x, \sigma, \lambda)$ on the Gabor functions, namely,

$$\alpha^G(x, \sigma, \lambda) = \int dx' G(x - x', \sigma, \lambda)f(x').$$

It was soon recognized that the scale scope and the wavelength are not really independent. Namely, to assess the presence of a local oscillatory mode, the scale scope must be greater than a wavelength, but also it should not be greater than two or three wavelengths to avoid losing the precise localization. As a matter of fact, the relation between these two parameters can be fixed to a constant and only one of the two will be varied independently. In the particular case of the Gabor functions, and assuming $\lambda = \kappa\sigma$, we have

$$G(x, \sigma, \lambda) = \sin(x/\lambda)e^{-(1/2)(x^2/\sigma^2)} = \sin(x/\kappa\sigma)e^{-(1/2)(x^2/\sigma^2)} \\ = G(x/\sigma, 1, \kappa).$$

Hence, if we define $g(x) = G(x, 1, \kappa)$, we are projecting on $g(x/\sigma)$. Namely, we are only interested in the projections on dilated versions of the kernel $g(x)$, because all the dependence on the scale scope enters as a scaling factor on the variable. In a more general framework, a wavelet is any function ψ that can be used to fully represent any function $f(x)$ by means of the projections on the resized and recentered versions of it. The wavelet projection of $f(x)$ on ψ at a position x and a scale r is given by

$$\alpha^\psi(x, r) \equiv \int dx' \frac{1}{r} \psi\left(\frac{x - x'}{r}\right) f(x').$$

If the wavelet ψ is *admissible* [i.e., $\int dk |\hat{\psi}|^2(k)/|k| < \infty$, where $\hat{\psi}(k)$ stands for the Fourier transform of ψ and admissibility implies that ψ must have zero mean], any function $f(x)$ can be fully reconstructed from its wavelet projections (Daubechies 1992), in the following way:

$$f(x) = \int dx' \int \frac{dr}{r^2} \alpha^\psi(x', r) \psi\left(\frac{x - x'}{r}\right).$$

Such a representation is interesting for analyzing transitions in signals (Mallat and Huang 1992; Mallat and Zhong 1992), but although complete it is extremely redundant. Notice that we are converting a function depending on a single-space variable $f(x)$ into a function depending on two variables, $\alpha^\psi(x, r)$. To produce a

more efficient representation, some positions x and scales r must be selected. For some particular, well-chosen wavelets (Daubechies 1992), positions $x = 2^{-j}k$ and scales $r = 2^{-j}$ (where j, k are integer numbers) fully represent the signal. For these wavelets the continuous representation of functions $f(x)$ above becomes a discretized version:

$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \alpha_{jk} \psi_{jk}(x),$$

where the functions $\psi_{jk}(x)$ are defined as

$$\psi_{jk}(x) = 2^{j/2} \psi(2^j x - k).$$

This type of discretized representation is known as a wavelet dyadic representation and is connected with the multiresolution theory of signals (Daubechies 1992; Mallat 1999). Notice that the coefficients α_{jk} are not the projections of $f(x)$ on ψ_{jk} , but on its dual wavelet basis $\tilde{\psi}_{jk}$ (Daubechies 1992). For the sake of simplicity, we will concentrate our attention on those wavelets that coincide with their dual, that is, when the dyadic representation defines an orthonormal basis and the coefficients α_{jk} coincide with the projections on $\psi_{jk}(x)$, namely,

$$\alpha_{jk} = \langle \psi_{jk}, f \rangle \equiv \int dx \psi_{jk}(x) f(x).$$

Orthonormal wavelets are very convenient, as there are fast algorithms to decompose a function into its wavelet coefficients and back, by means of the so-called quadrature mirror filters (QMFs; Daubechies 1992; Mallat 1999). Essentially, a QMF consists of the wavelet function $\psi(x)$ and a unity function $\phi(x)$, which is orthogonal to ψ at any coarser scale,

$$\langle \psi_{jk}, \phi_{j'k'} \rangle = 0 \quad j \neq j'.$$

The unity function is used to obtain the approximation to the function $f(x)$ at a given scale, while the higher-resolution details are furnished by the wavelet coefficients. At any finite scale $r = 2^{-j_0}$ we can expand any function $f(x)$ as

$$f(x) = a_{j_0}(x) + \sum_{j=j_0}^{\infty} d_j(x) = \sum_{k=-\infty}^{\infty} \langle \phi_{j_0k}, f \rangle \phi_{j_0k}(x) \\ + \sum_{j=j_0}^{\infty} \sum_{k=-\infty}^{\infty} \langle \psi_{jk}, f \rangle \psi_{jk}(x).$$

The second term in the sum above is the sum of all the details of scale j_0 or finer. Each detail level $d_j(x)$ is represented by the contributions of the wavelet decomposition at the scale j , namely,

$$d_j(x) = \sum_{k=-\infty}^{\infty} \langle \psi_{jk}, f \rangle \psi_{jk}(x).$$

The first term in the wavelet decomposition above, $a_{j_0}(x)$, is the approximation of $f(x)$ at the scale j_0 . It represents a coarsened version of the function, from which only coarser details have not yet been extracted. Notice that in fact the approximation term can also be expressed as the sum of the details coarser than j_0 :

$$a_{j_0}(x) = \sum_{j=-\infty}^{j_0-1} d_j(x).$$

As a direct consequence of the equations above, we can conclude that

$$a_j(x) = a_{j-1}(x) + d_{j-1}(x).$$

This relation is very useful to define fast, high-performance wavelet decompositions of real data. When analyzing, signals can be progressively coarsened by extracting the detail level at each resolution; when reconstructing, each approximation level is improved by adding the corresponding detail level. Once the numerical weights of the filters associated with ψ and ϕ are given, fast iterative algorithms can be defined [for a more detailed explanation, the reader is encouraged to consult Mallat (1999)].

For our study we are not interested in processing 1D signals but 2D scalar fields of vorticity, so the wavelet scheme we have presented so far needs to be generalized. Continuous wavelet transforms can also be calculated at any dimensionality and they can be used to reconstruct the signal when admissible wavelets are used. Besides, it is possible to subsample scales and positions to obtain nonredundant representations of data. However, to define a dyadic representation, more than a single basis wavelet must be used: simple algebraic reasoning shows that $2^d - 1$ different wavelets, where d is the dimension of the domain space, are required. For 2D scalar signals we will hence need $2^2 - 1 = 3$ different wavelets Ψ_1 , Ψ_2 , and Ψ_3 , and the generalized dyadic expansion of a 2D function $f(\mathbf{x})$ now reads

$$f(\mathbf{x}) = \sum_{r=1}^3 \sum_j \sum_{\mathbf{k}} \alpha_{rj\mathbf{k}} \Psi_{rj\mathbf{k}}(\mathbf{x}),$$

where $\Psi_{rj\mathbf{k}}(\mathbf{x}) = 2^j \Psi_r(2^j \mathbf{x} - \mathbf{k})$, $\mathbf{k} = (k_1, k_2)$, j, k_1, k_2 are integer numbers. As for the 1D case, it is possible to construct orthonormal bases such that

$$\langle \Psi_{rj\mathbf{k}}, \Psi_{r'j'\mathbf{k}'} \rangle = \int d\mathbf{x} \Psi_{rj\mathbf{k}}(\mathbf{x}) \Psi_{r'j'\mathbf{k}'}(\mathbf{x}) = \delta_{rr'} \delta_{jj'} \delta_{\mathbf{k}\mathbf{k}'},$$

and so

$$\alpha_{rj\mathbf{k}} = \langle \Psi_{rj\mathbf{k}}, f \rangle = \int d\mathbf{x} \Psi_{rj\mathbf{k}}(\mathbf{x}) f(\mathbf{x}).$$

To find three functions $\{\Psi_r\}_{r=1,2,3}$ that define an orthonormal wavelet basis is a complicated task that requires a good knowledge of wavelet theory. Fortunately, nowadays there exist complete libraries of wavelet bases with optimized numerical implementations that furnish a wide spectrum of functions to choose, according to some regularity conditions to be imposed (Mallat 1989, 1999; Daubechies 1992; Sweldens 1996). An easy way to construct a basis is to start from a 1D QMF (ϕ, ψ) and to construct a 2D QMF ($\Phi, \Psi_1, \Psi_2, \Psi_3$) in this way:

$$\begin{aligned} \Phi(x_1, x_2) &= \phi(x_1)\phi(x_2) \\ \Psi_1(x_1, x_2) &= \phi(x_1)\psi(x_2) \\ \Psi_2(x_1, x_2) &= \psi(x_1)\phi(x_2) \\ \Psi_3(x_1, x_2) &= \psi(x_1)\psi(x_2). \end{aligned}$$

This kind of 2D QMF is called “separable,” as we can separate the contribution of each coordinate variable. In a way analogous to the approximation-detail decomposition in 1D, we have

$$f(\mathbf{x}) = a_{j_0}(\mathbf{x}) + \sum_{r=1}^3 \sum_{j=j_0}^{\infty} d_{rj}(\mathbf{x}),$$

where the details are now defined for each wavelet type r and each scale j ,

$$d_{rj}(\mathbf{x}) = \sum_{\mathbf{k}} \alpha_{rj\mathbf{k}} \Psi_{rj\mathbf{k}}(\mathbf{x}),$$

and the approximation term is given by

$$\alpha_{j_0}(\mathbf{x}) = \sum_{\mathbf{k}} \langle \Phi_{j_0\mathbf{k}}, f \rangle \Phi_{j_0\mathbf{k}}(\mathbf{x}) = \sum_{r=1}^3 \sum_{j=-\infty}^{j_0-1} d_{rj}(\mathbf{x}).$$

As before, we have an iterative relation between consecutive scales:

$$a_j(\mathbf{x}) = a_{j-1}(\mathbf{x}) + \sum_r d_{r,j-1}(\mathbf{x}).$$

For a finite image of real data, as scale increases (j decreases), the number of wavelet coefficients decreases (because each wavelet coefficient is associated with a wavelet with a larger dispersion and less wavelets are necessary to cover the image). It is thus usual to represent all the wavelet coefficients arranged into the corresponding detail images and all of them forming a single image, called the multiresolution representation (see Fig. 3). In this kind of representation the wavelet coefficients for increasing scales occupy decreasing size subimages, always arranged around the diagonal of the image.

c. Implementation of CVS

Wavelet representations of data separate structures at different scales. It is then natural to think that in an appropriate wavelet basis, the motion could be concentrated on a few wavelet modes, which will better describe the whole dynamics. This is the essence of the CVS technique.

The first step in CVS is to represent the vorticity field in an appropriate dyadic orthonormal wavelet basis. Once vorticity is expanded as wavelet contributions, the field is split in a part concerned by the greatest amplitude contributions (coherent part) and a part gathering the rest (incoherent part). CVS is designed as the application of a threshold on the wavelet coefficients α_{rjk} . In the original derivation by Farge et al. (1999), the threshold Θ is set by applying Donoho and Johnstone's (1994) criterion, $\Theta = 2\sqrt{Z \ln N}$, where Z is the total enstrophy, $Z = \frac{1}{2}\langle \omega, \omega \rangle$, and N is the resolution of the image, in the sense of the total number of pixels. We hence split the vorticity into two terms, the coherent part ω_c and the incoherent part ω_i , in this way:

$$\omega = \omega_c + \omega_i = \sum_{|\alpha_{rjk}| > \Theta} \alpha_{rjk} \Psi_{rjk}(\mathbf{x}) + \sum_{|\alpha_{rjk}| \leq \Theta} \alpha_{rjk} \Psi_{rjk}(\mathbf{x}).$$

The coherent part is intended to select the most enstrophic vortices only, while the incoherent part is formed by the remains of the vorticity field. The value of the threshold Θ is not very far from the average value of α_{rjk} , but as the vorticity distribution is very skewed, the amount of wavelet coefficients in the coherent part is usually very small [less than 1% in the experiences carried out over direct numerical simulations (DNSs; Farge and Schneider 2001)]. Even so, the coherent part is a good approximation of the whole fluid in both DNSs and laboratory experiments in rotating turbulent flows: the coherent part accounts for 99% of the energy and 85% of the enstrophy (similar amounts in laboratory experiments; see Farge and Schneider 2001; Beta et al. 2003a,b; Ruppert-Felsot et al. 2005). Notice that Donoho and Johnstone's criterion was designed for optimizing the code budget in image transmission, which explains how it mainly describes the dominant features. Regarding the incoherent part, it has been shown to be noiselike and unstructured: the associated flow exhibits short streamlines with very small velocities. It is now clear why CVS is so interesting: we can deal with a complex flow with a large number of degrees of freedom by reducing it to its coherent part, which is a good approximation of the fluid motion but contains about 100 times less degrees of freedom and provides a description based on physical entities such as vortices. CVS appears as a good tool for pattern

identification, physical assessment, and improved forecasting (as evolution equations are enormously reduced).

As we pointed out before, the choice of the wavelet basis is crucial in the performance of CVS: if the wavelets in the basis are good approximators of real vortices, less wavelet coefficients will be needed and the vorticity field will be described with greater accuracy, particularly for flows dominated by vortex structures. Of course it is very unlikely that a general function could describe all possible vortex geometries, but just a good approximation can be rather satisfactory. In addition, some additional properties are generally required, for instance, that the basis acts as a good interpolation scheme or it has a high enough degree of differentiability. The bases we will implement in this paper will satisfy those different requirements. We will make use of three different families of wavelet bases: the Haar family, Daubechies family, and Battle-Lemarié family. The Haar family consists of a single basis, Haar basis, which is the simplest basis one could consider: it is a single step. Daubechies family consists of Daubechies wavelets of the order p ; the order indicates the degree of differentiability of the wavelet. Daubechies wavelets are the most compact wavelets at the given differentiability degree. Finally, the Battle-Lemarié family consists of Battle-Lemarié (or spline) wavelets of the order p , where the order indicates the degree of spline interpolation the wavelet is performing; Battle-Lemarié wavelets are equivalent to spline interpolators. The original works on CVS used $p = 6$ Battle-Lemarié basis, that is, the wavelet basis performing spline interpolation of the order 6. In our work we will show the results for Battle-Lemarié bases of the orders $p = 2$, $p = 3$, and $p = 6$ for Daubechies bases of the same orders and also the results for the Haar basis.

4. Experimental performance of CVS on altimetric sea surface height

In Fig. 1 we show 1 of the 15 maps of velocity fields that we have processed. For preprocessing, we will work with the divergence-free velocity field, that is, the one obtained after computing the vorticity ω and then applying Biot–Savart's law to recover the velocity. In principle we should obtain the same vector as we initially have, but in practice there is a small deviation, caused by small numerical divergences in the original data; we will thus keep the processed velocity, which by construction is divergence-free and stable when passing to vorticity and back. Anyway, both fields are very similar: the standard measure (in signal processing) of the peak signal-to-noise ratio (PSNR) gives values above

FIG. 1. (left) Original data velocities. (right) Divergence-free approximation (PSNR with respect to original data: 33.86 dB).

30 dB in all maps (which correspond to high-quality reconstructions).

The second step is to compute the vorticity $\omega = \partial_x v - \partial_y u$. As observed in Fig. 2, large positive and negative values of the vorticity can easily be identified with cyclonic and anticyclonic eddies, respectively. In principle a simple threshold on the absolute value of the vorticity can be used to separate the strongest eddies, but as discussed in Jeong and Hussain (1995), it is not a sufficient condition to educe vortex structures and more robust principles should be employed (Jeong and Hussain 1995; Hua and Klein 1998; Isern-Fontanet et al. 2003).

In Figs. 3 and 4 we can see the results of the multi-resolution decomposition of the vorticity field on the Haar, $p = 6$ Daubechies and $p = 6$ Battle-Lemarié bases. Also shown are the results of applying Donoho–Johnstone’s criterion to the associated wavelet decomposition. As the figures show, there are only a few coefficients that are finally kept at each scale layer; these coefficients mainly come from the greatest scales (those closest to the bottom left corner).

Once the wavelet coefficients are selected, we can construct the vorticity field corresponding to the coherent part of the flow; see some examples of this field in Fig. 5, compared to the total field. As shown in the figure, coherent vorticity fields obtained with smoother wavelets resemble the original field more. The incoherent field can be easily obtained by a simple subtraction: $\omega_i = \omega - \omega_c$.

In Fig. 6 we show the examples of the coherent velocities associated with the vorticity fields in Fig. 5. Analogous to what happens with vorticities, the incoherent velocity can be obtained by simple subtraction: $\mathbf{v}_i = \mathbf{v} - \mathbf{v}_c$.

Figures only serve for a qualitative assessment, but we need more quantitative measures. To obtain a quan-

FIG. 2. Vorticity field associated with the velocity in Fig. 1; brighter points correspond to greater positive values while darker points mean large negative values; intermediate gray is associated with small absolute values of vorticity.

FIG. 3. (left) First step in the multiresolution decomposition in the Haar basis of the vorticity represented in Fig. 2. The image is subdivided in three detailed parts corresponding to (clockwise from top left) the wavelet coefficients for each one of the wavelets in the basis and an approximation part, which contains the contributions for the larger scales still not resolved. The approximation is a lower-resolution version of the original image, for which the analysis can be iterated. (right) Complete multiresolution decomposition of the vorticity in the Haar basis; the separation process is iterated over the successive approximation images.

FIG. 4. (top) Multiresolution decompositions of the vorticity for different bases (Haar, $p = 6$ Daubechies, and $p = 6$ Battle-Lemarié). (bottom) Wavelet coefficients in the associated coherent parts.

FIG. 5. (top left) Total vorticity field (same as in Fig. 2) and coherent vorticities obtained after processing with (top right) Haar basis, (bottom left) $p = 6$ Daubechies basis, and (bottom right) $p = 6$ Battle-Lemarié basis.

titative comparison of the performance of each basis, in Table 1 we have summarized the results of the application of all the tested bases to one of the samples in our dataset; similar results were obtained for the other maps considered. To characterize the performance of CVS we have computed five parameters.

- N_c : number of wavelet coefficients in the coherent part according to Donoho and Johnstone's criterion. In the table we give this parameter as a percentage over the total amount of wavelet coefficients. Although this parameter is not used to assess the quality

of a particular basis to perform CVS, it is evident that a basis will attain a better performance than another if, for the same other quality parameters, N_c is smaller.

- Z_c and Z_i : coherent and incoherent enstrophies. Recalling that we can express the vorticity ω as the sum $\omega = \omega_c + \omega_i$, and the total enstrophy is $Z = \frac{1}{2}\langle\omega, \omega\rangle$, we construct the coherent enstrophy as the enstrophy associated with the coherent part ($Z_c = \frac{1}{2}\langle\omega_c, \omega_c\rangle$) and analogously for the incoherent enstrophy ($Z_i = \frac{1}{2}\langle\omega_i, \omega_i\rangle$). Because of the orthogonality of the wavelet basis used in the decomposition, the coherent vorticity is orthogonal to the incoherent vorticity, so

FIG. 6. (top left) Divergence-free velocity field (same as in Fig. 1) and coherent velocities obtained after processing with (top right) Haar basis, (bottom left) $p = 6$ Daubechies basis, and (bottom right) $p = 6$ Battle-Lemarié basis.

$\langle \omega_c, \omega_i \rangle = 0$ and hence $Z = Z_c + Z_i$. In the table we furnish the percentage of enstrophy over the total enstrophy (Z) represented by the coherent and the incoherent parts; indeed, the quantities shown there always sum up 100%. We consider that a wavelet basis is more suited to describe the behavior of the

flow when Z_c approaches 100% of Z . In *both DNSs and laboratory experiments* it was observed that Z_c was about 85% of Z .

- E_c and E_i : coherent and incoherent energies. From the coherent and incoherent vorticities we can construct the associated velocity fields using Biot–

TABLE 1. Results for summer 1996.

First realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	1.28	1.49	1.46	1.64	1.46	1.49	1.43
% Z_c	70.0	79.3	76.6	79.6	79.3	78.8	78.1
% Z_i	30.0	20.7	23.4	20.4	20.7	21.2	21.9
% E_c	150	95.1	73.8	67.5	67.9	62.1	63.4
% E_i	130	67.5	59.1	42.3	41.4	42.8	40.4
Second realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	1.37	1.46	1.25	1.67	1.46	1.04	1.61
% Z_c	68.9	79.0	75.3	76.8	78.6	79.0	77.9
% Z_i	31.1	21.0	24.7	23.2	21.4	21.0	22.1
% E_c	145	98.1	77.0	71.7	67.8	61.7	64.7
% E_i	128	64.3	54.3	41.8	41.6	40.9	39.9
Third realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	1.37	1.34	1.04	1.46	1.82	1.13	1.73
% Z_c	71.8	80.4	79.6	78.3	83.8	83.6	81.2
% Z_i	28.2	19.6	20.4	21.7	16.2	16.4	18.8
% E_c	144	99.7	83.8	71.3	80.9	69.3	75.6
% E_i	121	65.4	49.5	43.0	32.3	32.7	35.0
Fourth realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	1.37	1.13	1.13	1.67	1.34	1.13	1.49
% Z_c	71.6	81.2	82.5	80.4	80.8	84.0	82.1
% Z_i	28.4	18.8	17.5	19.6	19.2	16.0	17.9
% E_c	149	96.7	81.2	68.8	72.1	68.4	66.8
% E_i	130	62.3	48.5	43.3	35.5	36.1	33.4
Fifth realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	1.22	0.98	1.01	1.58	1.16	0.98	1.28
% Z_c	71.7	77.7	77.9	81.6	82.6	81.0	81.9
% Z_i	28.3	22.3	22.1	18.4	17.4	19.0	18.1
% E_c	151	92.8	69.2	71.9	69.5	63.0	65.6
% E_i	131	71.6	54.2	39.4	35.1	41.3	34.4

Savart's law. However, although $\mathbf{v} = \mathbf{v}_c + \mathbf{v}_i$, in general $\langle \mathbf{v}_c, \mathbf{v}_i \rangle \neq 0$ and so the energies are not additive: $E = \frac{1}{2} \langle \mathbf{v}^2 \rangle$, $E_c = \frac{1}{2} \langle \mathbf{v}_c^2 \rangle$, $E_i = \frac{1}{2} \langle \mathbf{v}_i^2 \rangle$, but $E \neq E_c + E_i$. In the table, we give the percentage over the total energy (E) represented by the coherent and the incoherent energies. The quantities do not sum up to 100% any longer but in general a greater amount. We consider that the basis is describing better the system when the sum of percentages approaches 100%, that is, when the velocities are mutually orthogonal. We will consider a basis to be efficient if, in addition to the mutual orthogonality of the coherent and incoherent velocities, the value of E_c approaches 100% of E . For comparison, let us say here that in 2D DNSs it

is observed that $E_c + E_i \approx E$, where E_c amounted to about 99% of E .

5. Discussion

After analyzing our results, the first conclusion we can extract is that for any wavelet basis, Donoho and Johnstone's criterion gives values of N_c that are really small, at most 2% of the total amount of wavelet coefficients, and usually much less. In addition, the coherent enstrophy Z_c for all the bases has large values, representing at least 65% of the total enstrophy and reaching values of about 80% for the most efficient bases, similar to what was observed for 2D DNSs. Our results

are also comparable with the level of reconstruction found in the analysis of laboratory data where on average about 2%–3% of large-amplitude coefficients were retained in the coherent fields, which accounted for about 85% of the total enstrophy of the flow. Thus, also in an oceanographic context, the coherent vorticity field is a rather good representation of the total field, as 1% of the vorticity coefficients are responsible for more than two-thirds of the total enstrophy; this result is very robust disregarding the particular basis used. Of course some bases seem to be more appropriate, and not surprisingly Daubechies and Battle-Lemarié wavelets of higher orders attain the best performances, while the Haar basis, which is a very rudimentary one, is always the worst basis.

Concerning energies, however, our results are less convincing. Several bases possess coherent and incoherent velocities that are by no means mutually orthogonal; the most exaggerated example is the Haar basis, but even low-order Daubechies present very significant deviations from mutual orthogonality. Only $p = 6$ Daubechies wavelets, and in general Battle-Lemarié wavelets of any order (although performance rises with higher values of p), have coherent and incoherent velocities close to mutual orthogonality, and this property is almost perfectly verified for the $p = 6$ Battle-Lemarié basis. But even in the best cases the coherent energy does not amount to more than about 70% of the total energy. It seems that CVS is missing some relevant information on oceanic motion, much more than what could be expected from 2D DNS results (in DNS $p = 6$ Battle-Lemarié attained 99% of coherent energy).

With respect to the choice of basis, it is evident that the Haar basis must be discarded: it is too rough and its squared shape tends to produce artificial squared vorticities and velocity fields consisting of streamlines too aligned with Cartesian axes. Comparing Daubechies and Battle-Lemarié, we observe that the second family is more efficient than the first one, and even low-order Battle-Lemarié wavelets attain relatively good qualities, better than their Daubechies counterparts. Hence, we conclude that for an efficient implementation of CVS, wavelet bases offering a good interpolation scheme (as the splines induced by Battle-Lemarié wavelets) are more convenient than bases associated with compact, regular wavelets. The reason can probably be due to the presence of important boundary conditions for the data under study, which induce sharp discontinuities that need to be conveniently smoothed away. For the highest-order bases (see Fig. 6), some deformation on the shape of the eddies when comparing the coherent velocity with the divergence-free ve-

locity is observed, although it does not seem to be as crucial as the loss of some streams not directly associated with vortices—a loss that according to the background of the method seems rather reasonable.

We have also explored a refinement of the Donoho–Johnstone’s criterion, according to what is discussed in Farge et al. (1999). Instead of fixing the threshold as $\Theta = 2\sqrt{Z \ln N}$, an iterative optimization of the threshold is performed. Taking into account that the goal is to split the vorticity in a coherent, non-Gaussian part and an incoherent, Gaussianly distributed part, the iterative algorithm consists of readapting the threshold Θ using the current coherent enstrophy until the incoherent vorticity is close to a Gaussian distribution. To control the closeness of the incoherent part to Gaussianity, Farge et al. proposed measuring the kurtosis [$\kappa = (\langle \omega_i^4 \rangle / \langle \omega_i^2 \rangle^2) - 3$] and some odd order moments and stopping the algorithm when these quantities are close enough to zero. We have tried to do the same, but we have found that in our case the algorithm did not converge to a nonzero Θ : for all the wavelets and configurations, Θ dropped to zero. We observed that the critical quantity was always the kurtosis κ , which initially decreases monotonically but eventually starts increasing monotonically, while the other quantities continued to decrease. Hence, we modified the algorithm so that it stopped when the kurtosis increased for the first time. In Table 2 we summarize the results for this refined algorithm.

All the conclusions regarding the quality of the different wavelet bases still hold when this refined choice of the threshold is applied. From these new results we can draw three conclusions: (i) there is a considerable improvement in the quality of the coherent part: with this refinement, the coherent part typically accounts for more than 95% of the energy and of the enstrophy; (ii) this improvement implies a considerable increase in the amount of wavelet coefficients of the coherent part, sometimes reaching to 10% of the total amount; and (iii) the kurtosis is never negligible, which makes the interpretation of the incoherent part as a Gaussian noiselike contribution quite doubtful. When we look at which coefficients were selected for the coherent part when the threshold is refined (Fig. 7) we see that almost any coefficient at the lower resolutions (larger scales) is now included in the coherent vorticity, which makes the observed increase in quality reasonable but renders the method nonphysically significant: higher-resolution details account for lower-scale movements and thus they obviously transport less energy and less enstrophy. As a consequence, the separation in *coherent + incoherent* parts becomes something too close to a *low-pass approximation + high-pass details* decomposition, with no

TABLE 2. Results for summer 1996, using the refined choice of the threshold Θ .

First realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	8.46	5.45	5.99	8.76	3.76	10.6	10.2
% Z_c	94.7	95.5	96.8	98.7	93.9	99.3	99.2
% Z_i	5.3	4.5	3.2	1.3	6.1	0.7	0.8
% E_c	130.7	111.7	101.4	97.1	88.3	97.5	96.2
% E_i	49.4	33.3	19.3	6.4	18.0	3.9	4.0
κ	2.69	2.50	2.37	1.53	1.84	1.10	0.75
Second realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	8.76	3.43	5.84	6.08	8.08	8.14	8.44
% Z_c	94.9	91.3	96.3	96.8	98.5	98.9	98.5
% Z_i	5.1	8.7	3.7	3.2	1.5	1.1	1.5
% E_c	126.1	105.2	99.8	94.7	95.7	96.4	94.6
% E_i	42.9	42.1	19.0	10.9	6.8	4.9	5.6
κ	2.86	3.28	2.22	1.27	1.37	1.18	0.51
Third realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	8.91	5.10	7.72	12.1	10.5	3.40	8.17
% Z_c	95.8	95.8	98.5	99.4	99.4	95.3	98.7
% Z_i	4.2	4.2	1.5	0.6	0.6	4.7	1.3
% E_c	122.7	109.0	101.4	98.5	97.9	88.9	95.7
% E_i	37.4	29.0	11.5	4.0	3.6	15.2	4.8
κ	2.47	3.02	2.22	1.90	1.25	1.54	0.88
Fourth realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	8.52	5.25	9.12	10.5	7.96	10.5	8.32
% Z_c	95.5	96.6	98.9	99.2	99.0	99.6	98.8
% Z_i	4.5	3.4	1.1	0.8	1.0	0.4	1.2
% E_c	127.3	110.2	100.3	97.6	96.6	97.7	95.7
% E_i	45.4	28.2	9.4	5.1	5.5	3.2	5.0
κ	2.82	3.15	2.51	1.56	1.49	1.09	1.04
Fifth realization							
Wavelet	Haar	Dau2	Dau3	Dau6	BM2	BM3	BM6
% N_c	13.3	3.19	5.63	8.73	5.84	8.32	5.84
% Z_c	97.8	92.8	97.3	98.8	98.1	99.1	97.8
% Z_i	2.2	7.2	2.7	1.2	1.9	0.9	2.2
% E_c	116.5	107.6	100.8	96.8	94.6	96.7	92.6
% E_i	25.7	42.0	18.0	6.2	8.5	5.0	8.4
κ	2.11	3.09	2.58	1.27	1.50	1.20	1.01

relevant physical interpretation. We thus prefer to keep as a reference the first choice of threshold, for which a small number of coefficients describes an important extent of the fluid, although the incoherent part is always non-Gaussian and is still affected by some relevant physical modes of motion.

6. Conclusions

We have seen that CVS is an interesting tool, not only in the original context in which it was developed

(DNS on 2D turbulence) but also in the more applied framework of physical oceanography for the study of mesoscale flows in the QG approximation. We have seen that the most efficient wavelet bases give rise to coherent flows that are defined by a small number of degrees of freedom and comprise three-fourths of the total flow enstrophy and two-thirds of the total flow energy. An important consequence is that with a quite small number of coefficients we are able to capture a large part of the dynamic content in these SLA maps.

FIG. 7. Coherent wavelet coefficients obtained applying (left) Donoho–Johnstone’s threshold and (right) the refined threshold. The wavelet was $p = 6$ Battle–Lemarié.

We hence expect that the ability of wavelet decomposition to efficiently compress the signal could serve to develop new approaches in numerical simulations of ocean circulation and to improve data assimilation techniques (Jameson et al. 2002). In addition, CVS seems to be the optimal way to detect and extract coherent structures, as an alternative to other techniques (Luo and Jameson 2002; Isern-Fontanet et al. 2003, 2006a; Lilly et al. 2003; Morrow et al. 2004). In particular, the clear advantage of CVS over similar techniques applied to the sea surface height fields (Luo and Jameson 2002) lies in the fact that CVS works on the vorticity field, which is more objective in the sense of being independent of the inertial frame and can be better compressed because it is more localized in space (Farge et al. 2001).

However, some aspects of CVS deserve improvement, and in this sense we must remark that there is a considerable loss of energy in oceanic coherent flows with respect to total energy, something that was not observed in the application of CVS to DNS. Our experiences show that the incoherent part is not Gaussianly distributed, and any refinement of the threshold used in the separation is not able to solve this issue and in fact does not lead to a real, physically significant decomposition. It seems that a part of the motion cannot be explained with this kind of decomposition and thus another decomposition scheme should be tried. In fact, if we could separate the coherent and incoherent parts as non-Gaussian and Gaussian parts, we will see that the non-Gaussian part is not only due to the effect of vortices, but also to jets and other mesoscale structures.

Hence, a good separation thresholding scheme should take into account this diversity in the structures responsible for the motion. We want to remark that Donoho–Johnstone’s criterion is a general thresholding, issued from general principles in signal processing but that does not take into account the physics of the flow. In Ruppert-Felsot et al. (2005), the authors propose a thresholding strategy in which the coefficients are selected in order to provide a low-entropy representation of the flow. This thresholding removes the necessity of performing an adaptive search, and with it the authors are also able to separate their vorticity fields (obtained from laboratory experiments of turbulent rotating flows) into incoherent Gaussian and coherent non-Gaussian parts. This leads us to believe that for oceanic flows a better criterion with a more direct physical interpretation should be implemented.

In addition, to start a specific research program on good separation schemes on oceanographic data, we cannot diminish the importance of adapting the wavelet bases to the specific particularities of oceanographic data. One of these particularities, in contrast to numerical simulations and laboratory experiments of turbulence, is the existence of irregular boundary conditions of important length. Such boundaries impose sharp adjustments on the flow, which must vanish beyond the coastline, and affect the quality of a method that relies on a fixed array of sampled coefficients. Algorithms with an adaptive grid, such as adaptive wavelet bases (Beta et al. 2003b) or second generation wavelets (Yuen et al. 2004), seem more appropriate to deal with the coastline, although the computational expense can

increased by a large amount if the boundaries are rather complex. A different approach would be to use free grids, as for instance the ones provided by *a trous* wavelet representations (Mallat 1999); however, so far there is no implementation of CVS in this framework.

Another aspect that seems desirable to improve in CVS is the determination in a less heuristic way of the optimal basis for the representation of oceanic flows or particular realizations. The results show important variability in performance when one or another basis is used, and we must be sure that results attain the best adaptation to data in order to infer general conclusions on the flow dynamics, for instance. For that, it is convenient to systematize the search and selection of the representation basis. Our results have shown that bases performing high-order interpolation seem to adapt better to oceanic data, probably due to boundary conditions once more; but we cannot be sure if a different basis could do even better. If a large enough database of oceanic data is available, some statistical search submitted to an optimization criterion (in a similar procedure to the one used to obtain empirical orthogonal modes; Preisendorfer 1988) should be possible. In that sense, some works on optimal coding of images have revealed that under certain conditions optimal wavelet bases exist and they can be learned from collections of images (Turiel and Parga 2000); an analogous application must be researched.

Once those technical issues (boundary conditions and optimal basis) are solved, we could go on discussing some implications on a more physical context. The results here produced evidence that vorticity fields in the western Mediterranean derived from sea level altimetric observations have a great contribution from the coherent part of the field. This suggests that eddies in this basin play a central role in the vorticity field, in agreement with recent results with the same dataset but using a partition flow technique based on the Okubo–Weiss parameter (Isern-Fontanet et al. 2006a,b). When the flow is separated into a contribution from the eddy field and another from the background, most of the non-Gaussian character of the PDF for the velocity field is due to the eddy field while the PDF for the background is mostly Gaussian but still contains a small non-Gaussian shape (Isern-Fontanet et al. 2006b). Although our analysis is limited by the amount of processed maps, the results using CVS point in the same direction. As commented on before, the refined use of the threshold implementing the separation of the flow into a Gaussian and a non-Gaussian did not converge. Visual assessment of our results and the nonnegligible non-Gaussian character of the incoherent part indicate that there is an intrinsic part of ocean motion that cannot be

associated with vortices, in contrast with what happens in 2D turbulence, but we cannot so far give a precise estimate on its relative importance. We should also mention that the procedures to elaborate the sea level anomaly maps may introduce long-range correlations that may explain the lack of convergence in separating the coherent from incoherent part of the flow. These may arise from small effects due to the temporal detrending and the aliasing effects due to the space–time coverage of altimeters.

As future work, it could be interesting to extend the CVS description to include other entities than vortices, such as jets, etc. It is also interesting to notice that the theoretical foundations of the CVS are still valid for the study of 3D turbulence (Farge et al. 2001), so this study could be extended for cases in which the quasigeostrophic approximation was no longer valid provided 3D data are available, which is not the case except for ocean numerical simulations.

Acknowledgments. This is a contribution to the IMAGEN (Spanish R+D Plan: Grant REN2001-0802-C02-02) and MERSEA projects (Grant EU AIP3-CT-2003-502885). A. Turiel is funded by a Ramon y Cajal fellowship from the Spanish Ministry of Education. The altimeter products were produced by Ssalto/Duacs and distributed by AVISO with support from CNES.

REFERENCES

- Armi, L., D. Hebert, and N. Oakey, 1989: Two years in the life of a Mediterranean salt lens. *J. Phys. Oceanogr.*, **19**, 354–370.
- AVISO, 1996: Corrected sea surface heights products. AVISO User Handbook, ed. 2.0, AVI-NT-011-311-CN, 201 pp.
- Ayoub, N., P. LeTraon, and P. D. Mey, 1998: A description of the Mediterranean surface variable circulation from combined ERS-1 and TOPEX/POSEIDON altimetric data. *J. Mar. Syst.*, **18**, 3–40.
- Babiano, A., C. Basdevant, B. Legras, and R. Sadourny, 1987: Vorticity and passive-scalar dynamics in two-dimensional turbulence. *J. Fluid Mech.*, **183**, 379–397.
- Benzi, R., S. Patarnello, and P. Santangelo, 1988: Self-similar coherent structures in two-dimensional decaying turbulence. *J. Phys. A: Math. Gen.*, **21**, 1221–1237.
- Beta, C., K. Schneider, and M. Farge, 2003a: Wavelet filtering to study mixing in 2D isotropic turbulence. *Commun. Nonlinear Sci. Numer. Simul.*, **8**, 537–545.
- , —, —, and H. Bockhorn, 2003b: Numerical study of mixing of passive and reactive scalars in two-dimensional turbulent flows using orthogonal wavelet filtering. *Chem. Eng. Sci.*, **58**, 1463–1477.
- Bracco, A., J. H. LaCasce, C. Pasquero, and A. Provenzale, 2000a: The velocity distribution of barotropic turbulence. *Phys. Fluids*, **12**, 2478–2488.
- , —, and A. Provenzale, 2000b: Velocity probability density functions for oceanic floats. *J. Phys. Oceanogr.*, **30**, 461–474.
- , J. von Hardenberg, A. Provenzale, J. B. Weiss, and J. C. McWilliams, 2004: Dispersion and mixing in quasigeostrophic

- turbulence. *Phys. Rev. Lett.*, **92**, 084501, doi:10.1103/PhysRevLett.92.084501.
- Chong, M., A. Perry, and B. Cantwell, 1990: A general classification of three-dimensional flow field. *Phys. Fluids A*, **2**, 765–777.
- Daubechies, I., 1992: *Ten Lectures on Wavelets*. CBMS-NSF Series in Applied Mathematics, Vol. 61, Capital City Press, 357 pp.
- Davis, A., A. Marshak, and W. Wiscombe, 1994: Wavelet-based multifractal analysis of non-stationary and/or intermittent geophysical signals. *Wavelet Transforms in Geophysics*, E. Foufoula-Georgiou and P. Kumar, Eds., Academic Press, 249–298.
- Davis, R., 1987: Modeling eddy transport of passive tracers. *J. Mar. Res.*, **45**, 635–666.
- Donoho, D., and I. Johnstone, 1994: Ideal spatial adaptation by wavelet shrinkage. *Biometrika*, **81**, 425–455.
- Elmhahidi, D., A. Provenzale, and A. Babiano, 1993: Elementary topology of two-dimensional turbulence from a Lagrangian viewpoint and single-particle dispersion. *J. Fluid Mech.*, **257**, 533–558.
- Farge, M., 1992: Wavelet transforms and their applications to turbulence. *Annu. Rev. Fluid Mech.*, **24**, 395–457.
- , and K. Schneider, 2001: Coherent Vortex Simulation (CVS): A semi-deterministic turbulence model. *Flow Turbul. Combust.*, **66**, 393–426.
- , —, and N. Kevlahan, 1999: Non-Gaussianity and coherent vortex simulation for two-dimensional turbulence using an adaptive orthogonal wavelet basis. *Phys. Fluids*, **11**, 2187–2201.
- , G. Pellegrino, and K. Schneider, 2001: Coherent vortex extraction in 3D turbulent flows using orthogonal wavelets. *Phys. Rev. Lett.*, **87**, 054501, doi:10.1103/PhysRevLett.87.054501.
- Font, J., J. Isern-Fontanet, and J. Salas, 2004: Tracking a big anticyclonic eddy in the Algerian basin. *Sci. Mar.*, **68**, 331–342.
- Gille, S., and S. G. Llewellyn Smith, 2000: Velocity probability density functions from altimetry. *J. Phys. Oceanogr.*, **30**, 125–136.
- Gupta, V., and E. Waymire, 1993: A statistical analysis of mesoscale rainfall as a random cascade. *J. Appl. Meteor.*, **32**, 251–267.
- Haller, G., 2005: An objective definition of a vortex. *J. Fluid Mech.*, **525**, 1–26.
- Holmes, P., J. L. Lumley, and G. Berkooz, 1996: *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*. Cambridge University Press, 420 pp.
- Hooker, S., J. Brown, and A. Kirwan, 1995: Kinematics of warm-core dipole ring. *J. Geophys. Res.*, **100**, 24 797–24 809.
- , R. Mied, J. Brown, and A. Kirwan, 1997: Remote sensing of dipole ring. *IEEE Trans. Geosci. Remote Sens.*, **35**, 1394–1399.
- Hua, B., and P. Klein, 1998: An exact criterion for the stirring properties of nearly two-dimensional turbulence. *Physica D*, **113**, 98–110.
- Isern-Fontanet, J., E. García-Ladona, and J. Font, 2003: Identification of marine eddies from altimetry. *J. Atmos. Oceanic Technol.*, **20**, 772–778.
- , —, and —, 2006a: Vortices of the Mediterranean Sea: An altimetric perspective. *J. Phys. Oceanogr.*, **36**, 87–103.
- , —, —, and A. García-Olivares, 2006b: Non-Gaussian velocity probability density functions: An altimeter perspective of the Mediterranean Sea. *J. Phys. Oceanogr.*, **36**, 2153–2164.
- , A. Turiel, E. García-Ladona, and J. Font, 2007: Microcanonical multifractal formalism: Application to the estimation of ocean surface velocities. *J. Geophys. Res.*, **112**, C05024, doi:10.1029/2006JC003878.
- Jameson, L., T. Waseda, and H. Mitsudera, 2002: Scale utilization and optimization from wavelet analysis for data assimilation: Sugo iwadai. *J. Atmos. Oceanic Technol.*, **19**, 747–758.
- Jeong, J., and F. Hussain, 1995: On the identification of a vortex. *J. Fluid Mech.*, **285**, 69–94.
- Jiménez, J., 1996: Algebraic probability density tails in decaying isotropic two-dimensional turbulence. *J. Fluid Mech.*, **313**, 223–240.
- Kamenkovich, I., A. Koshlyakov, and A. Monin, Eds., 1986: *Synoptic Eddies in the Ocean*. D. Reidel Publishing, 433 pp.
- LaCasce, J., 2005: On the Eulerian and Lagrangian velocity distributions in the North Atlantic. *J. Phys. Oceanogr.*, **35**, 2327–2336.
- Larnicol, G., P. LeTraon, N. Ayoub, and P. D. Mey, 1995: Mean sea level and surface circulation variability of the Mediterranean Sea from 2 years of TOPEX/POSEIDON altimetry. *J. Geophys. Res.*, **100** (C12), 385–396.
- , N. Ayoub, and P. L. Traon, 2002: Major changes in Mediterranean Sea level variability from 7 years of TOPEX/Poseidon and ERS-1/2 data. *J. Mar. Syst.*, **33**, 63–89.
- LeTraon, P., and F. Ogor, 1998: ERS-1/2 orbit improvement using TOPEX/POSEIDON: The 2 cm challenge. *J. Geophys. Res.*, **103** (C4), 8045–8057.
- , F. Nadal, and N. Ducet, 1998: An improved mapping method of multisatellite altimeter data. *J. Atmos. Oceanic Technol.*, **15**, 522–534.
- Lilly, J., P. Rhines, F. Schott, K. Lavender, J. Lazier, U. Send, and E. d'Asaro, 2003: Observations of the Labrador Sea eddy field. *Prog. Oceanogr.*, **59**, 75–176.
- Llewellyn Smith, S. G., and S. Gille, 1998: Probability density functions of large-scale turbulence in the ocean. *Phys. Rev. Lett.*, **81**, 5249–5252.
- Luo, J., and L. Jameson, 2002: A wavelet-based technique for identifying, labeling, and tracking of ocean eddies. *J. Atmos. Oceanic Technol.*, **19**, 381–390.
- Mallat, S., 1989: A theory for multiresolution signal decomposition: The wavelet representation. *IEEE Trans. Pattern Anal. Mach. Intell.*, **11**, 674–693.
- , 1999: *A Wavelet Tour of Signal Processing*. 2d ed. Academic Press, 637 pp.
- , and W. L. Huang, 1992: Singularity detection and processing with wavelets. *IEEE Trans. Inf. Theory*, **38**, 617–643.
- , and S. Zhong, 1992: Characterization of signals from multiscale edges. *IEEE Trans. Pattern Anal. Mach. Intell.*, **14**, 710–732.
- McWilliams, J., 1984: The emergence of isolated coherent vortices in turbulent flow. *J. Fluid Mech.*, **146**, 21–43.
- , 1990: The vortices of two-dimensional turbulence. *J. Fluid Mech.*, **219**, 361–385.
- Millot, C., 1999: Circulation in the western Mediterranean Sea. *J. Mar. Syst.*, **20**, 423–442.
- Morrow, R., F. Birol, D. Griffin, and J. Sudre, 2004: Divergent pathways of cyclonic and anti-cyclonic ocean eddies. *Geophys. Res. Lett.*, **31**, L24311, doi:10.1029/2004GL020974.
- Muzy, J. F., E. Bacry, and A. Arneodo, 1991: Wavelets and multifractal formalism for singular signals: Application to turbulence data. *Phys. Rev. Lett.*, **67**, 3515–3518.

- Nihoul, J., and B. Jamart, Eds., 1989: *Mesoscale/Synoptic Coherent Structures in Geophysical Turbulence*. Elsevier Oceanography Series, Vol. 50, Elsevier, 841 pp.
- Okubo, A., 1970: Horizontal dispersion of floatable particles in the vicinity of velocity singularities such as convergences. *Deep-Sea Res.*, **17**, 445–454.
- Olson, D., 1991: Rings in the ocean. *Annu. Rev. Earth Planet. Sci.*, **19**, 283–311.
- Pascual, A., B. Nardelli, G. Larnicol, M. Emelianov, and D. Gomis, 2002: A case of an intense anticyclonic eddy in the Balearic Sea (western Mediterranean). *J. Geophys. Res.*, **107**, 3183, doi:10.1029/2001JC000913.
- , Y. Faugere, G. Larnicol, and P.-Y. LeTraon, 2006: Improved description of the ocean mesoscale variability by combining four satellite altimeters. *Geophys. Res. Lett.*, **33**, L02611, doi:10.1029/2005GL024633.
- Pasquero, C., A. Provenzale, and A. Babiano, 2001: Parametrization of dispersion in two-dimensional turbulence. *J. Fluid Mech.*, **439**, 279–303.
- Preisendorfer, R. W., 1988: *Principal Component Analysis in Meteorology and Oceanography*. Elsevier, 425 pp.
- Provenzale, A., 1999: Transport by coherent barotropic vortices. *Annu. Rev. Fluid Mech.*, **31**, 55–93.
- Puillat, I., I. Taupier-Letage, and C. Millot, 2002: Algerian eddies lifetime can near 3 years. *J. Mar. Syst.*, **31**, 245–259.
- Rhines, P., 1979: Geostrophic turbulence. *Annu. Rev. Fluid Mech.*, **11**, 401–441.
- Robinson, A., Ed., 1983: *Eddies in Marine Science*. Springer-Verlag, 609 pp.
- Roux, S. G., A. Arneodo, and N. Decoster, 2000: A wavelet-based method for multifractal image analysis. III. Applications to high-resolution satellite images of cloud structure. *Eur. Phys. J. B*, **15**, 765–786.
- Ruppert-Felsot, J. E., O. Praud, E. Sharon, and H. L. Swinney, 2005: Extraction of coherent structures in a rotating turbulent flow experiment. *Phys. Rev. E*, **72**, 016311, doi:10.1103/PhysRevE.72.016311.
- Schorghofer, N., and S. Gille, 2002: Statistics of velocity gradients in two-dimensional Navier–Stokes and ocean turbulence. *Phys. Rev. E*, **65**, 026307, doi:10.1103/PhysRevE.65.026307.
- Siegel, A., and J. B. Weiss, 1997: A wavelet-packet census algorithm for calculating vortex statistics. *Phys. Fluids*, **9**, 1988–1999.
- Sweldens, W., 1996: The lifting scheme: A custom-design construction of biorthogonal wavelets. *Appl. Comput. Harmonic Anal.*, **3**, 186–200.
- Turiel, A., and N. Parga, 2000: Multifractal wavelet filter of natural images. *Phys. Rev. Lett.*, **85**, 3325–3328.
- , J. Grazzini, and H. Yahia, 2005a: Multiscale techniques for the detection of precipitation using thermal IR satellite images. *IEEE Trans. Geosci. Remote Sens. Lett.*, **2**, 447–450.
- , J. Isern-Fontanet, E. García-Ladona, and J. Font, 2005b: Multifractal method for the instantaneous evaluation of the stream function in geophysical flows. *Phys. Rev. Lett.*, **95**, 104502, doi:10.1103/PhysRevLett.95.104502.
- Weiss, J., 1991: The dynamics of enstrophy transfer in two-dimensional hydrodynamics. *Physica D*, **48**, 273–294.
- , and J. McWilliams, 1993: Temporal scaling behavior of decaying two-dimensional turbulence. *Phys. Fluids*, **5**, 608–621.
- Yuen, D., G. Erlebacher, O. V. Vasilyev, D. Goldstein, and M. Fuentes, 2004: Role of wavelets in the physical and statistical modelling of complex geological processes. *Pure Appl. Geophys.*, **161**, 2231–2244.