Strength evolution of geomaterials in the octahedral plane under non-isothermal and unsaturated conditions

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ABSTRACT

Current geomechanical applications imply non-isothermal processes of unsaturated geomaterials, in most cases following stress paths different than the classical triaxial compression often used in laboratory testing. Though the effects of temperature, suction and stress path direction (Lode's angle) on the strength of geomaterials have been investigated independently, the integrated analysis combining the three effects has not been performed yet. In this paper, we formulate a thermo-plastic constitutive model for unsaturated conditions that accounts for the Lode's angle on the strength of geomaterials. The yield surface evolves shrinking for increasing temperature, expanding for increasing suction and has its maximum strength for triaxial compression and the minimum for triaxial extension. We highlight the importance of accounting for temperature, suction and Lode's angle on the evolution of the strength through examples that can be related to geo-energy applications. Numerical results show that not considering these effects may give rise to misleading predictions of the strength of geomaterials.

Keywords: Lode's angle, plane strain, thermo-plasticity, constitutive modeling, thermo-hydro-mechanical couplings

INTRODUCTION

The strength of geomaterials is known to be dependent on the stress path direction in the octahedral plane, i.e., on the Lode's angle (Potts and Gens, 1984). The Lode's angle can be accounted for in existing constitutive models using, for example, the van Eekelen formulation (van Eekelen, 1980). However, deriving the parameters that describe the strength of geomaterials as a function of Lode's angle is difficult because the strength of soils is usually derived from conventional triaxial tests at the laboratory scale. Despite their appellative, most of the times conventional triaxial tests are to be intended as biaxial, since only two components are controlled simultaneously (vertical stress and the horizontal stress, equal in all directions and applied through confinement). To account for plane strain conditions, 'true triaxial' tests, in which the three principal stresses, σ'_1 , σ'_2 and σ'_3 , are controlled simultaneously, are required (e.g. Makhnenko and Labuz, 2015). Since the actual strength is lower for all stress paths different than a compressive triaxial stress path ($\sigma'_2 = \sigma'_3$) (Lee, 1970; Peric et al., 1992; Alshibli et al., 2003; Wanatowski and Chu, 2007; Makhnenko and Labuz, 2014), strength is usually overestimated in many geo-engineering problems that involve stress paths different than triaxial compression (Potts and Gens, 1984), such as landslides, shallow foundations and tunnel excavations.

In recent years, new geomechanical applications, including high-level nuclear waste disposal, energy piles and geologic carbon storage, which imply temperature and suction variations, have arisen because of the growing interest in geo-energies. This adds further complexity to the strength evolution of geomaterials because, apart from the effect of Lode's angle, the strength also evolves with temperature and suction changes. On the one hand, the yield limit is enhanced as suction increases (Gens et al., 2006; Sheng, 2011) and on the other hand, the yield surface shrinks for increasing temperatures (Hueckel and Baldi, 1990; Hueckel and

Borsetto, 1990; Laloui and Cekerevac, 2003). Though some efforts have been devoted to determining the strength of soils for stress paths different than triaxial compression (e.g. Nanda and Patra, 2015), the integrated analysis of the combined effect of temperature, suction and stress path direction on the strength of geomaterials has not been performed (e.g. Xie and Shao, 2012; Zhang et al., 2012).

This lack of integrated analysis of the strength of non-isothermal unsaturated geomaterials represents a big limitation to accurately predict the behavior of geomaterials in the current applications that are of interest. The objective of this paper is to provide a qualitative understanding of the effects of temperature, suction and stress path direction on the strength of geomaterials. To do so, a non-isothermal constitutive model for unsaturated geomaterials that accounts for the stress path direction is first presented. Next, we determine the evolution of the strength as a function of temperature, suction and stress path direction considering several examples that can be related to geo-energy applications. Finally, we discuss the implications of this study and draw some conclusions.

MODEL FORMULATION

We describe a sophisticated constitutive model that captures the main characteristics of the non-isothermal behavior of unsaturated geomaterials. This constitutive model is based on the work of Hujeux (1979) and its extensions to unsaturated non-isothermal conditions (Laloui and François, 2009; François and Laloui, 2008; Di Donna and Laloui, 2015). It, not only integrates non-linear thermoelasto-plasticity with a water retention curve model that includes hysteresis between the drying and wetting paths, but also incorporates the Lode's angle to account for a stress path dependent strength in the octahedral plane.

This constitutive model for unsaturated geomaterials under non-isothermal conditions is formulated in terms of the generalized effective stress approach (Nuth and Laloui, 2008a; Kim et al., 2013)

$$\boldsymbol{\sigma}' = (\boldsymbol{\sigma} - p_g \mathbf{I}) + S_l (p_g - p_l) \mathbf{I} = \boldsymbol{\sigma} - \alpha_b (S_g p_g + S_l p_l) \mathbf{I}, \quad (1)$$

where $\boldsymbol{\sigma}'$ is the effective stress tensor, $\boldsymbol{\sigma}$ is the total stress tensor, $s = p_g - p_l$ is suction, p_l is liquid pressure, p_g is gas pressure, $\mathbf{I}$ is the identity matrix, α_b is Biot coefficient, S_l is the liquid saturation degree and S_g is the gas saturation degree. The degrees of saturation of the two phases present in the pore volume satisfy

$$S_g + S_l = 1. \quad (2)$$

Changes in the effective stress will induce deformation of the geomaterial. Deformation can be either elastic or plastic, where the classical decomposition of the incremental total strain holds valid

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^e + d\boldsymbol{\varepsilon}^p, \quad (3)$$

where $d\boldsymbol{\varepsilon}$ is the total strain increment tensor, $d\boldsymbol{\varepsilon}^e$ is the non-linear thermo-elastic strain increment tensor and $d\boldsymbol{\varepsilon}^p$ is the thermo-plastic strain increment tensor. Elastic strain will occur when the stress state falls inside the yield surface. Additionally, plastic strain will occur when the stress state lays on the yield surface. We adopt the sign convention of geomechanics, i.e., stress and strain are positive in compression and negative in extension.

The rate of elastic strain is given by

$$d\boldsymbol{\varepsilon}^e = \mathbf{E}^{-1} d\boldsymbol{\sigma}' - \boldsymbol{\beta}_T dT, \quad (4)$$

where $\mathbf{E}$ is the mechanical stiffness tensor, β_T is the thermal expansion coefficient tensor and T is temperature. Thus, elastic strain can be due to changes in the total stress and/or fluid pressure (first term of Eq. (4)) and in temperature (second term of Eq. (4)).

A non-linear elastic theory is employed and the elastic moduli that compose the mechanical stiffness tensor $\mathbf{E}$ (Appendix A) depend on the effective stress state as

$$K = K_{ref} \left(\frac{p'}{p'_{ref}} \right)^{n_e}, \quad (5)$$

$$G = G_{ref} \left(\frac{p'}{p'_{ref}} \right)^{n_e}, \quad (6)$$

where K and G are the bulk and shear moduli, respectively, K_{ref} and G_{ref} are the reference bulk and shear moduli, respectively, measured at the reference mean effective stress p'_{ref} , p' is the mean effective stress and n_e is a material parameter controlling the non-linearity of the elastic law.

The thermal expansion coefficient tensor β_T depends on both temperature and effective stress state, which, assuming an isotropic thermal expansion, reads

$$\beta_T = \frac{1}{3} \beta'_s \mathbf{I}, \quad (7)$$

where the volumetric thermal expansion coefficient of the solid skeleton β'_s is given by (Laloui, 1993)

$$\beta'_s = \beta'_{s_0} \left(1 - \frac{T - T_0}{100} \right) \frac{p'_{cr0}}{p'}, \quad (8)$$

132 where β'_{s_0} is the reference thermal expansion coefficient at a reference temperature T_0 and

133 p'_{c_0} is the initial critical state pressure at reference temperature T_0 .

134 If the effective stress state reaches the yield surface, plastic strain occurs. This irreversible
135 strain is induced by two coupled dissipative processes

$$136 \quad d\boldsymbol{\varepsilon}^p = d\boldsymbol{\varepsilon}^{p,iso} + d\boldsymbol{\varepsilon}^{p,dev}, \quad (9)$$

137 where $d\boldsymbol{\varepsilon}^{p,iso}$ and $d\boldsymbol{\varepsilon}^{p,dev}$ are the components of the plastic strain tensor deriving from the
138 isotropic and deviatoric plastic mechanisms, respectively. Each of these mechanisms are
139 activated when their respective yield surface is reached, its magnitude is proportional to a
140 plastic multiplier and its direction is given by the flow rule as

$$141 \quad d\boldsymbol{\varepsilon}^{p,iso} = \lambda_{iso}^p \frac{\partial g_{iso}}{\partial \boldsymbol{\sigma}'}, \quad (10)$$

$$142 \quad d\boldsymbol{\varepsilon}^{p,dev} = \lambda_{dev}^p \frac{\partial g_{dev}}{\partial \boldsymbol{\sigma}'}, \quad (11)$$

143 where λ_{iso}^p and λ_{dev}^p are the plastic multipliers of the isotropic and deviatoric mechanisms,
144 respectively, and g_{iso} and g_{dev} are the plastic potential functions of the isotropic and
145 deviatoric mechanisms, respectively.

146 The yield surface of each mechanism is given by

$$147 \quad f_{iso} = p' - p'_c r_{iso} = 0, \quad (12)$$

$$148 \quad f_{dev} = q - Mp' \left(1 - b \ln \left(\frac{p'd}{p'_c} \right) \right) r_{dev} = 0, \quad (13)$$

149 where f_{iso} and f_{dev} are the yield surface of the isotropic and deviatoric mechanisms,
150 respectively, r_{iso} and r_{dev} are the degrees of plastification of the isotropic and deviatoric

mechanisms, respectively, p'_c is the preconsolidation pressure, q is the deviatoric stress, M is the slope of the critical state line in the $(q - p')$ plane, b is a material parameter and d is

$$d = \frac{p'_{c_0}}{p'_{cr_0}}, \quad (14)$$

where p'_{c_0} is the preconsolidation pressure at the reference temperature T_0 and p'_{cr_0} is the initial critical pressure at the reference temperature T_0 .

The mean effective stress and the deviatoric stress read

$$p' = \frac{1}{3}(\sigma'_{11} + \sigma'_{22} + \sigma'_{33}), \quad (15)$$

$$q = \sqrt{3J_{2D}} = \sqrt{\sigma'^2_{11} + \sigma'^2_{22} + \sigma'^2_{33} - \sigma'_{11}\sigma'_{22} - \sigma'_{22}\sigma'_{33} - \sigma'_{33}\sigma'_{11} + 3(\sigma'^2_{12} + \sigma'^2_{23} + \sigma'^2_{13})}, \quad (16)$$

respectively, where $J_{2D} = 1/2 \text{tr}(\mathbf{s}:\mathbf{s})$ is the second invariant of the deviatoric effective stress tensor $\mathbf{s} = \boldsymbol{\sigma}' - p'\mathbf{I}$ and σ'_{ij} is the ij th component of the effective stress tensor, where $i, j = 1, 2, 3$.

The preconsolidation pressure couples the two plastic mechanisms because it appears in both yield surfaces (recall Equations (12) and (13)). The preconsolidation pressure, which is the main hardening parameter, depends on the volumetric plastic strain, ε_v^p , temperature and suction as (Laloui and Cekerevac, 2003)

$$p'_c = \begin{cases} p'_{c_0} \exp(\beta \varepsilon_v^p) [1 - \gamma_T \log(T/T_0)], & \text{if } s < s_{e_H} \\ p'_{c_0} \exp(\beta \varepsilon_v^p) [1 - \gamma_T \log(T/T_0)] [1 + \gamma_s \log(s/s_{e_H})], & \text{if } s \geq s_{e_H} \end{cases}, \quad (17)$$

where β is the plastic compressibility modulus, i.e., the slope of the linear relationship $\varepsilon_v^p - \ln p'_c$, γ_T and γ_s are two material parameters for the thermal and suction evolution,

respectively, s is suction and s_{e_H} is the entry pressure of the non-wetting phase, i.e., the value of suction at which desaturation starts. The plastic compressibility modulus β is a function of suction as $\beta = \beta_0 + \omega(s - s_{e_H})$, if $s > s_{e_H}$, where β_0 is the value of β at suction equal or lower than the entry value and ω is a material parameter. Thus, the yield surfaces expressed in Eq. (12) and (13) should be expressed as a function of the variables $q, p', \varepsilon_v^p, T$ and s (Figure 1). The degrees of plastification allow a progressive evolution of the yield surfaces during loading as implied in the bounding surface plasticity theory. Their expressions are

$$r_{iso} = r_{iso}^e + \frac{\varepsilon_v^{p,iso}}{c + \varepsilon_v^{p,iso}}, \quad (18)$$

$$r_{dev} = r_{dev}^e + \frac{\varepsilon_d^p}{a + \varepsilon_d^p}, \quad (19)$$

where a and c are material parameters, r_{iso}^e and r_{dev}^e define the size of the elastic nuclei of the isotropic and deviatoric mechanisms, respectively, $\varepsilon_v^{p,iso}$ is the isotropic part of the volumetric plastic strain and ε_d^p is the deviatoric plastic strain. Previous to reaching the final yield surface, the model produces plastic hardening, while the softening behavior can be observed once the state of stress reaches the final yield surface on the “dry” side and the volumetric plastic strain becomes negative.

The deviatoric yield surface was formulated in Equation (13) as depending on two invariants of the effective stress tensor, q and p' . This implies assuming that the projection of the yield surface on the octahedral plane is circular. Such a circular yield surface in the octahedral plane involves that yield conditions are reached at the same deviatoric stress regardless of the direction of the stress path (Lode’s angle). However, plasticity is stress path dependent both for granular (e.g. Lade and Duncan, 1973) and fine-grained soils (e.g. Lade and Musante,

1978) as well as for rocks (e.g. Colmenares and Zoback, 2002; Makhnenko et al., 2015). To overcome such limitations, the yield surface can be formulated in terms of three invariants of stress. The new invariant is the circular coordinate in the deviatoric plane and it is known as Lode's angle, ϑ , and it varies from -30° to 30° . The Lode's angle is defined as

$$\sin(3\vartheta) = -\frac{3\sqrt{3}}{2} \frac{J_{3D}}{J_{2D}^{3/2}}, \quad (20)$$

where $J_{3D} = 1/3 \text{tr}(\mathbf{s}^3) = 1/3 \left(\sum_{i=1,2,3} (\sigma'_{ii} - p')^3 + (\sigma'_{11} - p')(\sigma'^2_{12} + \sigma'^2_{13} - 2\sigma'^2_{23}) + (\sigma'_{22} - p')(\sigma'^2_{12} + \sigma'^2_{23} - 2\sigma'^2_{13}) + (\sigma'_{33} - p')(\sigma'^2_{13} + \sigma'^2_{23} - 2\sigma'^2_{12}) + 6\sigma'_{12}\sigma'_{13}\sigma'_{23} \right)$ is the third invariant of the deviatoric effective stress tensor.

According to the given definition, Lode's angle equals 30° if the stress path is triaxial in compression, -30° if the stress path is triaxial in extension and an intermediate value for plane strain and plane stress conditions. Though a circular yield surface works well for triaxial stress paths in compression, it generally overestimates the strength for other stress paths. The Mohr-Coulomb yield surface, which shape is an irregular hexagon, is more accurate than the circular surface, but the corners are difficult to handle numerically (Potts and Gens, 1984) if an associated flow rule is employed (the direction of the plastic strain is non defined in the corners and proper techniques must be employed to overcome this issue). Among the different models proposed to reproduce experimental results providing a smooth yield surface, we adopt the van Eekelen (1980) formulation to define the coefficient M due to its versatility and consistency

$$M = 3\sqrt{3}a_L (1 + b_L \sin(3\vartheta))^{n_L}, \quad (21)$$

where a_L , b_L and n_L are three material parameters defined as (Barnichon, 1998)

$$a_L = \frac{r_c}{(1+b_L)^{n_L}} \quad (22)$$

and

$$b_L = \frac{(r_c/r_e)^{1/n_L} - 1}{(r_c/r_e)^{1/n_L} + 1}, \quad (23)$$

where

$$r_c = \frac{1}{\sqrt{3}} \frac{2 \sin \phi'_c}{(3 - \sin \phi'_c)} \text{ and } r_e = \frac{1}{\sqrt{3}} \frac{2 \sin \phi'_e}{(3 + \sin \phi'_e)}, \quad (24)$$

where ϕ'_c and ϕ'_e are the friction angles in triaxial compression and extension, respectively.

These friction angles may vary with temperature and if they are equal, the van Eekelen

surface coincides with the inscribed Mohr-Coulomb yield surface at Lode's angle equal to

$\pm 30^\circ$, yielding a M coefficient of $M = 6 \sin \phi' / (3 - \sin \phi')$ for compression and

$M = 6 \sin \phi' / (3 + \sin \phi')$ for extension. If no experimental data of triaxial tests in extension is

available, a good approximation could be assuming the extension friction angle equal to the

one in compression because it generally leads to small errors (Di Donna, 2014). The

parameter a_L must be positive, $-1 < b_L < 1$ and $b_L n_L > 0$. As in the original work of (van

Eekelen, 1980) the following conditions ensure convexity of the curve in the octahedral plane

$$\begin{aligned} |b_L| &\leq \frac{1}{|9n_L + 1|} & \text{if } n_L \geq 0 \text{ or } n_L \leq -\frac{3}{11} \\ |b_L| &\leq \frac{1}{2} \left[\frac{4 + 13n_L}{(1 + n_L)(1 - 9n_L^2)} \right]^{1/2} & \text{if } -\frac{3}{11} \leq n_L \leq 0, \end{aligned} \quad (25)$$

The parameter n_L , in order to ensure the convexity of the surface, has its optimum value of -

0.229 (van Eekelen, 1980) (see Figure 2, where the M parameter is plotted in the octahedral

plane for a compression angle of 30° and several extension angles). With this value of n_L , the second condition implies that

$$|b_L| \leq \frac{1}{2} \left[\frac{4 + 13n_L}{(1 + n_L)(1 - 9n_L^2)} \right]^{1/2} = 0.7925, \quad (26)$$

therefore, imposing restrictions on the values that the friction angle in extension and in compression can take (recall Equation (23)). Furthermore, from Equation (26) and assuming that $n_L = -0.229$, it is found that the ratio between coefficients r_c and r_e must satisfy the following condition

$$r_c / r_e \leq 1.6385. \quad (27)$$

Figure 3 shows the limiting values of the friction angle in extension as a function of the value of the friction angle in compression. To ensure convexity, all possible combinations of the two parameters must lie above the limiting curve and below the bisection line of the plane, where $\phi'_c = \phi'_e$. For values of ϕ'_c above 46.55° the yield surface becomes non-convex and at this point, to ensure convexity, one must have $\phi'_c = \phi'_e$. Above such limit the friction angle in extension would need to be greater than the one in compression. The maximum strength difference between compression and extension is reached when ϕ'_c is between 20° and 30° .

Figures 4 and 5 illustrate the yield surface in the octahedral plane as a function of temperature and suction, respectively. The yield surface does not have a circular shape due to the strength dependency on Lode's angle. While the yield surface shrinks as temperature increases (Figure 4), it maintains a constant size while the suction is lower than the entry pressure of the non-wetting phase, but it expands for higher values of suction.

248 The flow rule of the isotropic mechanism is associated, i.e., the plastic potential equals the
 249 yield surface, while the deviatoric flow rule is non-associated. The plastic potentials have the
 250 following expressions

$$251 \quad g_{iso} = p' - p'_c r_{iso} = 0, \quad (28)$$

$$252 \quad g_{dev} = q - \frac{\alpha}{\alpha - 1} M p' \left[1 - \frac{1}{\alpha} \left(\frac{p' d}{p'_c} \right)^{\alpha - 1} \right] = 0, \quad (29)$$

253 where α is a non-associativity parameter. Nevertheless, the form of the plastic potentials is
 254 unimportant and what matters is their derivative with respect to stress because it determines
 255 the flow rules

$$256 \quad d\boldsymbol{\varepsilon}^{p,iso} = \lambda_{iso}^p \frac{\partial g_{iso}}{\partial \boldsymbol{\sigma}'} = \frac{\lambda_{iso}^p}{3} \mathbf{I}, \quad (30)$$

$$257 \quad d\boldsymbol{\varepsilon}^{p,dev} = \lambda_{dev}^p \frac{\partial g_{dev}}{\partial \boldsymbol{\sigma}'} = \lambda_{dev}^p \left[\frac{\partial q}{\partial \boldsymbol{\sigma}'} + \frac{\alpha}{3} \left(M - \frac{q}{p'} \right) \mathbf{I} - \frac{\partial M}{\partial \boldsymbol{\sigma}'} \frac{\alpha}{\alpha - 1} p' \left[1 - \frac{1}{\alpha} \left(\frac{p' d}{p'_c} \right)^{\alpha - 1} \right] \right]. \quad (31)$$

258 The plastic multipliers λ_{iso}^p and λ_{dev}^p are determined from the Prager's consistency condition,
 259 which reads

$$260 \quad \begin{cases} d\mathbf{F} = \frac{\partial \mathbf{F}}{\partial \boldsymbol{\sigma}'} : d\boldsymbol{\sigma}' + \frac{\partial \mathbf{F}}{\partial \boldsymbol{\pi}} \cdot \frac{\partial \boldsymbol{\pi}}{\partial \boldsymbol{\lambda}^p} \cdot \boldsymbol{\lambda}^p \leq 0, \\ d\mathbf{F}^t \cdot \boldsymbol{\lambda}^p \geq 0, \quad \boldsymbol{\lambda}^p \geq 0 \end{cases}, \quad (32)$$

261 where $d\mathbf{F}$ is the differential yield surface vector, $\mathbf{F}$ is the yield surfaces vector, $\boldsymbol{\pi}$ is the
 262 internal variable vector and $\boldsymbol{\lambda}^p$ is the plastic multiplier vector. The internal variables of the
 263 isotropic and deviatoric mechanisms are the preconsolidation pressure p'_c through the
 264 volumetric plastic strain ε_v^p and the degree of plastification of each mechanism, r_{iso} and r_{dev} ,
 265 respectively. All the derivatives, as well as the plastic multipliers, are detailed in Appendix A.

Once the thermo-mechanical stress-strain model is defined, we define the water retention model. This water retention model includes a hysteretic retention curve (Figure 6) and a yield surface (Nuth and Laloui, 2008b).

Changes in suction will induce changes in the degree of saturation, which can be either elastic or plastic

$$dS_l = dS_l^e + dS_l^p, \quad (33)$$

where dS_l^e and dS_l^p are the elastic and plastic increments in liquid saturation degree, respectively. The elastic part is defined as

$$dS_l^e = \frac{ds}{K_H s / s_{e_H}}, \quad (34)$$

where K_H is the elastic modulus associated with suction strain and s_{e_H} is the updated entry pressure of the non-wetting phase. The entry pressure is a function of the volumetric strain and temperature as

$$s_{e_H} = s_e \left(1 - \theta_T \log \left(\frac{T}{T_0} \right) \right) (1 + c_H \varepsilon_v), \quad (35)$$

where s_e is the reference entry pressure of the non-wetting phase, ε_v is the volumetric strain and θ_T and c_H are material parameters describing the evolution of the entry pressure with respect to temperature and volumetric deformation, respectively.

The yield surface of the degree of saturation for both the drying and wetting paths is expressed as (Nuth and Laloui, 2008b)

$$f_H = \left\| \ln \left(\frac{s}{s_D} \right) + \frac{1}{2} \ln \left(\frac{s_{D_0}}{s_{e_H}} \right) \right\| - \frac{1}{2} \ln \left(\frac{s_{D_0}}{s_{e_H}} \right), \quad (36)$$

285 where s_D is the actual drying yield suction and s_{D_0} is the reference yield suction. s_D is
 286 analogous to the preconsolidation pressure of the mechanical stress-strain model. Thus, s_D
 287 evolves with suction and its initial value is $s_{D_0} \cdot 1/2 \ln(s_{D_0}/s_{e_H})$ is the radius of the yield
 288 surface, which is assumed to be constant, and controls the distance between the main drying
 289 and wetting curves. Since s_{e_H} evolves with the volumetric strain, the reference yield suction
 290 has to be updated to keep the hysteresis of the retention curve constant

$$291 \quad s_{D_0} = s_{D_i} \frac{s_{e_H}}{s_e}, \quad (37)$$

292 where s_{D_i} is the initial reference yield suction. The actual drying yield suction is given by

$$293 \quad s_D = s_{D_0} \exp(\beta_H dS_l^p), \quad (38)$$

294 where β_H is the coefficient of compressibility for the plastic part of the degree of saturation.

295 If the yield surface is reached, plastic increment of degree of saturation occurs. Since the flow
 296 rule is associated, this increment reads

$$297 \quad dS_l^p = \lambda_H^p \frac{\partial f_H}{\partial s}, \quad (39)$$

298 where λ_H^p is the plastic multiplier for the water retention model. The plastic multiplier is
 299 derived from the consistency equation

$$300 \quad \begin{cases} df_H = \frac{\partial f_H}{\partial s} ds + \frac{\partial f_H}{\partial \pi_H} \cdot \frac{\partial \pi_H}{\partial \lambda_H^p} \cdot \lambda_H^p \leq 0 \\ df_H \cdot \lambda_H^p \geq 0, \quad \lambda_H^p \leq 0 \end{cases}, \quad (40)$$

301 where π_H are the internal variables for the water retention model, i.e., dS_l^p . The derivation of
 302 the plastic multiplier is detailed in Appendix B.

If the soil is saturated, i.e., $s \leq s_{e_H}$, or the degree of saturation equals its residual value, i.e., $S_l = S_{l_{res}}$, then the degree of saturation remains constant and equals 1 or $S_{l_{res}}$, respectively.

EVOLUTION OF THE STRENGTH WITH TEMPERATURE, SUCTION AND STRESS PATH DIRECTION

We present some examples of the evolution of the strength with temperature, suction and stress path direction (recall Figures 4 and 5). In the first set of examples, which is based on the CO₂ storage site at In Salah, Algeria, we consider a clay rich shale whose properties are shown in Table 1. From a purely mechanical point of view, shales are usually approached with combined plastic-damage models (as in e.g. Chen et al., 2010; Shojaei et al., 2014; Parisio et al., 2015), although the theory of plasticity itself has been proven effective as well in modelling their complex deformation behavior (Salager et al., 2013). In the present work focus is addressed toward the purely plastic models that could be representative of a range of geomaterials that not necessarily include damageable behavior. Further extensions in the framework of Continuum Damage Mechanics are possible and foreseeable when focus is placed toward quasi-brittle materials like shale or other sedimentary rocks.

The initial stress state, pore water pressure and temperature are assumed to be those at In Salah, Algeria, i.e., $\sigma'_v = 26.5$ MPa, $\sigma'_H = 31.9$ MPa, $\sigma'_h = 12.7$ MPa, $p_l = 18.0$ MPa and $T = 95$ °C (Morris et al., 2011). We consider two cases, one in which an increase in pore water pressure of up to 12 MPa at constant temperature is applied and another in which, simultaneously, an increase in pore water pressure of up to 12 MPa and a temperature decrease of up to 45 °C are applied. For this particular example, saturated conditions are

maintained during the whole analysis, so temperature effects on shale strength are emphasized. In both cases, we analyze the effect of considering or not Lode's angle.

Figure 7 clearly illustrates the difference in the size of the yield surface when considering or not Lode's angle. Failure conditions are reached for an increase in pore water pressure of 2.3 MPa if the effect of Lode's angle on the strength is taken into account. However, not considering Lode's angle leads to safe geomechanical conditions even for an increase in pore water pressure of 12 MPa. This significant difference in the increase in pore water pressure necessary to reach failure conditions highlights the importance of using formulations that take into account Lode's angle, especially when the stress path is far from triaxial compression (see Figure 8). Thus, polyaxial triaxial tests at the laboratory scale including temperature and suction changes should also be employed for a full characterization of the geomechanical behavior of reservoir rocks, given the implications for safety analyses.

Figure 9 shows the effect of cooling on the strength of the considered geomaterial. The strength increases as temperature drops and thus, a higher increase in pore water pressure can be induced before reaching the yield surface. For the case in which Lode's angle is taken into account (Figure 9b), the yield surface is reached for a pore water pressure increase of 4.9 MPa, instead of the 2.3 MPa in the isothermal case. When not considering Lode's angle effect on the strength, the increase in pore water pressure that could be reached is even higher, which may lead to the false impression that injection conditions are safe. These examples could be representative of the processes that occur in geologic carbon storage. The caprock, which is a low permeability formation with high entry pressure will experience a pressure buildup that will progressively increase during the duration of the CO₂ injection (Rutqvist et al., 2010), which will last several decades. This pressure buildup will be due to water flow, but not CO₂ flow, because if the caprock integrity is maintained, CO₂ will not be able to

penetrate into the caprock due to capillarity (Vilarrasa and Carrera, 2015). Furthermore, CO₂ reaches the storage formation at a lower temperature than that corresponding to the geothermal gradient, which will eventually cool down the caprock (Paterson et al., 2008).

Since cooling increases the size of the yield surface, the reduction in the strength caused by the stress path direction is partially compensated (Figure 9). Thus, in the vicinity of the injection well, where cooling will occur, a larger increase in pore water pressure may be sustained without undergoing yielding. However, cooling will induce a thermal stress reduction that should be considered in rock stability analysis (Vilarrasa et al., 2015a). Farther away, outside of the cooled region, the increase in pore water pressure induced by injection will be smaller and thus, failure conditions will be less likely to occur even though the strength will not be increased by cooling.

In the second set of examples, we consider an expansive soil with the properties shown in Table 2. We consider an initial stress state with $\sigma'_v = 7.0$ MPa, $\sigma'_H = 1.0$ MPa, $\sigma'_h = 0.5$ MPa. We assume that the initial temperature equals 20 °C and that the initial suction of the soil is 3 MPa, which equals the initial entry pressure and thus, the initial water saturation degree is 1.0. We consider the effect of considering or not Lode's angle in two situations. One in which the suction remains constant while temperature progressively increases (temperature increment of 100 °C) and another in which both suction and temperature increase (suction and temperature increment of 3 MPa and 100 °C, respectively). The problem is solved in plane strain conditions at the material point level, with the out of plane direction coinciding with the maximum horizontal stress. The initial stress state is close to triaxial compression (Lode's angle of 26°). However, the symmetry of the problem causes an increase of the intermediate stress due to induced thermal and suction stresses that progressively brings the stress state far from triaxial compression.

Figures 10 and 11 plot the stress path in the temperature – mean effective stress plane and suction – mean effective stress plane, respectively, for the two cases, i.e., temperature increment and simultaneous increment in temperature and suction. The mean effective stress slightly increases due to the thermal stress induced by heating (Figures 10a and 11a). When suction increases along with temperature (Figures 10b and 11b), an additional increase in the mean effective stress takes place (recall Equation (1)). In both cases, the stresses induced by heating and drying become larger in the out of plane direction than in the other two principal directions because of the symmetry of the problem, which causes a reduction of Lode's angle. The Lode's angle becomes -5° and 11° when only heating occurs and when both temperature and suction increase, respectively. The larger reduction in Lode's angle when only heating occurs is due to the fact that the maximum and minimum principal stresses remain constant, while the intermediate increases due to the induced thermal stress. By contrast, all stresses become higher when suction also increases (recall Equation (1)), so the relative difference between the principal stresses remains smaller in this case.

Figure 10a shows that, when there are no suction changes, the yield surface is first reached around 82°C when considering Lode's angle, but around 92°C when not considering Lode's angle. Thus, not considering Lode's angle may result in an overestimation of the maximum reachable temperature before failure occurs. Note that the yield surface shrinks as temperature increases (recall Figures 1 and 4). This reduction in the size of the yield surface is the reason why the expansive soil eventually yields. The deviatoric yield surface is first reached (points *A* and *B*). Furthermore, the isotropic yield surface is also reached around 110°C when considering Lode's angle (point *B'*). For temperatures higher than the yield temperature, the yield surface moves following the stress path with hardening while plastic strain accumulates. Figures 10a and 10b show the isotropic yield surface when the deviatoric yield surface is first reached (point *B* at 82°C and point *A* at 92°C), the isotropic yield surface when it is first

reached when considering Lode's angle (point B' at 109 °C) and the final isotropic yield surface (points A' and B'' at 120 °C).

Figure 10b illustrates the isotropic yield surface when suction increases together with temperature in the temperature – mean effective stress plane at the onset of plasticity, which occurs around 101 °C when considering Lode's angle (point D) and around 109 °C when not considering Lode's angle (point C), and the final isotropic yield surfaces (points C' and D'). The initial yield surface (at point I) coincides with the one shown in Figure 10a for the initial yield. However, suction strengthens the geomaterial, increasing the size of the yield surface (recall Figures 1 and 5). Thus, the onset of failure occurs at a higher mean effective stress. Nevertheless, since the mean effective stress increases with suction, if the suction increment is very pronounced, plasticity could start at a lower temperature than when suction remains constant because the resulting stress path points more directly towards the yield surface.

Figure 12 represents the volumetric strain as a function of temperature for the two analyzed examples. The expansive soil dilates as it is heated. For the case where suction remains constant, this dilation is elastic until the onset of plasticity (points A and B). Between A and A' and B and B'' , in addition to the elastic expansion, there is a thermal consolidation that causes an irreversible volume decrease of the soil, which reduces the rate of expansion and even causes compaction (between B' and B''). When both suction and temperature increase, the expansion of the expansive soil is smaller because the soil shrinks as suction increases, i.e., water saturation degree decreases. Furthermore, strain evolution is not linear because the soil becomes stiffer as suction increases (recall Equations (5) and (6)). In both cases, considering Lode's angle leads to larger plastic strain than when not considering Lode's angle, which again highlights the importance of accounting for the combined effect of temperature, suction and Lode's angle on the strength of geomaterials.

Another interesting process is that when the soil is heated at constant suction, the degree of saturation slightly decreases. This decrease is due to the coupling between the thermo-mechanical stress-strain model and the water retention model, in which the entry pressure is a function of temperature and soil deformation (recall Equation (35)). A temperature increase and a dilation of the soil cause a decrease of the entry pressure of the non-wetting phase. Since the initial suction of the soil coincides with the initial entry pressure, the heating of the soil at constant suction leads to a decrease in the entry pressure that causes the desaturation of the soil.

The second set of examples (Figures 10-12) could be related to the processes that will occur in nuclear waste disposal. The high-level nuclear waste will release heat over hundreds of thousands of years as a result of the continuing nuclear decay of the radioactive material that it contains. This heat will cause expansion of the bentonite buffer, which, if not fully allowed to increase its volume, will generate thermally induced stresses. Furthermore, the temperature rise will initially dry the bentonite surrounding the canisters, increasing suction (Sanchez et al., 2012; Vilarrasa et al., 2015b). In the long-term, groundwater will eventually saturate the whole backfill, but this saturation process may take decades or even longer due to the dual structure nature of expansive soils (Sanchez et al., 2012; Vilarrasa et al., 2015b) and to the low rate of water inflow coming from the low permeability host rock.

The absolute values of temperature and increase in pore water pressure that lead to failure conditions in the presented examples should be interpreted qualitatively because we are not modeling the whole 3D problem. However, the relative effect of temperature, suction and stress path direction on strength evolution is clearly highlighted by these examples. To properly model whether failure conditions may be reached or not in practical applications, a complete characterization of geomaterials, including the combined effect of temperature,

suction and stress path direction should be carried out. Such testing programs represent new challenges for the entire geomechanics community, as clearly one can understand the difficulties of performing laboratory investigations with proper combinations of the mentioned effects. However complicated and challenging, such testing programs constitute the starting point towards increasing safety margins and reducing uncertainties related with the most demanding geomechanical applications, such as, e.g., nuclear waste storage, geologic carbon storage, unconventional reservoir and deep geothermal exploitation.

CONCLUSIONS

A constitutive model for unsaturated geomaterials that includes non-isothermal effects and the effect of the stress path direction on the strength has been presented. The yield surface shrinks for increasing temperature, expands for increasing suction and has its maximum value for triaxial compression and its minimum value for triaxial extension. Temperature and suction changes and a stress path direction different than triaxial compression are usually found in most geo-energy and geo-engineering applications. All these effects affect the strength of geomaterials. We have shown through examples that can be related to geologic carbon storage and nuclear waste disposal that not considering the combined effect of temperature, suction and stress path direction may originate misleading results that are far from what will actually happen. We conclude that constitutive models for geomaterials that include the combined effect of temperature, suction and stress path direction on the yield surface evolution should be used to obtain realistic results and accurately describe coupled thermo-hydro-mechanical processes.

APPENDIX A: CONSISTENCY EQUATIONS OF THE THERMO-MECHANICAL PART OF THE MODEL

We detail in this Appendix how to obtain the plastic multipliers from the thermo-mechanical part of the model from the Prager's consistency condition

$$\begin{cases} \mathbf{dF} = \frac{\partial \mathbf{F}}{\partial \boldsymbol{\sigma}'} : \mathbf{d}\boldsymbol{\sigma}' + \frac{\partial \mathbf{F}}{\partial \pi} \cdot \frac{\partial \pi}{\partial \lambda^p} \cdot \lambda^p \leq 0 \\ \mathbf{dF}' \cdot \lambda^p \geq 0, \quad \lambda^p \geq 0 \end{cases} \quad (\text{A1})$$

Taking into account that the effective stress increment in response to a prescribed strain increment can be expressed as

$$\mathbf{d}\boldsymbol{\sigma}' = \mathbf{E} : (\mathbf{d}\boldsymbol{\varepsilon} + \boldsymbol{\beta}_T dT - \mathbf{G} \cdot \lambda^p), \quad (\text{A2})$$

where $\mathbf{G} = \partial \mathbf{g} / \partial \boldsymbol{\sigma}'$ is the flow direction matrix and $\mathbf{g}$ is the plastic potential vector, the consistency condition can be rewritten as

$$\begin{cases} \mathbf{dF} = \mathbf{J} : \mathbf{E} : \mathbf{d}\boldsymbol{\varepsilon} + \mathbf{J} : \mathbf{E} : \boldsymbol{\beta}_T dT - (\mathbf{H} + \mathbf{J} : \mathbf{E} : \mathbf{G}) \cdot \lambda^p \leq 0 \\ \mathbf{dF}' \cdot \lambda^p \geq 0, \quad \lambda^p \geq 0 \end{cases}, \quad (\text{A3})$$

where $\mathbf{J} = \partial \mathbf{F} / \partial \boldsymbol{\sigma}'$ and $\mathbf{H} = -\partial \mathbf{F} / \partial \pi \cdot \partial \pi / \partial \lambda^p$ is the hardening moduli matrix. The elastic stiffness matrix has the form

$$\mathbf{E} = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ & K + \frac{4}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ & & K + \frac{4}{3}G & 0 & 0 & 0 \\ & \text{Symmetric} & & 2G & 0 & 0 \\ & & & & 2G & 0 \\ & & & & & 2G \end{bmatrix}, \quad (\text{A4})$$

and the derivative of the yield surfaces vector with respect to the effective stress tensor is

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_{iso}}{\partial \sigma'_{11}} & \frac{\partial f_{iso}}{\partial \sigma'_{22}} & \frac{\partial f_{iso}}{\partial \sigma'_{33}} & \frac{\partial f_{iso}}{\partial \sigma'_{12}} & \frac{\partial f_{iso}}{\partial \sigma'_{13}} & \frac{\partial f_{iso}}{\partial \sigma'_{23}} \\ \frac{\partial f_{dev}}{\partial \sigma'_{11}} & \frac{\partial f_{dev}}{\partial \sigma'_{22}} & \frac{\partial f_{dev}}{\partial \sigma'_{33}} & \frac{\partial f_{dev}}{\partial \sigma'_{12}} & \frac{\partial f_{dev}}{\partial \sigma'_{13}} & \frac{\partial f_{dev}}{\partial \sigma'_{23}} \end{bmatrix}, \quad (A5)$$

where the derivatives of the isotropic yield surface are

$$\begin{aligned} \frac{\partial f_{iso}}{\partial \sigma'_{11}} &= \frac{\partial f_{iso}}{\partial \sigma'_{22}} = \frac{\partial f_{iso}}{\partial \sigma'_{33}} = \frac{1}{3}, \\ \frac{\partial f_{iso}}{\partial \sigma'_{12}} &= \frac{\partial f_{iso}}{\partial \sigma'_{13}} = \frac{\partial f_{iso}}{\partial \sigma'_{23}} = 0, \end{aligned} \quad (A6)$$

and the derivatives of the deviatoric yield surface are obtained using the chain rule

$$\frac{\partial f_{dev}}{\partial \boldsymbol{\sigma}'} = \frac{\partial f_{dev}}{\partial q} \frac{\partial q}{\partial \boldsymbol{\sigma}'} + \frac{\partial f_{dev}}{\partial p'} \frac{\partial p'}{\partial \boldsymbol{\sigma}'} + \frac{\partial f_{dev}}{\partial \sin(3\vartheta)} \frac{\partial \sin(3\vartheta)}{\partial \boldsymbol{\sigma}'}, \quad (A7)$$

where

$$\begin{aligned} \frac{\partial f_{dev}}{\partial q} &= 1, \\ \frac{\partial f_{dev}}{\partial p'} &= M \left[b - 1 + b \ln \left(\frac{p'd}{p'_c} \right) \right] r_{dev}, \\ \frac{\partial f_{dev}}{\partial \sin(3\vartheta)} &= -3\sqrt{3}a_L b_L n_L (1 + b_L \sin(3\vartheta))^{n_L-1} p' \left(1 - b \ln \left(\frac{p'd}{p'_c} \right) \right) r_{dev}, \end{aligned} \quad (A8)$$

and

$$\frac{\partial q}{\partial \sigma'_{ij}} = \begin{cases} \frac{3}{2q} (\sigma'_{ij} - p'), & \text{if } i = j \\ \frac{3\sigma'_{ij}}{q}, & \text{if } i \neq j \end{cases}, \quad (A9)$$

$$\frac{\partial p'}{\partial \sigma'_{ij}} = \frac{\delta_{ij}}{3}, \quad (A10)$$

$$\frac{\partial \sin(3\vartheta)}{\partial \boldsymbol{\sigma}'} = -\frac{3\sqrt{3}}{2J_{2D}^{3/2}} \left((\mathbf{s}^2)^t - \frac{2}{3} J_{2D} \mathbf{I} - \frac{3J_{3D}\mathbf{s}}{2J_{2D}} \right),$$

or

$$\begin{aligned} \frac{\partial \sin(3\vartheta)}{\partial \sigma'_{ii}} &= -\frac{3\sqrt{3}}{2J_{2D}^{3/2}} \left[\frac{1}{3} \left(2(\sigma'_{ii} - p')^2 - (\sigma'_{jj} - p')^2 - (\sigma'_{kk} - p')^2 + \sigma'^2_{ij} + \sigma'^2_{ik} - 2\sigma'^2_{jk} \right) - \frac{3J_{3D}}{2J_{2D}} (\sigma'_{ii} - p') \right], \\ \frac{\partial \sin(3\vartheta)}{\partial \sigma'_{ij}} &= -\frac{3\sqrt{3}}{2J_{2D}^{3/2}} \left[-2\sigma'_{ij} (\sigma'_{kk} - p') + 2\sigma'_{jk} \sigma'_{ki} - \frac{3J_{3D}}{2J_{2D}} 2\sigma'_{ij} \right], \text{ if } i \neq j \end{aligned} \quad (A11)$$

where δ_{ij} is the delta Kronecker.

The hardening moduli matrix has the form

$$\mathbf{H} = \begin{bmatrix} -\frac{\partial f_{iso}}{\partial \lambda_{iso}^p} & -\frac{\partial f_{iso}}{\partial \lambda_{dev}^p} \\ -\frac{\partial f_{dev}}{\partial \lambda_{iso}^p} & -\frac{\partial f_{dev}}{\partial \lambda_{dev}^p} \end{bmatrix}, \quad (A12)$$

where

$$\begin{aligned} -\frac{\partial f_{iso}}{\partial \lambda_{iso}^p} &= -\frac{\partial f_{iso}}{\partial r_{iso}} \frac{\partial r_{iso}}{\partial \varepsilon_v^p} \frac{\partial \varepsilon_v^p}{\partial \lambda_{iso}^p} - \frac{\partial f_{iso}}{\partial p'_c} \frac{\partial p'_c}{\partial \varepsilon_v^p} \frac{\partial \varepsilon_v^p}{\partial \lambda_{iso}^p} = p'_c \frac{(1 - (r_{iso} - r_{iso}^e))^2}{c} + p'_c \beta r_{iso}, \\ -\frac{\partial f_{iso}}{\partial \lambda_{dev}^p} &= -\frac{\partial f_{iso}}{\partial r_{dev}} \frac{\partial r_{dev}}{\partial \varepsilon_d^p} \frac{\partial \varepsilon_d^p}{\partial \lambda_{dev}^p} - \frac{\partial f_{iso}}{\partial p'_c} \frac{\partial p'_c}{\partial \varepsilon_v^p} \frac{\partial \varepsilon_v^p}{\partial \lambda_{dev}^p} = p'_c \beta r_{iso} \alpha \left(M - \frac{q}{p'} \right), \\ -\frac{\partial f_{dev}}{\partial \lambda_{iso}^p} &= -\frac{\partial f_{dev}}{\partial r_{iso}} \frac{\partial r_{iso}}{\partial \varepsilon_v^p} \frac{\partial \varepsilon_v^p}{\partial \lambda_{iso}^p} - \frac{\partial f_{dev}}{\partial p'_c} \frac{\partial p'_c}{\partial \varepsilon_v^p} \frac{\partial \varepsilon_v^p}{\partial \lambda_{iso}^p} = Mp'b\beta r_{dev}, \\ -\frac{\partial f_{dev}}{\partial \lambda_{dev}^p} &= -\frac{\partial f_{dev}}{\partial r_{dev}} \frac{\partial r_{dev}}{\partial \varepsilon_d^p} \frac{\partial \varepsilon_d^p}{\partial \lambda_{dev}^p} - \frac{\partial f_{dev}}{\partial p'_c} \frac{\partial p'_c}{\partial \varepsilon_v^p} \frac{\partial \varepsilon_v^p}{\partial \lambda_{dev}^p} = \\ &= Mp' \left(1 - b \ln \left(\frac{p'd}{p'_c} \right) \right) \frac{(1 - (r_{dev} - r_{dev}^e))^2}{a} + Mp'b\beta \alpha \left(M - \frac{q}{p'} \right) r_{dev}, \end{aligned} \quad (A13)$$

where the derivatives of the degrees of plastification of the isotropic and deviatoric

mechanisms are $\partial r_{iso} / \partial \varepsilon_v^p = (1 - (r_{iso} - r_{iso}^e))^2 / c$ and $\partial r_{dev} / \partial \varepsilon_d^p = (1 - (r_{dev} - r_{dev}^e))^2 / a$,

respectively. The flow direction matrix is

$$\mathbf{G} = \begin{bmatrix} \frac{\partial g_{iso}}{\partial \sigma'_{11}} & \frac{\partial g_{iso}}{\partial \sigma'_{22}} & \frac{\partial g_{iso}}{\partial \sigma'_{33}} & \frac{\partial g_{iso}}{\partial \sigma'_{12}} & \frac{\partial g_{iso}}{\partial \sigma'_{13}} & \frac{\partial g_{iso}}{\partial \sigma'_{23}} \\ \frac{\partial g_{dev}}{\partial \sigma'_{11}} & \frac{\partial g_{dev}}{\partial \sigma'_{22}} & \frac{\partial g_{dev}}{\partial \sigma'_{33}} & \frac{\partial g_{dev}}{\partial \sigma'_{12}} & \frac{\partial g_{dev}}{\partial \sigma'_{13}} & \frac{\partial g_{dev}}{\partial \sigma'_{23}} \end{bmatrix}^t, \quad (A14)$$

where the derivatives of the isotropic plastic potential are

$$\begin{aligned} \frac{\partial g_{iso}}{\partial \sigma'_{11}} &= \frac{\partial g_{iso}}{\partial \sigma'_{22}} = \frac{\partial g_{iso}}{\partial \sigma'_{33}} = \frac{1}{3}, \\ \frac{\partial g_{iso}}{\partial \sigma'_{12}} &= \frac{\partial g_{iso}}{\partial \sigma'_{13}} = \frac{\partial g_{iso}}{\partial \sigma'_{23}} = 0, \end{aligned} \quad (A15)$$

and the derivatives of the deviatoric plastic potential are obtained using the chain rule

$$\frac{\partial g_{dev}}{\partial \boldsymbol{\sigma}'} = \frac{\partial g_{dev}}{\partial q} \frac{\partial q}{\partial \boldsymbol{\sigma}'} + \frac{\partial g_{dev}}{\partial p'} \frac{\partial p'}{\partial \boldsymbol{\sigma}'} + \frac{\partial g_{dev}}{\partial \sin(3\vartheta)} \frac{\partial \sin(3\vartheta)}{\partial \boldsymbol{\sigma}'}, \quad (A16)$$

where

$$\begin{aligned} \frac{\partial g_{dev}}{\partial q} &= 1, \\ \frac{\partial g_{dev}}{\partial p'} &= \alpha \left(M - \frac{q}{p'} \right), \\ \frac{\partial g_{dev}}{\partial \sin(3\vartheta)} &= -3\sqrt{3}a_L b_L n_L (1 + b_L \sin(3\vartheta))^{n_L-1} \frac{\alpha}{\alpha-1} p' \left(1 - \frac{1}{\alpha} \left(\frac{p'd}{p'_c} \right)^{\alpha-1} \right), \end{aligned} \quad (A17)$$

and the derivatives of the stress variables with respect to the effective stress tensor are given by Eqs. (A9) – (A11).

The products of these matrices become

$$\mathbf{A} = \mathbf{J} : \mathbf{E} : d\boldsymbol{\varepsilon} = \begin{pmatrix} K\varepsilon_v \\ \frac{3G}{q} \sum_{i=1,2,3} \varepsilon_{ii} (\sigma'_{ii} - p') + \frac{6G}{q} (\varepsilon_{12}\sigma'_{12} + \varepsilon_{13}\sigma'_{13} + \varepsilon_{23}\sigma'_{23}) + jK\varepsilon_v + L_f \end{pmatrix}, \quad (A18)$$

where $j = M[b - 1 + b \ln(p'd / p'_c)]r_{dev}$ and L_f is the contribution due to the Lode's angle,

which reads

$$\begin{aligned}
L_f &= -\frac{3\sqrt{3}G}{J_{2D}^{3/2}} \frac{\partial f_{dev}}{\partial \sin(3\vartheta)} (L_{f1} + L_{f2} + L_{f3} + L_{f4} + L_{f5} + L_{f6}), \\
L_{f1} &= \varepsilon_{11} \left[\frac{1}{3} \left(2(\sigma'_{11} - p')^2 - (\sigma'_{22} - p')^2 - (\sigma'_{33} - p')^2 + \sigma'^2_{12} + \sigma'^2_{13} - 2\sigma'^2_{23} \right) - \frac{3J_{3D}}{2J_{2D}} (\sigma'_{11} - p') \right], \\
L_{f2} &= \varepsilon_{22} \left[\frac{1}{3} \left(2(\sigma'_{22} - p')^2 - (\sigma'_{33} - p')^2 - (\sigma'_{11} - p')^2 + \sigma'^2_{12} + \sigma'^2_{23} - 2\sigma'^2_{13} \right) - \frac{3J_{3D}}{2J_{2D}} (\sigma'_{22} - p') \right], \\
L_{f3} &= \varepsilon_{33} \left[\frac{1}{3} \left(2(\sigma'_{33} - p')^2 - (\sigma'_{11} - p')^2 - (\sigma'_{22} - p')^2 + \sigma'^2_{13} + \sigma'^2_{23} - 2\sigma'^2_{12} \right) - \frac{3J_{3D}}{2J_{2D}} (\sigma'_{33} - p') \right], \\
L_{f4} &= \varepsilon_{12} \left[-2\sigma'_{12} (\sigma'_{33} - p') + 2\sigma'_{13}\sigma'_{23} - \frac{3J_{3D}}{J_{2D}} \sigma'_{12} \right], \\
L_{f5} &= \varepsilon_{13} \left[-2\sigma'_{13} (\sigma'_{22} - p') + 2\sigma'_{12}\sigma'_{23} - \frac{3J_{3D}}{J_{2D}} \sigma'_{13} \right], \\
L_{f6} &= \varepsilon_{23} \left[-2\sigma'_{23} (\sigma'_{11} - p') + 2\sigma'_{12}\sigma'_{13} - \frac{3J_{3D}}{J_{2D}} \sigma'_{23} \right],
\end{aligned} \tag{A19}$$

$$\mathbf{B} = \mathbf{J} : \mathbf{E} : \boldsymbol{\beta}_T dT = \begin{pmatrix} K\beta'_s dT \\ jK\beta'_s dT \end{pmatrix}, \tag{A20}$$

$$\mathbf{J} : \mathbf{E} : \mathbf{G} = \begin{pmatrix} K & K\alpha \left(M - \frac{q}{p'} \right) \\ jK & \bar{H} + L_{\bar{H}} \end{pmatrix}, \tag{A21}$$

$$\text{where } \bar{H} = \frac{9G}{2q^2} \sum_{i=1,2,3} (\sigma'_{ii} - p')^2 + \frac{18G}{q^2} (\sigma'^2_{12} + \sigma'^2_{13} + \sigma'^2_{23}) + jK\alpha \left(M - \frac{q}{p'} \right) \text{ and}$$

$$L_{\bar{H}} = L_{\bar{H}_1} + L_{\bar{H}_2} + L_{\bar{H}_3},$$

$$L_{\bar{H}_1} = -\frac{3\sqrt{3}}{2J_{2D}^{3/2}} \frac{3G^2}{q} \frac{\partial g_{dev}}{\partial \sin(3\vartheta)} \left[\begin{aligned} & (\sigma'_{11} - p')^3 + (\sigma'_{22} - p')^3 + (\sigma'_{33} - p')^3 - \\ & - \sigma'^2_{12} (\sigma'_{33} - p') - \sigma'^2_{13} (\sigma'_{22} - p') - \sigma'^2_{23} (\sigma'_{11} - p') - \\ & - \frac{3J_{3D}}{2J_{2D}} [(\sigma'_{11} - p')^2 + (\sigma'_{22} - p')^2 + (\sigma'_{33} - p')^2] \end{aligned} \right],$$

$$L_{\bar{H}_2} = L \left\{ \begin{aligned} & \left[\frac{1}{3} (2(\sigma'_{11} - p')^2 - (\sigma'_{22} - p')^2 - (\sigma'_{33} - p')^2 + \sigma'^2_{12} + \sigma'^2_{13} - 2\sigma'^2_{23}) - \frac{3J_{3D}}{2J_{2D}} (\sigma'_{11} - p') \right]^2 + \\ & + \left[\frac{1}{3} (2(\sigma'_{22} - p')^2 - (\sigma'_{33} - p')^2 - (\sigma'_{11} - p')^2 + \sigma'^2_{12} + \sigma'^2_{23} - 2\sigma'^2_{13}) - \frac{3J_{3D}}{2J_{2D}} (\sigma'_{22} - p') \right]^2 + \\ & + \left[\frac{1}{3} (2(\sigma'_{33} - p')^2 - (\sigma'_{11} - p')^2 - (\sigma'_{22} - p')^2 + \sigma'^2_{13} + \sigma'^2_{23} - 2\sigma'^2_{12}) - \frac{3J_{3D}}{2J_{2D}} (\sigma'_{33} - p') \right]^2 \end{aligned} \right\},$$

$$L_{\bar{H}_3} = L \left\{ \begin{aligned} & \left[-2\sigma'_{12} (\sigma'_{33} - p') + 2\sigma'_{13} \sigma'_{23} - \frac{3J_{3D}}{J_{2D}} \sigma'_{12} \right]^2 + \\ & + \left[-2\sigma'_{13} (\sigma'_{22} - p') + 2\sigma'_{12} \sigma'_{23} - \frac{3J_{3D}}{J_{2D}} \sigma'_{13} \right]^2 + \\ & + \left[-2\sigma'_{23} (\sigma'_{11} - p') + 2\sigma'_{12} \sigma'_{13} - \frac{3J_{3D}}{J_{2D}} \sigma'_{23} \right]^2 \end{aligned} \right\},$$

(A22)

$$\text{where } L = \frac{27G}{2J_{2D}^3} \frac{\partial f_{dev}}{\partial \sin(3\vartheta)} \frac{\partial g_{dev}}{\partial \sin(3\vartheta)}.$$

$$\mathbf{C} = \mathbf{H} + \mathbf{J} : \mathbf{E} : \mathbf{G} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}. \quad (\text{A23})$$

To obtain the values of the plastic multipliers, we have to solve the consistency equations

$$\begin{cases} dF_{iso} = A_1 + B_1 - C_{11} \lambda_{iso}^p - C_{12} \lambda_{dev}^p = 0 \\ dF_{dev} = A_2 + B_2 - C_{21} \lambda_{iso}^p - C_{22} \lambda_{dev}^p = 0 \end{cases}. \quad (\text{A24})$$

If only one yield surface is active at a time, when the isotropic plastic mechanism is active, the plastic multipliers are

$$\lambda_{iso}^p = \frac{A_1 + B_1}{C_{11}} = \frac{K\varepsilon_v + K\beta'_s dT}{p'_c \left(1 - (r_{iso} - r_{iso}^e)\right)^2 / c + p'_c \beta_r r_{iso} + K}, \lambda_{dev}^p = 0, \quad (A25)$$

and when the deviatoric plastic mechanism is active, the plastic multipliers are

$$\lambda_{iso}^p = 0, \lambda_{dev}^p = \frac{A_2 + B_2}{C_{22}} = \frac{\frac{3G}{q} \sum_{i=1,2,3} \varepsilon_{ii} (\sigma'_{ii} - p') + \frac{6G}{q} (\varepsilon_{12} \sigma'_{12} + \varepsilon_{13} \sigma'_{13} + \varepsilon_{23} \sigma'_{23}) + jK (\varepsilon_v + \beta'_s dT) + L_f}{C_{22}}, \quad (A26)$$

where

$$C_{22} = Mp' \left(1 - b \ln(p'd/p'_c)\right) \left(1 - (r_{dev} - r_{dev}^e)\right)^2 / a + Mp'b\beta\alpha(M - q/p')r_{dev} + \bar{H} + L_{\bar{H}}. \quad (A27)$$

If the two plastic mechanisms were active at the same time, the stress state would be on the intersection between the two yield surfaces.

APPENDIX B: CONSISTENCY EQUATIONS OF THE WATER RETENTION MODEL

We detail in this Appendix how to obtain the plastic multiplier of the water retention model from the consistency equation

$$\begin{cases} df_H = \frac{\partial f_H}{\partial s} ds + \frac{\partial f_H}{\partial \pi_H} \cdot \frac{\partial \pi_H}{\partial \lambda_H^p} \cdot \lambda_H^p \leq 0 \\ df_H \cdot \lambda_H^p \geq 0, \quad \lambda_H^p \leq 0 \end{cases}. \quad (B1)$$

Since the internal variable of the water retention model is the plastic increment of the degree of saturation, the derivatives that appear in the consistency equation read

$$\begin{aligned} \frac{\partial f_H}{\partial s} &= \text{sign} \left(\ln \left(\frac{s}{s_D} \right) + \frac{1}{2} \ln \left(\frac{s_{D_0}}{s_{e_H}} \right) \right) \frac{1}{s} \\ \frac{\partial f_H}{\partial d S_l^p} &= \text{sign} \left(\ln \left(\frac{s}{s_D} \right) + \frac{1}{2} \ln \left(\frac{s_{D_0}}{s_{e_H}} \right) \right) \left(-\frac{1}{s_D} \right) s_D \beta_H. \\ \frac{\partial d S_l^p}{\partial \lambda_H^p} &= \frac{\partial f_H}{\partial s} \end{aligned} \quad (B2)$$

Thus, the consistency equation results in

$$df_H = \text{sign} \left(\ln \left(\frac{s}{s_D} \right) + \frac{1}{2} \ln \left(\frac{s_{D_0}}{s_{e_H}} \right) \right) \frac{ds}{s} - \beta_H \frac{1}{s} \lambda_H^p = 0, \quad (\text{B3})$$

and therefore, the plastic multiplier is

$$\lambda_H^p = \text{sign} \left(\ln \left(\frac{s}{s_D} \right) + \frac{1}{2} \ln \left(\frac{s_{D_0}}{s_{e_H}} \right) \right) \frac{ds}{\beta_H}. \quad (\text{B4})$$

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REFERENCES

- Alshibli, K.A., Batiste, S.N., Sture, S. Strain localization in sand: plane strain versus triaxial compression. *Journal of Geotechnical and Geoenvironmental Engineering*, **129**(6), 483-494 (2003).
- Barnichon, J.D. Finite element modeling in structural and petroleum geology. *PhD thesis, University of Liege, Faculte des Sciences Appliquees* (1998).
- Chen, L., Shao, J.F., Huang, H.W. Coupled elastoplastic damage modeling of anisotropic rocks. *Computers and Geotechnics*, **37**(1), 187-194 (2010).
- Colmenares, L.B., Zoback, M.D. A statistical evaluation of intact rock failure criteria constrained by polyaxial test data for five different rocks. *Int. J. of Rock Mec. and Min. Sci.*, **39**(6), 695-729 (2002).

572 Di Donna, A. Thermo-mechanical aspects of energy piles. PhD Thesis, École Polytechnique
573 Fédérale de Lausanne (2014).

574 Di Donna A., Laloui, L. Numerical analysis of the geotechnical behaviour of energy piles.
575 *International Journal for Numerical and Analytical Methods in Geomechanics*, DOI:
576 10.1002/nag.2341, **39**(8), 861-888 (2015).

577 François, B., Laloui, L. ACMEG-TS: A constitutive model for unsaturated soils under
578 non-isothermal conditions. *International Journal for Numerical and Analytical*
579 *Methods in Geomechanics*, **32**(16), 1955-1988 (2008).

580 Gens, A., Sánchez, M., Sheng, D. On constitutive modelling of unsaturated soils. *Acta*
581 *Geotechnica*, **1**, 137–147 (2006).

582 Hueckel, T., Baldi, G. Thermoplasticity of saturated clays: experimental constitutive study.
583 *Journal of Geotechnical Engineering*, **116**(12), 1778-1796 (1990).

584 Hueckel, T., Borsetto, M. Thermoplasticity of saturated soils and shales: constitutive
585 equations. *Journal of Geotechnical Engineering*, **116**(12), 1765-1777 (1990).

586 Hujeux, J.C. Calcul numérique de problèmes de consolidation elastoplastique. Ecole Centrale,
587 Paris, 1979.

588 Kim, J., Tchelepi, H.A., Juanes, R. Rigorous coupling of geomechanics and multiphase flow
589 with strong capillarity. *SPE Journal*, **18**(06), 1123-1139 (2013).

590 Lade, P.V., Duncan, J.M. Cubical triaxial tests on cohesionless soil. *Journal of the Soil*
591 *Mechanics and Foundation Division*, **99**(SM10), 793–812 (1973).

592 Lade, P.V., Musante, H.M. Three-dimensional behavior of remolded clay. *Journal of the*
593 *Geotechnical Engineering Division*, **104**(2), 193-209 (1978).

594 Laloui, L. (1993). Modélisation du comportement thermo-hydro-mécanique des milieux
595 poreux anélastique. PhD Thesis, Ecole Centrale de Paris.

596 Laloui L., Cekerevac. C. Thermo-plasticity of clays: an isotropic yield mechanism.
 597 *Computers and Geotechnics*, **30**(8), 649-660 (2003).

598 Laloui, L., François, B.. ACMEG-T: A Soil Thermo-Plasticity Model. *Journal of Engineering*
 599 *Mechanics*, **135**(9), 932-944 (2009).

600 Lee, K.L. Comparison of plane strain and triaxial tests on sand. *Journal of the Soil Mechanics*
 601 *and Foundations Division*, **96**(3), 901-923 (1970).

602 Makhnenko, R.Y., Labuz, J. Plane strain testing with passive restraint. *Rock Mechanics and*
 603 *Rock Engineering*, **47**(6), 2021-2029 (2014).

604 Makhnenko, R.Y., Labuz, J. Dilatant hardening of fluid-saturated sandstone. *Journal of*
 605 *Geophysical Research: Solid Earth*, **120**, doi:10.1002/2014JB011287, (2015).

606 Makhnenko, R.Y., Harvieux, J., Labuz, J.F. Paul-Mohr-Coulomb Failure Surface of Rock in
 607 the Brittle Regime. *Geophysical Research Letters*, doi: 10.1002/2015GL065457
 608 (2015).

609 Morris, J.P., Hao, Y., Foxall, W., McNab, W. A study of injection-induced mechanical
 610 deformation at the In Salah CO₂ storage project. *International Journal of Greenhouse*
 611 *Gas Control* **5**(2), 270-280 (2011).

612 Nanda, S., Patra, N.R. Determination of soil properties for plane strain condition from the
 613 triaxial tests results. *International Journal for Numerical and Analytical Methods in*
 614 *Geomechanics*, DOI: 10.1002/nag.2337 (2015).

615 Nuth M., Laloui L. Effective Stress Concept in Unsaturated Soils: Clarification and
 616 Validation of a Unified Framework. *International Journal of Numerical and*
 617 *Analytical Methods in Geomechanics*, **32**(7), 771-801 (2008a).

618 Nuth, M., Laloui, L. Advances in modelling hysteretic water retention curve in deformable
 619 soils. *Computers and Geotechnics*, **35**(6), 835-844 (2008b).

620 Paterson, L., Lu, M., Connell, L., Ennis-King, J.P. Numerical modeling of pressure and
621 temperature profiles including phase transitions in carbon dioxide wells. In *SPE*
622 *Annual Technical Conference and Exhibition*, Denver, 21-24 September (2008).

623 Parisio, F., Samat, S., Laloui, L. Constitutive analysis of shale: a coupled damage plasticity
624 approach. *Int. J. Solids Struct.* (2015), <http://dx.doi.org/10.1016/j.ijsolstr.2015.08.003>.

625 Peric, D., Runesson, K., Sture, S. Evaluation of plastic bifurcation for plane strain versus
626 axisymmetry. *Journal of Engineering Mechanics*, **118**(3), 512-524 (1992).

627 Potts, D.M., Gens, A. The effect of the plastic potential in boundary value problems involving
628 plane strain deformation. *International Journal for Numerical and Analytical Methods*
629 *in Geomechanics*, **8**(3), 259-286 (1984).

630 Rutqvist, J., Vasco, D.W., Myer, L. Coupled reservoir-geomechanical analysis of CO₂
631 injection and ground deformations at In Salah, Algeria. *International Journal of*
632 *Greenhouse Gas Control*, **4**(2), 225-230 (2010).

633 Salager, S., François, B., Nuth, M., Laloui L. Constitutive analysis of the mechanical
634 anisotropy of Opalinus Clay. *Acta Geotechnica*. **8**(2), 137-154 (2013).

635 Sánchez, M., Gens, A., Olivella, S. THM analysis of a large-scale heating test incorporating
636 material fabric changes. *Int. J. Numer. Anal. Meth. Geomech.*, **36**, 391–421 (2012).

637 Sheng, D. Review of fundamental principles in modelling unsaturated soil behaviour.
638 *Computers and Geotechnics*, **38**(6), 757-776 (2011).

639 Shojaei, A., Taleghani, A.D., Li, G. A continuum damage failure model for hydraulic
640 fracturing of porous rocks. *International Journal of Plasticity*, **59**, 199-212 (2014).

641 van Eekelen, H.A.M. Isotropic yield surfaces in three dimensions for use in soil mechanics.
642 *International Journal for Numerical and Analytical Methods in Geomechanics*, **4**(1),
643 89-101 (1980).

- Vilarrasa, V., Carrera, J. Geologic carbon storage is unlikely to trigger large earthquakes and reactivate faults through which CO₂ could leak. *Proceedings of the National Academy of Sciences*, **112(19)**, 5938-5943 (2015).
- Vilarrasa, V., Rutqvist, J., Rinaldi, A.P. Thermal and capillary effects on the caprock mechanical stability at In Salah, Algeria. *Greenhouse Gases: Science and Technology*, 5: 449-461 (2015a).
- Vilarrasa, V., Rutqvist, J., Blanco Martin, L., Birkholzer, J. Use of a dual structure constitutive model for predicting the long-term behavior of an expansive clay buffer in a nuclear waste repository. *ASCE International Journal of Geomechanics*, doi: 10.1061/(ASCE)GM.1943-5622.0000603 (2015b).
- Wanatowski, D., Chu, J. Drained behaviour of Changi sand in triaxial and plane-strain compression. *Geomechanics and Geoengineering*, **2(1)**, 29-39 (2007).
- Xie, S.Y., Shao, J.F. Experimental investigation and poroplastic modelling of saturated porous geomaterials. *International Journal of Plasticity*, **39**, 27-45 (2012).
- Zhang, S., Leng, W., Zhang, F., Xiong, Y. A simple thermo-elastoplastic model for geomaterials. *International Journal of Plasticity*, **34**, 93-113 (2012).

TABLES

Table 1. Material parameters of a clay rich shale based on properties of Opalinus clay adapted from Salager et al. (2013).

Property	Value
Reference bulk modulus, K_{ref} (MPa)	1500
Reference shear modulus, G_{ref} (MPa)	1250
Material parameter n_e (-)	0.15
Reference thermal expansion coefficient, β'_{s_0} ($^{\circ}\text{C}^{-1}$)	$2.0 \cdot 10^{-5}$
Initial critical state pressure, p'_{cr_0} (MPa)	25.0
Parameter d (-)	1.2
Parameter b (-)	0.8
Plastic compressibility modulus at zero suction β_0 (-)	43
Material parameter ω (MPa^{-1})	0
Material parameters for the thermal evolution γ_T (-)	0.7
Material parameters for the suction evolution γ_s (-)	0.25
Material parameter a (-)	0.003
Material parameter c (-)	0.02
Size of the elastic nucleus of the isotropic mechanism, r_{iso}^e (-)	1.0
Size of the elastic nucleus of the deviatoric mechanism, r_{dev}^e (-)	1.0
Non-associativity parameter, α (-)	1.0
Friction angle in triaxial compression, ϕ'_c ($^{\circ}$)	25

Friction angle in triaxial extension, ϕ'_e (°)	23
Lode's angle parameter n_L (-)	-0.229

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671 Table 2. Material parameters of an expansive clay adapted from François and Laloui (2008).

Property	Value
Reference bulk modulus, K_{ref} (MPa)	16.0
Reference shear modulus, G_{ref} (MPa)	3.5
Material parameter n_e (-)	1.0
Reference thermal expansion coefficient, β'_{s_0} (°C ⁻¹)	$6.8 \cdot 10^{-4}$
Initial critical state pressure, p'_{cr_0} (MPa)	12.0
Parameter d (-)	1.5
Parameter b (-)	1.0
Plastic compressibility modulus at zero suction β_0 (-)	14.3
Material parameter ω (MPa ⁻¹)	10^{-6}
Material parameters for the thermal evolution γ_T (-)	0.87
Material parameters for the suction evolution γ_s (-)	1.1
Material parameter a (-)	0.001
Material parameter c (-)	0.02

Size of the elastic nucleus of the isotropic mechanism, r_{iso}^e (-)	0.99
Size of the elastic nucleus of the deviatoric mechanism, r_{dev}^e (-)	0.99
Non-associativity parameter, α (-)	1.0
Friction angle in triaxial compression, ϕ'_c (°)	30
Friction angle in triaxial extension, ϕ'_e (°)	25
Lode's angle parameter n_L (-)	-0.229
Reference entry pressure of the non-wetting phase, s_e (MPa)	3.0
Elastic modulus associated with suction strain, K_H (MPa)	∞
Coefficient of compressibility for the plastic part of the degree of saturation, β_H (MPa)	8.64
Material parameter θ_T (-)	0.1
Material parameter c_H (-)	1.15

FIGURES

Figure 1. Effect of (a) temperature, (b) suction and (c) volumetric plastic strain on the shape of the yield surface of the non-isothermal constitutive model for unsaturated geomaterials.

Figure 2. Yield surface accounting for the Lode's angle in the octahedral plane for $n_L = -0.229$, a friction angle in compression of 30° and several friction angles in extension.

Figure 3. Range of possible values of the friction angle in extension as a function of the value of the friction angle in compression to guarantee convexity of the yield surface.

Figure 4. 3D representation of the yield surface in the octahedral plane as a function of temperature. The yield surface shrinks for increasing temperatures.

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703 Figure 5. 3D representation of the yield surface in the octahedral plane as a function of
 704 suction. The yield surface maintains its size while suction is lower than the entry pressure,
 705 but it expands for higher values of suction.

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709 Figure 6. Hysteretic retention curve used in the water retention model.

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713 Figure 7. Stress path due to a fluid pressure increase of 12 MPa in the deviatoric stress – mean
 714 effective stress plane. Both the yield surface when not considering (circular yield) and
 715 when considering Lode's angle (yield locus VE, referring to van Eekelen (1980)
 716 formulation) are plotted.

Figure 8. Yield surface in the octahedral plane that causes failure when considering Lode's angle (VE: van Eekelen (1980) formulation) at a mean effective stress of 21.4 MPa (see Figure 7). The circular yield surface, which does not consider Lode's angle, is shown for comparison.

Figure 9. Stress path due to a fluid pressure increase of 12 MPa and a temperature decrease of 45 °C in the deviatoric stress – mean effective stress plane when (a) not considering Lode's angle and (b) considering Lode's angle. Cooling increases the size of the yield surface. The initial yield surface (before cooling) is shown for comparison.

Figure 10. Stress path in the temperature – mean effective stress plane for (a) a 100 °C increment in temperature and (b) a simultaneous 100 °C and 3 MPa increment in temperature and suction, respectively. Both the isotropic yield surface at the onset of failure and the final isotropic yield surface are indicated.

Figure 11. Stress path in the suction – mean effective stress plane for (a) a 100 °C increment in temperature and (b) a simultaneous 100 °C and 3 MPa increment in temperature and suction, respectively. Both the isotropic yield surface at the onset of failure and the final yield surface are indicated.

Figure 12. Volumetric strain as a function of temperature considering (Lode) and not considering (no Lode) Lode's angle for the case in which only temperature increment of 100°C is applied (T) and for the case in which a simultaneous temperature increment of 100 °C and suction increment of 3 MPa are applied (T and s). The letters in the Figure can be correlated to the letters appearing in Figures 10 and 11.