Evolution of nuclear shapes in medium mass isotopes from a microscopic perspective

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The self-consistent Hartree-Fock-Bogoliubov approximation based on the finite-range and density-dependent Gogny interaction (parametrization D1S) has been used to generate potential energy curves in various chains of Pd, Xe, Ba, Nd, Sm, Gd, and Dy isotopes. The evolution of shapes with the number of nucleons is studied and a search for signatures of E(5) and X(5) critical point symmetries is also conducted. The results are compared with those obtained in the framework of other mean-field approximations to the nuclear many-body problem. The role of triaxiality in nuclei where this degree of freedom could be relevant is considered and discussed.

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I. INTRODUCTION

The systematic description of low-energy nuclear properties using simple ideas is one of the central aims of theoretical and experimental nuclear structure studies. The spontaneous symmetry breaking mechanism of the mean field, leading to the concept of deformed nuclear shapes [1,2], has played an important role in that respect as it allows for an easy interpretation of many low-lying nuclear excited states [3]: if the ground state shows a well-deformed axially symmetric shape, then rotational bands built on the ground-state configuration are expected to appear. However, when the nuclear shape of the ground state corresponds to a spherical configuration, then typical signatures of vibrational states with $E_{4+}/E_{2+} \approx 2$ show up in the spectrum. Going a step further, the constrained mean-field approximation provides a more physical picture by exploring the energy of the nucleus as a function of the quadrupole deformation parameters β_2 and γ . The energy shows a landscape [the so-called potential energy curve (PEC) or surface (PES)] with maxima and minima. The absolute minimum corresponds to the ground state and, if the minimum is deep enough as to constraint the dynamics of the deformation degrees of freedom in the vicinity of the minimum, we talk about well-defined ground-state deformations that lead, according to the position of the minimum, to the basic situations described above. Intermediate situations with several minima and/or not deep enough (or shallow) wells lead to different patterns in the low-lying spectrum that are not easy to interpret. To get a simple interpretation in some of those intermediate situations Iachello has recently [4,5] considered very simple analytical forms for the β_2 - γ PES that allows an analytic or almost analytic evaluation of the nuclear spectrum and related properties such as $B(E2)$ transition probabilities; up to an overall scale factor. In the so-called E(5) dynamical symmetry model the potential energy surface is assumed to be square well in the β_2 degree of freedom and constant along the γ direction. This situation corresponds to a vibrational nucleus that is, in addition, unstable in the γ direction and the

corresponding spectrum should be something intermediate in between the predictions of the IBM [6] for the U(5) (spherical system) and O(6) (γ -unstable) limits. The other dynamical symmetry studied by Iachello is referred to as X(5) and it corresponds to a transition from U(5) to SU(3) (prolate axially symmetric system). In this case the potential energy surface is assumed to be of the form $u(\beta_2) + v(\gamma)$, where the potential u is again a square-well potential, whereas v is a parabola with a minimum at $\gamma = 0^\circ$. Several candidates have already been proposed experimentally for both E(5) and X(5) symmetries. For example, the nuclei ^{134}Ba [7], ^{102}Pd [8], ^{108}Pd [9], and $^{128-130}\text{Xe}$ [10] have been identified as candidates for E(5), whereas ^{152}Sm [11], ^{150}Nd [12], ^{154}Gd [13], and ^{156}Dy [14] provide examples of X(5) symmetry.

The best way to obtain realistic PES of nuclei is by means of constrained mean-field calculations with realistic (although phenomenological) interactions (or energy density functionals) like Skyrme or Gogny in the nonrelativistic side or relativistic Lagrangians. These realistic interactions usually provide a good experimental agreement with nuclear properties all over the nuclear chart, but they can produce different results for magnitudes not directly observable like the PES. Therefore, it is always interesting to check the predictions of different interactions concerning the PES as an agreement among them will imply that the phenomena under study is very likely to show up in nature. The possibility of carrying out systematic PES calculations over extended regions of the nuclear chart has opened the door to analyze whether nuclei are rotational, vibrational or they have to be catalogued as belonging to intermediate situations with shallow and/or coexisting minima. In the latter case, it is interesting to examine in which nuclei the assumptions of relatively flat potentials in E(5) and X(5) are justified as in those cases the dynamical symmetry models will provide a simple interpretation of the spectrum.

Pioneer calculations, establishing a link between dynamical symmetry models and microscopic theories, have been carried out first using the relativistic mean-field approximation [15–17] to study potential energy curves for isotopic chains in which the occurrence of critical point symmetries has been predicted. It has been shown that particular isotopes

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exhibit relatively flat PECs over an extended range of the deformation parameter. Nevertheless, the behavior of the PECs, and particularly the height of the barriers centered around (with a maximum at) the spherical configuration that are in between the prolate and oblate configurations (spherical barriers, hereafter), are quite sensitive to the relativistic interaction used, as well as to the pairing treatment [15–17]. The sensitivity of the PECs to the force can be seen in Ref. [15], where several parametrizations of the mean-field Lagrangian were compared. The sensitivity to the treatment of pairing in relativistic calculations also becomes apparent comparing the energy of the spherical barriers predicted for Sm isotopes in Refs. [15] and [17] using the parametrization NL3, which is perhaps the best parameter set ever fitted for the relativistic mean-field Lagrangian, and different choices for pairing correlations. Let us remark that, at variance with Ref. [15], the results presented in Ref. [17] were obtained using a self-consistent treatment of pairing correlations based on a finite-range Brink-Boeker interaction in the pairing channel and, as a main effect, a reduction of the energy of the spherical barriers is observed.

Therefore, the question arises whether a similar situation occurs when using nonrelativistic effective interactions to study candidates for critical point symmetries. Such systematic nonrelativistic studies were, to a large extent, missing. Only recently has the first step been undertaken using the self-consistent Skyrme Hartree-Fock + BCS mean-field approximation [2,18]. The calculations [19] were based on the code EV8 [20] using the parametrization SLy4 [21] of the Skyrme interaction in the particle-hole (ph) channel together with a zero-range and density-dependent interaction in the pairing channel [22] (with strength $g = 1000 \text{ MeV fm}^3$). Different combinations of mean-field and pairing treatments were also considered in Ref. [19]. As a result of this study, it was found that the energy of the spherical barriers strongly depend on the details of the calculations, especially on the pairing force. It was also found that the assumption of flat potentials in the E(5) critical point symmetry model is supported by the calculations in $^{108,110}\text{Pd}$, $^{128,130}\text{Xe}$, and $^{130,132}\text{Ba}$. However, in the case of X(5) it was found that the rare-earth isotopes with $N = 88$ and 90 show a transitional behavior that could be interpreted in terms of X(5) but flat PECs were not found, in agreement with previous calculations [15–17].

All the microscopic studies carried out so far, both relativistic [15–17] and nonrelativistic [19], have been based on very powerful and state-of-the-art mean-field approximations. Nevertheless, in all of them, the mean-field and pairing channels have been calculated using different forces. Given the dependence on pairing of many properties, as discussed above, it seems reasonable to extend the calculations to the case of an interaction where the same interaction provides both the ph and pairing correlations. Such is the case of the finite-range and density-dependent Gogny interaction [23] that is the effective nucleon-nucleon force used in the full-fledged Hartree-Fock-Bogoliubov (HFB) calculations presented in this article (full-fledged in contrast with the HF + BCS approximation used in other calculations mentioned before). In particular, the present study has been performed using the parametrization D1S [24] of the Gogny interaction.

In addition to the interest that this study could have by itself, it could also be considered, together with the previous calculations of Refs. [17,19], as part of an ongoing effort to explore the shape of the PES over an extended range of nuclei to search for those intermediate situations in which either good candidates to be described by simple dynamical symmetry models can be found or admit more elaborated treatments beyond the mean field. Therefore, in the present study we will use the HFB approximation with the Gogny interaction to study systematically the PEC and PES of several isotopic chains to check for the occurrence of the above-mentioned intermediate situations not corresponding either to well-deformed or spherical systems. In addition, we go a step beyond previous works and consider simultaneously both β_2 and γ degrees of freedom in the corresponding energy surfaces discussing the potential in the β_2 and γ directions.

We will consider Pd, Xe, and Ba isotopic chains as candidates to show PEC patterns characteristic of the E(5) model and Nd, Sm, Gd, and Dy isotopic chains as possible candidates to show the X(5) patterns. A comparison with similar results obtained in Ref. [19] with the parametrization SLy4 of the Skyrme force [21] plus a zero range pairing [22] will also be considered.

The article is organized as follows. In Sec. II we present a brief description of the theoretical formalism (i.e., HFB) used to obtain the main ingredient of the present study, i.e., the PEC and PES for the considered isotopic chains. For a more detailed account the reader is referred to the corresponding literature (see, for example, Refs. [23] and [25]). Section III contains our results with a discussion of PECs and also of PESs for selected nuclei taken as illustrative examples in the present study. In particular, in this section, and in addition to axially symmetric calculations, we will also explore the role played by the γ degree of freedom when searching candidates for critical point symmetries. Finally, Sec. IV is devoted to the concluding remarks.

II. THEORETICAL FRAMEWORK

To compute both PECs and PESs we have used the (constrained) HFB mean-field method together with the parametrization D1S [24] of the Gogny interaction [23]. Two sets of calculations have been considered: first, calculations preserving axial symmetry have been carried out for all the nuclei considered to explore the different possibilities. The restriction to axial symmetry is dictated by computational complexity considerations. The results obtained here are the same as the ones of the compilation of axially symmetric PECs performed by the Bruyères-Le-Châtel group [26]. In a second step, calculations exploring also the triaxial degree of freedom have been carried out. The triaxial calculations are more involved than the axial ones (computationally, around a factor 60 more expensive) and this fact restricts them to relevant candidates only. In both types of calculations, axial and triaxial, the solution of the HFB equations has been reformulated as a minimization process and the so-called gradient method [27] has been used to locate the minimum. The gradient method has the advantage over the usual way of solving the HFB

FIG. 1. Potential energy curves obtained with the Gogny D1S force (full lines) for the isotopes of Pd. Along with each of them, the quantity $E(-\beta_2)$ is plotted (dashed line) to stress the similarities or differences between the prolate and oblate sides. The results obtained with the Skyrme interaction SLy4 and a δ -pairing force with strength $g = 1000 \text{ MeV fm}^3$ [19] are also plotted as dotted lines.

equations (i.e., successive iterations method) that the handling of constraints is much more easily implemented. This is a nice feature for the triaxial calculation as four constraints have to be handled at the same time (proton and neutron number and the two deformation parameters β_2 and γ). As it is customary in all the mean-field calculations with the Gogny force, we have subtracted the kinetic energy of the center-of-mass motion from the Routhian to be minimized to ensure that the center

FIG. 2. The same as Fig. 1 but for Xe isotopes. In this case no Skyrme interaction results are included.

of mass is kept at rest. We have also dealt with the exchange Coulomb energy in the Slater approximation and neglected the contribution of the Coulomb interaction to the pairing field. As mentioned above, we have used the parametrization D1S [24] of the Gogny interaction [23]; this parameter set was adjusted to reproduce basic nuclear matter parameters, the binding energies of several magic and semimagic nuclei and to have a more reasonable value of the surface energy coefficient $a_s = 19 \text{ MeV}$. The latter was considered to have a better reproduction of the fission barrier properties of ^{240}Pu .

The advantage of the Gogny interaction over other alternatives, like Skyrme or relativistic interactions, is that its finite

range allows a consistent treatment of pairing correlations with the same interaction. The interaction by itself produces the natural cutoff for the pairing matrix elements (i.e., the same effective nucleon-nucleon interaction can be used in both the ph and pairing channels). As already discussed in the introduction the role of pairing in the topology of the PEC is crucial and therefore considering the Gogny force represents another kind of interaction as compared to Skyrme or relativistic interactions.

The HFB wave functions have been expanded in a harmonic oscillator (HO) basis containing enough number of shells to grant convergence of the results for all values of the mass quadrupole moment considered and for all the nuclei studied. The results for the energy obtained in the calculations preserving axial symmetry will be presented as a function of the quadrupole deformation parameter β_2 defined as

$$\beta_2 = \sqrt{\frac{4\pi}{5}} \frac{Q_{20}}{A\langle r^2 \rangle} \quad (1)$$

where the mean square radius $\langle r^2 \rangle$ and the quadrupole moment $Q_{20} = \langle z^2 - \frac{1}{2}(x^2 + y^2) \rangle$ are computed with the HFB wave functions. Please, notice that the definition of Q_{20} used here is a factor of 2 smaller than the usual definition. In the triaxial case we will use as parameters for representation the quadrupole moment Q_0 and the γ deformation parameter. The latter is defined as

$$\tan(\gamma) = \frac{Q_{22}}{Q_{20}} \quad (2)$$

in terms of the multipole moment $Q_{22} = \langle x^2 - y^2 \rangle$ and Q_{20} . The former is defined as $Q_0 = \sqrt{Q_{20}^2 + Q_{22}^2}$ and is the one to be used instead of Q_{20} in the formula for the β_2 deformation parameter in the triaxial case.

III. DISCUSSION OF THE RESULTS

The main outcome of our constrained HFB study preserving axial symmetry is presented in the form of multipanel plots of

the PECs as a function of the deformation parameter β_2 for the nuclear species considered. In Figs. 1 and 2 we show the PECs as functions of the deformation parameter β_2 for the nuclei $^{96-118}\text{Pd}$ and $^{118-140}\text{Xe}$. We have also computed the PECs of the barium isotopes $^{118-140}\text{Ba}$ but they are not shown here due to their similarity to the ones of the xenon isotopes with the same number of neutrons (two units less of mass number). The Ba and Xe results for the same neutron number show the same shape except for a slightly higher spherical maximum (see below). The chains shown in Figs. 1 and 2, as well as the one corresponding to the Ba chain, contain E(5) candidates found from systematic studies on available data on energy levels, E_2 , E_1 , and $M1$ strengths [7,8,10,28,29]. In Figs. 1 and 2, we have also plotted the quantity $E(-\beta_2)$ (dashed line) to stress the similarities or differences between the curve in the prolate ($\gamma = 0^\circ$) and oblate ($\gamma = 60^\circ$) side. The dashed line should be understood just as a device to get a hint on the possible dependence of the energy on the triaxial deformation parameter γ .

We observe several spherical isotopes associated to magic numbers: ^{96}Pd ($N = 50$), $^{134,136}\text{Xe}$ ($N = 80$ and 82), and $^{136,138}\text{Ba}$ ($N = 80$ and 82). Apart from those, the other nuclei present a maximum for the spherical configuration ($\beta_2 = 0$) that is located between a prolate and an oblate minima. The prolate minimum is always deeper than the oblate one, and the latter is very shallow in some cases. The excitation energy of the oblate minimum with respect to the prolate one is rather small in all the cases and never exceeding 1.5 MeV. This is at variance with other nuclei where it is common the existence of a prolate well that is much deeper than the oblate one. However, the similar behavior of both the prolate and oblate sides of the PECs around sphericity [which is manifest by considering the dashed lines corresponding to $E(-\beta_2)$] indicates that most of the nuclei considered are going to be rather γ unstable as it is very likely that the prolate and oblate minima are connected through a flat path through the γ degree of freedom. This means a great amount of shape mixing between the prolate and oblate configurations and therefore a static picture like the mean field, not including the dynamics on the quadrupole

FIG. 3. The height of the spherical barrier separating the prolate and oblate minima in the considered PEC's is plotted for the Pd, Xe, and Ba isotopes as a function of the mass number A . The results obtained here (marked as Gogny D1S) are compared to the ones already reported in Ref. [19] and obtained using the Skyrme SLy4 interaction and with a pairing interaction strength $g = 1000 \text{ MeV fm}^3$. For more details, see the main text.

FIG. 4. (Color online) Contour plot of the potential energy surface of the isotopes ^{102}Pd , ^{108}Pd , and $^{112-118}\text{Pd}$ in a β_2 - γ plane like representation. Instead of the β_2 deformation parameter we have used the total mass quadrupole moment $Q_0 = \sqrt{Q_{20}^2 + Q_{22}^2}$ in units of barns. For the mass numbers considered a Q_0 value of 10 b corresponds to $\beta_2 = 0.61$. The contour lines correspond to the following scheme: (a) from $\epsilon_{\min} + 0.25$ MeV to $\epsilon_{\min} + 2$ MeV alternating dotted and dashed contours every 0.25 MeV (the contour $\epsilon_{\min} + 0.25$ MeV is plotted in blue, the contours $\epsilon_{\min} + 0.5$ MeV and $\epsilon_{\min} + 0.75$ MeV are plotted in green, and the ones corresponding to $\epsilon_{\min} + 1$ MeV and $\epsilon_{\min} + 1.25$ MeV are plotted in red; the rest are in black); (b) from $\epsilon_{\min} + 2.5$ MeV to $\epsilon_{\min} + 4.5$ MeV the contour lines are plotted every 0.5 MeV as full lines; (c) from $\epsilon_{\min} + 5$ MeV to $\epsilon_{\min} + 10$ MeV the contour lines are plotted again every 0.5 MeV but now as dashed lines; and, finally, (d) from $\epsilon_{\min} + 11$ MeV to $\epsilon_{\min} + 20$ MeV contour lines are depicted every 1 MeV as full lines. For each Q_0 - γ contour plane an inset has been added corresponding to the energy along the axially symmetric shapes (i.e., $\gamma = 0^\circ$ for positive Q_0 and $\gamma = 60^\circ$ for negative ones).

degree of freedom, will not be able to reproduce the main characteristics of the ground state and lowest-lying excited states of those nuclei. However, for those nuclei showing flat PEC in the sense of the $E(5)$ dynamical symmetry model we can still expect a simple description in terms of the model. Taking into account that the energy scale for collective states like quadrupole vibrations is of the order of 1 MeV we can state that a “flat PEC” is a potential energy curve that shows a region where the changes in energy over a characteristic β_2 interval (typically around 0.2–0.3 in this region) do not exceed that typical energy scale of one MeV. According to this definition we can talk about flat PEC and therefore possible candidates to show a pattern similar to the one of $E(5)$ in the nuclei $^{112-118}\text{Pd}$. To less extend we can identify also the “flat PEC” pattern in the nuclei $^{130,132,138}\text{Xe}$ and $^{132-134,140}\text{Ba}$.

To make a connection with previous work [19], we have also plotted in Fig. 1 the results obtained in Ref. [19] for the Skyrme interaction SLy4 and a pairing strength of $g = 1000 \text{ MeV fm}^3$ as dotted lines. Note the similarity between the two sets of curves as they present the same maxima and minima located at the same β_2 values as well as the same asymptotic behavior for large deformation. The most remarkable difference (see below) is the height of the spherical maximum that is higher for Skyrme than for Gogny and it is related to their different pairing interactions. The same pattern can also be observed for the Xe and Ba isotopes considered.

In Fig. 3, we compare the height of the spherical barrier separating the prolate and oblate minima in the PECs considered (to be denoted E_B) and obtained in the present work with the Gogny interaction for Pd, Xe, and Ba isotopes with the ones already reported in Ref. [19] with the pairing strength $g = 1000 \text{ MeV fm}^3$ and the parametrization SLy4 of the Skyrme interaction in the ph channel. For the considered isotopes, the predicted E_B energies are systematically lower than the ones obtained in Ref. [19] with $g = 1000 \text{ MeV fm}^3$. This lower values of E_B obtained with the Gogny D1S force are a direct consequence of the different pairing interaction used in this case and as compared to the one of Skyrme. The values of E_B are uncorrelated with the position of maxima and minima in the PEC but they can change the shape of the PEC and make it more or less flat depending on their value. However, we observe that the predictions of both Skyrme and Gogny forces follow the same isotopic trends for the considered nuclei. Finally, it is worth mentioning that the energy barriers are higher in the Ba than in the Xe isotopes for the Gogny force results in spite of their similar PEC topography.

To confirm the suitability of the predicted candidates to show PEC patterns similar to the one of the $E(5)$ dynamical symmetry model and to confirm the expectations of γ instability in some cases, we have carried out calculations constraining the Q_0 - γ degrees of freedom (instead of the more usual β_2 deformation parameter we use for these plots the Q_0 quadrupole moment) for several of the nuclei previously considered and the results are presented in Figs. 4 and 5 for the Pd isotopes and Figs. 6 and 7 for the Xe and Ba ones. With this kind of calculations we can analyze the behavior along the γ degree of freedom of the PES.

In Fig. 4 contour plots for the PESs of the even mass isotopes ^{102}Pd , ^{108}Pd , and $^{112-118}\text{Pd}$ as a function of the Q_0

FIG. 5. (Color online) The HFB energy is plotted as a function of the triaxial deformation parameter γ (in degrees) for several values of Q_0 ranging from 1 b to 10 b in steps of 1 b. The curve corresponding to $Q_0 = 1 \text{ b}$ is plotted with the shortest dashed line and for increasing values of Q_0 the length of the dash increases. This is so up to $Q_0 = 6 \text{ b}$ where the pattern repeats again and, to improve the legibility, the lines are thicker and in light gray (red on line). The zero of the energy scale refers to the energy of the minimum.

and γ variables are presented. Along with each contour plot we have also plotted a two-dimensional cut corresponding to axially symmetric shapes that is used as a reference to identify maxima and minima. The first two isotopes have been chosen because there are experimental indications [8,9] that point them as possible examples of the $E(5)$ symmetry. The remaining isotopes are usually considered as γ soft and in principle should not be considered as candidates of $E(5)$ but their axial PECs pointed to a $E(5)$ -like Q_0 - γ potential energy surface. The Pd isotopes considered have axial PECs that are rather flat around sphericity and this manifest in the Q_0 - γ plane as a rather flat region around $Q_0 = 0$ characterized by a low density of the contour lines. We also observe arcs of circles centered around the spherical point that implies constant potentials in the γ direction. It is also interesting to notice that all the oblate minima of the nuclei considered in this figure are not true minima but saddle points in the Q_0 - γ surface.

From this figure we immediately realize that the depicted Pd isotopes are rather γ soft for low values of Q_0 up to $Q_0 \approx 5 \text{ b}$ ($\beta_2 \approx 0.3$) and some of them could be potential candidates to show the $E(5)$ dynamical symmetry. However, by looking at

FIG. 6. (Color online) Same as Fig. 4 but for the $^{128-132}\text{Xe}$ and $^{130-134}\text{Ba}$ nuclei. For the mass numbers considered a Q_0 value of 10 b corresponds to $\beta_2 = 0.47$.

the Q_0 - γ plane plots it is hard to get a quantitative idea of how close they are to the E(5) limit. Therefore, we complement Fig. 4 with another plot, presented in Fig. 5, where the energy is depicted as a function of γ (in degrees) for different values of Q_0 . This representation is nothing but the energy of Fig. 4 along circular paths of radius Q_0 and is introduced with the sole purpose to give a more quantitative view of the results of Fig. 4. In the panels of Fig. 5 we clearly observe how, in most

of the cases, the curves for Q_0 up to 4 b ($\beta_2 = 0.25$) are flat and bunched together reflecting the potential assumed for the E(5) symmetry (independent of γ and flat around $Q_0 = 0$). From $Q_0 = 5$ b on the energy differs from the E(5) assumptions but it is not clear from the plot the effect of such deviation on the spectrum. From Fig. 5 we can conclude that the lightest Pd isotope considered ^{102}Pd does not show a PES pattern fully compatible with the one expected from the E(5) symmetry.

FIG. 7. (Color online) Same as Fig. 5 but for the $^{128-132}\text{Xe}$ and $^{130-134}\text{Ba}$ nuclei and the Q_0 values that now range from 1.2 b up to 12 b in steps of 1.2 b.

However, the other five isotopes considered there, namely ^{108}Pd and $^{112-118}\text{Pd}$, are reasonably good candidates to show the E(5)-like behavior.

The same triaxial calculations have been repeated for selected Xe and Ba isotopes and the results are presented in Fig. 6 for the Q_0 - γ plane and Fig. 7 for the energy plotted as a function of γ for different choices of Q_0 . The plots presented for these two sets of isotopes are rather similar to the ones already discussed for the Pd case and from them we can conclude that the Xe and Ba isotopes studied can be

FIG. 8. The same as Fig. 1 but for Sm isotopes.

considered as reasonable examples of the PECs expected for the E(5) symmetry. The closest to the E(5) casuistic are ^{130}Xe and ^{132}Ba . It is worth mentioning that the Q_0 - γ energy surfaces for the Xe and Ba isotopes with the same number of neutrons are similar but are not as close as the case when only the axially symmetric cuts are considered.

Next, we consider several rare-earth isotopic chains (like Nd, Sm, Gd, and Dy) where candidates exhibiting X(5) behavior have been identified [11,12,30–33]. The nuclei considered can always be grouped in isotones (same number of neutrons) and it turns out that the PECs for all the members of an isotonic chain are very similar. For this reason, as in

FIG. 9. The energy barriers obtained in the present work for Nd, Sm, Gd, and Dy isotopes are compared with the ones already reported in Ref. [19] with the pairing strength $g = 1000 \text{ MeV fm}^3$.

FIG. 10. (Color online) Q_0 - γ energy planes for several isotopes of the Nd and Sm. See the caption of Fig. 4 for a detailed explanation. For the mass numbers considered in this plot a Q_0 value of 10 b corresponds to $\beta_2 = 0.39$.

the Ba isotopes case, we have chosen not to plot all of them but give only one example corresponding to the chain of Sm isotopes that is shown in Fig. 8. A transition is observed from spherical (^{146}Sm) toward clearly prolate $^{154-160}\text{Sm}$ where the prolate minimum is more than 5 MeV deeper than the oblate one and the height of the spherical maximum also exceeds 5 MeV. The isotopes in between, namely ^{148}Sm ($N = 86$),

^{150}Sm ($N = 88$), and ^{152}Sm ($N = 90$) show a transitional behavior with wide minima on the prolate side and additional minima on the oblate sector at 0.82, 1.62, and 3.30 MeV excitation energy, respectively. These excitation energies are slightly smaller than the values of 1.21, 2.59, and 3.58 MeV reported in Ref. [19] for these nuclei. The same is also true for the energy of spherical barriers of 1.27 MeV in

FIG. 11. (Color online) Q_0 - γ energy planes for several isotopes of the Gd and Dy. See the caption of Fig. 4 for a detailed explanation. For the mass numbers considered in this plot a Q_0 value of 10 b corresponds to $\beta_2 = 0.37$.

^{148}Sm , 3.43 MeV in ^{150}Sm , and 6.67 MeV in ^{152}Sm obtained with the Gogny D1S force and to be compared with the values of 2.00, 4.81, and 7.95 MeV reported in Ref. [19]. Finally, the nucleus ^{146}Sm is spherical with a somewhat flat minimum. The PECs shown in Fig. 8 agree qualitatively well with the ones obtained in previous studies [15,17,19,33] as can be corroborated with the comparison made between our

results and the ones of Ref. [19] that are explicitly shown in Fig. 8.

For the isotopes of Nd, and in the same way as in the case of Sm isotopes, the transitional behavior appears for the $N = 86$ – 90 isotopes, 146 – ^{150}Nd . The same holds true for Gd and Dy isotopes for which the transitional behavior is found for $A = 150, 152$, and 154 as well as $A = 152, 154$, and 156 ,

respectively. In agreement with previous calculations [17,19], our results, based on the parametrization D1S of the Gogny interaction, do not predict flat PECs for the $N = 90$ isotopes, casting some doubts, at least in the framework of microscopic mean-field approximations, about the possible realization of the X(5) dynamical symmetry in the isotopes considered.

In Fig. 9, we compare the energies of the spherical barriers in between the prolate and oblate minima (E_B) obtained in the present work for Nd, Sm, Gd, and Dy isotopes with the ones already reported in Ref. [19] with the parametrization SLy4 of the Skyrme interaction in the ph channel and $g = 1000 \text{ MeV fm}^3$. We observe the same isotopic trend for both the Gogny and Skyrme forces (i.e., increasing excitation energy of the spherical barriers for increasing mass number A). As in the previous Pd, Xe, and Ba isotopes, the values of E_B obtained with the Gogny force are smaller than the ones of Skyrme SLy4 and the differences seem to be rather independent on the mass number of the isotopes. Therefore, the differences become irrelevant for the heavier isotopes and become more important for the transitional ones.

For the X(5) dynamical symmetry the potential along the β_2 degree of freedom is of the square-well type and to this potential another one $v(\gamma)$ is simply added. We can extract some information about the assumptions of the X(5) symmetry by comparing the shapes of the PECs in the prolate and oblate sides; as the former corresponds to $\gamma = 0^\circ$, whereas the latter one corresponds to $\gamma = 60^\circ$. To facilitate the comparison we have plotted again in Fig. 8 the quantity $E(-\beta_2)$ as a dashed curve along with the PEC. The assumption of the X(5) dynamical symmetry about the γ dependence would mean in our context that the full and dashed curves should have the same shape and should be just displaced by an amount corresponding to the difference $v(\gamma = 60) - v(\gamma = 0)$. This is reasonably true in most of the cases for large values of deformation but it is not true around the spherical configuration. This is not surprising as the X(5) assumption is necessarily ill behaved around $\beta_2 = 0$ due to the “polar” topology of the β_2 and γ variables (for $\beta_2 \rightarrow 0$ all possible values of γ do not longer belong to an arc but they collapse to a point). Taking this peculiarity into account we could conclude that many nuclei could correspond to the γ dependence of the X(5)-like potential energy surface and the isotopes with $N = 88, 90$, and 92 would be the more close. We have to say that the other assumption, namely almost flat potential energy surfaces around the Q_0 at equilibrium, is not satisfied in any of the examples shown.

To check the validity of this conclusion we have carried out triaxial calculations for those nuclei with $N = 88, 90$, and 92 and the results are shown in Figs. 10 and 11 in the form of Q_0 - γ energy surfaces. The first remarkable fact is the similarity of the results for all nuclei that is especially accurate for those nuclei with the same number of neutrons. From this similarity we conclude that the deformation properties in this region are quite insensitive to the number of protons and vary rather smoothly with the number of neutrons. Checking the validity of the X(5) symmetry implications in the Q_0 - γ PESs is not easy due mainly to the expected behavior around $Q_0 = 0$ and a consequence of the “polar” topology of the Q_0 - γ plane. For this reason we have also included in Fig. 12

FIG. 12. (Color online) Same as Fig. 7 but for the $^{148-152}\text{Nd}$, $^{150-154}\text{Sm}$, $^{152-156}\text{Gd}$, and $^{154-158}\text{Dy}$ nuclei. In this case the different curves correspond to Q_0 values ranging from 1.2 b up to 14.4 b in steps of 1.2 b.

a representation of the PES that is complementary to the one of Figs. 10 and 11 and in which we have plotted the energy as a function of γ for different values of Q_0 . If the behavior as a function of γ of the PES were the one of the X(5) dynamical symmetry model then we would expect in those nuclei a bunch of parabolas corresponding each of them to different values of Q_0 . However, we observe that for Q_0 deformations smaller than the one of the prolate minimum the energy is almost independent of γ (implying the appearance of horizontal lines), whereas for Q_0 values in the vicinity of the prolate minimum the energy $E(Q_0, \gamma)$ shows a parabolic behavior as a function of γ . This is particularly true for low γ angles but tends to a flat behavior as γ approaches 60° . For big-enough values of Q_0 the parabolic behavior is no longer valid. From these results we can conclude that any of the PES studied fully show the typical pattern of the X(5) dynamical symmetry. However, the isotopes of Nd and Sm and especially ^{152}Sm are the ones that show resemblance with the PES of the X(5) model. This is in agreement with the experimental findings.

IV. CONCLUSIONS

In this article we have carried out a systematic mean-field exploration of the PECs in various series of isotopic chains containing coexisting prolate and oblate minima that could be described in simple terms by means of the E(5) or X(5) dynamical symmetry models. We used the HFB framework based on the finite-range and density-dependent Gogny interaction (parametrization D1S). For large-scale exploratory calculations, axial symmetry was kept as self-consistent symmetry. This restriction was relaxed for selected candidates and the triaxial degree of freedom was considered in those cases.

In good agreement with previous relativistic [17] and nonrelativistic [19] studies, we found that the assumption of flat potentials in the PEC, which is characteristic of the E(5) critical point symmetry model, is supported by the results of the present study in $^{108,110}\text{Pd}$ and to some extent in $^{128-132}\text{Xe}$ and $^{130-134}\text{Ba}$. Those nuclei have already been suggested as examples of E(5) based on experimental data. However, though the rare-earth isotopes with $N = 88$ and 90 show a transitional behavior that could be interpreted in terms of X(5) symmetry, we do not find the characteristic flat PECs around the minima for them.

A combined study of the potential energy surfaces in both β_2 and γ degrees of freedom has been performed on the best-suited candidates for critical symmetries. As a result of this analysis, we find that the relatively flat potentials in

the β_2 direction for the Pd, Xe, and Ba candidates to the E(5) symmetry are accompanied by relatively flat potentials in the γ direction at the equilibrium Q_0 values. However, in the examples studied in Nd, Sm, Gd, and Dy isotopes corresponding to possible X(5) realizations, we find that at equilibrium the γ potentials exhibit a quadratic behavior centered at $\gamma = 0$ that becomes flat as γ approaches 60° . We find flat patterns for smaller Q_0 values. All of this, reinforces the assignment of the already suggested isotopes ^{108}Pd , $^{130,132}\text{Xe}$, and $^{132-134}\text{Ba}$ as suitable candidates for E(5) behavior. It also points out other examples like $^{112-118}\text{Pd}$ that display the characteristics of the potentials assumed for E(5). In the case of the rare-earth candidates for X(5) studied, we find a shape transition from spherical to well-deformed nuclei, but the realization of the X(5) symmetry in terms of flat (quadratic) patterns in the β_2 (γ) potentials is uncertain.

The results of the present study, together with the ones already obtained in Refs. [17] and [19], allow us to conclude that all the available state-of-the-art mean-field approximations, both relativistic and nonrelativistic, support the assumption of flat potentials compatible with the ones postulated in the E(5) dynamical symmetry model for some selected isotopes. However, we have not found satisfactory examples of PES showing a pattern similar to the one of the X(5) model. In those cases, as we are in the realm of PES showing shape coexistence, the above result implies that a full microscopic analysis is needed to describe low-energy dynamics.

It has to be mentioned that effects beyond mean field like the restoration of the rotational symmetry can substantially modify the shape of the PES as the rotational energy correction has a nontrivial behavior with deformation (see, for example, Ref. [34] for an example with the Gogny D1S force). Also dynamical correlations expressed in the form of deformation dependent collective inertias can modify in an effective way (that is, when a transformation to a constant inertia framework is performed) the shape of the PES. Both effects acting alone or in combination could eventually lead to PES similar to the ones of X(5) but the effort to go there is of the same order as to fully solve the dynamical problem and therefore the use of simple models is not really necessary. In this respect the work of Ref. [35] is rather representative.

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