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**Comparison of different methods to estimate the soil sorptivity from an upward
infiltration curve**

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Abstract

The soil sorptivity, S , which is defined as a measure of the capacity of a porous medium to absorb or desorb liquid by capillarity, is commonly estimated under laboratory conditions from upward infiltration measurements. The objective of this work is to compare different methods to estimate S from a single upward infiltration curve obtained from both theoretical and experimental soils. An additional analysis of the influence of synthetic infiltration noise on the estimation of S was also performed on the theoretical soils. Five different methods were compared: Short Time model for horizontal infiltration (ST), the Cumulative Linearization method (CL) and the Differentiated Linearization (DL) linear regressions models, Short-time (SIM) methods that use the simplified Haverkamp et al (1994) model, and Complete-time (CIM) upward infiltration method that uses the quasi-analytical Haverkamp et al (1994) function. Since finite soil columns were considered, the saturated hydraulic conductivity needed to estimate S with the Haverkamp et al. (1994) model was calculated from an overpressure step at the end of the water absorption process, using the Darcy's law. The methods were contrasted on four theoretical and six sieved experimental soils, ranging from sand to clay textures. Although all methods showed acceptable estimates of S on clean theoretical upward infiltration curves, the ST, SIM and CIM were the methods that gave significant ($p < 0.001$) regression analysis on noised infiltration curves, and only SIM and CIM presented a relative error $< 1\%$. From these results we can conclude that although acceptable approaches of S were obtained with the simplest ST method, the CIM procedure was the most accurate method to estimate S in both clean and noised theoretical and experimental upward infiltration curves.

Keywords: Soil hydraulic properties; Hydraulic conductivity; Water retention curve.

INTRODUCTION

Estimation of the soil hydraulic properties is of paramount importance within the soil hydrological research field. Among the different soil hydraulic properties, we have the sorptivity, S , which is defined as a measure of the capacity of a medium to absorb or desorb liquid by capillarity (Philip, 1957). This parameter can be analytically calculated as a function of the initial and final soil water content and the diffusivity (Parlange et al., 1975). For horizontal infiltration, Philip (1957) reduced this absorption term to a linear relationship between cumulative infiltration, I , and the square root of time, t , such as $I = S t^{1/2}$.

On non-horizontal water flow, the soil sorptivity is commonly estimated at the very early stages of infiltration, where suction or capillarity forces prevail over gravity. Thus, for instance, in cement and concrete researches, where water flow is mainly controlled by the capillarity, S is commonly estimated from an upward infiltration process using the Philip (1957) model (Pitroda and Umrigar, 2013; Zhou, 2014). However, the $I = S t^{1/2}$ equation must be carefully considered in soils with important gravity flow effects (i.e. sands), where the water flow can rapidly be dominated by soil hydraulic conductivity, K . In this case, the chosen time interval is likely to strongly influence the calculated S (Bonnell and Williams, 1986). To avoid these problems, Moret-Fernández and Latorre (2016) used the quasi-analytical solution of the Haverkamp et al. (1994) model to accurately estimate S from an upward infiltration curve. These authors demonstrated that during the early-medium time stages of the infiltration the S was quasi-independent of the β parameter of the Haverkamp et al. (1994) model that is defined as an integral shape constant depending on the soil diffusivity, the hydraulic conductivity function and the initial and final volumetric water content.

Alternatively, S can be estimated from downward infiltration measurements using a disc infiltrometer with single disc and infiltration tension (Minasny and McBratney, 2000; Bagarello and Iovino, 2003). In this case, the simplified or the complete quasi-analytical solution of the Haverkamp et al. (1994) model can be used. When the simplified Haverkamp et al. (1994) model is employed,

methods based on linear regressions of the infiltration curve with respect to $t^{1/2}$ can be applied (Vandervaere et al., 2000; Minasny and McBratney, 2000). In contrast, if the quasi-analytical Haverkamp et al. (1994) model is employed, more complex numerical analyses are required (Latorre et al., 2015).

Comparison between upward and downward infiltration procedures to estimate soil S under laboratory conditions highlights four reasons for which upward infiltration measurements are preferable over downward process: (i) difficulties to hold the disc infiltrometer on the soil cylinder; (ii) collapsing of the soil macropores due to infiltrometer weight (Moret and Arrúe, 2005), which can disturb the estimations of the soil hydraulic properties; (iii) loss of hydraulic contact between infiltrometer disc and soil surface as the soil becomes saturated due to partial soil collapse; and (iv) provide some information not available in downward processes. It is to say, while upward infiltration, due to the capillary rise processes, can detect the influence of smaller porous, the advance of the wetting front in the downward infiltration may mask the influence of these pores, probably because the wetting front advance is faster than the capillary movement.

The soil sorptivity is also an interesting soil hydraulic parameter, since it allows estimating related hydraulic properties. For instance, Moret-Fernández and Latorre (2016) demonstrated that the parameters of the van Genuchten (1980) water retention curve could be estimated from the saturated hydraulic conductivity measured by Darcy's law and the S and β parameters estimated from the inverse analysis of an upward infiltration curve. Thus, given the importance of S to estimate the soil hydraulic properties from upward infiltration measurements, the objective of this work is to compare different models to estimate S from a single upward infiltration measurement. To this end, five different models running from the simplest Philip (1957) equation to the quasi-exact Haverkamp et al. (1994) formulation adapted to an upward infiltration process were compared on theoretical and experimental soils.

2. MATERIAL AND METHODS

2.1. Theory

The 1-D upward cumulative infiltration, I (L), for a homogeneous, infinite length soil column with uniform initial water content can be described by the following equation derived from the quasi-exact Haverkamp et al. (1994) model (Moret-Fernández and Latorre, 2016)

$$\frac{2(1-\beta)\Delta K^2}{S_0^2}t = \frac{2\Delta K(I + K_i t)}{S_0^2} - \ln \left\{ \frac{1}{\beta} \exp \left[\frac{2\beta\Delta K(I + K_i t)}{S_0^2} \right] + 1 - \frac{1}{\beta} \right\} \quad (1)$$

where t is time (T), $S = S_0(\theta_0, \theta_i)$ ($L T^{-0.5}$) is the sorptivity, θ_0 ($L^3 L^{-3}$) is the volumetric water content at the infiltration boundary, θ_i ($L^3 L^{-3}$) is the initial volumetric water content, $\Delta K = K_i - K_0$, with K_0 ($L T^{-1}$) and K_i ($L T^{-1}$) hydraulic conductivity corresponding to θ_0 and θ_i , respectively, and β is an integral shape parameter. The respective initial and boundary conditions for upward infiltration are

$$\begin{aligned} z = 0, t > 0, \theta &= \theta_s \\ z \geq 0, t = 0, \theta &= \theta_i \\ z \rightarrow \infty, t > 0, \theta &= \theta_i \end{aligned} \quad (2)$$

where z is a vertical coordinate (L) positive upward, and θ_s ($L^3 L^{-3}$) and θ_i ($L^3 L^{-3}$) are the saturated and initial volumetric water content, respectively. In our case, θ_0 and K_0 correspond to the saturated volumetric water content, θ_s ($L^3 L^{-3}$), and the saturated hydraulic conductivity, K_s ($L T^{-1}$), respectively. Although Eq. (1) is suitable for those soils (sand to silt) where the β parameter ranges between 0.3 and 1.7 (Lassabatere et al., 2009), Moret-Fernández and Latorre (2016) reported that during the medium time stages of an upward infiltration S was not affected by β , and consequently Eq. (1) could be applied to all type of soils, including clays.

For early to intermediate infiltration time and assuming $K_n \rightarrow 0$, Eq.(1) results (Haverkamp et al., 1994)

$$I = S\sqrt{t} - \frac{2-\beta}{3} K_s t \quad (3)$$

which can be expressed as

$$I = C_1\sqrt{t} - C_2t \quad (4)$$

where

$$C_1 = S \quad (5)$$

and

$$C_2 = \frac{2-\beta}{3} K_s \quad (6)$$

Using algebraic combinations or the derivation with respect to the square root of time, Eq. (4) can be respectively expressed as:

$$\frac{I}{\sqrt{t}} = C_1 - C_2\sqrt{t} \quad (7)$$

$$\frac{dI}{d\sqrt{t}} = C_1 - 2C_2\sqrt{t} \quad (8)$$

The value of the constants C_1 and C_2 are estimated to calculate sorptivity and hydraulic conductivity. For this purpose, different fitting procedures which use Eq. (4) or its time-derived version can be adopted. These methods were referred as CL for Cumulative Linearization (Eq. 7) and DL for Differentiated Linearization (Eq. 8) (Vandervaere et al., 2000).

For horizontal infiltration or absorption processes, Eq. (1) is reduced to the Philip (1957) one-term model

$$I = S\sqrt{t} \quad (9)$$

This simple equation is commonly applicable during the very early time of upward or downward infiltrations, where suction or capillarity forces prevail over gravity.

Parlange (1975) demonstrated that, for homogeneous, uniform initial water content and infinite length soil column, the soil sorptivity could be expressed as

$$S^2(\theta_s, \theta_i) = \int_{\theta_i}^{\theta_s} D(\theta) [\theta_s + \theta - 2\theta_i] d\theta \quad (11)$$

where $D(\theta)$ ($L^2 T^{-1}$) is the diffusivity defined by Klute (1952) as

$$D(\theta) = K(\theta) \frac{dh}{d\theta} \quad (12)$$

According to van Genuchten (1980) the soil diffusivity can be expressed as

$$D(S_e) = \frac{(1-m)K_s}{\alpha m(\theta_s - \theta_r)} S_e^{1/2-1/m} \left[\left(1 - S_e^{1/m}\right)^{-m} + \left(1 - S_e^{1/m}\right)^m - 2 \right] \quad (13)$$

$$S_e = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} \quad (14)$$

where θ_r ($L^3 L^{-3}$) denotes the residual volumetric water content, respectively, and α (L^{-1}), n , and $m=(1-1/n)$ are empirical parameters of the van Genuchten (1980) water retention curve model.

Combining Eq. (11) and Eq. (13) we obtain

$$S^2 = \frac{(1-m)K_s}{\alpha m(\theta_s - \theta_r)} \int_{\theta_i}^{\theta_s} [\theta_s + \theta - 2\theta_i] S_e^{1/2-1/m} \left[\left(1 - S_e^{1/m}\right)^{-m} + \left(1 - S_e^{1/m}\right)^m - 2 \right] d\theta \quad (15)$$

This equation allows estimating S from the saturated hydraulic conductivity and the water retention curve parameters.

For saturated and steady state condition, the water flow is described by the Darcy's law (Lichtner et al., 1996)

$$q = -K_s \frac{dH}{dz} \quad (16)$$

where q [$L T^{-1}$] is the water flux density, $H=h+z$ is the total head and h (L) is the matric component of soil water potential.

2.2. Sorptivity estimation

2.2.1. Methods

In a 1-D upward infiltration experiment on a finite soil column, K_s can be easily estimated from an overpressure step at the end of the wetting process, according to Eq. (16). Under these conditions, five different fitting methods to estimate S can be used:

- 1.- For very short times (i.e. 10 s), method ST: a linear regression of Eq. (9)
- 2.- For short to intermediate times (i.e. 100 s), CL method, that is linear regression of Eq.(7)
- 3.- Alternatively to the previous method, DL method, that is linear regression of Eq.(8)
- 4.- Once calculated the K_s , the SIM method estimates S from the best fitting between Eq. (3) and experimental infiltration curve.
- 5.- As in the fourth method, but using the Eq. (1), CIM.

Details of the ST, CL and DL methods can be found in Minasny and McBratney (2000) and Vandervaere et al. (2000). Next a more detailed description of the SIM and CIM methods is presented.

SIM method

This method is valid for early to intermediate infiltration times and assumes a known K_s measured by the Darcy law (Eq. 16). Given that at short infiltration times S is independent of β (Moret-Fernández and Latorre, 2016), an average β value of 0.6 was selected (Angulo-Jaramillo et al. 2000). The S was estimated by minimizing an objective function, Q , that represents the difference between Eq.(3) and experimental cumulative upward infiltrations data

$$Q = \sum_{i=1}^n ((I_i - I(S, t_i)) \Delta t_i)^2 \quad (17)$$

where n is the number of measured (I, t) values. To this end, a global optimization search (Pardalos and Romeijn, 2002) was employed.

CIM method

This method requires solving Eq.(1) for upward infiltrations, using, to this end, the same procedure described by Latorre et al. (2015) for downward infiltration processes. Only infiltration times between $t = 0$ and the time just before the wetting front arrives at the top of the soil column can be considered. The S was estimated by minimizing an objective function, Q (Eq. 17), that represents the difference between Eq.(1) and experimental upward infiltration data. In this case, the K_s was measured by Darcy and the β value calculated according to the Moret-Fernández and Latorre (2016) procedure. Unlike to SIM, β can not considered as constant value since at intermediate-long time, β has a significant influence on the upward infiltration curve (Moret-Fernández and Latorre, 2016).

2.3. Numerical Experiments

The theoretical upward infiltration curves used to calculate S were generated with the HYDRUS-1D software (Simunek et al., 1996). A sand, loam, silt and clay soils as estimated by Carsel and Parrish (1988) were used. The soil hydraulic parameters of the theoretical soils are summarised in Table 1 (Simunek et al., 2008). The soil volume was discretized with a 1-D mesh of 1001 cells. The minimum time step used in the simulations was 0.001 s. The initial pressure head of the homogeneous and isotropic column was $-1.66 \cdot 10^6$ cm, and the tension at the base of the soil column was 0 cm. Atmospheric conditions with a maximal tension of 0 cm was imposed at the top boundary. According to HYDRUS-1D, this condition does not allow water to build up on the surface, preventing water overpressures on the soil surface. At the end of the soil water absorption process, once the wetting front arrived to the top soil, an overpressure step of 5 cm was simulated during 10 min. This additional step was subsequently used to calculate the saturated hydraulic conductivity, K_s , by Eq. (16). The synthetic upward infiltrations generated by HYDRUS were used to estimate, with the different methods, the soil sorptivity. These values were subsequently compared with the theoretical S numerically calculated with Eq. (15), after applying the K_s , α and n values of Table 1.

Under laboratory conditions, air bubbling in the sorptivimeter water reservoir can affect the measurement of the cumulative upward infiltration. To study the influence of air bubbling on the S estimation, a synthetic noise was added to the synthetic infiltration curves calculated in the previous step. A preliminary experiment performed with a ± 0.5 psi pressure transducer installed in a 2.0 cm-diameter water reservoir and connected to a 2.5 cm-diameter soil cylinder resulted to an upward infiltration measurement uncertainty of ± 0.05 mm ($SD = 0.05$ mm). This uncertainty source was propagated to the synthetic curve in form of undesired “noise”. The resultant upward infiltration curves were used to test the viability of the ST, CL, DL, SIM and CIM methods. The sensitivity analysis for ST, CL and DL methods was determined from the regression coefficient, which gives information about the points dispersion, and the slope and the significance of the corresponding regression lines. For SIM and CIM procedures, the sensitivity analysis was performed around each inverse solution as part of a first order uncertainty analysis (Hanness et al., 1991).

2.4. Experimental measurements

2.4.1. Sorptivimeter

The soil sorptivity of experimental soils was estimated with a sorptivimeter (Moret-Fernández and Latorre, 2016). This consisted on a porous base (5 cm internal diameter -i.d.- and 0.7 cm high) contained in an aluminum receptacle of 10 cm diameter (Fig. 1). The top of the porous base was covered with a 20 μ m pore size nylon mesh, which was hermetically closed against an aluminium receptacle with an O-ring plus an aluminium ring (1 cm thick and 10 and 5 cm external and internal diameter, respectively). The bottom of the aluminium receptacle was connected to a Mariotte water supply reservoir (30 cm high and 2.0 cm i.d.). A ± 0.5 psi differential pressure transducer (PT) (Microswitch, Honeywell), connected to a datalogger (CR1000, Campbell Scientist Inc.), was installed at the bottom of the water-supply reservoir (Casey and Derby, 2002).

To setup the sorptivimeter, the porous base plus nylon mesh should be previously saturated, and all air trapped between the nylon mesh and the porous base removed. The measurements started when the soil to be analysed, which was contained in a stain steel cylinder (5 cm- internal diameter -i.d.- and 5 cm-high) closed by the base with a 20 μm pore size mesh, was placed on the saturated porous base, and finishes when the wetting front arrives at the soil surface. Because the wetting front is not steep, as in the case of capillary medium, the presence of a zero flux boundary condition (top of sample) in close vicinity to the infiltrating surface (bottom of sample) could influence the upward flow of water and, consequently, application of Eq. (1), valid for (semi-) infinite homogeneous soil profile. For the reason, the final part of the cumulative upward infiltration was omitted from the Eq. (1) optimization. Once the wetting front arrived to the soil column surface, an overpressure step was applied by raising the water reservoir to a desired height. The saturated hydraulic conductivity was calculated from the overpressure section of the cumulative absorption curve according to Eq.(16). The initial water content was measured gravimetrically, and the final water content was calculated as the sum of the initial water content plus the water absorbed by the soil at the time that a water sheet appears on the top of the cylinder. Once the experiment finished, the cylinder was disassembled and the final water content (θ_s) was again measured gravimetrically.

2.4.2. Laboratory testing

Six different soils were employed: two different sand (80- 160 and 250-500 μm particle size, respectively) (Ex-sand fine and Ex-sand coarse), a 2-mm sieved loam (Ex-Loam), clay loam (Ex-Clay-Loam) and silt-gypseous soils and a 0.25-mm sieved clay (Ex-Clay) soil. Except for the sand, where the fast absorption process led to use a 10 cm high cylinder, a 5 cm high cylinder was employed for the remaining soils. Textural and chemical characteristics of the experimental soils are summarized in Table 2.

The S calculated in the experimental soils with the different upward infiltration analysis were compared to the corresponding S values calculated with Eq. (15) from measured the K_s , α and n parameters estimated in the same soils using the modified multiple tension upward infiltration method (Moret-Fernández et. al, 2016). To this end, a negative, zero and an overpressure head steps were applied to the soil cores. The negative pressure heads ranged from -15 cm for sand to -35 to -40 cm for the remaining soils. The overpressure from the soil surface varied from +2cm to +5 cm.

3.- RESULTS AND DISCUSSION

Table 1 shows the theoretical S values calculated numerically (Eq. 15) for synthetic sand, loam, silt and clay soils, using the corresponding K_s , α and n parameters. The results showed that the quasi-exact Haverkamp et al. (1994) model (Eq. 1) modified for an upward infiltration was suitable for all type of soil (including clay soil) when medium infiltration times were considered (Fig. 2). The graphical analysis of the different methods used to estimate S on a theoretical loam soil demonstrated that all methods exhibited good behaviour when clean theoretical curves were used (Fig. 3). The SIM and CIM techniques, with an error of 0.22%, yielded the most accurate results (Table 3). The accuracy to estimate S with the other methods followed the gradient $CL > ST > DL$, and the corresponding error was in all cases less than 2% (Table 3). The ability to estimate S decreases when synthetic noise was included in the infiltration curves (Fig. 3). Under this new scenario, only the ST, SIM and CIM procedures gave significant ($p < 0.001$) regression lines, and only SIM and CIM presented a relative error $< 1\%$ (Table 3). Similarly as observed by Latorre et al. (2015) with disc infiltrometers, the synthetic noise applied to the soil water absorption curves had an important influence on the DL method, which resulted on a significant decrease of R^2 (Table 3, Fig. 3c). This problem is due to Eq. (8) is very sensitive to anomalous changes in the derivative of the water absorption curve. This limitation, however, vanished on the CIM and SIM methods, where the synthetic noise had an irrelevant effect (Fig. 3d and e).

The sensitivity analysis of the SIM method calculated for a theoretical loam soil with and without synthetic noise (Table 1) and water absorption times of 120s showed an unique and well defined minimum (Fig. 4a). In both cases, the S values within a confidence interval of $0.05 \text{ mm s}^{-0.5}$ ranged from 0.365 to $0.38 \text{ mm s}^{-0.5}$. These results indicate that this procedure can be a satisfactory method to approach the S value. The sensitivity analysis applied of the CIM method also showed in both clean and noised curves accurate estimations of S (Fig. 4b). However, compared to SIM, the interval of confidence of S within the $0.05 \text{ mm s}^{-0.5}$ error reduced between 0.365 and $0.37 \text{ mm s}^{-0.5}$, which indicates that CIM allowed even best estimations of the soil sorptivity. These differences could be attributed to the longer infiltration time (up to 500 s) applied on the CIM method.

Comparison between the theoretical S and the corresponding values estimated with the different methods from synthetic upward infiltration curves showed that under clean curve conditions all methods gave good estimations of S (Fig. 5). The results however changed on noised curves, where the CIM method gave the best estimations of S . While acceptable estimations of S were also obtained with ST, CL and SIM, the DL method gave not-significant results. However, due to no significant regression analysis of the CL method applied on the noised infiltration curves were obtained (Table 3), this method may be questioned. From these analyses we can conclude that the CIM method was the best option to accurate estimations of S , and the ST procedure, which resulted to be the simplest method, could be considered as a good approach to estimate the soil sorptivity.

The saturated hydraulic conductivity and the van Genuchten (1980) water retention curve parameters measured with the multiple tension method for the different experimental soils are summarized in Table 2. Except for the DL method, acceptable relationships were obtained between the S values estimated with ST, DL, SIM and CIM and the corresponding values calculated from the hydraulic parameters estimate with the multiple tension method (Fig. 6). Overall, the SIM and CIM method were the most significant, showing the highest R^2 and the best slope of the regression function (Table 4).

4.- CONCLUSIONS

This work, that compares different methods to estimate S from theoretical and experimental upward infiltration curves, demonstrated that (i) the simplest Philip (1957) equation for horizontal infiltration allows good approaches of S , and (ii) the CIM procedure, which calculates S from the best fitting between the experimental upward infiltration and the quasi-exact Haverkamp et al. (1994) model, allowed for all type of soils the best estimations of S . These results are very interesting for those methods in which accurate estimate of S are needed.

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Figure captions

Figure 1. Sorptivimeter scheme

Figure 2. Comparison between the upward infiltration curves (I) simulated by HYDRUS-1D for the synthetic sand, loam, silt and clay soils and those generated for the same soils with the Haverkamp et al. (1994) model (Eq. 1) using the soil hydraulic properties of Table 1.

Figure 3. Graphical analysis of the (a) ST, (b) CL, (c) CL, (d) SIM and (e) CIM methods applied to a theoretical loam upward infiltration curve without (circles) and with (crosses) synthetic noise.

Figure 4. Sensitivity analysis of the (a) SIM and (b) CIM methods applied on clean (red line) and noised (symbol) upward infiltration curves for a synthetic loam soil and infiltration times of 120 and 500 s, respectively.

Figure 5. Comparison between the theoretical sorptivity and the corresponding values estimated with the ST, CL, CL, SIM and CIM methods from clean (white symbols) and noised (grey symbols) synthetic upward infiltration curves generated by HYDRUS-1D. Regression lines for the clean and noised upward infiltration curves are also showed

Figure 6. Comparison between the sorptivity values calculated for the six experimental soils from hydraulic properties measured with the multiple-tension upward infiltration method (S_{MT}) and the corresponding values ($S_{Infiltration}$) estimated with the ST, CL, CL, SIM and CIM methods from the upward infiltration curves.

This work compares five methods to estimate S from a single exfiltration curve

Methods run from simple approaches (ST) to analytical models (CIM)

The methods were compared on theoretical and experimental soils

Acceptable approaches were obtained with the ST method

The CIM was the most accurate method to estimate S in all soils

Table 1. Values of initial (θ_i), saturated (θ_s) and residual (θ_r) water content, α and n parameters of the van Genuchten (1980) water retention curve, saturated hydraulic conductivity (K_s) and the sorptivity (S) (Eq. 15) calculated from the soil hydraulic properties fro the different theoretical soils.

	θ_i	θ_s	θ_r	α	n	K_s	S
		cm ³ cm ⁻³		cm ⁻¹		mm s ⁻¹	mm s ^{-0.5}
<i>Sand</i>	0.045	0.43	0.045	0.145	2.68	8.25 10 ⁻²	1.521
<i>Loam</i>	0.078	0.43	0.078	0.036	1.56	2.88 10 ⁻³	0.367
<i>Silt</i>	0.034	0.46	0.034	0.016	1.37	6.93 10 ⁻⁴	0.238
<i>Clay</i>	0.068	0.38	0.068	0.008	1.09	5.55 10 ⁻⁴	0.076

Table 2. Values of initial (θ_i), saturated (θ_s) and residual (θ_r) water content, saturated hydraulic conductivity (K_s), α and n parameters of the van Genuchten (1980) water retention curve estimated with the multiple-tension method, soil textural properties and organic carbon and gypsum content of the different experimental soils.

	ρ_b	θ_i	θ_r	θ_s	α	n	K_s	Sand	Silt	Clay	Organic carbon	Gypsum
	g cm ⁻³		cm ³ cm ⁻³		cm ⁻¹		mm s ⁻¹			g kg ⁻¹		
<i>Ex-coarse sand</i>	1.50	0.02	0.02	0.43	0.084	3.20	0.285	1000	-	-	-	-
<i>Ex-fine sand</i>	1.54	0.02	0.02	0.43	0.113	2.71	0.127	1000	-	-	-	-
<i>Ex-loam</i>	1.06	0.01	0.01	0.53	0.100	1.52	0.018	280	470	250	11.7	-
<i>Ex-clay loam</i>	1.32	0.02	0.02	0.49	0.124	1.43	0.105	205	497	298	19.9	-
<i>Ex-silt-gypseous</i>	0.95	0.02	0.02	0.46	0.042	1.23	0.018	316	591	129	1.5	703
<i>Ex-Clay</i>	1.30	0.02	0.02	0.42	0.011	1.36	0.003	151	344	465	12.4	-

Table 3. Theoretical sorptivity (S) calculated on a synthetic loam soil and the corresponding S calculated with the different methods ¹ (ST, CL, DL, SIM, CIM) from a theoretical upward infiltration curve (without and with noise) simulated with HYDRUS. R^2 and p are the regression coefficient and the significance for the corresponding linear regression of dataset, respectively. E indicates the relative error between the theoretical and the estimated S .

	<i>Water absorption curves without synthetic noise</i>				<i>Water absorption curves with synthetic noise</i>			
	S	R^2	p	E	S	R^2	p	E
	mm s ^{-0.5}			%	mm s ^{-0.5}			%
Theoretical	0.367				-			
ST	0.365	0.99	<0.001	0.79	0.341	0.95	<0.001	7.13
CL	0.368	0.99	<0.001	0.49	0.392	0.16	0.06	6.75
DL	0.372	0.99	<0.001	1.31	0.403	0.02	0.56	9.75
SIM	0.368	0.99	<0.001	0.22	0.365	0.97	< 0.001	0.60
CIM	0.368	0.99	<0.001	0.22	0.366	0.98	< 0.001	0.32

¹ ST, short time method
CL linear regression of the dataset ($t^{0.5}, I/t^{0.5}$)
DL, linear regression of the dataset ($t^{0.5}, dI/dt^{0.5}$)
SIM, numerical fitting of Eq. (3)
CIM, numerical fitting of Eq. (1)

Table 4. Regression analysis for the relationship between the sorptivity S calculated (Eq. 15) for the six experimental soils from the hydraulic properties (Table 2) estimated with the upward infiltration multiple tension method (MT) and the corresponding values estimated from the inverse analysis of a single upward infiltration curve ¹

	Regression function	R^2	p cm^{-1}
<i>ST</i>	$S = 1.15 S_{MT} - 0.132$	0.94	0.0010
<i>CL</i>	$S = 1.10 S_{MT} - 0.20$	0.92	0.0230
<i>DL</i>	$S = 1.86 S_{MT} + 0.48$	0.76	0.0230
<i>SIM</i>	$S = 1.01 S_{MT} - 0.01$	0.93	0.0020
<i>CIM</i>	$S = 1.06 S_{MTC} - 0.10$	0.95	0.0008

¹ ST, short time method
CL linear regression of the dataset ($t^{0.5}, I/t^{0.5}$)
DL, linear regression of the dataset ($t^{0.5}, dI/dt^{0.5}$)
SIM, numerical fitting of Eq. (3)
CIM, numerical fitting of Eq. (1)

Figure 1.

Figure 2.

Figure 3_Moret-Fernandez

Figure 3

Figure 4.

Figure 5_Moret-Fernandez

Figure 5.

Figure 6.