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Operator $[\sigma(1)\times\sigma(2)]^{\lambda}t_+(1)t_+(2)$: Signature selection rules

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It is shown that in the single j shell model, the double beta decay matrix element of the operator $[\sigma(1)\times\sigma(2)]^{\lambda}t_+(1)t_+(2)$ from the ground state of the magic nucleus ^{48}Ca to the 2_1^+ state of ^{48}Ti vanishes. This is due to a signature selection rule. Other matrix elements in this region are also discussed. The asymptotic Nilsson model is also discussed, especially for $\lambda=2$. The relation of this operator to double beta decay operators is discussed. The possible confusion of $(j)^n$ states with intruder states is also considered.

I. INTRODUCTION

Our purpose is to study some properties of the operators $[\sigma(1)\times\sigma(2)]^{\lambda}t_+(1)t_+(2)$ with $\lambda=0$, 1, and 2. These operators obviously have an association with the double beta decay process, although the full operators in that theory are more complicated. We nevertheless believe that it would be fruitful to study these somewhat simpler operators and try to get a feel for how the matrix elements depend on the nuclear structure. We will also review some of the developments since the pioneering work of Primakoff and Rosen.[1]

For $\lambda=0$ there is a very close association with the allowed double beta decay from $J=0^+$ to $J=0^+$ with $\Delta T=2$. For $\lambda=2$ the above operator is relevant to the kinematically hindered two neutrino (2ν) decay and to the kinematically allowed neutrinoless (0ν) decay, although for the latter case the operator is much more complicated, as shown by Vergados,[2–5] Doi *et al.*,[6,7] and Haxton and Stephenson.[8]

In previous work on ground state double beta decay transitions $J=0^+\to J=0^+$ it was noted that ^{48}Ca is a rather special case[9,10] as compared with heavier open shell nuclei such as ^{76}Ge, ^{82}Se, and ^{130}Tl. For the latter nuclei it was noted by Zamick and Auerbach (ZA) (Ref. 10) that the large matrix elements obtained by Haxton, Stephenson, and Strottman (HSS) (Refs. 11 and 12) could be explained as being due to pairing coherence. This coherence pertains only to ground state to ground state transition and does not extend to excited 0^+ or 2^+ states. On the other hand for the ^{48}Ca case there are cancellations. This will be discussed later.

At the time of this writing there seems to be a consensus among most theorists that the matrix elements for medium-heavy open shell nuclei are large and in some sense easy to calculate. One even gets such large matrix elements in the interacting-boson approximation (IBA) (Ref. 13). Unfortunately the experimental evidence seems to indicate that the matrix elements are 5 to 10 times smaller than theory. This includes both the geochemical experiments of Kirsten *et al.*[14] and the most recent laboratory experiments of Moe.[15]

Just how this problem will be resolved is not clear. Grotz and Klapdor[16] have suggested that for the case of 2ν decay the closure approximation is not good and that one gets significantly reduced matrix elements by summing explicitly over intermediate states and putting in the correct energy denominators. However, Vogel and Fisher[17] (VF) have recently also done these nonclosure calculations and do not get significant changes for most nuclei (although for ^{76}Ge they do get a reduction of a factor of 2). They suggest that Grotz and Klapdor[16] use effective interactions which lead to unusually small pairing gaps. Indeed, both ZA (Ref. 10) and VF (Ref. 17) have shown that getting the right pairing gap is critical.

Getting back to ^{48}Ca, the double beta decay here at first looked promising because of the large energy release and because of the simple configurations involved. However the calculated matrix elements were found to be rather small. This was explained qualitatively by ZA (Ref. 10) as being due to a K selection rule, previously introduced by Lawson[18] to explain single beta decay. Since they are small they are sensitive to small configuration admixtures and as shown first by Suboi *et al.*[19] and then by Brown[20] the closure approximation is not reliable. Concerning excited states, Vergados *et al.*,[2–5] who had done much of the important early work on nuclear structure effects in double beta decay, noted that the matrix elements to excited 0^+ states can be larger than to the ground state. This may be due to the fact that the change in the K quantum numbers is less.

In striking contrast to the $0\to0$ transition, the $0\to2$ transition is "allowed" only in the neutrinoless case. Roughly speaking for an allowed two neutrino emission the electrons have to come out in s states. However to form spin two the electrons have to have their spins parallel. This violates the Pauli principle. In the neutrinoless case one of the electrons has to be in a p state, i.e., $p_{3/2}$. But the virtual neutrino can have a high energy. It has been shown by Rosen[21] and Doi *et al.*[6,7] that the $0\to2$

matrix element depends on left-right coupling terms but not on a finite neutrino mass.

Although there are several operators for the $0\to 2$ 0ν transition, in the approximation in which $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$ is replaced by R_0 $(=r_0A^{1/3}r_0=1.2$ fm) the only thing that survives is $[\sigma(1)\times\sigma(2)]^{\lambda=2}t_{+}(1)t_{+}(2)$. This approximation was considered early on by Primakoff and Rosen.[1] Vergados[3] has shown though that the exact matrix elements can differ considerably from this approximation.

II. TRANSITIONS FROM CALCIUM TO TITANIUM

We calculate the matrix elements

$$M=\left\langle I_f\left|\left|\sum_{i<j}[\sigma(i)\times\sigma(j)]^{\lambda}t_{+}(i)t_{+}(j)\right|\right|I_i\right\rangle$$

for transitions between calcium isotopes and titanium isotopes. (Note for $\lambda=0$

$$[\sigma_1\sigma_2]^{\lambda=0}=-\frac{1}{\sqrt{3}}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\ .$$

Usually in the literature the matrix elements of $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ are quoted for the $J=0$ to $J=0$ transitions.) We will consider only the case where $I_i=0$. Our convention is $|t_{+}\text{neutron}\rangle=|\text{proton}\rangle$.

In the single j shell model $(f_{7/2})$ the calcium states of given angular momentum and seniority occur only once. We will require the two particle coefficients of fractional parentage

$$(j^nI_0v_0(j^2)I_0\,|\,\}j^{(n+2)}0)\ .$$

These are tabulated in many places. It is instructive to note that one can read these off from the wave functions of McCullen, Bayman, and Zamick,[22] as given in the Princeton University report. One simply looks at the wave functions of states of higher isospin for the titanium isotopes $[T=T(\text{ground})+2]$. The tables list the coefficients $D^{\lambda}(L_pL_nv)$, the probability amplitudes that the protons couple to angular momentum L_p and the neutrons to angular momentum L_n and seniority v.

For beta decay problems it is convenient to introduce an isospin variable so that the wave function for a titanium isotope with n neutrons in the $f_{7/2}$ shell becomes

$$\psi=\frac{1}{\left[\dfrac{(n+2)!}{2!n!}\right]^{1/2}}\mathscr{A}_{(12)(3\cdots n+2)}\times\sum_{L_pL_n}D^{\lambda}(L_pL_n)[(j^2)^{L_p}(j^n)^{L_n}]^{\lambda}\times\chi_{1/2}(1)\chi_{1/2}(2)\chi_{-(1/2)}(3)\cdots\chi_{-(1/2)}(n+2)\ .$$

Here $\chi_{-(1/2)}$ is a neutron state and $\chi_{1/2}$ a proton state. The operator t_{+} is such that $t_{+}\chi_{-(1/2)}=\chi_{1/2}$.

The matrix element can be shown to be

$$M=\sqrt{(2\lambda+1)}\left[\frac{(n+1)(n+2)}{2}\right]^{1/2}\frac{(j+1)(2j+1)}{j}(-1)^{\lambda}\times\sum_{L_pL_n}D^{\lambda}(L_pL_n)(2L_p+1)^{1/2}((j^n)L_n(j^2)L_n\,|\,\}j^{(n+2)}0)\begin{Bmatrix}1&1&\lambda\\ j&j&L_n\\ j&j&L_p\end{Bmatrix},\ j\equiv f_{7/2}\ ,$$

where we have introduced the usual nine j symbol. We note that only even values of L_n and L_p enter. This is obvious for ^{44}Ti (two protons and two neutrons) and ^{48}Ti (two protons and two neutron holes) since for two identical fermions in a single j shell the total angular momentum must be even to satisfy the Pauli principle. For ^{46}Ti it is possible for four neutrons to couple to angular momentum $L_n=5$ but the two particle coefficient of fractional parentage (cfp) only allows values of L_n equal to L_p so that the $L_n=5$ term never enters.

For completeness consider the case $\lambda=1$. For such a state we must have $L_p=L_n$ (even). The nine j symbol is

$$\begin{Bmatrix}1&1&1\\ j&j&L_n\\ j&j&L_n\end{Bmatrix}.$$

This must vanish because the second and third row are identical and the sum of all the nine angular momenta is odd. Hence *the matrix element vanishes for $\lambda=1$.* (However this operator is not associated with double beta decay for $\lambda=1$.)

We next specialize to ^{48}Ca$\to^{48}$Ti. One can easily show that the coefficients of fractional parentage are the following:

$$((j^2)L_n(j^6)L_n\,|\,\}(j^8)0)=\left[\frac{2L_n+1}{28}\right]^{1/2},\ L_n\text{ even}\ .$$

We find for $\lambda=0$ or 2

$$M(48)=\frac{(j+1)(2j+1)}{j}\sqrt{(2\lambda+1)}\times\sum_{L_pL_n}\sqrt{(2L_p+1)(2L_n+1)}\begin{Bmatrix}1&1&\lambda\\ j&j&L_n\\ j&j&L_p\end{Bmatrix}\times D^{\lambda}(L_pL_n)\ .$$

The coefficient multiplying $D^{\lambda}(L_pL_n)$ is symmetric under the interchange of L_p and L_n (for $\lambda=0$ and 2). What about $D^{\lambda}(L_pL_n)$ itself?

As discussed by one of us (L.Z.) at an Argonne symposium in 1964,[23,24] the findings of McCullen, Bayman, and Zamick[22] were that for a system with the same number of protons and neutron holes the wave functions were such that for some states $D^{\lambda}(L_pL_n)=+D^{\lambda}(L_nL_p)$, while for the rest $D^{\lambda}(L_pL_n)=-D^{\lambda}(L_nL_p)$. For ^{48}Ti a signature quantum number s was introduced such that

$$D^{\lambda}(L_p,L_n)=(-1)^{\lambda+s}D^{\lambda}(L_n,L_p) .$$

The signature quantum number has also been considered by Lawson[25] and Ogawa.[26]

It has been shown[24,25] that this signature property leads to several selection rules. For example, for the electric quadrupole operator $e_p\sum r^2Y_2=e_n\sum r^2Y_2$. The $B(E2)$ between states of opposite signature is proportional to $(e_p+e_n)^2$, and between states of the same signature to $(e_p-e_n)^2$. The quadrupole moment of the 2^+ state is proportional to e_p-e_n.

This signature selection rule in the context of double beta decay is relevant to the transition $0^+\to2^+$. (The $\lambda=0$ states necessarily all have even signature, so nothing interesting happens here.)

It turns out that the 2_1^+ state of ^{48}Ti in the single j shell calculation has odd signature, but the 2_2^+ state has even signature. [This is consistent with the fact that the $B(E2)$ to the 2_1^+ state is much stronger than to the 2_2^+ state. The former goes as $(e_p+e_n)^2$, the latter as $(e_p-e_n)^2$.]

It is now clear that the matrix element $M(48)$ $0_1\to2_1$ will *vanish* because everything multiplying $D(L_pL_n)$ is symmetric while $D(L_pL_n)$ itself is antisymmetric.

Thus the nonobservance of the $0_1\to2_1$ transition cannot immediately be used to deduce that the left-right coupling term is zero. The transition is strongly suppressed by nuclear structure effects.

Amusingly, Vergados also obtained some matrix elements which were zero for the $J=0\to J=2$ transitions in his early 1976 paper.[4] He did not however comment on the reason for their vanishing. He did the calculations with two interactions, one realistic but bare, the other effective. The first interaction gave a zero result for the second 2^+ state but not the first. The effective interaction did the opposite. Clearly the result with the effective interaction is the right one. There is no way that the first 2^+ state could have even signature. The electric quadrupole transition to the first 2^+ state is *strong* and to the second 2^+ state *weak*. Thus the first 2^+ state must have odd signature (and it turns out that the second 2^+ state has even signature).

We have compiled in Table I a few other matrix elements (M) of the operator $[\sigma(1)\times\sigma(2)]^{\lambda}t_+(1)t_+(2)$ using the $f_{7/2}$ wave functions of McCullen, Bayman, and Zamick (MBZ).[22] Many of these matrix elements are suppressed so one expects that configuration mixing will be important. Nevertheless it is useful to calculate these in a simple model (even for cases where double beta decay competes with allowed single beta decay) just to try to get a feel for the systematic behavior.

TABLE I. Matrix elements of the operator $[\sigma(1)\times\sigma(2)]^{\lambda}t_+(1)t_+(2)$ in the single j shell model.

Final nucleus	Transition	M
^{44}Ti	$0_1\to0_1$	0.318(-0.550)[a]
^{46}Ti	$0_1\to0_1$	-0.175(0.303)
^{48}Ti	$0_1\to0_1$	-0.104(0.180)
	$0_1\to0_2$	0.416(-0.721)
^{44}Ti	$0_1\to2_1$	0.051
^{46}Ti	$0_1\to2_1$	-0.011
	$0_1\to2_2$	0.157
^{48}Ti	$0_1\to2_1$	zero
	$0_1\to2_2$	0.056

[a]The numbers in parentheses are for the operator $\sigma(1)\cdot\sigma(2)t_+(1)t_+(2)$.

In the case of $0\to0$ transitions we also list in parentheses the matrix element for the operator $\sigma_1\cdot\sigma_2t_+(1)t_+(2)$ since that is what is usually given in the literature.

Concerning the $0_1\to2_1$ transition in the single j shell model, we see that not only is it zero in ^{48}Ti, but it is very weak in ^{46}Ti; ^{44}Ti, though somewhat larger, is still on the whole not strong. In the process of calculating the matrix elements one can see a lot of cancellation of various terms.

We see that the $0_1\to2_2$ transition matrix element is larger in general than $0_1\to2_1$. There will of course be a suppression of this process due to loss of phase space.

This is in agreement with previous work of Haxton and Stephenson,[8] who state very explicitly that the $0_1\to2_1$ transition was suppressed and it would be essentially impossible to extract a meaningful parameter for the coupling of left-handed and right-handed currents.

It is instructive to consider a detailed example which shows the strong cancellation of the Primakoff-Rosen term[1] for the $0\to2^+$ transition. We consider $0\to2_1$ in ^{46}Ti and write down in Table II the contributions for each value of L_p and L_n to the matrix element of $[\sigma(1)\times\sigma(2)]^{\lambda=2}t_+(1)t_+(2)$.

We see that the sum is much smaller than the individual terms. The cancellation is almost complete for this

TABLE II. Contributions of each (L_pL_n) component in the wave function of the 2_1^+ state of ^{46}Ti to the matrix element $[\sigma(1)\times\sigma(2)]^{\lambda=2}t_+(1)t_+(2)$.

L_p	L_n	
0	2	-0.4588
2	0	0.7771
2	2	-0.2101
2	4	0.0368
4	2	-0.2273
4	4	0.0375
4	6	0.0486
6	4	-0.0262
6	6	0.0117
Sum		-0.0107

case. It is obvious that the matrix element will be supersensitive to configuration mixing and that this is a very unfavorable case for extracting the parameter that couples the left- and right-handed currents.

We now consider the transitions $0\to 0$ in more detail. The $0\to 0_1$ was already considered by Zamick and Auerbach (ZA),[10] who noted that a K selection rule, introduced by Lawson[8] for single beta decay, also applies here and leads to a suppression of the transition. Roughly speaking, to go from the ^{48}Ca ground state to the ^{48}Ti ground state one takes two neutrons from $K=\frac{7}{2}$ states and puts them into $K=\frac{1}{2}$ orbits. But $\Delta K=\pm 1$ or 0 is all that is allowed for the Gamow-Teller operator.

The expression for the matrix element $0\to 0$ is simple. We give it here for $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ rather than $[\sigma(1)\times\sigma(2)^0]$.

$$M_{(\sigma_1\cdot\sigma_2)}=\frac{1}{2j^2}\left[\frac{N(N-1)}{2}\right]^{1/2}\times\sum_{L\ (\text{even})}D^0(LL)_f D^0(LL)_i L(L+1)\ .$$

In the above expression N refers to the total number of $f_{7/2}$ *nucleons,* e.g., eight for ^{48}Ti. The $D^0(LL)_f$ are the coefficients describing the final state wave function. The $D^0(LL)_i$ can be regarded either as the two particle fractional parentage coefficients previously introduced, or as the coefficients describing the wave function of the $J=0$ state in the titanium isotope which is the double analog of the ground state of the corresponding calcium isotope, e.g., in ^{48}Ti i refers to the $T=4$, $J=0$ (unique) state, while f is either the $T=2$, $J=0$ ground state (for $0_1\to 0_1$) or the $T=2$, $J=0$ first excited state (for $0_1\to 0_2$).

Note that in the original ZA paper we had in the above expression not $L(L+1)$ but $[L(L+1)-2j(j+1)]$. However the $j(j+1)$ term will vanish because of the orthonormal property $\sum_L D^0(LL)_i D^0(LL)_f=\delta_{fi}$. [For transitions involving $\Delta T=0$, e.g., ^{42}Ca$\to^{42}$Ti the $2j(j+1)$ must be left in.]

This new way of writing the expression perhaps changes the physical description of why the matrix element is suppressed. The remark has often been made that the $L=0$ term and $L=2$ term cancel. But in this way of writing things the $L=0$ term vanishes. The $L=2$ and $L=4$ terms add coherently. There is a cancellation from $L=6$ however.

The large matrix element to the 0_2 state of ^{48}Ti makes this a promising candidate for the experimental technique in which one looks for γ rays which follow the double beta decay to excited states.

One motive for looking for transitions to excited 0^+ states is that the phase space factors for 2ν and 0ν decays are quite different. If $\widetilde{T}_0$ is the energy release in units of the electron rest energy then the energy dependence of 2ν decay is given by a factor

$$K_{2\nu}=\widetilde{T}_0^7(1+\widetilde{T}_0/2+\widetilde{T}_0^2/9+\widetilde{T}_0^3/90+\widetilde{T}_0^4/1980)\ .$$

The corresponding factor of 0ν decay is

$$K_{0\nu}=\widetilde{T}_0^5/15+2\widetilde{T}_0^4/3+8\widetilde{T}_0^3/3+4\widetilde{T}_0^2+2\widetilde{T}_0\ .$$

For the ground state transition we have $\widetilde{T}_0=8.343$, whereas for the transition to the 0_2^+ state at 2.997 MeV the value of $\widetilde{T}_0$ is 2.478. The values of the above quantities are the following:

$$\text{ground state: } K_{2\nu}=6.13\times 10^7\ ,\quad K_{0\nu}=7.77\times 10^3\ ,$$

$$0_2^+\ \text{state: } K_{2\nu}=1.78\times 10^3\ ,\quad K_{0\nu}=1.01\times 10^2\ .$$

We see that the 2ν factor decreases much more rapidly with decreasing $\widetilde{T}_0$ than does the corresponding 0ν factor. Thus decay to an excited 0^+ state would enhance the 0ν decay relative to the 2ν decay. In fact we have

$$\frac{K_{2\nu}(0_2)}{K_{2\nu}(0_1)}=2.91\times 10^{-5},\quad \frac{K_{0\nu}(0_2)}{K_{0\nu}(0_1)}=1.31\times 10^{-2}\ .$$

Just to get a feeling for the numbers we use the previously calculated matrix elements $M_{\rm GT}(0_1)=-0.104$, $M_{\rm GT}(0_2)=0.416$. We recognize that because of strong cancellations they cannot be taken too seriously. Nevertheless we calculate the ratio of transition rates

$$\frac{\omega_{2\nu}(0_2)}{\omega_{2\nu}(0_1)}=4.65\times 10^{-4},\quad \frac{\omega_{0\nu}(0_2)}{\omega_{0\nu}(0_1)}=0.21\ .$$

With the above matrix elements the 2ν transition to the second 0_2 state is strongly suppressed but the 0ν transition is down by only a factor of 5.

This indicates that it is worthwhile to look for neutrinoless double beta decay to the first excited state of ^{48}Ti. A possible scenario is then that the ground state transition is overwhelmingly suppressed by nuclear structure effects but the transition to the 0_2 state is not.

However one must worry about whether the 0_2 state is really basically a $(f_{7/2}^8)$ or even $(fp)^8$ state, or rather an intruder state. Indeed Lawson[25] has suggested that the 0_2^+ state in ^{48}Ti consists of two protons excited from the sd shell giving one a configuration $^{50}\text{Cr}\times(sd)^{-2}$. If this were indeed the configuration then the double beta decay matrix element would be very strongly suppressed. Lawson[25] has made a quite reasonable energy estimate of the energy of the intruder state and finds it even lower than 3 MeV. He used arguments similar to those of Bansal and French and Zamick.[28] Furthermore the pure $(f_{7/2}^8)$ 0_2^+ state comes at 5 MeV using MBZ matrix elements.[22] On the other hand Haxton[29] claims that a full $(fp)^8$ calculation brings the 0_2 state to about the right energy to be associated with the 3 MeV state in ^{48}Ti.

The relative strengths in transfer reactions to various 0^+ states are given in Table III.

We are indebted to Sherr for pointing out that the 0_2 state gets about 10% of the strength in all the above transfer reactions.[30] This is not inconsistent with a model in which the 0_2 is indeed dominantly an intruder state. Such as $^{50}\text{Cr}\times(sd)^{-2}$, but with about 10% admixture of the unperturbed ground state configuration.

TABLE III. Relative strengths in various transfer reactions to 0^+ states in ^{48}Ti.

	E	(t,p)	(α,p)	(p,t)	(d,t)
0_1	0	100	100	100	100
0_2	3.00	11	10	13	7
0_3	3.82	<2		7	
0_4	4.59	25	10	17	
0_5	4.97	73	100	<2.3	

However, there are other data which we feel more compellingly lead to the conclusion that the 0_2 state is basically an $(fp)^8$ state. This is the strong γ decay to the 2^+ state. Whereas the decay of the 2_1^+ to ground has a strength of 12.9 Weisskopf units (W.u.), the 0_2 to 2_1 decay has a strength of 16.7 W.u., as shown by Bardin *et al.*[32] and by Kavaloski and Kossler.[33]

Such a strong transition would be impossible if the state was basically $^{50}\mathrm{Cr}\times(sd)^{-2}$. In the vibrational model the ratio $\Gamma_{0_2\to 2_1}/\Gamma_{2_1\to 0_1}=2$ and the above empirical results are not too far from this limit.

Concerning the transfer data there is an amusing result in the single j shell model[22] which prevents the one nucleon transfer (d,t) from going to the excited state. The values of $D^J(L_pL_n)$ are the same for the $J=0$ states of ^{48}Ti and for the $J=\frac{7}{2}$ states of ^{49}Ti. For example, the ground state of ^{48}Ti is

$$\psi=0.91[00]^0-0.40[22]^0-0.02[44]^0+0.15[66]^0$$

and the wave function of the lowest state of ^{49}Ti is

$$\psi=0.91[\tfrac{7}{2}0]^{7/2}-0.40[\tfrac{7}{2}2]^{7/2}-0.02[\tfrac{7}{2}4]^{7/2}+0.15[\tfrac{7}{2}6]^{7/2}\,.$$

This has the consequence that in the reaction ^{49}Ti(d,t)^{48}Ti the spectroscopic strength in the single j shell model will all go to the ground state. One need not therefore worry that there is no strength to an excited f^8 state.

Summarizing this part, the calculated double beta decay matrix element to the 0_2 state is relatively large unless the state is dominantly an intruder state. The $E2$ transition of 0_2 to 2_1 is strong and this seems to argue against a dominant intruder state. The transfer data are somewhat inconclusive, although a more careful quantitative analysis might lead to more definitive results. Perhaps the last word has not yet been said on the subject, but it looks promising that the 0_2 state has a configuration which is dominantly $(fp)^8$ and that the double beta decay matrix element to this state is reasonably strong. In that case the technique of looking for γ rays would be profitable.

Amusingly, Sherr has noted that Alburger's measurements[34] of the highly forbidden single beta decay of ^{48}Ca also sets limits on the double beta decay to the 0_2 state. This is because the Alburger experiment[34] would have detected a γ ray not only from a ^{48}Sc decay but also from ^{48}Ti.

III. CALCULATIONS FOR OPEN SHELL NUCLEI

We next consider the $J=0^+\to 2^+$ double beta decay in another extreme limit—the asymptotic Nilsson scheme. To this end it is convenient to write out the operators in the M scheme.

$$[\sigma\times\sigma]_0^0=-\frac{1}{\sqrt{3}}[4S_0(1)S_0(2)+2S_+(1)S_-(2)+2S_-(1)S_+(2)]\,,$$

$$[\sigma\times\sigma]_0^2=\sqrt{2/3}[4S_0(1)S_0(2)-S_+(1)S_-(2)-S_-(1)S_+(2)]\,,$$

where

$$S_+|\downarrow\rangle=|\uparrow\rangle,\quad S_-|\uparrow\rangle=|\downarrow\rangle\,.$$

We assume the $J=2^+$ state is a member of the ground state $K=0$ band. We evaluate the above operators in the intrinsic state, following closely the previous work of ZA.[10]

In the asymptotic Nilsson model[10,27] we take two nucleons from the state $K_\nu\bar{K}_\nu$ and put them into the state $K_\pi\bar{K}_\pi$, where in more detail

$$|K\rangle=|NM_z\Lambda\Sigma\rangle\,,$$

$$|\bar{K}\rangle=(-1)^{l+(1/2)+\Lambda+\Sigma}|NM_z-\Lambda-\Sigma\rangle\,.$$

We list in Table IV the values of the matrix elements of these operators in the three cases that were previously considered by ZA.[10] We can then imagine a smearing of these matrix elements because of pairing. The expression for the matrix element

$$M=\sum_{K_\pi K_\nu}U(K_\pi)V(K_\pi)U(K_\nu)V(K_\nu)\times\langle(1-P_{12})K_\pi\bar{K}_\pi[\sigma_1\times\sigma_2]_0^\lambda K_\nu\bar{K}_\nu\rangle\,.$$

The product of the U's and V's is always positive. For $\lambda=0$ the matrix elements are all of the same sign. Hence we get a strong pairing coherence.

For $\lambda=2$ however the cases $\Sigma_\pi=\Sigma_\nu$ and $\Sigma_\pi=\Sigma_\nu\pm 1$ have opposite signs and there will be strong cancellations. In fact for $\Lambda\neq 0$ there are four possibilities which are listed in Table V.

Half the cases are positive, the other half equal but of opposite sign. Thus we expect when the smearing due to the pairing force is taken into account the answer will be very close to zero.

TABLE IV. Matrix elements in the asymptotic Nilsson model for $\lambda=0$ and $\lambda=2$.

	$[\sigma\times\sigma]_0^0$	$[\sigma\times\sigma]_0^2$
(1) $\Sigma_\pi=\Sigma_\nu$, $\Lambda\neq 0$	$1/\sqrt{3}$	$-\sqrt{2}/\sqrt{3}$
(2) $\Sigma_\pi=\Sigma_\nu$, $\Lambda=0$	$3/\sqrt{3}$	0
(3) $\Sigma_\pi=\Sigma_\nu\pm 1$, $\Lambda\neq 0$	$2/\sqrt{3}$	$\sqrt{2}/\sqrt{3}$

TABLE V. Matrix elements of the operator $[\sigma(1)\times\sigma(2)]^{\lambda=2}t_+(1)t_+(2)$ in the asymptotic Nilsson scheme.

Proton state	Neutron state	Matrix element
$\Lambda\uparrow$	$\Lambda\uparrow$	$-\sqrt{2}/\sqrt{3}$
$\Lambda\downarrow$	$\Lambda\downarrow$	$-\sqrt{2}/\sqrt{3}$
$\Lambda\downarrow$	$\Lambda\uparrow$	$\sqrt{2}/\sqrt{3}$
$\Lambda\uparrow$	$\Lambda\downarrow$	$\sqrt{2}/\sqrt{3}$

IV. IMPROVED EXPRESSION FOR FINITE DEFORMATIONS

We show here that in the Nilsson pairing approximation the expression for the $0^+\rightarrow 0^+$ ground state to ground state matrix element for double beta decay is negative definite even at finite deformation. The matrix element can be expressed as

$$\langle K_\pi \bar{K}_\pi | \sigma_1\cdot\sigma_2 | K_\nu \bar{K}_\nu - \bar{K}_\nu K_\nu\rangle = \langle K_\pi \sigma_1 K_\nu\rangle\langle \bar{K}_\pi \sigma_2 \bar{K}_\nu\rangle - \langle K_\pi \sigma_1 \bar{K}_\nu\rangle\langle \bar{K}_\pi \sigma_2 K_\nu\rangle \ .$$

If $\mathcal{T}$ is the operator for time reversal then

$$\mathcal{T}|K\rangle = |\bar{K}\rangle,\ \ \mathcal{T}|\bar{K}\rangle = -|K\rangle \ ,$$

and

$$\mathcal{T}\sigma^\dagger\mathcal{T}^{-1} = -\sigma \ .$$

Hence the following relations are obtained:

$$\langle \bar{K}_\pi \sigma \bar{K}_\nu\rangle = -\langle K_\pi \sigma K_\nu\rangle^* \ ,$$

$$\langle \bar{K}_\pi \sigma K_\nu\rangle = \langle K_\pi \sigma \bar{K}_\nu\rangle^* \ .$$

Hence the value of the double beta decay matrix element is the following:

$$M = -\sum_{K_\pi K_\nu}(UV)_{K_\pi}(UV)_{K_\nu} \times[\,|\langle K_\pi \sigma K_\nu\rangle|^2 + |\langle K_\pi \sigma \bar{K}_\nu\rangle|^2] \ .$$

This is negative definite. This expression shows why the results are rather insensitive to deformation, e.g., Haxton and Stephenson obtained essentially the same result in a pairing calculation at a finite deformation as in the asymptotic Nilsson limit.

Of course the above expression holds only to the extent that the Nilsson pairing model is valid, and is not expected to work near closed shells.

V. EXCITED 0^+ STATES IN OPEN SHELL NUCLEI FOUR QUASIPARTICLE STATES

The first excited 0^+ states in open shell nuclei clearly have more complex structure than the ground states. In the vibrational nuclei they may form parts of two phonon triplets and in the rotational region band heads for beta vibrational bands. In the decay $^{130}Te\rightarrow{}^{130}Xe$ the 0_2^+ state is at a much higher energy ($E=1.79$ MeV) than the other members of the phonon triplet ($E=1.12$ MeV for the 2_2^+ state and $E=1.20$ MeV for the 4_1^+ state). This unfortunately cuts down the phase space considerably and makes this a bad candidate for experiment. In the decay $^{76}Ge\rightarrow{}^{76}Se$, however, the 0^+ state is at an energy $E=1.12$ MeV which is consistent with it being a two phonon state. This nucleus is favorable for experimental investigation.

Let us here consider a model in which the 0_2^+ state consists of four quasiparticles. This is consistent with the 0_2^+ state consisting of two quadrupole phonons coupled to an overall angular momentum zero. Each phonon is a linear combination of several two quasiparticle states. (While it is true that one can form a 0^+ state from only two quasiparticles the energy of such a state would be somewhat higher, i.e., roughly equal to the pairing gap energy.)

If $a_K^\dagger$ and a_K are normal creation and destruction operators then the transformation to quasiparticles is given by

$$a_K^\dagger = U_K\alpha_K^\dagger + V_K\alpha_{\bar{K}} \ ,$$

$$a_{\bar{K}}^\dagger = U_K\alpha_{\bar{K}}^\dagger - V_K\alpha_K \ .$$

The 0_2^+ state is written as follows:

$$|0_2^+\rangle = \sum_{\substack{ab\\a'b'}}{}'[\,d^0_{ab,a'b'}(\alpha_a^\dagger\alpha_{\bar{a}'}^\dagger)_\pi(\alpha_b^\dagger\alpha_{\bar{b}'}^\dagger)_\nu + e^0_{ab,a'b'}(\alpha_a^\dagger\alpha_b^\dagger)_\pi(\alpha_{\bar{b}'}^\dagger\alpha_{\bar{a}'}^\dagger)_\nu + f^0_{ab,a'b'}(\alpha_{\bar{a}}^\dagger\alpha_{\bar{b}}^\dagger)_\pi(\alpha_{b'}^\dagger\alpha_{a'}^\dagger)_\nu]\,|0\rangle \ ,$$

where $\sum'$ means the sum over states $\langle K\rangle$ but not $\langle\bar{K}\rangle$.

The states a, b, a', and b' are all near the Fermi surface. The superscript 0 indicates that we are coupling the four quasiparticles to total angular momentum zero.

For brevity we introduce the notation

$$\langle K_1K_2\overline{\sigma_1\sigma_2}K_3K_4\rangle = \langle K_1K_2\sigma_1\cdot\sigma_2K_3K_4\rangle - \langle K_1K_2\sigma_1\cdot\sigma_2K_4K_3\rangle \ .$$

Suppose first that we have only a single four quasiparticle state with $d^0_{ab,a'b'}=1$, $e^0=f^0=0$. The expression for the matrix element would be

$$M^{0_2} = -\langle a\bar{a}'|\overline{\sigma_1\sigma_2}|b\bar{b}'\rangle(U_aU_{a'})_\pi(V_bV_{b'})_\nu \ .$$

This is very similar in form to the ground state matrix element except for the fact that there is *no sum* over states. Clearly then this matrix element will be much smaller than that for the ground state. It may even be zero if the orbits a and b and a and b' (or $\bar{a}'$ and $\bar{b}'$ and $\bar{a}'$ and b) differ in their radial quantum numbers. In the most favorable but highly unlikely scenario $U_a=U_{a'}=V_b=V_{b'}\approx 1/\sqrt{2}$ so that $M^{0_2}=-\frac{1}{4}\langle a\bar{a}'\overline{\sigma_1\sigma_2}b\bar{b}'\rangle$ and, further, the neutron and proton states would have to have the same radial quantum numbers.

The general expression for the matrix element is

$$M^{U_2}=\sum_{\substack{ab\\a'b'}}{}'[-d^0_{ab,a'b'}\langle a\bar{a}'\,|\,\overline{\sigma_1\sigma_2}\,|\,bb'\rangle(U_aU_{a'})_\pi(V_bV_{b'})_\nu$$
$$+(e^0_{ab,a'b'}+f^0_{ab,a'b'})\langle ab\,|\,\overline{\sigma_1\sigma_2}a'b'\rangle$$
$$\times(U_aU_b)_\pi(V_{a'}V_{b'})_\nu]\ .$$

Since the wave function must be normalized to unity the coefficients d, e, and f are all less than unity. In contrast to the ground state transition there is no particular coherence in the sum over states. The matrix element is therefore expected to be very small.

Thus far no one has claimed to see neutrinoless double beta decay. There have been measurements of the $J=0\rightarrow J=2$ transition in ^{76}Ge by Fiorini *et al.*,[35–37] and more recently by a Spanish-French collaboration Leccia *et al.*[38] The most recent work was by Avignone *et al.*,[39] who considered the neutrinoless decay of ^{76}Ge to the ground state of ^{76}Se. Extensive work on the calcium decay was done by Wu's group.[40] Of course the geochemical results of Kirsten[14] (the anomalous abundance of ^{130}Xe in a ^{130}Te sample) show that double beta decay, be it 0ν or 2ν, has occurred.

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