Nonclassical phase of the electromagnetic field in a nonstationary dielectric

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The quantum state of the electromagnetic field propagating in a nonstationary dielectric can acquire a phase shift that arises from modifications in the quantum fluctuations of the field. The shift could be observed, even for quite weak modifications, as a fringe displacement in an interference experiment.
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There has been recently an increased interest in quantum phenomena in nonstationary media. Most of the recent work addresses the possibility of photon generation in simple models of dielectrics and a dielectric cavity with a time-varying index of refraction [1]. In this context methods for producing an appropriately large rate of change of the refractive index have also been anticipated, e.g., in a rapidly growing plasma produced by short optical pulses [2] or by pulse-induced photoconductivity in the transparent region of a semiconductor [3].

The phase of a quantized electromagnetic field, on the other hand, is a subject of continued interest. Much of the work has dealt with finding a phase operator [4], or a specific phase measurement [5], that is consistent with the phase properties of the corresponding classical field. The electromagnetic field also exhibits various topological phases [6] that only depend on the path followed by the system in some parameter space and various experimental realizations have been implemented [7].

In this paper we are concerned with another type of nonclassical phase phenomenon associated with a redistribution of the quantum fluctuations of the electromagnetic field in a time-dependent dielectric. The quantum state of the field acquires a time-dependent phase factor as the refractive index of the medium varies in time. We show that this phase factor includes, besides a well-known always present contribution, an additional term that is exhibited only for specific modulation parameters of the medium and specific quantum states of the field. This extra contribution originates from the transient nonlinear excitation of two-photon processes in the medium and it is nonclassical in nature. Conditions for its occurrence and measurability are discussed within a simplified model of a uniform, lossless, nondispersive and nonstationary dielectric.

The electromagnetic field in such a medium can be described by Maxwell's equations in the absence of sources. The first-order Maxwell's equations can be replaced by the second-order wave equation for the vector potential, i.e., $\nabla^2\mathbf{A}(\mathbf{r},t)-c^{-2}\partial_t[\varepsilon(t)\partial_t\mathbf{A}(\mathbf{r},t)]=0$, where $\varepsilon(t)$ is the dielectric permittivity of the medium. For a uniform and bounded medium whose boundaries are held fixed, the vector potential $\mathbf{A}(\mathbf{r},t)$ can be decomposed in terms of a discrete set $\{\lambda\}$ of mode functions having independent space $\mathbf{A}_\lambda(\mathbf{r})$ and time $q_\lambda(t)$ dependencies [1]. In the following we will consider a dielectric cavity where the presence of the cavity only serves the purpose of providing good frequency and spatial discrimination. The wave equation for $\mathbf{A}(\mathbf{r},t)$ coincides with the Euler equations obtained from a Lagrangian $L[q_\lambda(t),\partial_t q_\lambda(t),t]$; in the adiabatic limit the relative rate of change of the dielectric permittivity is smaller than the frequency of the dielectric cavity modes, i.e., $\varepsilon^{-1}(t)\partial_t\varepsilon(t)<\Omega_\lambda(t)$ and the Lagrangian can be written as [1]

$$L_{\mathrm{ad}}[q_\lambda(t),\partial_t q_\lambda(t),t]=\tfrac{1}{2}\sum_\lambda\,[(\partial_t q_\lambda(t))^2-\Omega_\lambda^2(t)q_\lambda^2(t)]. \tag{1}$$

Here $\Omega_\lambda(t)=\omega_\lambda/\sqrt{\varepsilon(t)}$ and the ω_λ's are the cavity vacuum-frequencies. From Eq. (1) a Hamiltonian $H_{\mathrm{ad}}[q_\lambda(t),p_\lambda(t),t]$ can be derived which permits a straightforward quantization of the field,

$$\hat{H}(t)=\tfrac{1}{2}\sum_\lambda\,[\hat{p}_\lambda^2+\Omega_\lambda^2(t)\hat{q}_\lambda^2], \tag{2}$$

so that the radiation field in our model of nonstationary dielectric is described by an infinite set of uncoupled harmonic oscillators with time-dependent frequencies. The $\hat{q}_\lambda$'s and $\hat{p}_\lambda$'s are hermitian (Heisenberg) operators with the usual algebra $[\hat{q}_\lambda,\hat{p}_{\lambda'}]=i\hbar\,\delta_{\lambda,\lambda'}$.

We proceed to represent $\hat{H}(t)$ as an element of a time-dependent SO(2,1) Lie algebra that enables us to disclose the intrinsic time-dependent nonlinearity of this Hamiltonian. This is done, for a single mode, by introducing the explicitly time-dependent Bose annihilation and creation operators,

$$\begin{aligned}\hat{a}(t)&=\frac{1}{\sqrt{2\hbar}}\left[\frac{1}{\rho(t)}-i\dot{\rho}(t)\right]\hat{q}+i\,\frac{\rho(t)}{\sqrt{2\hbar}}\,\hat{p}\\&=\frac{\rho^{-1}(t)+\Omega(t_0)\rho(t)-i\dot{\rho}(t)}{2\Omega(t_0)^{1/2}}\,\hat{a}(t_0)\\&\quad+\frac{\rho^{-1}(t)-\Omega(t_0)\rho(t)-i\dot{\rho}(t)}{2\Omega(t_0)^{1/2}}\,\hat{a}^\dagger(t_0)\\&\equiv\mu(t)\hat{a}(t_0)+\nu(t)\hat{a}^\dagger(t_0),\end{aligned} \tag{3}$$

and its Hermitian conjugate for $\hat{a}^\dagger(t)$, where $\hat{a}^\dagger(t_0)$ and $\hat{a}(t_0)$ are the unperturbed field operators. The Hamiltonian in Eq. (2) then turns into the *bi*-quadratic form [8],

$$\hat{H}(t)=\hbar w_1(t)[\hat{a}^\dagger(t)\hat{a}(t)+\tfrac{1}{2}]+\frac{\hbar}{2}[w_2^*(t)\hat{a}^{\dagger 2}(t)+\text{H.c.}]. \quad (4)$$

The time-dependent frequencies $w_{1,2}(t)$ can be derived directly from Eqs. (2), (3), and (4), while $\rho(t)$ is an auxiliary real function. The significance of $\rho(t)$ is discussed within the theory of the exact invariants for time-dependent harmonic oscillators as it is reviewed, e.g., in [9]. From the Heisenberg invariance condition, $[\hat{a}^\dagger(t)\hat{a}(t)+\frac{1}{2}]$ is an invariant for $\hat{H}(t)$ when $\rho(t)$ satisfies the nonlinear differential equation,

$$\ddot{\rho}(t)+\Omega^2(t)\rho(t)=\rho^{-3}(t). \quad (5)$$

With the initial conditions $\dot{\rho}(t_0)=0$ and $\rho(t_0)=\Omega_0^{-1/2}=\varepsilon_0^{1/4}/\omega^{1/2}$, the invariant represents the initial Hamiltonian of the system. Here $\varepsilon_0\equiv\varepsilon(t_0)$ and $\Omega_0\equiv\Omega(t_0)$ denote, respectively, the unperturbed values of the permittivity and cavity frequency. From the new representation (4), it is clear that a change in the permittivity of the medium originates time-dependent two-photon processes characterized by the term $\hat{a}^{\dagger 2}(t)$ and dependent on nonvanishing values of $\nu(t)$. These processes are responsible for distortions of the field quantum fluctuations.

The Bogolubov transformation with time-dependent coefficients in Eq. (3) leaves the commutator invariant since $|\mu(t)|^2-|\nu(t)|^2=1$. Under this condition such a transformation can be implemented by the unitary operator

$$\hat{U}^\dagger(t,t_0)=e^{i\varphi_\mu(t)\hat{a}^\dagger(t_0)\hat{a}(t_0)}\exp[e^{-i\delta\varphi(t)/2}\times\cosh^{-1}|\mu(t)|\hat{a}^{\dagger 2}(t_0)-\text{H.c.}], \quad (6)$$

which transforms unperturbed energy eigenstates $|n,t_0\rangle$ into time-dependent ones with the same eigenvalues according to $|n,t\rangle=\hat{U}(t,t_0)|n,t_0\rangle$ (Von Neumann theorem). The coefficients $\mu(t)\equiv|\mu(t)|e^{i\varphi_\mu(t)}$ and $\nu(t)\equiv|\nu(t)|e^{i\varphi_\nu(t)}$ have been separated into their time-dependent magnitudes and phases with $\delta\varphi(t)=\varphi_\mu(t)-\varphi_\nu(t)$.

We shall now adopt the Lewis and Riesenfeld approach to construct the exact time-dependent photon states for our specific system. According to their pioneering work [9], for a radiation field characterized by the explicit time-dependent Hamiltonian (2) and a Hermitian invariant, the general state of the field at time t can be expanded in terms of the invariant eigenfunctions $|n,t\rangle$ as

$$|\psi,t\rangle=\sum_n c_n e^{i\alpha_n(t)}|n,t\rangle,$$

where

$$\alpha_n(t)=-(n+\tfrac{1}{2})\int_{t_0}^{t}dt'\rho^{-2}(t') \quad (7)$$

is the *Lewis phase*, and the c_n's, are arbitrary complex constants fixed by the initial conditions on the field. We will be interested, in particular, in the evolution of the phase of the state of the field: the overall change of phase acquired during the evolution of $|\psi,t_0\rangle$ onto $|\psi,t\rangle$ is obtained from the overlap between these two states. For a cavity field initially in the number state $|n_0,t_0\rangle$ ($c_n=\delta_{n,n_0}$) this overlap can be evaluated with the help of Eqs. (6 and 7) to yield

$$_{n_0}\langle\psi,t|\psi,t_0\rangle_{n_0}=A_{n_0}(t)e^{-i\alpha_{n_0}(t)+in_0\varphi_\mu(t)}\equiv A_{n_0}(t)e^{-i\chi_{n_0}(t)} \quad \text{(number state)}. \quad (8)$$

Using this result and a generating function technique we can then derive the overlap for an initial coherent state ($c_n=\beta^n\exp[-|\beta|^2/2]/\sqrt{n!}$), i.e., as well

$$_\beta\langle\psi,t|\psi,t_0\rangle_\beta=A_\beta(t)\exp[-i\alpha_0(t)-i|\beta|^2\sin 2\alpha_0(t)]\times\exp\left(+i\frac{|\beta|^2|\nu(t)|}{2|\mu(t)|}\{\sin\delta\varphi(t)+\sin[4\alpha_0(t)+\delta\varphi(t)]\}\right)\equiv A_\beta(t)\exp\{-i[\chi_\beta^C(t)+\chi_\beta^{\text{NC}}(\nu,t)]\}\equiv A_\beta(t)e^{-i\chi_\beta(t)} \quad \text{(coherent state)}. \quad (9)$$

The complex amplitude $\beta=|\beta|e^{i\phi_\beta}$ contains the initial phase ϕ_β of the field, which is taken to be equal to π, and the initial average number of photons $|\beta|^2$ proportional to the field intensity. The explicit expression for the real amplitudes $A_{n_0}(t)$ and $A_\beta(t)$ will not be given here. When no change of permittivity takes place $[\varepsilon(t)\equiv\varepsilon_0]$ Eqs. (8) and (9) reduce to known results [10]. When a change occurs, instead, the first two terms of the phase mismatch for a coherent state, i.e., $\chi_\beta^C(t)$, describe an always present standard contribution arising from the evolution of the field in a nonstationary medium. This classical contribution to the mismatch $\chi_\beta(t)$ only depends on the Lewis phase for a fixed field intensity. There is, in addition, the term $\chi_\beta^{\text{NC}}(\nu,t)$ proportional to ν, which becomes important for suitable modulation parameters of the dielectric cavity as we will demonstrate later. Physically, this last term arises from distortions of the quantum fluctuations due to the transient excitation of a two-photon state of the field [cf. Eq. (3)]. In this sense, we regard $\chi_\beta^{\text{NC}}(\nu,t)$ as a nonclassical contribution to the mismatch $\chi_\beta(t)$. An analogous term does not appear in $\chi_{n_0}(t)$ since in the adiabatic limit $\varphi_\mu(t)$ is very very small and the Lewis phase is essentially the only significant contribution to the mismatch for a number state. The most interesting feature is that the nonclassical component $\chi_\beta^{\text{NC}}(\nu,t)$ can be separated out from the rest, and this will be illustrated with the following example.

Consider a time variation of the permittivity in the form (Fig. 1 inset)

$$\varepsilon(t)=\varepsilon_0\left\{1-s\cosh^{-2}\left[2\pi\left(\frac{t}{\tau}-\frac{1}{2}\right)\right]\right\}\left(s\equiv 1-\frac{\varepsilon_m}{\varepsilon_0}\right), \quad (10)$$

where $S\equiv 1-(\varepsilon_m/\varepsilon_0)$. Here τ denotes the length of a full modulation of the dielectric, while s denotes the maximum relative change of its permittivity between the initial and minimum value ε_0 and ε_m, respectively. The phase difference $\chi_\beta(t)$ accumulated by a coherent state at some time t is completely determined by the form of $\rho(t)$: In the adiabatic limit, from Eq. (5), one has $\rho_{\text{ad}}(t)=\Omega^{-1/2}(t)=\varepsilon^{1/4}(t)/\omega^{1/2}$. With the help of Eqs. (3) and (7) the coefficients of the Bogolubov transformation and the Lewis phase α_0 are

$$|\nu(t)|=\frac{1}{2}\left\{\frac{[\varepsilon_0^{1/2}-\varepsilon^{1/2}(t)]^2}{\varepsilon_0^{1/2}\varepsilon^{1/2}(t)}+\left(\frac{\pi s}{\Omega_0\tau}\right)^2\times\frac{\varepsilon_0}{\varepsilon^{3/2}(t)}\frac{\sinh^2\left[2\pi\left(\frac{t}{\tau}-\frac{1}{2}\right)\right]}{\cosh^6\left[2\pi\left(\frac{t}{\tau}-\frac{1}{2}\right)\right]}\right\}^{1/2}=\sqrt{|\mu(t)|^2-1}, \tag{11}$$

$$\left\{\begin{matrix}\varphi_\mu(t)\\ \varphi_\nu(t)\end{matrix}\right\}=-\arctan\left\{\frac{\pi s}{\Omega_0\tau}\frac{\varepsilon_0\varepsilon^{-1/2}(t)}{[\varepsilon_0^{1/2}\pm\varepsilon^{1/2}(t)]}\times\frac{\sinh\left[2\pi\left(\frac{t}{\tau}-\frac{1}{2}\right)\right]}{\cosh^3\left[2\pi\left(\frac{t}{\tau}-\frac{1}{2}\right)\right]}\right\}, \tag{12}$$

and

$$\alpha_0(t)=-\frac{\Omega_0\tau}{4\pi}\ln\left[z_2+\sqrt{\frac{\varepsilon_m}{\varepsilon_0}+z_2^2}\bigg/ z_1+\sqrt{\frac{\varepsilon_m}{\varepsilon_0}+z_1^2}\right], \tag{13}$$

where $z_1=\sinh[2\pi t_0\tau^{-1}-\pi]$ and $z_2=\sinh[2\pi t\tau^{-1}-\pi]$. With the help of these results an analytical expression for $\chi_\beta(t)$ in Eq. (9) is obtained that is most conveniently plotted in Fig. 1. The black curve in the upper frame represents the whole shift $\chi_\beta(t)$ over an interval of time τ while the gray curve represents its *classical* component $\chi_\beta^C(t)$. The curve A in the lower frame, given by the difference of the previous two curves, represents instead $\chi_\beta^{NC}(\nu,t)$. It is important to observe that this *nonclassical* contribution is substantial for time scales τ of the order of those associated with the photon cavity frequency and for large relative changes of the dielectric permittivity. For slower time modulations and not as big changes in the permittivity, $\chi_\beta^{NC}(\nu,t)$ becomes smaller and smaller and this is illustrated by the examples (B-C-D) in the lower frame of Fig. 1. Steep enough modulation depths and fast enough modulation speeds of the refractive index can boost quantum fluctuations into real photons whose effect is memorized through an additional (quantum) contribution to the characteristic (classical) phase associated with the evolution of the field in a nonstationary dielectric.

The nonclassical component of $\chi_\beta(t)$ could, in principle, be detected by an interference experiment that would produce at time t a superposition between a coherent state of the field propagating through the modulated dielectric of the cavity and a reference state. One could, e.g., split a coherent light beam in two, pass one of them through free space (*out*) and the other through the medium (*in*) while this is being modulated, and combine them again. One possible arrangement is sketched in the inset of Fig. 2. The out-part of the incident beam propagates in the free space outside the cavity while the in-part propagates through the resonant cavity

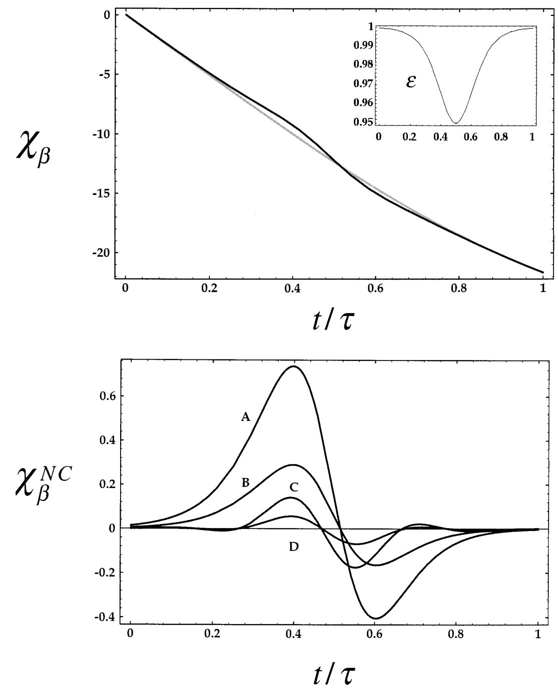

FIG. 1. Phase shift χ_β (solid curve) acquired by a coherent state of the field as it propagates through a nonstationary dielectric cavity (upper frame). The cavity unperturbed frequency Ω_0 and the duration τ of the modulation of the dielectric are such that $\Omega_0\tau=1$. The average number of photons in the field is $|\beta|^2=25$ with a maximum 5% decrease of the permittivity from an initial nominal value $\varepsilon_0=1$ (inset). The shift χ_β comprises a *classical* (gray curve) and a *nonclassical* component χ_β^{NC}, which is plotted separately in the lower frame for (A) $\Delta\varepsilon/\varepsilon_0=5\%$, $\Omega_0\tau=1$; (B) $\Delta\varepsilon/\varepsilon_0=2\%$, $\Omega_0\tau=1$; (C) $\Delta\varepsilon/\varepsilon_0=5\%$, $\Omega_0\tau=10$; and (D) $\Delta\varepsilon/\varepsilon_0=2\%$, $\Omega_0\tau=10$.

filled with a linear and lossless dielectric that has an externally prescribed time-varying permittivity.

There are a number of intriguing suggestions by which sufficiently large rates of change of the refractive index could be achieved, namely, in a rapidly growing plasma produced by sudden gas ionization [2] or in the transparent region of a semiconductor slab in a regime of virtual photoconductivity [3]. Indeed the photoionization of gas by short optical pulses may turn the gas in a cell into a plasma with a substantial sudden drop of the index of refraction below unit [11,12]. Likewise in a semiconductor slab, the excitation of electron-hole pairs by subpicosecond optical pulses can make its refractive index drop far below its static value in a very brief period of time [3,12]. Newly developed techniques for the ultrafast optical excitation of solids are also promising. These allow for the generation of coherent phonon oscillations that modulate the dielectric function of various types of semiconductors on THz time scales suggesting a sound scheme for attaining rapid changes of the refractive index in dielectric materials [13]. Current far-infrared cavity technology and the latest developments in subpicosecond semiconductor pumping could well provide the dielectric modulation depths and speeds required to observe an appreciable effect.

The relative phase difference $\Delta\Phi(t)$ between the phase $\chi_\beta^{in}(t)$ and $\chi_\beta^{out}(t)$ accumulated by the states of the electromagnetic field in the two portions of the incident beam between t_0 and t can be retrieved from the beating signal at the

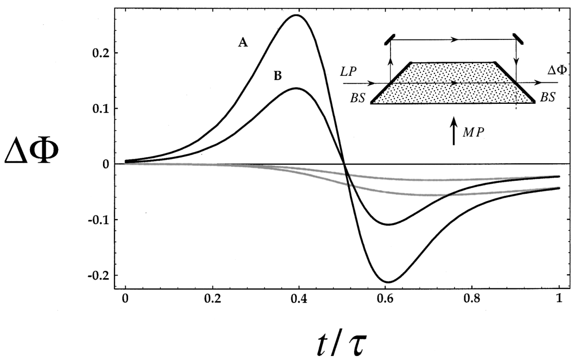

FIG. 2. Interference shift $\Delta\Phi$ [Eq. (14)] (solid curve) and its classical component $\Delta\Phi^C$ (gray curve) when the field average number of photons is (A) $|\beta|^2=50$ and (B) $|\beta|^2=25$. All curves correspond to $\Delta\varepsilon/\varepsilon_0=1\%$ and $\Omega_0\tau=1$. No modulation $[\varepsilon(t)\equiv\varepsilon_0\to 1]$ would result into an identically vanishing shift. Schematic interferometric setup (inset) for the measurement of the shift $\Delta\Phi$. The *in* and *out* parts of the coherent laser pump beam (LP) are divided and then brought together by the two beam splitters (BS) placed at both side ends of the dielectric cavity. The synchronous modulation of the material inside the cavity is realized by an external pump (MP) impinging on the open side end of the cavity.

output port of the cavity where the two portions of the beam recombine. The shift can be expressed as

$$\begin{aligned}\Delta\Phi(t)\cong{}&\alpha_0^{\rm in}(t)-\alpha_0^{\rm out}(t)+2|\beta|^2\cos[\alpha_0^{\rm in}(t)+\alpha_0^{\rm out}(t)]\\&\times\sin[\alpha_0^{\rm in}(t)-\alpha_0^{\rm out}(t)]+|\nu(t)|\,\frac{|\beta|^2}{2}\\&\times\{\sin\varphi_\nu(t)+\sin[\varphi_\nu(t)-4\alpha_0^{\rm in}(t)]\}\\\equiv{}&\Delta\Phi^C(t)+\chi_\beta^{\rm NC}(\nu,t),\end{aligned}\tag{14}$$

where the initial and final times t_0 and t are determined by the synchronous modulation of the material inside the cavity. One attempt to retrieve the nonclassical term $\chi_\beta^{\rm NC}(\nu,t)$ consists of focusing on the dispersion that it induces on the evolution of $\Delta\Phi(t)$ when the intensity is held fixed [14]. For a suitable modulation speed τ of the refractive index, a series of measurements of $\Delta\Phi(t)$ at various times t could resolve its dispersive shape even when the modulation depth s is quite small. This is shown in Fig. 2: each point on the black curves corresponds to a single measurement. The dispersion, produced by a sign flip of $\Delta\Phi(t)$ about $\tau/2$, is a signature of the nonclassical component of $\chi_\beta(t)$. In fact, for the system parameters used here $\alpha_0^{\rm out}(t)>\alpha_0^{\rm in}(t)$ and $\Delta\Phi^C(t)$ (gray curves) is negative and essentially monotonic [15]; yet, $\Delta\Phi(t)$ (black curves) undergoes a substantial jump about $\tau/2$ because in this case the magnitude of $\nu(t)$, i.e., that of $\chi_\beta^{\rm NC}(\nu,t)$, is non-negligible and the phase of $\nu(t)$ is antisymmetric around this point. The asymmetry of $\varphi_\nu(t)$ originates from the change in the slope of the permittivity about its minimum point. More importantly, it is crucial to observe that for appropriate intensities the phase $\chi_\beta^{\rm NC}(\nu,t)$ could be made large enough to be measurable. Figure 2 shows the enhancement of this nonclassical shift with increasing intensities, even for the rather small modulation depth of the permittivity considered in this case.

The realization of large and fast changes in the refractive index of a material able to produce sizeable distortions in the quantum fluctuations of the electromagnetic field is still an open problem and a major challenge [12]. We show that a suitable modulation of the refractive index can produce distortions of the field quantum fluctuations, the effects of which are memorized through a nonclassical contribution to the phase of a light beam passing through the medium. We suggest an interference scheme for the measurement of this nonclassical shift which we anticipate to be appreciable even for rather modest changes of the refractive index.

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[14] A more straightforward attempt to retrieve $\chi_\beta^{\rm NC}$ consists in measuring the shift (14) for various field intensities over a fixed time interval $[t_0,\tilde{t}]$ with $\tilde{t}<\tau$. This would yield the Lewis phase $\tilde{\alpha}_0^{\rm in}$ ($\tilde{\alpha}_0^{\rm out}$ is known). The slope of $\Delta\tilde{\Phi}$ vs $|\beta|^2$ that one would measure in the absence of $\tilde{\chi}_\beta^{\rm NC}(\nu)$ could then be inferred and departures from its actual measured value would then be a signature of the nonclassical component of $\tilde{\chi}_\beta$. This procedure is not entirely satisfactory, however, since it requires the knowledge of $\tilde{\alpha}_0^{\rm out}$ which has to be provided by the theory in this case.

[15] Values of $\tau\gg\Omega_0^{-1}$ (same s) would also make $\Delta\Phi(t)$ oscillatory due to large deviations of $\alpha_0^{\rm in}(t)$ from $\alpha_0^{\rm out}(t)$: in this case $\Delta\Phi(t)\cong\Delta\Phi^C(t)$ and the dispersion of the interference shift would not be a signature of $\chi_\beta^{\rm NC}(\nu,t)$.