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Non-supersymmetric tachyon-free type-II and type-I closed strings from RCFT

B. Gato-Rivera[a,b,1] and A.N. Schellekens[a,b,c]

[a] *NIKHEF Theory Group, Kruislaan 409, 1098 SJ Amsterdam, The Netherlands*

[b] *Instituto de Matemáticas y Física Fundamental, CSIC, Serrano 123, Madrid 28006, Spain*

[c] *IMAPP, Radboud Universiteit, Nijmegen*

Abstract

We consider non-supersymmetric four-dimensional closed string theories constructed out of tensor products of $N = 2$ minimal models. Generically such theories have closed string tachyons, but these may be removed either by choosing a non-trivial partition function or a suitable Klein bottle projection. We find large numbers of examples of both types.

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[1] Also known as B. Gato

Supersymmetry is invaluable as a calculational tool in field theory and string theory, but it is still not clear whether nature makes use of this virtue. Interesting empirical evidence is the apparent convergence of the gauge couplings if gauginos and higgsinos are taken into account in their running. Additional circumstantial evidence is the fact that low energy supersymmetry automatically produces roughly the required amount of dark matter. On the other hand, the usual argument that supersymmetry is "required to stabilize the gauge hierarchy" is in need of reassessment in view of the string theory landscape. If the cosmological constant does not require any symmetry to "remain" small, why would there be such a requirement for the much smaller gauge hierarchy?

On the other hand, one might argue that supersymmetry is an essential ingredient of string theory. However, the main ingredient of one-loop finiteness of closed strings is not supersymmetry but modular invariance. Supersymmetry is sufficient (but not necessary) to remove tachyons and the resulting divergence. It has been known since 1986 that one-loop finite, tachyon-free non-supersymmetric heterotic strings can easily be built in ten [1][2] as well as in four [3][4] dimensions. Despite these examples, there is a surprisingly widespread belief that absence of tachyons in string theory requires supersymmetry. Another misconception we recently learned about, and that finds its origin in lack of familiarity with the early literature, is that string theory was once believed to predict only negative cosmological constants, and that the positive observational result led to a re-consideration. In fact the issue of the cosmological constant in non-supersymmetric (and of course tachyon-free) string theories was studied in several papers following the work of G. Moore on "Atkin-Lehner" symmetry [5], and examples with both signs of the cosmological constant were found, for example in [6]. Non-supersymmetric String Theories are usually divergent at higher loops (see however [7] for a set of exceptions and [8] for further discussion), but these divergencies can be attributed to massless dilatons, which are unacceptable anyway. Of course we are aware of the problems one encounters if one leaves the supersymmetry highway at an earlier exit, but all of those problems have to be confronted at some stage anyway, and the scenery may well be worth the price.

At present there is a huge literature on non-supersymmetric string constructions and their implications, although this is still dwarfed by the amount of work on supersymmetric constructions. An essential issue is the appearance of tachyons, and many mechanisms have been proposed for getting rid of them [7][9]. In this paper we will construct non-supersymmetric tachyon-free type-II and type-I closed strings from tensor products of $N = 2$ minimal models. The models we consider are all obtained by tensoring M minimal $N = 2$ models with a four-dimensional NSR sector, and imposing world-sheet supersymmetry. The latter is, as always, done by extending the chiral algebra with all pairs of world-sheet superfields ("alignment currents") of the $M + 1$ factors. Note that since we wish to construct fermionic strings, world-sheet supersymmetry is essential. However, this can also be achieved using $N = 1$ building blocks. The reason we limit ourselves to $N = 2$ here is simply that most of the necessary algorithms are already directly available and optimized, because of previous work in supersymmetric strings [10].

The resulting theory $\mathcal{G}$ has a chiral algebra consisting of the separate $N = 2$ algebras of the building blocks, extended by the alignment currents. This theory is tachyonic. On top of this we may add the following algebraic structures:

1. A (further) extension $\mathcal{E}$ of the chiral algebra.

2. A modular invariant partition function (MIPF), denoted $\mathcal{M}$.

3. An orientifold choice, denoted $\mathcal{I}$.

Note that a MIPF can be of automorphism or extension type, or combinations thereof. Even if it is purely of extension type, it still plays a different role than $\mathcal{E}$. The extension $\mathcal{E}$ defines the CFT used in the rest of the construction, in the sense that only primaries, characters, O-planes (and eventually boundary states) will be used that respect all of the symmetries of that extension. The extension in the MIPF projects out closed string states that are non-local with respect to it, but O-planes and boundary states that violate it are still permitted. Hence the symmetry is respected in the bulk, but not necessarily by boundaries and crosscaps. The introduction of orientifold planes is a first step towards type-I strings, of which we only consider the closed sector here. These unoriented closed string theories will in general have tadpoles due to closed string one-point functions on the crosscap. In some cases, these tadpoles may be cancelled by introducing non-vanishing boundary state (Chan-Paton) multiplicities, but that possibility will not be investigated here. Our main interest is to find out if it is possible to obtain a closed string sector that is completely free of tachyons, and how often that occurs.

In principle, all three kinds of algebraic modifications listed above might remove the tachyons. An obvious way to get rid of all the tachyons is to choose a space-time supersymmetric chiral algebra extension $\mathcal{E}$. In general, there are many of these, but they can all be obtained as a basic extension by a spin-1 operator related to the space-time supercurrent, and further extensions on top of that. This is guaranteed to work, but will produce supersymmetric strings and therefore not of interest here. Hence we will choose a non-supersymmetric $\mathcal{E}$. This leaves us with four distinct possibilities for removing the tachyons:

- 1. The action of the non-supersymmetric $\mathcal{E}$ itself.

- 2a. A MIPF including a space-time supersymmetric extension.

- 2b. A non-supersymmetric MIPF.

- 3. An orientifold choice.

These four possibilities are the subject of our present investigation. Note that tachyon-free models in class 2a are guaranteed to exist, but unlike the supersymmetric $\mathcal{E}$ case, they do not lead automatically to supersymmetric open strings, and have not been considered before in this context. This case will enable us to study type-I models with supersymmery in the bulk, but not on the branes.

The total number of combinations $\mathcal{G}$, $\mathcal{E}$, $\mathcal{M}$ and $\mathcal{I}$ at our disposal is gigantic. The number of choices for $\mathcal{G}$ is 168 (and larger still if we were to allow $N = 1$ building blocks).

The number of choices for $\mathcal{E}$ is typically of order 100 to 1000, of which just one has been considered in [10], where a search was done for standard-model-like supersymmetric open string spectra. For each $\mathcal{E}$, the number of MIPFs $\mathcal{M}$ is of order 10 to 1000, and for each $\mathcal{M}$ the number of $\mathcal{I}$ is of order 10. We limit ourselves here to simple current extensions, MIPFs and orientifolds, which are the only ones for which we have the required calculational tools available [11]. Furthermore we limit ourselves to left-right symmetric MIPFs for the same reason. Hence the oriented theories we get are of type IIB.

In addition, some practical limits must be imposed to make this project manageable. In [10], the supersymmetry extension $\mathcal{E}$ plays several rôles: it automatically removes tachyons, simplifies the tadpole conditions by combining NS and R tadpoles, drastically reduces the number of primaries and boundary states to a reasonably sized set, and last (and perhaps also least), it gives rise to a phenomenologically interesting supersymmetric spectrum. Of these four features, the first, second and last are inevitably lost in the present setting, but the third is essential if we ever want to explore standard-model-like boundary state combinations. For example, the supersymmetric extension of the tensor product $(3,3,3,3,3)$ (related to the "Quintic") has 4000 primaries, but without that extension it has 400000; for the tensor product $(6,6,6,6)$ these numbers are respectively 9632 and 2458624. In the latter case the corresponding BCFT has 2458624 boundaries, for the charge conjugation MIPF, out of which one has to choose three or four to make a standard model configuration. For this reason we consider here only extensions that yield at most 4000 primaries. Even at this stage, without considering boundary states, a larger number is hard to deal with because the number of candidate tachyons as well as the number of MIPFs tends to increase with the number of primaries.

Another issue that we have to deal with is that of permutation symmetry of identical $N=2$ factors in a given tensor product and outer automorphisms thereof. Each $N=2$ factor has an outer automorphism, namely charge conjugation. If these symmetries survive the extension $\mathcal{E}$, then they act on the MIPFs and relate them to each other, as $M \rightarrow P^{-1}MP$, where P is the symmetry and M the modular matrix. MIPFs related in this way give rise to the same physics, and therefore should be identified. The supersymmetry extension has the advantage of removing the outer automorphisms, because the separate outer automorphisms of each factor act on the corresponding component of the supercurrent, changing it from $(S,\ldots,S,S,S,\ldots,S)$ to $(S,\ldots,S,S',S,\ldots,S)$. Here S and S' are the two mutually conjugate Ramond simple currents of each factor, with conformal weight $c/24$. These current combinations are always non-local with respect to each other for any number of S', except if all components are flipped simultaneously. But the product of all outer automorphisms corresponds to charge conjugation in the full theory, and charge conjugation acts trivially on the MIPF: $CMC = M$. This leaves the permutation symmetry, which is never broken by the supersymmetric extension, and can be at most the permutation group of 9 elements for the tensor product $(1)^9$. This can still be handled, with some difficulty. In the non-supersymmetric case the situation is more complicated. First of all we have to mod-out the action of all these symmetries on the extensions. For the supercurrent this is easy: clearly $(S,\ldots,S,S,S,\ldots,S)$ is equivalent to all other choices obtained by replacing any S by S'. Once we have selected an extension we have to consider the surviving symmetry. Now, the extension can break the automorphisms as well as the permutation symmetry, but in the worst case none of them is broken, and we end up with a symmetry group of $2^9 \times 9!$ elements for the tensor product $(1)^9$.

As was mentioned before, the total number of tensor combinations is 168. We have scanned all these models for solutions of type 1, 2a, 2b and 3, and no solutions of type 1 were found. In other words, the only way to get rid of tachyons by means of a pure extension $\mathcal{E}$ is to choose the supersymmetric extension. Regarding solutions of the other three types, we will not list all the ones of type 2a, since their existence is essentially automatic: any simple current that is local with respect to the supercurrent $(S, \ldots, S, S, S, \ldots, S)$ can be used to generate a non-supersymmetric extension $\mathcal{E}$ that allows a supersymmetric MIPF. Usually such currents exist, although they do not always reduce the number of primaries below our practical upper limit of 4000. The existence of solutions of type 2b and 3, on the other hand, is non-trivial. We present our results for these cases in the table. The organization of this table is as follows. Column 1 lists, in an obvious notation, the tensor product of the N=2 minimal models. Column 2 lists the total number of extensions, including the supersymmetric ones, and column 3 lists the number of extensions which are non-supersymmetric and have 4000 or fewer primaries. These are the ones considered in this paper. Column 4 lists the total number of MIPFs for the extensions considered. The last two columns specify the number of tachyon-free models of oriented and unoriented type. Obviously the oriented ones may still be orientifold projected, and since they are already tachyon-free, all their orientifolds will be tachyon-free as well. Hence the total number of closed sectors of type-I strings consists of the numbers in column 6, plus the ones in column 5 multiplied with the number of orientifolds.

The reason that the last two columns contain four entries is as follows. Apart from supersymmetric extensions of the chiral algebra or in the MIPF, one may also encounter extensions that correspond to an embedding of the NSR fermions in more than 4 dimensions. Note that in a type-IIB theory, N=4 space-time supersymmetry implies such an extension to D=6, whereas N=8 implies an extension to D=10. In non-supersymmetric type-IIB string theories, extensions to 6, 8 and 10 dimensions may occur, and in a few cases even to 12 and 14 dimensions. In the table we have indicated this by giving in each field in the last two columns four numbers, indicating respectively the number of cases with an NSR sector embedded in 4, 6, 8 and 10 dimensions. The larger dimensions occur very rarely (D=12 for the tensor product $1^5 4^2$, and D=12 and D=14 for $1^7 4$ and 1^9). Note that the bosons X^μ describe a four-dimensional target space in all these cases. We do not distinguish between higher dimensional embeddings for the extensions $\mathcal{E}$ or the MIPFs in the table. Both occur, but only the latter are of interest for future purposes, namely finding chiral open string spectra: if the extension $\mathcal{E}$ is higher-dimensional all spectra will be automatically non-chiral.

There is some small overcounting for a few of the free-field based models, namely tensor products containing one or more factors $k = 1$ or $k = 2$. This happens because not all permutation symmetries were taken into account for the tensor product 1^9, and also because in a few other cases there are some degeneracies that are particular for free-field models and that have not been taken into account. The table only contains tensor products for which non-supersymmetric tachyon-free spectra were found corresponding to solutions of type 2b and 3. Tensor products with high values of the $N = 2$ minimal model parameter k are absent mainly because their extensions violate our upper limit of 4000 on the number of primaries.

In conclusion: we have compiled a large database of non-supersymmetric tachyon-free type-IIB and type-I closed strings. We think that the existence of these theories is of interest in its own right. Furthermore, in the future these theories will serve as a starting point to study non-supersymmetric tachyon-free open string theories, as has been done for the supersymmetric open string models in [10] and [12]. The hope would be to find non-supersymmetric realizations of the standard model spectrum. This is still a huge challenge, because in addition to the closed string tachyons, we will have to deal with open string tachyons as well as separate Neveu-Schwarz and Ramond tadpoles. However, the fact that the non-supersymmetric, non-tachyonic closed string database is so huge may give us a chance to achieve that goal.

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Table 1: *Summary of tachyon-free solutions of type 2b and 3*

Tensor	Ext.	<= 4000	MIPFs	Oriented	Unoriented
(1,10,22,22)	303	19	158	0,0,0,0	1,0,0,0
(2,6,8,38)	538	68	2384	0,0,0,0	4,0,0,0
(2,6,10,22)	1046	142	2982	0,0,0,0	2,0,0,0
(2,6,14,14)	733	158	5064	0,0,0,0	19,0,0,0
(2,10,10,10)	261	48	2088	0,0,0,0	6,0,0,0
(3,6,6,18)	88	16	188	0,0,0,0	1,0,0,0
(4,4,6,22)	398	46	1512	0,0,0,0	9,0,0,0
(4,4,8,13)	70	8	90	0,0,0,0	2,0,0,0
(4,4,10,10)	319	62	3238	0,0,0,0	79,0,0,0
(4,6,6,10)	378	95	3410	0,0,0,0	48,0,0,0
(6,6,6,6)	191	80	5254	74,0,0,0	238,0,0,0
(1,1,4,6,22)	316	85	3218	0,0,0,0	1,0,0,0
(1,1,4,7,16)	191	57	946	0,0,0,0	2,0,0,0
(1,1,4,10,10)	222	54	1748	0,0,0,0	12,0,0,0
(1,1,6,6,10)	88	26	1348	0,0,0,0	31,0,0,0
(1,2,2,7,16)	147	46	1376	0,0,0,0	4,0,0,0

Continued on next page

Table 1 – continued from previous page

Tensor	Ext.	<= 4000	MIPFs	Oriented	Unoriented
(1,2,2,10,10)	341	126	10180	22,4,0,0	198,0,0,0
(1,2,2,6,22)	768	245	17468	20,0,0,0	73,0,0,0
(1,2,4,4,10)	463	188	26508	4,0,0,0	340,0,0,0
(1,2,4,6,6)	374	178	24364	20,0,0,0	370,26,0,0
(1,4,4,4,4)	192	74	5292	68,0,0,0	241,14,0,0
(2,2,2,3,18)	216	88	6092	130,66,0,0	0,0,0,0
(2,2,2,4,10)	1133	557	223978	2264,520,0,0	6334,784,0,0
(2,2,2,6,6)	1155	644	271198	1808,356,0,0	8988,1256,0,0
(2,2,3,3,8)	63	26	816	0,0,0,0	4,0,0,0
(2,2,4,4,4)	333	130	33804	72,48,0,0	635,40,0,0
(3,3,3,3,3)	12	3	14	0,0,0,0	1,0,0,0
(1,1,1,1,5,40)	36	10	162	0,12,0,0	0,0,0,0
(1,1,1,1,7,16)	123	61	1160	15,16,0,0	0,0,0,0
(1,1,1,1,8,13)	36	12	186	0,6,0,0	0,0,0,0
(1,1,1,1,10,10)	78	29	1208	16,24,0,0	1,1,0,0
(1,1,1,1,6,22)	108	35	892	0,8,0,0	0,0,0,0
(1,1,1,2,4,10)	228	106	8888	16,24,0,0	39,3,0,0
(1,1,1,2,6,6)	88	43	3652	0,0,0,0	0,16,0,0
(1,1,1,4,4,4)	197	113	8534	430,95,0,0	395,78,0,0
(1,1,2,2,2,10)	216	100	16972	408,148,0,0	676,0,0,0
(1,1,2,2,4,4)	265	164	49008	160,120,0,0	396,172,0,0
(1,2,2,2,2,4)	546	403	388155	2912,1583,0,387	4180,1564,0,0
(2,2,2,2,2,2)	754	617	2112682	17680,12560,0,1942	105653,43836,6818,4202
(1,1,1,1,1,2,10)	56	31	2984	28,52,0,0	0,0,0,0
(1,1,1,1,1,4,4)	120	80	8668	270,200,26,0	97,86,0,0
(1,1,1,1,2,2,4)	126	82	12832	0,84,32,0	27,50,4,0
(1,1,1,2,2,2,2)	120	91	38228	0,448,0,186	0,416,0,0
(1,1,1,1,1,1,1,4)	60	41	4426	218,190,95,0	9,11,8,0
(1,1,1,1,1,1,2,2)	35	24	2838	0,18,24,0	0,0,0,0
(1,1,1,1,1,1,1,1,1)	289	202	161774	52058,17568,5359,0	41168,10292,3993,478

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