Vol. 8 (2015) *Acta Physica Polonica B Proceedings Supplement* No 3

GROUND-STATE PROPERTIES AND SYMMETRY ENERGY OF MG ISOTOPES WITH $A = 20$–36*

M.K. Gaidarov, A.N. Antonov

Institute for Nuclear Research and Nuclear Energy
Bulgarian Academy of Sciences
Sofia 1784, Bulgaria

P. Sarriguren

Instituto de Estructura de la Materia, IEM-CSIC
Serrano 123, 28006 Madrid, Spain

E. Moya de Guerra

Departamento de Fisica Atomica, Molecular y Nuclear
Facultad de Ciencias Fisicas, Universidad Complutense de Madrid
28040 Madrid, Spain

(Received November 10, 2015)

A comprehensive study of various ground-state properties of neutron-rich and neutron-deficient Mg isotopes with $A = 20$–36 is performed in the framework of the axially symmetric self-consistent Skyrme–Hartree–Fock plus BCS method. The correlation between the skin thickness and the characteristics related with the density dependence of the nuclear symmetry energy is investigated for the same isotopic chain following the theoretical approach based on the coherent density fluctuation model. The results of the calculations show that the behavior of the nuclear charge radii and the nuclear matter properties in the Mg isotopic chain is closely related with the nuclear deformation. We also study the emergence of an "island of inversion" at the neutron-rich ^{32}Mg nucleus proposed from the analyses of spectroscopic measurements of its low-lying energy spectrum and the charge r.m.s. radii of all magnesium isotopes in the *sd* shell.

DOI:10.5506/APhysPolBSupp.8.575
PACS numbers: 21.60.Jz, 21.65.Ef, 21.10.Gv, 27.30.+t

---

* Presented at the XXII Nuclear Physics Workshop "Marie and Pierre Curie", Kazimierz Dolny, Poland, September 22–27, 2015.

1. Introduction

Low-lying states of neutron-rich nuclei around the neutron number $N=20$ attract a great interest, as the spherical configurations associated with the magic number disappear in the ground states. For ^{32}Mg, from the observed population of the excited 0_2^+ state (found at 1.058 MeV) in the (t,p) reaction on ^{30}Mg, it is suggested [1] that the 0_2^+ state is a spherical one coexisting with the deformed ground state and that their relative energies are inverted at $N = 20$. Very recently, a new signature of an existence of "island of inversion" [2] has been experimentally tested by measuring the charge radii of all magnesium isotopes in the *sd* shell at ISOLDE-CERN [3] showing that the borderline of this island lies between ^{30}Mg and ^{31}Mg.

In the light of the new precise spectroscopic measurements of the neutron-rich ^{32}Mg nucleus, which lies in the much explored "island of inversion" at $N = 20$, in the present work (see also Ref. [4]), we aim to perform a systematic study of the nuclear ground-state properties of neutron-rich and neutron-deficient Mg isotopes with $A = 20$–36, such as charge and matter r.m.s. radii, two-neutron separation energies, neutron, proton, and charge density distributions, neutron (proton) r.m.s. radii and related with them thickness of the neutron (proton) skins. The new data for the charge r.m.s. radii [3] is a challenging issue to test the applicability of the mean-field description to light nuclei, thus expecting to understand in more details the nuclear structure revealed by them. The need of information for the symmetry energy in finite nuclei, even theoretically obtained, is a major issue because it allows one to constrain the bulk and surface properties of the nuclear energy-density functionals (EDFs) quite effectively. Therefore, following our recent works [5, 6], we analyze the correlation between the skin thickness and the characteristics related to the density dependence of the nuclear symmetry energy for the same Mg isotopic chain. Such an analysis may probe the accurate account for the effects of interactions in our method within the considered Mg chain, where the breakdown of the shell model could be revealed also by the nuclear symmetry energy changes. A special attention is paid to the neutron-rich ^{32}Mg nucleus by performing additional calculations modifying the spin-orbit strength of the effective interaction, to check theoretically the possible appearance of the "island of inversion" at $N = 20$.

2. Theoretical framework

The results of the present work have been obtained from self-consistent deformed Hartree–Fock calculations with density dependent Skyrme interactions [7] and pairing correlations. Pairing between alike nucleons has been included by solving the BCS equations at each iteration with a fixed pairing strength that reproduces the odd–even experimental mass differences [8]. We consider the Skyrme forces SLy4, Sk3, and SGII because they are among the most extensively used and are considered as standard references.

The spin-independent proton and neutron densities are given by [9, 10]

$$\rho\left(\vec{R}\right) = \rho(r,z) = \sum_i 2v_i^2 \rho_i(r,z)\,, \tag{1}$$

where r and z are the cylindrical coordinates of $\vec{R}$, v_i^2 are the occupation probabilities resulting from the BCS equations and ρ_i are the single-particle densities. The mean square radii for protons and neutrons are defined as

$$\left\langle r_{p,n}^2 \right\rangle = \frac{\int R^2 \rho_{p,n}\left(\vec{R}\right) d\vec{R}}{\int \rho_{p,n}\left(\vec{R}\right) d\vec{R}}\,, \tag{2}$$

and the r.m.s. radii for protons and neutrons are given by

$$r_{p,n} = \left\langle r_{p,n}^2 \right\rangle^{1/2}\,. \tag{3}$$

Having the neutron and proton r.m.s. radii [Eq. (3)], the neutron skin thickness is usually estimated as their difference

$$\Delta R = \left\langle r_n^2 \right\rangle^{1/2} - \left\langle r_p^2 \right\rangle^{1/2}\,. \tag{4}$$

The mean square radius of the charge distribution in a nucleus can be expressed as

$$\left\langle r_{\rm ch}^2 \right\rangle = \left\langle r_p^2 \right\rangle + \left\langle r_{\rm ch}^2 \right\rangle_p + (N/Z)\left\langle r_{\rm ch}^2 \right\rangle_n + r_{\rm CM}^2\,, \tag{5}$$

where $\langle r_p^2 \rangle$ is the mean square radius of the point proton distribution in the nucleus (2), $\langle r_{\rm ch}^2 \rangle_p = 0.80$ fm^2 [11] and $\langle r_{\rm ch}^2 \rangle_n = -0.12$ fm^2 [12] are the mean square charge radii of the charge distributions in a proton and a neutron, respectively, and $r_{\rm CM}^2$ is a small correction due to the center-of-mass motion [13]. Correspondingly, we define the charge r.m.s. radius $r_{\rm c} = \langle r_{\rm ch}^2 \rangle^{1/2}$.

We calculate the symmetry energy s and the neutron pressure p_0 of Mg isotopes using the coherent density fluctuation model (CDFM) [14, 15]. The CDFM makes the transition from the properties of nuclear matter to the properties of finite nuclei. The analysis of the latter has been carried out on the basis of the Brückner EDF for infinite nuclear matter [16, 17]. It can be shown in the CDFM that under some approximation the properties of finite nuclei can be calculated using the corresponding ones for asymmetric nuclear matter (ANM), folding them with the weight function $|f(x)|^2$. The latter can be obtained by using a known density distribution for a given nucleus (see, *e.g.* [4]). In the CDFM, the symmetry energy for finite nuclei and related with it pressure are given by

$$s = \int_0^\infty dx |f(x)|^2 s^{\mathrm{ANM}}(x)\,, \tag{6}$$

$$p_0 = \int_0^\infty dx |f(x)|^2 p_0^{\mathrm{ANM}}(x)\,. \tag{7}$$

3. Results and discussion

Here, we present results for some of the nuclear ground-state properties examined for Mg isotopes. The charge radius is related to the deformation and the isotope shifts of charge radii can be used to investigate the deformations in the isotopic chains. Our results for the squared charge radii differences $\delta\langle r_{\mathrm{c}}^2\rangle^{26,A} = \langle r_{\mathrm{c}}^2\rangle^A - \langle r_{\mathrm{c}}^2\rangle^{26}$ taking the radius of ^{26}Mg as the reference are compared in Fig. 1 with the experimental data [3]. In general, different Skyrme forces do not differ much in their predictions of charge r.m.s. radii of magnesium spanning the complete *sd* shell. The trend of the behavior of the experimental points and theoretical values strongly corresponds to the neutron shell structure. For $^{21-26}$Mg isotopes, the charge distribution is compressed due to the filling of the $d_{5/2}$ orbital and the charge radii do not fluctuate too much. The addition of more neutrons on either $s_{1/2}$ or $d_{3/2}$ in the range of $^{28-30}$Mg results in a fast increase of the radius. Finally, for

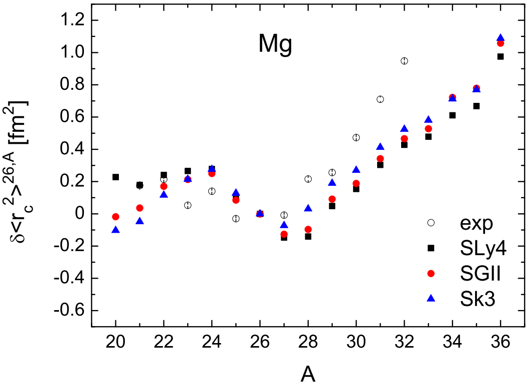

Fig. 1. Theoretical (with different Skyrme forces) and experimental [3] isotope shifts $\delta\langle r_{\mathrm{c}}^2\rangle$ of magnesium isotopes relative to ^{26}Mg.

isotopes beyond ^{30}Mg, where the "island of inversion" does exist in terms of the r.m.s. charge radius [3], the theoretical results underestimate clearly the experimental points. Obviously, an additional treatment is needed to understand in more details this specific region. We note the intermediate position of ^{27}Mg, where a minimum is observed in Fig. 1, since one of the neutrons added to ^{25}Mg fills the last $d_{5/2}$ hole and the other one populates the $s_{1/2}$ subshell.

As is well known, the spin-orbit interaction and the pairing correlations have influence on the deformation of nuclei. Therefore, we perform additional calculations for the ^{32}Mg nucleus by increasing the spin-orbit strength of the SLy4 effective interaction by 20%. The corresponding potential energy curve is illustrated in Fig. 2 together with the curve from the original SLy4 interaction leading to a spherical equilibrium shape in ^{32}Mg. As a result, we find strong prolate deformation for the intruder configuration ($\beta = 0.38$). This value of the quadrupole deformation is close to the value $\beta = 0.32$ found for the generator coordinate in Ref. [3], where a slight modification of the spin-orbit strength of the effective interaction for a better description of the "island of inversion" was also applied.

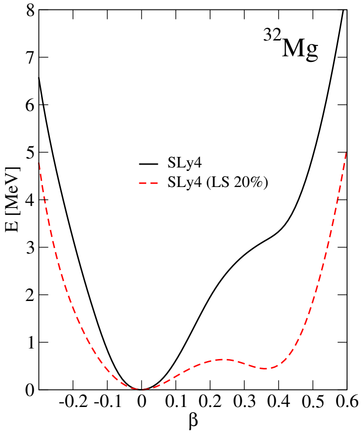

Fig. 2. (Color on-line) Potential energy curves of ^{32}Mg obtained from HF+BCS calculations with SLy4 force for the spherical case (solid/black line) and in the case when the spin-orbit strength of the effective SLy4 interaction is increased by 20% (dashed/red line).

The impact of these new modified calculations on the evolution of the charge radii, especially in the region of the Mg isotopic chain where an "island of inversion" is expected, is illustrated in Fig. 3. In addition to ^{32}Mg, we apply the same procedure also to ^{31}Mg nucleus in order to better establish the border of the island. A further increase of the charge radii of these isotopes is found. For ^{31}Mg, the charge r.m.s. radius increases from 3.117 fm to 3.154 fm and for ^{32}Mg, from 3.137 fm to 3.179 fm toward the experimentally extracted values for both nuclei indicated in Fig. 3. In general, it can be seen from Fig. 3 that the comparison between the new values that are very close to the experimental data [3] and the previously obtained values of the charge radii of ^{31,32}Mg isotopes can define a region associated with the "island of inversion" which is not seen in the HF+BCS theoretical method by using the original Skyrme force fitted to stable nuclei. Therefore, it seems important to investigate the dependence of the spin-orbit interaction to the isospin. However, one cannot discard other effects such as a proper treatment of the neutron pairing correlations, beyond mean field approaches and continuum effects that should be also considered in exotic nuclei approaching the neutron drip lines.

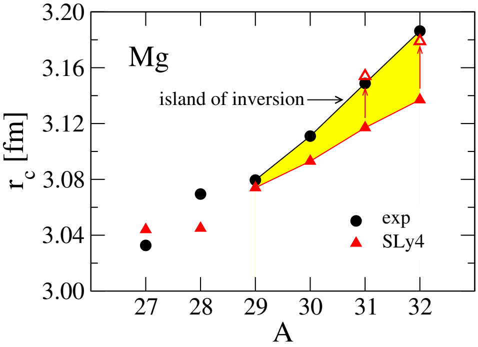

Fig. 3. (Color on-line) Theoretical (with the SLy4 Skyrme force) and experimental [3] r.m.s. charge radii r_c of Mg isotopes in the range of $A = 27$–32. The open (red) triangles represent the calculated values of r_c when the spin-orbit strength of the effective interaction is increased by 20%.

We show in Fig. 4 the correlation of the neutron-skin thickness ΔR [Eq. (4)] of Mg isotopes with the s and p_0 parameters extracted from the density dependence of the symmetry energy around the saturation density. It can be seen from Fig. 4 that, in contrast to the results obtained in Refs. [5,6], there is no linear correlation observed for the Mg isotopic chain. This behavior is valid for the three Skyrme parametrizations used in the calculations. Such a non-linear correlation of s and p_0 with the neutron skin thickness ΔR can be explained with the fact that stability patterns are quite irregular within this Mg isotopic chain, where anomalies in shell closures around $N = 20$ leading to increased quadrupole collectivity exist. Additionally, we find the same peculiarity at $A = 27$ from Fig. 1 exhibited in the case of the charge r.m.s. radii just reflecting the transition regions between different nuclear shapes of Mg isotopes in the considered chain and a small change in the behavior for nuclei heavier than ^{32}Mg, as well.

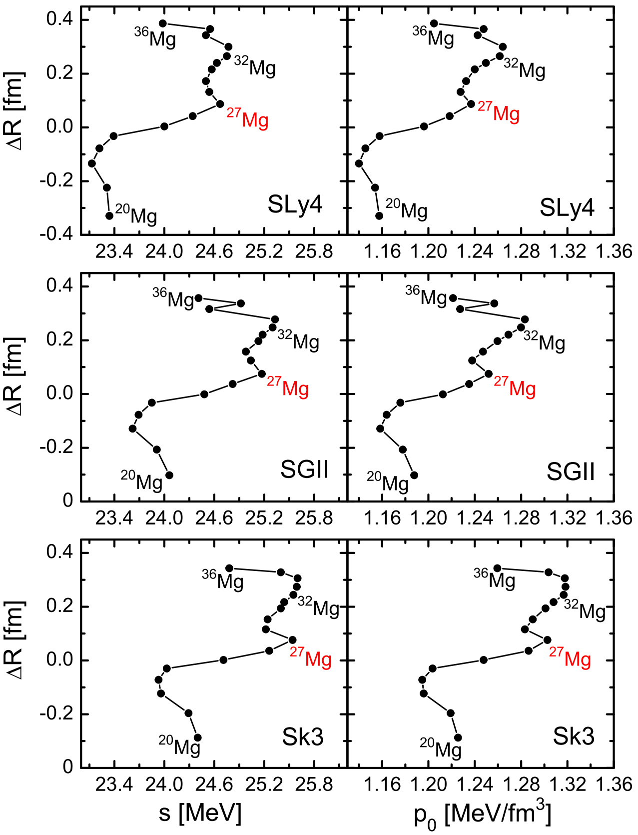

Fig. 4. HF + BCS neutron skin thicknesses ΔR for Mg isotopes as a function of the symmetry energy s and the pressure p_0 calculated with SLy4, SGII, and Sk3 forces.

4. Summary

We study nuclear properties of Mg isotopes by means of a theoretical approach to the nuclear many-body problem that combines the CDFM and the deformed HF+BCS method (with Skyrme-type density-dependent effective interactions). The isotopes investigated in this work go from the proton-drip-line nucleus ^{20}Mg up to ^{36}Mg that approaches the neutron-drip-line.

The charge and mass radii follow the trends observed in the experiment, with fluctuating values up to $A = 26$, and smoothly increasing values with A, beyond $A = 27$. These global properties are found to be rather similar to the three Skyrme forces. The observed tendency for ^{32}Mg to become deformed has been confirmed by repeating our calculations with SLy4 force modifying slightly the spin-orbit interaction. The correlations of the neutron skin thickness ΔR with the symmetry energy s and with the neutron pressure

p_0 do not exhibit linear behavior. They show up the same peculiarities at $A = 27$ that reflect the transition regions between different nuclear shapes of Mg isotopes in the considered chain.

To conclude, we would like to note that further study is necessary to prove theoretically the existence of an "island of inversion" probed by the REX-ISOLDE experiment. In particular, it is worth to perform calculations by including effects of tensor and three-body forces and exploring novel energy density functionals.

M.K.G. and A.N.A. are grateful for the support of the Bulgarian Science Fund under Contract No. DFNI-T02/19. E.M.G. and P.S. acknowledge support from MINECO (Spain) under Contracts FIS2011-23565 and FIS2014-51971-P.

REFERENCES

[1] K. Wimmer *et al.*, *Phys. Rev. Lett.* **105**, 252501 (2010).
[2] E.K. Warburton, J.A. Becker, B.A. Brown, *Phys. Rev. C* **41**, 1147 (1990).
[3] D.T. Yordanov *et al.*, *Phys. Rev. Lett.* **108**, 042504 (2012).
[4] M.K. Gaidarov, P. Sarriguren, A.N. Antonov, E. Moya de Guerra, *Phys. Rev. C* **89**, 064301 (2014).
[5] M.K. Gaidarov, A.N. Antonov, P. Sarriguren, E. Moya de Guerra, *Phys. Rev. C* **84**, 034316 (2011).
[6] M.K. Gaidarov, A.N. Antonov, P. Sarriguren, E. Moya de Guerra, *Phys. Rev. C* **85**, 064319 (2012).
[7] D. Vautherin, *Phys. Rev. C* **7**, 296 (1973).
[8] G. Audi, A.H. Wapstra, C. Thibault, *Nucl. Phys. A* **729**, 337 (2003).
[9] P. Sarriguren, M.K. Gaidarov, E. Moya de Guerra, A.N. Antonov, *Phys. Rev. C* **76**, 044322 (2007).
[10] E. Moya de Guerra *et al.*, *Nucl. Phys. A* **529**, 68 (1991).
[11] I. Sick, *Phys. Lett. B* **576**, 62 (2003).
[12] T.R. Gentile, C.B. Crawford, *Phys. Rev. C* **83**, 055203 (2011).
[13] J.W. Negele, *Phys. Rev. C* **1**, 1260 (1970).
[14] A.N. Antonov, V.A. Nikolaev, I.Zh. Petkov, *Bulg. J. Phys.* **6**, 151 (1979); *Z. Phys. A* **297**, 257 (1980); **304**, 239 (1982); *Nuovo Cim. A* **86**, 23 (1985); A.N. Antonov *et al.*, *Nuovo Cim. A* **102**, 1701 (1989); A.N. Antonov, D.N. Kadrev, P.E. Hodgson, *Phys. Rev. C* **50**, 164 (1994).
[15] A.N. Antonov, P.E. Hodgson, I.Zh. Petkov, *Nucleon Momentum and Density Distributions in Nuclei*, Clarendon Press, Oxford 1988; *Nucleon Correlations in Nuclei*, Springer-Verlag, Berlin–Heidelberg–New York 1993.
[16] K.A. Brueckner, J.R. Buchler, S. Jorna, R.J. Lombard, *Phys. Rev.* **171**, 1188 (1968).
[17] K.A. Brueckner, J.R. Buchler, R.C. Clark, R.J. Lombard, *Phys. Rev.* **181**, 1543 (1969).