Affirmative Action through Minority Reserves: An Experimental Study on School Choice*

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Abstract

Minority reserves are an affirmative action policy proposed by Hafalir *et al.* [8] in the context of school choice. In a laboratory experiment, we find that adding minority reserves to the GS and TTC mechanisms has positive effects on stability but is quite disappointing in terms of efficiency. Also GS induces higher rates of truth-telling by minority students and thus outclasses TTC.

Keywords: affirmative action; minority reserves; school choice; deferred acceptance; top trading cycles.

JEL-Numbers: C78, C91, C92, D78, I20.

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1 Introduction

In the last decade, matching theory has been employed to accommodate affirmative action in school choice. For instance, Abdulkadiroğlu and Sönmez [2] and Abdulkadiroğlu [1] consider a maximum quota on the number of students from the same group a school can admit. However, Kojima [11] shows that for the widely studied Gale-Shapley (GS) and Top Trading Cycles mechanisms (TTC) maximum quotas can be detrimental to the very minorities they are supposed to help. To overcome this problematic feature, Hafalir *et al.* [8] propose to reserve seats for minority students. Loosely speaking, schools give higher priority to minority students up to the point where these students are interested in filling the reserved seats. Hafalir *et al.* [8] adapt GS and TTC to minority reserves and show that these mechanisms preserve the property of strategy-proofness (no student can ever benefit by misrepresenting her preferences) and that they are an improvement over majority quotas.

Still, the theoretical results may easily break down in practice. It is well documented in the experimental literature on school choice that agents do not always realize that it is in their best interest to reveal their true preferences when confronted with strategy-proof mechanisms.¹ Since distinct strategy-proof mechanisms may be perceived differently, they could give rise to very different types of behavior and outcomes. Thus, it is the objective of our laboratory experiment to evaluate the effects of introducing minority reserves in GS and TTC.

2 The experiment

We consider the problem described in Table 1. Six students look for a seat at one of three schools. Four students — M_1 , M_2 , M_3 , and M_4 — belong to the *majority group*, and two students — m_1 and m_2 — form the *minority group*. Each of the three schools (s_1 , s_2 , and s_3) offers exactly two seats.

	Preferences						Priorities		
	M_1	M_2	M_3	M_4	m_1	m_2	s_1	s_2	s_3
Best match	s_1	s_1	s_3	s_3	s_2	s_2	m_1	M_3	M_1
Second best match	s_2	s_2	s_1	s_1	s_3	s_3	m_2	M_4	M_2
Third best match	s_3	s_3	s_2	s_2	s_1	s_1	M_4	M_2	m_2
Fourth best match							M_3	M_1	m_1
Fifth best match							M_1	m_1	M_3
Sixth best match							M_2	m_2	M_4

Table 1: Preferences of students over schools (left) and priority orderings of schools over students (right).

¹See, for instance, Calsamiglia *et al.* [3], Chen and Kesten [4], Chen and Sönmez [5], Featherstone and Niederle [6], Guillen and Hing [7], Klijn *et al.* [10], and Pais and Pintér [12].

We chose this problem for two reasons. First, in the absence of any positive discrimination, m_1 is in exactly the same situation as M_1 and M_3 (and m_2 faces exactly the same problem as M_2 and M_4). This symmetry helps us to evaluate the effect of minority reserves in a clear-cut way. Second, the unique Pareto-efficient outcome (assign each student to her top school) is obtained under all mechanisms only if submitted rankings satisfy stringent conditions. Essentially, subjects have to rank their best schools first. However, when subjects are not aware of strategy-proofness and look at their own priorities only, one cannot expect them to rank their best schools first, let alone to tell the truth. Yet, under minority reserves, m_1 can obtain a seat at her top school by ranking this school first independently of the behavior of the other subjects. This could reduce strategic uncertainty and induce all subjects to tell the truth more often.

Subjects assume the role of students, schools are not strategic players. Given the information in Table 1, the subjects' task is to submit a ranking over schools (not necessarily the true one) to be used by a central clearinghouse to assign students to schools. Subjects receive 12, 9, and 6 ECU if they obtain a seat in their top, second most, and least preferred school, respectively. Our two control treatments are the standard GS student proposing deferred acceptance (*GSs*) and the standard TTC mechanism (*TTCs*). Following Hafalir *et al.* [8], the modified mechanisms, denoted by *GSm* and *TTCm*, favor the minority group by obliging each school to reserve one seat to students from this group in case it is demanded.

Two matching mechanisms were played in each of the eight sessions. At the beginning of the session, subjects were anonymously matched into groups of six. Each subject received instructions for one of the four mechanisms and was told that she would play the game twice in different roles. Participants were also informed that after this first phase of two games they would take part in a second phase under a different matching mechanism. After completing the two games under the first mechanism, subjects received instructions for the second mechanism. The group assignment did not change and subjects took decisions in the same roles as before. Subjects were paid for their performance in one of the games (1 Euro per ECU earned). No feedback whatsoever was provided during the entire experiment. Subjects knew all procedures from the beginning. Sessions lasted about 75 minutes and subjects earned on average 12 Euro (including a 3 Euro show-up fee). In total, 175 subjects participated in the experiment.

3 Results

We analyze how the different mechanisms perform in terms of truth-telling. Afterwards, we study the implications of individual behavior on stability and welfare.

Result 1. *Minority reserves lead to more truth-telling only of m_1 under GS.*

Evidence on Result 1. Table 2 shows the distributions of submitted rankings and the probabilities of truth-telling. For example, the notation (2,3,1) indicates the ranking where a student lists her second most preferred school first, her least preferred school second, and her most preferred school last.

Mechanism	Submitted ranking						Truth-telling					
	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)	$\underline{M_1}$	M_2	$\underline{M_3}$	M_4	$\underline{m_1}$	m_2
<i>GSs</i>	0.39	0.02	0.34	0.11	0.02	0.12	<u>0.47</u>	0.37	<u>0.37</u>	0.27	<u>0.44</u>	0.37
<i>GSm</i>	0.39	0.07	0.26	0.09	0.06	0.13	<u>0.25</u>	0.30	<u>0.43</u>	0.14	<u>0.76</u>	0.44
<i>TTCs</i>	0.48	0.05	0.31	0.07	0.03	0.06	<u>0.53</u>	0.55	<u>0.50</u>	0.33	<u>0.58</u>	0.39
<i>TTCm</i>	0.34	0.06	0.33	0.11	0.05	0.11	<u>0.50</u>	0.42	<u>0.21</u>	0.07	<u>0.55</u>	0.30

Table 2: Distribution of submitted rankings (over all roles) and probability of truth-telling (by roles).

The left part of Table 2 reveals that many subjects do not realize that it is in their best interest to report preferences truthfully. Moreover, minority reserves do not increase the average truth-telling rate under GS and even lead to a drop under TTC. The effects of the different mechanisms and roles on truth-telling can be evaluated with the help of the right part of Table 2. First, due to the symmetry of the game, one would expect that the proportion of truth-telling of the minority students is the same as that of their majority counterparts in the absence of minority reserves. Mann-Whitney U tests confirm this intuition in all cases. Moreover, as predicted, m_1 tells the truth more frequently in *GSm* than in *GSs* ($p=0.0019$, two-sided). Also, she tells the truth more often than both M_1 ($p=0.0015$, two-sided) and M_3 ($p=0.0107$, two-sided) in *GSm* and more often than M_3 in *TTCm* ($p=0.0368$, two-sided). On the other hand, minority reserves do not positively affect the level of truth-telling of this student under TTC. Finally, there is no evidence for spillover effects as no other student tells the truth significantly more often under minority reserves. $\square$

Result 2. *Minority reserves increase stability.*

Evidence on Result 2. A matching is stable in *GSs* and *TTCs* if for any student, all the schools she prefers to the one she is assigned to have exhausted their capacity with students that have higher priority. Therefore, in a stable matching no student can form a “blocking pair” with any school. Hafalir *et al.* [8] develop for *GSm* and *TTCm* an alternative notion of stability

that gives less blocking power to majority students.² The probabilities that a stable matching is reached are 0.3471 in *GSs* and 0.0460 in *TTCs*. Using the notion of Hafalir *et al.* [8], the corresponding numbers for *GSm* and *TTCm* are 0.3553 and 0.2403, respectively. So, GS tends to be more stable than TTC and minority reserves increases stability for TTC while it almost does not affect GS.³ Also, with respect to the quality of the stable matchings reached we obtain that if one orders stable matchings according to average payoff, then the probability distribution over stable matchings delivering different average payoffs under *GSm* (*TTCs*) “almost” first-order stochastically dominates the distribution obtained under *GSs* (*TTCm*).⁴

Mechanism	Students							
	M_1	M_2	M_3	M_4	m_1	m_2	Majority	Minority
<i>GSs</i>	0.1938	0.1494	0.3878	0.0543	0.2301	0.1988	0.1963	0.2024
<i>GSm</i>	0.1972	0.1357	0.0941	0.0197	0.0960	0.0328	0.1117	0.0644
<i>TTCs</i>	0.3471	0.4674	0.3716	0.2721	0.2695	0.4913	0.3646	0.3804
<i>TTCm</i>	0.2813	0.3419	0.1931	0.0507	0.2162	0.0852	0.2168	0.1507

Table 3: Average probability of belonging to a blocking pair (standard definition for *GSs* and *TTCs* and notion of Hafalir *et al.* [8] for *GSm* and *TTCm*).

To complete the analysis on stability, we look into the expected probability of belonging to a blocking pair. Table 3 reveals, as expected, that in the standard mechanisms the average majority and the average minority student have roughly the same probability of being part of a blocking pair. This symmetry breaks down once minority reserves are implemented as the average probability with which a minority student belongs to a blocking pair is significantly reduced, while that of majority student is reduced to a lesser extent. $\square$

Result 3. *Minority reserves slightly increase (decrease) expected payoffs under GS (TTC). They benefit minority students, but harm majority students.*

Evidence on Result 3. Any successful discriminatory policy should increase the payoffs (or at least the average payoff) of minority students when minority reserves are introduced.

²We are grateful to an anonymous referee for the suggestion to compare the standard notion of stability for treatments *GSs* and *TTCs* with the modified notion of stability for treatments *GSm* and *TTCm*.

³Due to the very large number of recombinations, all pairwise comparisons are significant at $p = 0.01$. This is also true for the remaining analysis on stability and our final result on payoffs.

⁴In each comparison there is one instance in which a probability mass of less than 0.002 goes in the wrong direction.

Mechanism	Overall	Students						
		M_1	M_2	M_3	M_4	m_1	m_2	Minority
GSs	12.06 (3.38)	12.33 (4.59)	12.02 (1.99)	11.98 (5.55)	12.07 (1.58)	12.06 (4.25)	11.92 (2.20)	11.99 (6.65)
GSm	12.27 (5.05)	11.97 (8.09)	11.67 (5.05)	12.27 (3.93)	11.26 (2.94)	14.34 (3.19)	12.12 (1.31)	13.23 (3.20)
$TTCs$	12.70 (5.64)	13.44 (4.34)	12.24 (6.11)	13.10 (5.19)	12.01 (6.13)	13.52 (4.13)	11.90 (5.23)	12.70 (9.61)
$TTCm$	12.25 (5.26)	12.29 (6.88)	11.33 (5.18)	12.16 (5.80)	11.80 (2.48)	13.44 (5.69)	12.47 (2.99)	12.96 (8.27)

Table 4: Expected payoffs. In parentheses, we present variances.

Table 4 indicates that, on average, the introduction of minority reserves leads to slightly higher (lower) payoffs under GS (TTC). A closer inspection of the above table reveals that all majority students (except M_3 when we compare GSm with GSs) obtain lower expected payoffs once minority reserves are introduced. On the contrary, minority students (except m_1 when we compare $TTCm$ with $TTCs$) benefit from the introduction of minority reserves. In fact, if we consider minority students as a group, the expected rewards increase and their variance is reduced when minority reserves are introduced. Finally, a comparison of the full distributions of individual payoffs reveals that minority student m_1 's payoffs under GSm has first-order stochastic dominance over his payoff distribution under GSs and that the opposite holds for majority student M_4 . On the other hand, for M_1 , M_2 , and M_3 the payoff distribution under $TTCs$ dominates the distribution under $TTCm$.

□

4 Conclusion

We have analyzed experimentally the effects of minority reserves on the GS and the TTC mechanisms in a school choice problem. In contrast with Kawagoe *et al.* [9], our main finding highlights that adding minority reserves increases truth-telling only by some minority student in GS. The intuition that the GS mechanism performs better than the TTC mechanism in the considered problem is confirmed when one looks at stability and welfare. Finally, one should also keep in mind that our results were obtained in a simple setting with the flavor of a coordination game that puts pressure on the level of truth-telling but where minimum reserves could actually bring a clear improvement for all students in terms of efficiency, which we do not observe.

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