1 Wind induced energy-momentum distribution along the
2 Ekman-Stokes layer. Application to the Western
3 Mediterranean Sea climate.

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8 **Abstract**

9 Wind-wave interaction in the Western Mediterranean Sea is analyzed using
10 16 years of model data. The mass transport and energy distribution due
11 to wind and waves are integrated through the Ekman-Stokes layer and then
12 spatially and seasonally analyzed. The Stokes drift is estimated from an em-
13 pirical parameterization accounting for local surface wind and the significant
14 wave height. The impact of the Stokes drift depends on wind variability at
15 the ocean surface and also on the geographical configuration of the basin.
16 The Western Mediterranean Sea has on average a wind energy input two
17 times higher in winter than in summer, and the Stokes-Ekman mass trans-
18 port interaction term contributes approximately 10% to 15% of the total
19 wind induced transport, but at some locations the contribution is as much
20 as 40% or more.

21 *Keywords:* Wind-wave interaction, Stokes drift, Ekman-Stokes layer,
22 Mass transport, Wind energy input, Western Mediterranean Sea

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23 1. Introduction

24 Surface gravity waves have an associated current, the Stokes drift veloc-
25 ity resulting from the non-linearities of the wave orbital velocities (Stokes,
26 1847). The most accurate way to consider how this wave-induced cur-
27 rent interacts to the mean flow is still a subject of ongoing study (see *e.g.*
28 McWilliams et al. (2004); Weber et al. (2006); Mellor (2008)). From the
29 Eulerian standpoint, the Stokes-drift-induced-current is considered an addi-
30 tive term that interacts to the mean ageostrophic current, appearing in the
31 momentum equations as an external force, in the form of a vortex force or as
32 the Coriolis-Stokes force (McWilliams and Restrepo (1999); Smith (2006);
33 Uchiyama et al. (2010)).

34 Few previous studies estimated the effect of the Stokes drift on energy
35 and mass transport within the Ekman-Stokes layer in real scenarios. Liu
36 et al. (2007) studied at global scale the importance of the Stokes drift inter-
37 actions in relation to the wind input energy along the Ekman-Stokes layer.
38 They quantified the energy input to the subinertial motions, by deducing
39 an expression for the total wind input in a stationary wave field distinguish-
40 ing two terms: the direct wind and the wave induced components showing
41 that the wave energy component contributes around 12% in the total (direct
42 wind and wave) energy input. Wu and Liu (2008) studied the energy distri-
43 bution in the Antarctic Circumpolar Current finding that the wave energy
44 component accounted on average for a 22% of the total energy. Kantha
45 et al. (2009), estimated the role of the Stokes drift as an energy dissipator
46 at a global scale; they pointed out that although the Stokes drift penetrates
47 downwards only a few meters it enhances the turbulence affecting the ocean
48 mixed layer and contributing to the Langmuir circulation. Teixeira (2012)

49 developed a model to obtain the turbulent kinetic energy dissipation injected
 50 into the water by breaking waves and subsequently amplified due to its dis-
 51 tortion by the mean shear of the wind-induced current and straining by the
 52 Stokes drift of surface waves in the same mechanism as the one responsible
 53 for the Langmuir circulation. Tamura et al. (2012) estimated the impact of
 54 the Stokes drift on the wind induced transport along the Ekman-Stokes layer
 55 in the North Pacific ocean, showing that the value of the Stokes-drift com-
 56 puted from the wave significant height and peak period (bulk parameters,
 57 *i.e.* statistical estimations) underestimates the real value.

58 Conversely, the Stokes depth is overestimated by using the bulk expres-
 59 sion, which is inversely proportional to the wave number magnitude. The
 60 main drawback in the Stokes drift bulk expression is the absence of the local
 61 wind in the formulation neglecting the wave-current interaction. On the
 62 other hand, the bulk formulation assumes that waves are monochromatic
 63 and therefore do not take into account the rapid decay of a real broad wave
 64 spectra and overestimates the Stokes e-folding depth (Breivik et al., 2014).
 65 Recent global studies have computed directly the Stokes drift at global scale
 66 by using the full wave spectra (Harcourt and D’Asaro, 2008) or by using
 67 data measured from scatterometers (Liu et al., 2014). Both works focused
 68 on the mass transport distribution at the ocean surface analyzing the Stokes
 69 drift contribution from wind-sea and swell. Some empirical parameteriza-
 70 tions include the local surface wind in the Stokes drift (*e.g.* Li and Garrett
 71 (1993), Ardhuin et al. (2009)).

72 These works pointed out the importance of the effects of waves on the
 73 mass transport at the sea surface. For instance, the role of the Stokes
 74 drift in the advection of debris in certain regions has been already noticed
 75 (Kubota, 1994) as well as the dispersion of buoyant material by Langmuir

76 circulation (Nimmo Smith and Thorpe, 1999; Thorpe, 2009). However, to
77 the best of our knowledge, there are no previous studies quantifying the
78 influence of the Stokes drift in the energy and mass distribution in the
79 Western Mediterranean Sea. In fact, operational models used for Search and
80 Rescue and oil spill operations usually do not include the interaction between
81 wind induced currents and the wave term which is known to contribute
82 for the surface dynamics at the sub mesoscale. In this work we approach
83 the above wind-wave interaction trying to fill the gap in the knowledge of
84 the processes in the Western Mediterranean Sea by analyzing the vertically
85 integrated mass transport and energy distribution in the upper ocean.

86 This manuscript is divided as follows: Section 2 presents the dataset
87 (ocean wave model outputs and buoys) and the area of study; in Section 3
88 the wind induced energy and mass flux formulations are presented; in Section
89 4 main results are presented and discussed. In section 5 main conclusions
90 and limitations of the methodology are presented together with future work.

91 **2. Dataset and Study Area**

92 *2.1. Wind and Wave model*

93 The wave model implemented is the third generation spectral wave model
94 WAM (Komen et al., 1994). The model describes the evolution of two-
95 dimensional ocean wave spectra without any assumption on the spectral
96 shape by integrating the 2D transport of action density equation. Wind
97 forcing is a 3-hour wind field from ARPERA, covering 16-year period from
98 January 1st 1993 to December 31st 2008. ARPERA is a multi-decadal wind
99 hindcast from a dynamical downscaling of a coarser climatic model (ERA40).
100 Fields are provided every 3 hours with a spatial resolution of $\sim 0.16^\circ$. Here,

101 we use averaged daily fields centred at 12:00 UTC, being the period cov-
 102 ered long enough to capture the seasonal variability. Each model output
 103 includes the statistical information of the integrated spectra together with
 104 the surface wind velocity information at each grid point. Data analyzed con-
 105 sisted of 5840 daily fields of wind velocity, wind direction, and wave-related
 106 statistically averaged magnitudes (also called bulk parameters) such as the
 107 significant wave height, wave direction and mean and peak wave period.
 108 In addition, model outputs also include some bulk parameters separated
 109 by both wind-sea and swell wave components such as the wave significant
 110 height, the wave direction and the mean wave period that will be used in
 111 Section 4.5. Here we notice that since T_p is not provided in the model out-
 112 puts for wind-sea and swell components, we will estimate it by applying the
 113 following relationship between the mean (T_m) and the peak period (T_p) for
 114 the JONSWAP spectrum,

$$T_p \approx \frac{T_m}{1 - 0.532(\gamma + 2.5)^{-0.569}},$$

115 where γ is a factor that determines the concentration of the spectrum on
 116 the peak and varies between $[1, 7]$, with a mean value of $\gamma \approx 3$.

117 *2.2. Buoys*

118 Several deep water buoys from the Spanish Harbour Authority -Puertos
 119 del Estado- are available within the model domain. Temporal coverage and
 120 percentage of gaps differs from buoy to buoy, ranging from a couple of years
 121 to about a decade. The location of each buoy is marked with a black circle
 122 in Fig. 1. All buoys contain information about several parameters although
 123 for this study we only will use the wave significant height (H_s), mean wave
 124 period (T_m), wave peak period (T_p) and wind speed (W_{10}). Buoys are

distributed across areas with very different ocean and atmospheric dynamics. For instance, the buoy at “Cabo Gata” is placed on the Alboran Sea, very close to the quasi-permanent eastern anticyclonic gyre, a location where the wind direction is highly variable. On the contrary, the buoy at “Cabo Begur” is located in the middle of the Northern Current, where wind usually blows from North or North-West (Tramuntana or Mistral from the local wind names) specially from October to March.

Wave model data were calibrated in a previous work through a Root Mean Square Error (RMSE) minimization process using several buoys in the Western Mediterranean Sea (Martínez-Asensio et al., 2013). The relationship between calibrated and original H_s is given by $H_s^{\text{cal}} = \alpha H_s^\beta$, where α and β depend on the model grid point. In short, the calibration corrected a wave model underestimation of H_s .

2.3. Area of interest

WAM simulations cover mainly the Western Mediterranean Sea from 5° W (Strait of Gibraltar) to 15°E (Sicily Channel) (Fig. 1). Surface winds are characterized by a strong seasonality with intense northerly and north-westerly events in the Gulf of Lions during autumn and winter (well-known as Tramuntana and Mistral winds in the Mediterranean region). These winds result in a large fetch area in the Gulf of Lions. Waves propagate from there toward the South-East, reaching the Italian islands of Sardinia and Sicily and the Northern African coast. A secondary area of generation with a small fetch is at the Alboran Sea where strong winds, mainly during summer and autumn blow with large spatial variability. The nearly enclosed topography inhibits waves from propagating far. When westerly wind blows, remote long waves from the North Atlantic region enter through the Strait

151 of Gibraltar.

152 **3. Methods**

153 *3.1. Ekman-Stokes layer solution: Vertically integrated transport*

154 To analyze the effects of surface waves into the Ekman layer we average
 155 the momentum equations over wave periods considering a stationary flow
 156 in a slowly varying wave field (McWilliams et al., 1997; Lewis and Belcher,
 157 2004; Polton et al., 2005). Under these conditions, the ageostrophic motions
 158 are governed by,

$$\rho_w f \hat{\mathbf{k}} \times (\mathbf{u} + \mathbf{u}_s) = \frac{\partial}{\partial z} \left(\kappa \frac{\partial \mathbf{u}}{\partial z} \right), \quad (1)$$

159 being ρ_w the seawater density, f the Coriolis parameter, $\hat{\mathbf{k}}$ the unit vector
 160 in the z -direction, κ a constant vertical viscosity and where the Stokes-
 161 Coriolis force, $f \hat{\mathbf{k}} \times \mathbf{u}_s$ has been added to the Ekman model of the mean
 162 velocity $\mathbf{u}$. The classical assumption of constant κ has strong implications
 163 on the Ekman transport, restricting the Ekman spiral shape. In particular
 164 the surface Ekman current is forced to 45° to the right of the surface wind
 165 and the Ekman vertically integrated transport is directed 90° to the right
 166 of the surface wind (for the Northern Hemisphere case, to the left in the
 167 Southern Hemisphere) (Ekman, 1905). This approach is only acceptable for
 168 deep ocean (*i.e.* ocean depth $\gg$ Ekman depth) because κ cancels when
 169 integrating along the whole Ekman spiral. For the sake of clarity, the Ekman
 170 depth is usually defined as the depth at which the Ekman current have
 171 turned 180° relative to the direction at the surface. Adopting the complex
 172 form for the velocity vectors and assuming monochromatic waves, *i.e.*,

$$\mathbf{u}_s = u_s^x + i u_s^y = U_s e^{i\theta_w} e^{z/\delta_{St}} \equiv \mathcal{U}_s,$$

$$\mathbf{u} = u^x + i u^y \equiv \mathcal{U},$$

Eq. (1) can be rewritten as,

$$\frac{\partial^2 \mathcal{U}}{\partial z^2} - a\mathcal{U} = a\mathcal{U}_s, \quad (2)$$

where $a = i(f/\kappa)$ and δ_{St} being the Stokes e-folding depth (the depth at which Stokes drift current has decayed e) and θ_w the angle of waves measured counterclockwise from the west-east direction. The following boundary conditions apply for the mean flow,

$$\frac{\partial \mathcal{U}}{\partial z} = \frac{|\tau|}{\kappa \rho_w} e^{i\theta_u^{10}} \quad \text{at} \quad z = 0 \quad (3)$$

$$\mathcal{U} = 0 \quad \text{at} \quad z/\delta_{St} \text{ and } z/\delta_{Ek} = -\infty \quad (4)$$

where θ_u^{10} is the angle of the wind measured counterclockwise from the west-east direction and δ_{Ek} is the Ekman depth. The solution for the two-point boundary value problem of Eq. (2) subjected to (3)-(4) is:

$$\begin{aligned} \mathcal{U}(z) = e^{\frac{(1+i)z}{\delta_{Ek}}} & \left\{ \frac{(1-i)|\tau|e^{i\theta_u^{10}}}{\rho_w f \delta_{Ek}} + U_s e^{i\theta_w} c \left(\frac{(2-c^2) - i(2+c^2)}{c^4 + 4} \right) \right\} + \\ & + 2U_s e^{i\theta_w} e^{z/\delta_{St}} \left(\frac{ic^2 - 2}{4 + c^4} \right) \end{aligned} \quad (5)$$

where $c = \delta_{Ek}/\delta_{St}$ is the non-dimensional Ekman - Stokes number. The first term in Equation (5) is the Ekman current which can be rewritten in trigonometric form as (see *e.g.* Pond and Pickard (1983)),

$$\begin{aligned} U_{Ek} &= V_0 \cos(\theta_u^{10} - \pi[1/4 + |z|/\delta_{Ek}]) e^{z/\delta_{Ek}} \\ V_{Ek} &= V_0 \sin(\theta_u^{10} - \pi[1/4 + |z|/\delta_{Ek}]) e^{z/\delta_{Ek}} \end{aligned} \quad (6)$$

where (U_{Ek}, V_{Ek}) are defined for $z \leq 0$ and for the Northern Hemisphere. In Eq. (6), $V_0 = \frac{\sqrt{2}|\tau|}{\delta_{Ek}\rho_w|f|}$ is the Ekman current amplitude and $|\tau|$ is the modulus of the wind stress. For typical values in the Mediterranean Sea as

187 $|\boldsymbol{\tau}| = 0.1 \text{ N/m}^2$, $\delta_{\text{Ek}} = 30 \text{ m}$, $f = 10^{-4} \text{ s}^{-1}$, $\rho_w = 1029 \text{ kg/m}^3$, the surface
 188 Ekman currents amplitude take a value of $V_0 \approx 4.5 \text{ cm/s}$.

189 The total transport in the Ekman-Stokes layer can be expressed in com-
 190 plex notation $\mathcal{M}$ as,

$$\mathcal{M} = \int_{-\infty}^0 \mathcal{U}(z) dz = \mathbf{M}^{\text{Ek}} + \mathbf{M}^{\text{St-Ek}} + \mathbf{M}^{\text{St}}, \quad (7)$$

where,

$$\begin{aligned} \mathbf{M}^{\text{Ek}} &= \frac{-i|\boldsymbol{\tau}|}{f\rho_w} e^{i\theta_u^{10}} \\ \mathbf{M}^{\text{St-Ek}} &= \frac{(1-i)}{2} c^2 (\mathbf{U}_s \delta_{\text{St}}) e^{i\theta_w} \frac{[(2-c^2) - i(2+c^2)]}{c^4 + 4} \\ \mathbf{M}^{\text{St}} &= 2\mathbf{U}_s \delta_{\text{St}} e^{i\theta_w} \frac{(ic^2 - 2)}{4 + c^4} \end{aligned}$$

191 The zonal and meridional transport are $M_x = \Re\{\mathcal{M}\}$ and $M_y =$
 192 $\Im\{\mathcal{M}\}$. The first term, $\mathbf{M}^{\text{Ek}}$ is the vertically integrated wind induced
 193 Ekman transport, with a net transport of 90° to the right of the surface
 194 wind. The second term, $\mathbf{M}^{\text{St-Ek}}$ is the integrated transport resulting from
 195 the interaction between the Stokes-drift and the wind-induced current, while
 196 the third term, $\mathbf{M}^{\text{St}}$ is the Stokes transport due to the non-linear advection
 197 induced by waves.

198 To show the contribution of the terms that do not appear in the classical
 199 Ekman solution, Fig. 2 shows the theoretical distribution of $|\mathbf{M}^{\text{St}}|^2 = \frac{4(\mathbf{U}_s \delta_{\text{St}})^2}{4 + c^4}$
 200 (red line) and $|\mathbf{M}^{\text{St-Ek}}|^2 = \frac{c^4 (\mathbf{U}_s \delta_{\text{St}})^2}{(4 + c^4)}$ (blue line) as a function of c . Also
 201 the addition of both terms is shown for completeness (black line). As seen in
 202 Fig. 2 as c increases, the Stokes component (red line) reduces whereas the
 203 Stokes-wind term increases (blue line). In the absence of wind, the Stokes-
 204 Ekman term is zero so as $\delta_{\text{Ek}} = c = 0$ cancels, whereas the Stokes term
 205 will be the only contribution if waves are present, $\lim_{c \rightarrow 0} |\mathbf{M}^{\text{St}}|^2 = (\mathbf{U}_s \delta_{\text{St}})^2$.

206 As wind increases, the Ekman layer develops and c arises. At $c = \sqrt{2}$ the
 207 sum of the Stokes and Stokes-Ekman terms is maximum (sea black line).
 208 At this point the Ekman spiral is affected by the surface waves while the
 209 wave induced mass flux remains high. After this point the Stokes mass
 210 transport decays up to $c \sim 4$, where almost the asymptotic limit is reached:
 211 $\lim_{c \rightarrow \infty} |\mathbf{M}^{\text{St-Ek}}|^2 = (U_s \delta_{\text{St}})^2$. Contrarily, if $c \gg$ the Stokes drift term be-
 212 comes negligible respect to the Stokes-wind term and almost all mass trans-
 213 port results from the interaction between Stokes drift and the Ekman cur-
 214 rent. Considering that wind is ubiquitous in the real ocean -except in some
 215 limited areas such as the equatorial regions-, the Ekman depth is always
 216 there being larger than the Stokes depth.

217 3.2. Ekman-Stokes layer solution: Wind induced integrated energy

The energy distribution in the Ekman-Stokes layer can be readily ob-
 tained following Liu et al. (2007) and Wu and Liu (2008) by multiplying both
 sides of Eq. (1) by $\mathbf{u}$ and averaging over a wave period. Polton (2009) intro-
 duced a correction in the formulation by including an extra term accounting
 for the correlations between the Stokes-Coriolis and the mean current shear.
 Additionally, the expression of the vertically integrated energy was rewrit-
 ten in terms of the surface Stokes drift and the wind stress for the case of
 a steady wave field. Hereinafter we will use Polton's approach to compute
 the energy rate balance assuming a steady field. The total wave-averaged,
 depth-integrated energy is decomposed into wind-induced (E_w , mainly Ek-
 man currents) and wave-induced (E_s , mainly Stokes drift) components so as

$E = E_w + E_s$ and being,

$$E_w = \frac{|\boldsymbol{\tau}_0|^2}{\rho \delta_{Ek} f} + \boldsymbol{\tau}_0 \times \mathbf{U}_{s_0} \hat{\mathbf{k}} F_2(c) - \boldsymbol{\tau}_0 \cdot \mathbf{U}_{s_0} F_1(c) \quad (8)$$

$$E_s = \frac{1}{c} \left[\rho_w f \delta_{Ek} |\mathbf{U}_{s_0}|^2 F_2(c) + \boldsymbol{\tau}_0 \cdot \mathbf{U}_{s_0} + \boldsymbol{\tau}_0 \times \mathbf{U}_{s_0} \hat{\mathbf{k}} \right] \quad (9)$$

where $F_1(c) = \frac{c+2}{(c+1)^2+1}$ and $F_2(c) = \frac{c}{(c+1)^2+1}$. The surface wind stress is empirically obtained as a function of the wind velocity at 10 m height, W_{10} as $|\boldsymbol{\tau}_0| = \rho_a C_d W_{10}^2$ whereas the drag coefficient is $C_d = (0.75 + 0.067 W_{10}) \cdot 10^{-3}$ (Li and Garrett, 1993). The air density is constant and taken in this work as $\rho_a = 1.2 \text{ kg/m}^3$, whereas seawater density is $\rho_w = 1029 \text{ kg/m}^3$ (the average value for the Mediterranean Sea).

In this simplified model the wind energy input per unit of time and area (units in $\text{W} \cdot \text{m}^{-2}$) (the so-called energy rate in Polton (2009)), can be decomposed in two contributions (E_w and E_s) the first accounting for the wind-induced energy used to generate currents (first term in Eq. 8) and the energy exchange with waves (second and third terms in Eq. 8) and the second related with waves (indirect wind energy), where the first term in Eq. 9 represents the wave-induced Stokes drift and the other two terms result from the interaction between the Stokes drift and the surface wind. All possible combinations derived from the relative angle between wind and waves are shown in Fig. 3 where parameters and variables in Eq. 8 and Eq. 9 have been taken as constant (see caption of Fig. 3 for more details). From this figure we can notice that E and E_w are always positive whereas that E_s can be negative. Note that the second and third terms in both E_w and E_s redistribute the energy from wind to waves or viceversa. From Fig. 3 we see that E is maximum when the angle between wind and waves is $\theta = 90^\circ$ (clockwise notation) whereas E_w is maximum for $\theta = 135^\circ$ and the maximum for E_s corresponds to $\theta = 45^\circ$. This is explained as

241 follows; E_s is more favored when waves move parallel to the surface Ekman
 242 current ($\theta = 45^\circ$) since it can be reinforced receiving energy from wind. The
 243 opposite occurs when ($\theta = 225^\circ$) since wind (Ekman current) block wave
 244 propagation. If $\theta = 90^\circ$ the total energy increases because the vertically
 245 integrated Ekman transport is parallel to waves and momentum can be
 246 transferred from waves downwards the Ekman layer. On the other hand,
 247 waves can extract less energy from wind. Finally, if $\theta = 135^\circ$ waves cannot
 248 take energy from wind but they reinforce the Ekman transport being all
 249 energy available for E_w . The role of the Stokes-Coriolis term is therefore to
 250 redistribute the energy through the Ekman-Stokes layer but not contributing
 251 to a net energy production. The above energy balance only accounts for the
 252 energy budget below the troughs as already pointed out by Polton (2009).

253 3.3. Stokes drift estimation

254 For irrotational waves propagating in deep waters (*i.e.* $kh > \pi$ where
 255 $k = |\mathbf{k}| \equiv |k(\cos \theta, \sin \theta)|$ is the wave number and h the local water depth),
 256 the bulk Stokes drift at the surface is (Longuet-Higgins, 1953),

$$U_s^b = g^{-1} \omega_p^3 a^2 \quad (10)$$

257 where g is acceleration of gravity, $a \equiv H_s/2\sqrt{2}$ the wave amplitude and $\omega_p =$
 258 $2\pi/T_p$ the frequency at the peak period. Eq. (10) is valid for harmonic
 259 waves.

Ardhuin et al. (2009) obtained two parameterizations to compute the
 Stokes drift velocity based on model and buoy data that, contrary to the bulk
 expression, include the local surface wind effect (W_{10}). These expressions

are for buoy and model data respectively:

$$\begin{aligned} U_{s_b}^{Ad} &= 5.9 \cdot 10^{-4} \left(1.25 - 0.25 \left(\frac{0.5}{f_c} \right)^{1.3} \right) W_{10} W_{10}^m + 0.027 (H_s - 0.4), \\ U_{s_m}^{Ad} &= 5 \cdot 10^{-4} \left(1.25 - 0.25 \left(\frac{0.5}{f_c} \right)^{1.3} \right) W_{10} W_{10}^m + 0.025 (H_s - 0.4), \end{aligned} \quad (11)$$

where f_c refers to the cut-off frequency (~ 0.5 Hz from buoys), and W_{10}^m imposes an upper threshold for wind data that can be expressed as follows:

$$W_{10}^m = \begin{cases} W_{10} & \text{if } W_{10} < 14.5 [m s^{-1}], \\ 14.5 & \text{if } W_{10} \geq 14.5 [m s^{-1}]. \end{cases}$$

Then:

$$\mathbf{U}_s^{Ad}(z) = U_{s,0}^{Ad} e^{z/\delta_{St}} \left[\cos(\theta_w) \hat{\mathbf{i}} + \sin(\theta_w) \hat{\mathbf{j}} \right] \quad (12)$$

3.4. Ekman-Stokes layer depths

Stokes depth

The Stokes e-folding depth layer expression for a developed, stationary, and deep water monochromatic wave can be obtained from the Stokes drift exponential decay,

$$U_s = U_s(0) e^{2kz}.$$

where the subscript b denoting “bulk” expression has been omitted for clarity. Setting δ_{St} to be where U_s decays to $U_s(0)/e$, we have $\delta_{St} = 1/2k$. For deep water wave, *i.e.* $\omega^2 = gk$, the Stokes depth expressed with bulk parameters is, $\delta_{St}^{Bulk} = g/(2\omega^2)$. However, as shown by Tamura et al. (2012) (see their Fig. 1) this expression overestimates the Stokes depth. In fact, Breivik et al. (2014) stated that the bulk expression: “... *it is clear that the shear under a broad spectrum is much stronger than that of a monochromatic wave of intermediate wavenumber due to the presence of short waves whose associated Stokes drift quickly vanishes with depth. At the same time, the deep*

Stokes drift profile will be stronger than that of a monochromatic wave since the low-wavenumber components penetrate much deeper". In this context these authors proposed a modified vertical profile for the Stokes drift that approaches the exponential shape near the surface and goes as an asymptotic solution in the deep, as,

$$U_s^c = U_s(0) \frac{e^{2k_e z}}{1 - C k_e z},$$

being $k_e \sim k/3$ and $C \sim 8$. For $U_s^c = U_s(0)/e$ the e-folding depth is the root of the non-linear equation,

$$\delta_{St} - \frac{\log(1 - 8k_e \delta_{St}) - 1}{2k_e} = 0, \quad (13)$$

that can be solved numerically.

Ekman Depth

The Ekman depth can be expressed empirically as (see Csanady (1982) and Wu and Liu (2008)), $\delta_{Ek} = \epsilon u_w^*/f$, where u_w^* is the friction velocity $u_w^* = \sqrt{|\tau_0|/\rho_w}$, and ϵ is a non-dimensional constant ≈ 0.38 .

Let $u_a^* = \sqrt{|\tau_0|/\rho_a} = \sqrt{C_d} W_{10}$, we obtain,

$$\delta_{Ek} \simeq 0.38 \frac{W_{10}}{f} \sqrt{\frac{C_d \rho_a}{\rho_w}}, \quad (14)$$

which will be used in this work.

3.5. Stokes drift decomposition: wind-sea and swell

As a first approach the Stokes drift can be linearly decomposed as (see e.g. McWilliams et al. (2013)),

$$\mathbf{U}_{St} = \mathbf{U}_{St}^{sea} + \mathbf{U}_{St}^{swell}, \quad (15)$$

278 where it has been assumed that wind-sea and swell spectra are separable,
 279 with a marked different period resulting in a much narrower spectra for swell
 280 waves.

281 Previous works have estimated the Stokes drift separately for wind-sea
 282 and swell wave components by separating the spectrum through the defini-
 283 tion of a threshold frequency (for a more complete explanation of the method
 284 read Bidlot (2001)). However, due to the unavailability of the full spectra
 285 we are forced to estimate wind-sea and swell Stokes drift components di-
 286 rectly from the wave model bulk parameters presented in Section 2.1. Thus,
 287 in this work the Stokes drift wind-sea component, U_{St}^{sea} is computed using
 288 the Ardhuin parametrization (Eq. (11), because of the wind-sea dependence
 289 on local wind) whereas the swell component, U_{St}^{swell} is computed using the
 290 bulk approach (Eq. (10), swell waves are by definition independent of local
 291 wind). This division between wind-sea and swell when using the wave model
 292 output parameters is not very accurate as will be shown later. However, it
 293 is useful to better characterize spatially and seasonally if a region is more
 294 influenced by local wind waves or by remote swell waves.

295 If the Stokes drift is linearly decomposed as presented in Eq. (15), the
 296 solution of the Stokes-Ekman layer (Eq. 7) reads,

$$\mathbf{M} = \int_{-\infty}^0 \mathcal{U}(z) dz = \mathbf{M}_{Ek} + \mathbf{M}_{St-Ek}^{sea} + \mathbf{M}_{St-Ek}^{swell} + \mathbf{M}_{St}^{sea} + \mathbf{M}_{St}^{swell}, \quad (16)$$

297 where the different terms given by,

$$\begin{aligned}
\mathbf{M}_{\text{Ek}} &= \frac{-i|\boldsymbol{\tau}|}{f\rho_w} e^{i\theta_u^{10}} \\
\mathbf{M}_{\text{St-Ek}}^{\text{sea}} &= U^{\text{sea}} \delta_{\text{Ek}} e^{i\theta^{\text{sea}}} \frac{(1-i)}{2} c_{\text{sea}} \frac{(2-c_{\text{sea}}^2) - i(2+c_{\text{sea}}^2)}{4+c_{\text{sea}}^4} \\
\mathbf{M}_{\text{St-Ek}}^{\text{swell}} &= U^{\text{swell}} \delta_{\text{Ek}} e^{i\theta^{\text{swell}}} \frac{(1-i)}{2} c_{\text{swell}} \frac{(2-c_{\text{swell}}^2) - i(2+c_{\text{swell}}^2)}{4+c_{\text{swell}}^4} \quad (17) \\
\mathbf{M}_{\text{St}}^{\text{sea}} &= 2U^{\text{sea}} e^{i\theta^{\text{sea}}} \delta_{\text{St}}^{\text{sea}} \frac{(ic_{\text{sea}}^2 - 2)}{4+c_{\text{sea}}^4} \\
\mathbf{M}_{\text{St}}^{\text{swell}} &= 2U^{\text{swell}} e^{i\theta^{\text{swell}}} \delta_{\text{St}}^{\text{swell}} \frac{(ic_{\text{swell}}^2 - 2)}{4+c_{\text{swell}}^4},
\end{aligned}$$

298 where $c_{\text{sea}} = \delta_{\text{Ek}}/\delta_{\text{St}}^{\text{sea}}$ and $c_{\text{swell}} = \delta_{\text{Ek}}/\delta_{\text{St}}^{\text{swell}}$, where c_{sea} and c_{swell} refer to the
299 Ekman-Stokes numbers and $\delta_{\text{St}}^{\text{sea}}$ and $\delta_{\text{St}}^{\text{swell}}$ are the Stokes e-folding depths
300 for wind-sea and swell terms. Both depths have been estimated following
301 Eq. 13 by using the wave parameters for wind-sea and swell introduced in
302 Section 2.1 (mean wave direction, mean wave significant height and mean
303 wave period).

304 4. Results and Discussion

305 4.1. Stokes drift estimation

306 The Stokes-drift computed for the deep-water buoys (see locations in
307 Fig. 1) and wave model are compared in Table 1 with the statistical pa-
308 rameters defined in Appendix 1. The table shows the values of the Stokes
309 drift computed using the calibrated value of H_s (see Section 2.2) as well
310 as the non-calibrated value. Moreover, Stokes drift for both sets of data is
311 computed using the bulk formulation and the Ardhuin parametrization (Eq.
312 (11)). The most significant results that can be inferred from the Table 1
313 are:

- 314 • Stokes drift computed from the bulk expression is 50% smaller than
315 the one computed from the Ardhuin formulation,
- 316 • when using the Ardhuin expression, the correlation coefficients (CC)
317 between buoy and model data are substantially higher and the scatter
318 index (SI) a 50% lower, showing that results are less dispersed,
- 319 • the calibration deteriorates the Stokes drift estimations when using
320 the bulk formulation, increasing the root mean square error (RMSE)
321 as well as the dispersion (SI) and reducing the CC. A calibration of
322 T_p may also be necessary in order to improve the Stokes drift compu-
323 tation, which is beyond the scope of this paper.

324 Estimation of the Stokes drift velocity using Eq. 10 or Eq. 11 requires
325 accurate measurements of H_s , T_p and W_{10} . Tamura et al. (2012) compared
326 the Stokes drift velocity from wave model data and *in situ* measurements
327 with several buoys in the North Pacific by using different methods, *i.e.*:
328 integrating the model spectra, applying the wave bulk values Eq. (10) and
329 finally, using empirical parameterizations. They found that the Stokes drift
330 was systematically underestimated when using the bulk formulation, which
331 is based on the wave statistical estimators, H_s and T_p . In fact, they found a
332 much better agreement with the buoys if the spectrum was fully integrated
333 or when using empirical parametrizations rather than the bulk expression.
334 In more detail, together with the integration of the spectra, the use of the
335 parametrization of Ardhuin (either for buoy or model) turned out to be
336 the best approach. By including the local wind effect in computing the
337 Stokes drift, as is done in the Ardhuin expression, dispersion and RMSE are
338 reduced significantly, while correlation between buoys and model increased
339 dramatically.

340 The comparison provided in Table 1 is in agreement with the results
 341 of Tamura et al. (2012), confirming that the inclusion of the local wind, is
 342 critical to reproduce accurately the Stokes drift current. Despite this, Stokes
 343 drift estimated from model data tends to be underestimated compared to
 344 buoys. The distribution of calibrated (red dots) and non-calibrated (blue
 345 dots) Stokes drift velocities for each buoy is shown in Fig. 4. As seen,
 346 the Stokes drift velocity dispersion and magnitude varies greatly from buoy
 347 to buoy, reaching in some places as “Maó” and “Cabo Gata” values over
 348 0.15 m/s, being of the order (or even higher) than the geostrophic currents
 349 -given for instance in Fig. 8 of Poulain et al. (2012)-. Fig. 4 also shows the
 350 linear least squares fit for calibrated (solid orange line) and non-calibrated
 351 data (solid cyan line). In the remaining part of this paper, unless otherwise
 352 stated, Stokes drift velocity will be computed using the Ardhuin formulation
 353 with calibrated model data.

354 The seasonal behavior of the Stokes drift velocity is assessed by ana-
 355 lyzing its spatial distribution during winter (December-January-February)
 356 and summer (June-July-August) seasons (hereinafter DJF and JJA respec-
 357 tively). Fig. 5, top panels display the averaged Stokes drift at surface (black
 358 arrows) for DJF (left) and for JJA (right). Background color represents the
 359 angular deviation (in degrees) between the direction of the averaged surface
 360 wind fields (at 10 m) and the direction of the Stokes drift for the same pe-
 361 riods. Positive angular deviations indicate that wind is blowing to the left
 362 of the Stokes drift whereas negative angular deviations indicate that wind
 363 blows to the right of the surface Stokes drift. The magnitude of the Stokes
 364 drift during winter doubles the value obtained during summer (maxima in
 365 winter are around ~ 10 cm/s). Over the shelf of Italy and Spain wind and
 366 waves are on average in the opposite direction during winter. The central

and bottom rows in Fig. 5, represent winter and summer mean Stokes drift and angular deviations for the sea and swell Stokes drift components. The wind-sea component is much larger than the swell mainly around the generation area. However, in coastal areas the swell component can become large.

4.2. Ekman-Stokes layer estimation

The upper ocean depth where wind and wave induced currents interact to each other is known as the Ekman-Stokes layer. Other interactions can occur near the surface such as the formation of the Langmuir cells. However our numerical model does not resolve these small scale structures.

The averaged Ekman depth, δ_{EK} , for the analyzed period is displayed in Fig. 6, top left. Values of the Ekman layer range from 15 m in coastal areas to maxima around 40 m in the middle of the Gulf of Lions, a region characterized by strong winds blowing all year round (fetch region) and more intensely in late fall and winter (Ponce de León et al., 2016). As stated above the Stokes depth, δ_{St} , when computed through bulk parametrization overestimates the values more accurately estimated in other regions by previous works. The result of correcting the Stokes e-folding depth at each grid point is shown in Fig. 6 (top-right). As observed, the depth increases to the south with a clear gradient oriented with the direction of the main northerly winds with maximum values starting at the lee of Menorca Island.

Values for the Stokes depth are significantly improved when applying the Breivik correction as shown in Table 2. Temporal averaged Stokes depth in the whole basin is reduced to 0.6 m and the median diminishes 0.5 m approaching to the values of 1 – 3 m given in the literature (see for instance Tamura et al. (2012); Breivik et al. (2014)).

393 Solution of Eq. (13) is also used to obtain the wind-sea Stokes depth,
 394 δ_{St}^{sea} and the swell Stokes depth, δ_{St}^{swell} both displayed in Fig. 6. As seen, the
 395 averaged wind-sea Stokes depth layer takes values between 1 – 3 m, while the
 396 swell Stokes layer depth can be around 10 m (see Fig. 6 (bottom panels)).
 397 The higher depth for swell component is because its larger wavelengths that
 398 penetrate deeper into the ocean.

399 4.3. Wind Induced Energy Distribution

400 Following the energy rate decomposition presented in Section 3.2 and in
 401 Fig. 3, we estimate the wind induced energy, E_w , the wave induced energy,
 402 E_s and the total wind energy, $E = E_w + E_s$. These terms explain how wind
 403 energy is distributed along the Ekman-Stokes layer under a stationary wave
 404 field and assuming no dissipation mechanisms. In addition, cross terms
 405 involving wave-wind interaction (or wind-wave) are also considered (see Eq.
 406 (8) and Eq. (9)).

407 The mean total energy for DJF (left) and JJA (right) are depicted in
 408 the top panels of Fig. 7. The background color represents the wind energy
 409 input per unit of area and time ($W m^{-2}$). Most of the energy is concentrated
 410 in the Gulf of Lions, with values during winter near $10^{-2} W m^{-2}$ (Fig. 7,
 411 top-left). Energy contours present a similar distribution as the Stokes drift
 412 (Fig. 5, top left) since waves with larger wavelengths arrive to the North
 413 African coast (mainly coasts of Algeria and Tunisia) unimpeded by islands
 414 or land intrusions between the Gulf of Lions and the coast. Energy values
 415 decrease drastically during summer with maxima around $3 \cdot 10^{-3} W m^{-2}$.
 416 During summer the wind energy input in the Alboran Sea is as important
 417 as the input of energy in the Gulf of Lions.

418 Energy components for wind and waves are different (second and third

row panels of Fig. 7). Wind energy primarily produces Ekman currents (Fig.
 7, second row, left column for DJF). The energy contribution from waves is
 much smaller, as it is especially during JJA with maxima located far from
 the storm generation area indicating that the cyclogenetic events during this
 season have less duration and intensity. The ratio between the wave induced
 energy and the total energy is ~ 0.3 (see Fig. 7, bottom left) which suggests
 that waves are important as an energy input mechanism in the upper water
 column. This energy ratio decreases in summer to a 10% – 15%. According
 to Fig. 3 variations in the wave induced energy are due to two reasons: a
 higher relative angle between local wind and waves and, variations in the
 Stokes drift due to wind seasonality. Positive E_s values indicate that the
 angle between wind and waves are in between $[-45^\circ, 135^\circ]$, which is mainly
 occurring in the middle of the basin when wind and waves are more aligned
 (see Fig. 5, top-left). E_s can be also zero at locations where the angle
 between wind and waves is very variable (more common in coastal areas).
 For instance it is the case of the Alboran Sea where the rate of E_s to E is
 very low (Fig. 7, bottom panels).

Here we notice that, following the work of, *e.g.* D’Ortenzio et al. (2005),
 Ekman depth in summer can be much shallower due to the upward displace-
 ment of the mixed layer, inhibiting the downward wind-induced momentum
 and thus, blocking the Ekman currents. This effect enhances the seasonal
 variability modifying the relative importance of the Stokes drift contribution
 during summer.

Monthly averaged values (for the whole basin) are shown in Fig. 8 for
 the total energy rate (top panel), for the wind induced energy rate (middle
 panel) and for the wave induced energy rate (bottom panel). In the box plots
 the star refers to the mean and the horizontal line inside the boxes represents

446 the median, whose value is indicated on the top of each box. All energy
 447 terms show a marked seasonal behavior with maxima in December/January
 448 and minima during July/August given by the variability of energy input
 449 at the ocean surface along the year. Box plot size increases during winter
 450 with maximum values spatially localized with a large variability along the
 451 basin. These results are in accordance with the ones obtained by Wu and
 452 Liu (2008), who found that wave induced energy was above 20% of the total
 453 wind energy input in the Antarctic Circumpolar Current, and therefore of
 454 potential importance that has to be considered in ocean transport analysis.

455 4.4. *Wind Induced Mass Flux*

456 In this section the Stokes layer momentum solution (see Section 3.1) is
 457 analyzed in order to study the mass transport $\mathbf{M} = \mathbf{M}_{\text{Ek}} + \mathbf{M}_{\text{St-Ek}} + \mathbf{M}_{\text{St}}$
 458 (see Eq. 7). Fig. 9, top panel, shows the averaged $\mathbf{M}$ in the Ekman-Stokes
 459 layer for DJF (left) and JJA (right) where the arrow length indicates the
 460 mass flux in $\text{m}^2 \text{s}^{-1}$. Maxima during winter are around $4.0 \text{ m}^2 \text{s}^{-1}$ while
 461 during summer is of $1.5 \text{ m}^2 \text{s}^{-1}$. During DJF the averaged mass transport is
 462 towards the south/south-west being in JJA season anticyclonically deflected.
 463 Second and third rows in Fig. 9 display respectively the Ekman and Stokes
 464 components of the mass flux. The Ekman component clearly dominates the
 465 total mass transport, being two orders of magnitude larger than the Stokes
 466 transport. Mean Ekman transport is deflected around 90° rightward to the
 467 surface wind direction following the classical Ekman solution. However,
 468 the effects of waves produces a slight change in direction of the total mass
 469 transport. The Stokes-Ekman interaction term is depicted for DJF and JJA
 470 in the fourth row. Although this term is one order of magnitude smaller than
 471 the Ekman contribution is larger than the pure Stokes term contributing on

472 average for an $\sim 15\%$ of the total transport and is directed mostly to the
 473 south-east following the wind dominant direction. The median ratio between
 474 the Stokes mass transport terms and the total mass transport is depicted
 475 in Fig. 9, bottom panels, showing that the Stokes terms are much more
 476 important in DJF than in JJA (about 2-3 times larger) and that in some
 477 locations this contribution can be as high as 40% of the total mass flux.

478 The above spatial mass transport distribution is further analyzed by
 479 computing the modulus of the monthly transport as well as the Ekman,
 480 the Stokes and the combined Stokes-Ekman terms (Fig. 10). Box plots are
 481 computed by averaging the whole basin. Roughly, comparison between the
 482 different contributions provides $|M_{Ek}| \sim 10|M_{St-Ek}| \sim 100|M_{St}|$. Besides
 483 the seasonality in the transport, it is noticeable that during winter, variance
 484 is larger because of the large differences in transport in the basin. Stokes
 485 transport during March and April remains practically constant or slightly
 486 increases during April (see third panel in Fig. 10). This is due to the spring
 487 storms affecting the Mediterranean Sea during this month.

488 4.5. Mass flux for wind-sea and swell terms

489 In this work we assumed that the Stokes drift can be described by the
 490 linear combination of wind-sea and swell Stokes drift components (see Eq.
 491 15, Section 3.5). To infer the validity in this assumption, we compute the
 492 RMSE of the Stokes drift (Fig. 11, top panel) and the variance for the
 493 zonal and meridional velocity components independently (Fig. 11, central
 494 and bottom panels respectively) for DJF (left) and JJA (right) seasons.
 495 The mean RMSE value is about 1 cm/s at basin scale during summer, and
 496 2 cm/s during winter. RMSE is larger in the middle of the basin, mainly in
 497 the North African coast around the longitude of 5° E. In fact, the highest

498 RMSE is explained by the maximum variance in the meridional component
 499 (Fig. 11, bottom left).

500 The total mass transport and the contribution of each of the above de-
 501 scribed components for the Stokes (sea and swell) and the Stokes-Ekman
 502 (sea and swell) mass transport are shown in Fig. 12 (note that the pure
 503 Ekman transport is the same as in Fig. 9 and therefore not repeated). By
 504 this decomposition the total mass transport (Fig. 12, top panels) is slightly
 505 larger than the one obtained previously (Fig. 9, top panels). The pure
 506 Stokes component is dominated by the swell contribution (60 – 70%) (Fig.
 507 12, middle panels). The Stokes mass transport component shows, specially
 508 during DJF, a similar spatial pattern than the provided by the swell (com-
 509 pare Fig. 9, middle-left panel and Fig. 12, middle-left panel). Contrarily,
 510 the Stokes-Ekman interaction term is clearly dominated by the wind-sea
 511 component with a contribution of over 90% in the mass transport (compare
 512 Fig. 12, fourth and fifth rows). Recently, Carrasco et al. (2014) studied
 513 separately wind-sea and swell mass flux distribution at global scale for more
 514 than 50 years of data using a wave model reanalysis. These authors found
 515 that the swell dominates the transport around the equatorial ocean where
 516 winds are weak (these regions are called “swell pools” following references
 517 therein).

518 Seasonality of the above presented magnitudes are presented in Fig. 13
 519 for the total modulus of the mass transport as well as for the four other
 520 components. It is clear that the pure Stokes-swell component, $|\mathbf{M}_{\text{St}}^{\text{swell}}|$ is
 521 about 3 times the value of the wind-sea component, $|\mathbf{M}_{\text{St}}^{\text{sea}}|$. The wind-
 522 sea component in the Stokes-Ekman interaction term $|\mathbf{M}_{\text{St-Ek}}^{\text{sea}}|$ is about
 523 4 – 5 times larger than the swell component, $|\mathbf{M}_{\text{St-Ek}}^{\text{swell}}|$. This result is in
 524 agreement with the fact that the Stokes-Ekman interaction is mostly due to

525 the interaction between local wind and waves induced Stokes drift.

526 5. Conclusions

527 In this work the wind induced energy and mass flux have been estimated
528 for the Western Mediterranean Sea from model data. Both magnitudes have
529 been integrated along the Ekman-Stokes layer and spatially and seasonally
530 analyzed. The impact of the Stokes drift depends primarily on two as-
531 pects: first, the wind variability at the ocean surface (wave generation) and,
532 second, the geographical configuration of the basin, *i.e.* size, depth and
533 coastline profile (wave propagation), which are particularly complex in the
534 Mediterranean basin.

535 In the Western Mediterranean Sea, wind induced energy and mass trans-
536 port along the Ekman-Stokes layer show a marked seasonal character, being
537 higher during winter since wind is stronger and reduced to one half during
538 summer. In the north side of the basin, around the Gulf of Lions, there
539 is a well known cyclogenetic area where high waves are generated mainly
540 during late fall and winter. On the other hand along the coast of North
541 Africa the Stokes transport is higher, being primarily composed by the swell
542 components. At basin level, the Stokes-Ekman mass transport interaction
543 term is about a 10% – 15% of the total transport but largely depending on
544 the spatial location

545 The main drawback of this work is related to the statistical wave magni-
546 tudes and the empirical parametrizations applied, the assumption of station-
547 arity and the unavailability of the full wave spectra, the use of a constant
548 eddy viscosity model and the ageostrophy of the currents. However, our
549 results can be taken as a lower bound of the magnitudes presented. In addi-

tion, to solve the corrected Stokes depth requires considerable computational effort since they were estimated for hundreds of grid points and thousands of days. The different contributions from the swell and wind-sea components are analyzed by assuming that they can be linearly decomposed. The total mass transport following this decomposition is 10% higher.

Results emphasize the importance of including the Stokes drift in the estimation of the upper ocean transports. Inclusion of the Stokes-wind interaction terms is specially important for operational applications aimed to provide forecasts for oil spill and Search and Rescue operations. Neglecting those terms can result in errors in the surface velocity around 40% of the wind induced velocity, and the magnitude can be even higher than the geostrophic velocities obtained from altimetry. Another remarkable point is the role that, at local level, Stokes transport terms, mainly the swell component, can play in accumulating floating debris. The North of Africa, specially Algeria, but also Morocco, Tunisia and the South of Spain have been found to be sinks of marine debris. Other mechanisms for debris transport such as the advection by Langmuir circulation can also be included for a better understanding of the fate of pollutants at the subbasin scale.

Despite some previous studies about the role of the Stokes drift in the wind induced energy and momentum distribution, to our knowledge, this is the first time that it has been studied with some detail in the Mediterranean Sea.

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580 **Appendix 1**

581 In the Table 1 are displayed the following statistical magnitudes, being
 582 M_i and O_i are the model and real (buoy) observations respectively. N is the
 583 data length. The Scatter Index (S.I.) [%] is defined as,

$$SI = \frac{1}{O_{RMS}} \left[\frac{1}{N-1} \sum_{i=1}^N (M_i - \bar{M} - (O_i - \bar{O}))^2 \right]^{\frac{1}{2}}. \quad (18)$$

584 The Normalized mean Bias NB is,

$$NB = 100\% \times \frac{\sum_{i=1}^N (M_i - O_i)}{\sum_{i=1}^N O_i}. \quad (19)$$

585 The Root Mean Square Error (RMSE) between observations and model is:

$$RMSE = \left[\frac{1}{N} \sum_{i=1}^N (M_i - O_i)^2 \right]^{1/2}. \quad (20)$$

586 The Correlation Coefficient (CC) is calculated as:

$$CC = \frac{1}{N-1} \sum_{i=1}^N \left(\frac{M_i - \bar{M}}{\sigma_{M_i}} \right) \left(\frac{O_i - \bar{O}}{\sigma_{O_i}} \right) \quad (21)$$

587 where

$$O_{RMS} = \left[\frac{1}{N} \sum_{i=1}^N (O_i)^2 \right]^{1/2}. \quad (22)$$

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Buoy	Method	GAP	Mean (cm/s)	CC	NB	SI	RMSE
		(%)	(buoy, model)		(%)	(%)	(cm/s)
Cabo Begur	Bulk cal.	33.6	(- -, 3.81)	0.7	17.2	69.5	1.82
	Bulk noncal.	-	(3.21, 2.40)	0.81	-26.9	40.9	1.61
	Ardh. cal.	-	(- -, 5.28)	0.9	-2.4	33.4	1.65
	Ardh. noncal.	-	(4.92, 4.53)	0.9	-15.5	33.5	1.73
Cabo Palos	Bulk cal.	29.0	(- -, 3.37)	0.62	47.3	63.2	1.84
	Bulk noncal.	-	(2.27, 2.03)	0.72	-11.8	46.8	1.21
	Ardh. cal.	-	(- -, 3.51)	0.88	11.4	35.6	1.53
	Ardh. noncal.	-	(3.75, 2.93)	0.88	5.3	35.2	1.49
Dragonera	Bulk cal.	3.3	(- -, 2.71)	0.53	18.2	64.3	1.93
	Bulk noncal.	-	(2.32, 1.61)	0.64	-30.0	50.2	1.62
	Ardh. cal.	-	(- -, 3.51)	0.87	-3.4	32.9	1.59
	Ardh. noncal.	-	(3.61, 2.93)	0.87	-19.5	33.2	1.74
Cabo Gata	Bulk cal.	4.6	(- -, 2.31)	0.56	-1.2	60.9	1.87
	Bulk noncal.	-	(2.34, 1.38)	0.67	-40.0	49.9	1.78
	Ardh. cal.	-	(- -, 3.18)	0.86	-19.5	37.4	1.86
	Ardh. noncal.	-	(3.62, 2.62)	0.85	-29.9	37.7	2.07
Maó	Bulk cal.	40.8	(- -, 2.90)	0.58	34.1	59.2	1.27
	Bulk noncal.	-	(2.12, 1.74)	0.69	-20.78	46.9	0.97
	Ardh. cal.	-	(- -, 4.52)	0.87	-2.4	34.7	1.59
	Ardh. noncal.	-	(4.62, 3.84)	0.87	-17.3	35.1	1.64
Tarragona	Bulk cal.	6.6	(- -, 2.51)	0.56	9.3	65.7	1.98
	Bulk noncal.	-	(2.32, 1.47)	0.64	-35.7	52.3	1.75
	Ardh. cal.	-	(- -, 2.55)	0.85	-15.3	38.4	1.71
	Ardh. noncal.	-	(3.05, 2.05)	0.85	-31.7	38.7	1.91

Table 1: Statistical estimates for the Stokes drift computed by the bulk expression and Arduin (Ardh) parametrization for the buoys and for the calibrated (cal) and raw (noncal) model data. CC (correlation coefficient), NB (normalized mean bias), SI (percentage scatter index) and RMSE (root mean square). The percentage of missing data in each timeseries is provided in the GAP column. The mathematical definitions of the statistical estimators are given in Appendix 1.

Table 2: Stokes e-folding depth comparison.

Method	Mean [m]	Std [m]	Median [m]
Bulk	3.41	2.55	2.62
Breivik	2.80	2.07	2.12

Figure 1: Buoy locations (black circles) and western Mediterranean Sea bathymetry contour map. Units in m.

Figure 2: Mass transport modulus for Stokes term (red line), for Stokes-Ekman interaction (blue line) and for the addition of both terms (black line). It has been normalized by $(\delta_{\text{St}} U_s)$ as a function of $c = \delta_{\text{Ek}}/\delta_{\text{St}}$.

Figure 3: Total wind energy input (E , solid black line) and its redistribution between direct wind-induced energy rate (E_w from Eq. (8), solid red line) and indirect energy rate on wave terms (E_s from Eq. (9), solid blue line). Some parameters and/or variables have are assumed constant: $|\tau_0| = 0.1 \text{ N} \cdot \text{m}^{-2}$, $|\mathbf{U}_{s0}| = 0.1 \text{ m} \cdot \text{s}^{-1}$, $\delta_{Ek} = 25 \text{ m}$, $\delta_{St} = 2.5 \text{ m}$, $c = 10$, $f = 10^{-4} \text{ s}^{-1}$. θ is the angle between wind and waves in a clockwise sense. Horizontal black dashed line shows the zero energy line. Units in $\text{W} \cdot \text{m}^{-2}$.

Figure 4: Stokes drift and linear fits, U_{St} computed following Arduin et al. (2009) at each buoy for calibrated (red dots and orange solid line) and non calibrated (blue dots and cyan solid line) data as given in Martínez-Asensio et al. (2013).

Figure 5: Top row: averaged Stokes drift (black arrows) for DJF (left) and for JJA (right). Central row: averaged wind-sea component for the Stokes drift for DJF (left) and JJA (right). Bottom row: averaged swell component for the Stokes drift for DJF (left) and JJA (right). Vector length units in $\text{cm} \cdot \text{s}^{-1}$. Background color represents the angular deviation (in degrees) between the direction of the averaged surface wind field (at 10 m) and the direction of the Stokes drift.

Figure 6: Averaged Ekman depth (top-left). Averaged Stokes depth (top-right). Averaged wind-sea Stokes depth average (bottom-left). Averaged Stokes swell depth average (bottom-right). The Ekman depth is estimated according to Eq. (15) and the Stokes depths by solving Eq. (14). Period extends from 1993 to 2008. Units in m.

Figure 7: First row: total averaged energy rate. Second row: wind induced averaged energy rate. Third row: wave induced averaged energy rate. Units in $W \cdot m^{-2}$. Fourth row: ratio between wave induced energy rate and total energy rate. Left column for DJF and right column for JJA.

Figure 8: Top panel: box plot of monthly spatially averaged total energy rate. Middle panel: monthly spatially averaged wind induced energy rate. Bottom panel: monthly spatially averaged wave induced energy rate. Stars represent the mean value and the number on each box the median. Each box represents the interquartile range: Q_1 (percentile 25th) to Q_3 (percentile 75th) and upper and bottom whiskers are computed as: $Q_3 + 1.5 \cdot (Q_3 - Q_1)$ and $Q_1 - 1.5 \cdot (Q_3 - Q_1)$ respectively. Units in $W \cdot m^{-2}$.

Figure 9: First row: vertically integrated total mass flux. Second row: vertically integrated Ekman mass flux. Third row: vertically integrated Stokes mass flux. Fourth row: vertically integrated Stokes-Ekman interaction mass flux. Units in $\text{m}^2 \cdot \text{s}^{-1}$. Fifth row: median ratio between Stokes related mass transport (which is the addition of both, the pure Stokes and the Stokes-Ekman interaction term) and the total mass transport. Left column for DJF and right column for JJA.

Figure 10: First panel: box plot of the modulus of the monthly spatially averaged total mass transport. Second panel: modulus of the monthly spatially averaged Ekman mass transport. Third panel: modulus of the monthly spatially averaged Stokes. Fourth panel: modulus of the monthly spatially averaged Stokes-Ekman interaction mass transport. The numbers, symbols and box-plot quantiles are the same as in Figure 8.

Figure 11: Top panels present the Stokes drift velocity Root Mean Square Error (for total velocity). Middle and bottom panels show the variance (u and v components respectively). Left column for winter season (DJF), right column for summer (JJA). $\text{RMSE} = \sqrt{\frac{1}{n_t} \sum_{i=1}^{n_t} (\hat{u}_{\text{St}})^2 + (\hat{v}_{\text{St}})^2}$, being $\hat{u}_{\text{St}} = u_{\text{St}} - (u_{\text{St}}^{\text{sea}} + u_{\text{St}}^{\text{swell}})$ and $\hat{v}_{\text{St}} = v_{\text{St}} - (v_{\text{St}}^{\text{sea}} + v_{\text{St}}^{\text{swell}})$. Units in $\text{m} \cdot \text{s}^{-1}$ for RMSE and $\text{m}^2 \cdot \text{s}^{-2}$ for variance.

Figure 12: Mass depth integrated transport average along the Ekman-Stokes layer decomposing the Stokes drift velocity in wind-sea and swell components. The first row panels represent the total transport average in winter (DJF) and summer (JJA). The second row panels show the pure Stokes wind-sea transport. The third row panels the pure Stokes swell induced mass transport. The fourth row panels the Stokes-Ekman interaction wind-sea term. The last row is the Stokes-Ekman interaction swell component term.

Figure 13: First panel: box plot of the modulus of the monthly spatially averaged total mass transport. Second panel: modulus of the monthly spatially averaged pure Stokes wind-sea component. Third panel: modulus of the monthly spatially averaged pure Stokes swell term. Fourth panel: modulus of the monthly spatially averaged Stokes-Ekman interaction wind-sea mass transport. Fifth panel: modulus of the monthly spatially averaged Stokes-Ekman interaction swell component. The numbers, symbols and box-plot quantiles are the same as in Figure 8.