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Absence of cosmological constant problem in special relativistic field theory of gravity: one-loop renormalization group

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Abstract. There exists a nonlinear theory of gravity which is not structurally equivalent to general relativity and that, in the non-interacting limit, describes a free massless particle with helicity ± 2. We have recently shown that this theory can be understood as the result of self-coupling, in complete parallelism to the well-known case of general relativity. This special relativistic field theory of gravity exhibits a decoupling of vacuum zero-point energies of matter and passes all the known experimental tests in gravitation. It is explicitly demonstrated that there is no flow of the effective cosmological constant under the action of the renormalization group at one-loop level, while simple symmetry arguments show that this would continue to be true for higher-loop corrections. The important lesson is that just mild local assumptions concerning the nature of the particle mediating the gravitational interactions are enough to motivate theories which are free of the cosmological constant problem.

1. Introduction

The cosmological constant problem appears when trying to combine the formalism of quantum field theory with the curved spacetime notions of general relativity. One of the implications of this combination is a problematic radiative instability of the cosmological constant term under the renormalization group [1]. Finding a mechanism which can stabilize the value of the cosmological constant, while keeping intact the low-energy physics as to pass the stringent experimental tests on deviations from general relativity, is what is usually known as the cosmological constant problem [1, 2, 3].[1]

The mainstream school of thought is that this problem would be solved in the framework of a consistent theory of quantum gravity. It is also common the theoretical expectation that the would-be theory of quantum gravity should lead exactly to general relativity in the long-wavelength limit [4]. This motivates the search of different mechanisms which can surpass the cosmological constant problem while keeping the structure of general relativity practically intact. After a long search there is no compelling model and, moreover, they usually involve unknown

[1] Here we do not address the issue of determining the actual value of the cosmological constant by means of first-principles calculations, but to alleviate this tension.

physics [1]. With a totally different spirit, here we argue that there exists a solution to the cosmological constant problem that only rests on the well-settled principles of quantum field theory in flat spacetime, with the proposal that gravity is mediated by a helicity ± 2 graviton, and the equivalence principle as reformulated by Weinberg [5]. That only these mild local assumptions are necessary to propose a solution is the point that we would like to stress in this contribution. As we will see, the key feature is the construction of a nonlinear theory of gravity which exhibits an unbroken, anomaly-free Weyl symmetry at the semiclassical level.

Notation.– We use the metric convention $(-,+,+,+)$. Curvature-related quantities will be defined following Misner-Thorne-Wheeler's convention [6]. When the Minkowski metric η_{ab} is used, it is understood as written in a generic coordinate system. The covariant derivative ∇ is always related to the flat metric and $\mathrm{d}\mathscr{V}$ is the Minkowski volume element.

2. Self-coupling of a helicity ± 2 graviton

Within the classification of fundamental interactions in terms of the unitary representations of the Poincaré group [7], gravity is expected to be mediated by a particle corresponding to the massless spin-2 representation. The massless character of the representation means that only the states with helicity ± 2 are physical. This representation has a direct on-shell implementation in field theory (see *e.g.* [8] and references therein). Let us stress that this on-shell description is the only firm statement one can draw from the assumption that gravity is mediated by a helicity ± 2 graviton only, with no admixture of spin 1 or 0. The usual way to proceed is to enlarge the functional space as well as the gauge symmetry, so that the degrees of freedom are kept the same. There exist two of such extensions: the well-known Fierz-Pauli theory and Weyl transverse theory [9].

If we want to describe gravity we need to couple these linear theories to matter. Nowadays it is well understood that this coupling will be through the stress-energy tensor, implying that the resulting theory would be nonlinear [10]. Let us write the resulting action of the self-coupling procedure as

$$\mathscr{A} = \mathscr{A}_0 + \mathscr{A}_{\mathrm{I}}, \tag{1}$$

where $\mathscr{A}_0$ is the free part and $\mathscr{A}_{\mathrm{I}}$ is the self-interacting part we are going to solve for. Given this action, one would be able to obtain its stress-energy tensor. $\mathscr{A}_{\mathrm{I}}$ is then fixed by the requeriment of leading to this stress-energy tensor (obtained by Hilbert's prescription) as the source of the equations of motion. One just needs to expand $\mathscr{A}_{\mathrm{I}} = \sum_{n=1}^{\infty} \lambda^n \mathscr{A}_n$ in terms of the coupling constant λ and compare different orders in the coupling constant λ to obtain the set of iterative equations of the self-coupling problem [11]:

$$\frac{\delta \mathscr{A}_n}{\delta h^{ab}} = \lim_{\gamma \to \eta} \frac{\delta \mathscr{A}_{n-1}}{\delta \gamma^{ab}}, \qquad n \geq 1. \tag{2}$$

Notice that to obtain the right-hand side of these equations one needs to write the partial actions $\mathscr{A}_n$ in terms of an auxiliary metric γ_{ab}. In this procedure different choices can be made in the form of non-minimal couplings to the auxiliary metric [8].

It can be seen that general relativity is a solution to these iterative equations leading, at the lowest order, to Fierz-Pauli theory [8, 12, 13]. In complete parallelism, we have recently shown that Weyl transverse gravity is also a solution to these iterative equations, leading at the lowest order to Weyl transverse theory [14]. Weyl transverse gravity is defined by an action motivated by unimodular gravity:

$$\mathscr{A} = \frac{1}{\lambda^2} \int \mathrm{d}\mathscr{V} R(\hat{g}), \tag{3}$$

where $R(\hat{g})$ has the same functional form as the Ricci scalar of a metric $\hat{g}_{ab}$ whose determinant is constrained by the condition $\det(\hat{g}) = \det(\eta)$. However, for us $\hat{g}_{ab}$ is just a tensor field which lives in a flat background. To make this explicit, let us use the well-known fact [15] that one can express the Ricci scalar in terms of the covariant derivatives associated with the flat metric η_{ab}, denoted by ∇, and integrate by parts to put the action in the form:

$$\mathscr{A} = \frac{1}{4\lambda^2} \int \mathrm{d}\mathscr{V} \left[2\hat{g}_{bj}\delta^a_k\delta^i_c - \hat{g}^{ai}\hat{g}_{bj}\hat{g}_{ck}\right] \nabla_a \hat{g}^{bc} \nabla_i \hat{g}^{jk}. \tag{4}$$

Written in this form, it is clear that it describes a special relativistic field theory, albeit nonlinear. This form of the action is the natural one in the framework of the self-coupling problem. Starting with (3) is just convenient to make contact with our knowledge of general relativity but, indeed, one first obtains the action (4), which can be optionally supplemented with an irrelevant surface term to give (3); see [8] for an analogue discussion in the case of general relativity. The covariant notation we are using makes it clear that this theory is invariant under general coordinate transformations. Moreover, by construction this action is invariant under transverse diffeomorphisms, whose infinitesimal form is

$$\delta_\xi \hat{g}^{ab} = \mathcal{L}_\xi \hat{g}^{ab}, \qquad \nabla_a \xi^a = 0. \tag{5}$$

The field $\hat{g}_{ab}$ is given by

$$\hat{g}_{ab} = \kappa^{-1/4} g_{ab}, \tag{6}$$

where $\kappa := \det(g)/\det(\eta)$, so that the action is invariant under Weyl transformations, whose infinitesimal version is

$$\delta_\omega g_{ab} = \omega\, g_{ab}. \tag{7}$$

As a result, there is no distinction from the perspective of self-coupling between general relativity and the theory presented here. Both nonlinear theories lead to the kind of particle one expects to mediate the gravitational interaction in the non-interacting limit and can be explicitly constructed by self-coupling this particle. The only distinction between them is their internal symmetry group. In previous analyses such as [12, 16] it was always implicitly assumed that the gauge symmetry characteristic of a helicity ± 2 graviton is that of Fierz-Pauli theory. For example in Appendix B of [16], which is nowadays considered as the definite reference as far as the uniqueness of the construction leading to general relativity is concerned, the gauge transformations of Fierz-Pauli theory are extensively used. If one accepts this internal symmetry structure, then general relativity is the only consistent nonlinear theory one would find which preserves the notion of gauge invariance [8]. Nevertheless, this is an assumption which is not necessary from the perspective of special relativistic field theory and, moreover, it is ultimately motivated by the knowledge of the symmetry group of general relativity.

To complete the construction we need to consider the inclusion of matter. The integration of the matter part of the iterative equations is straightforward, although the result is not unique. But we can appeal to two principles to resolve its non-uniqueness. The first one is the quantum version of the equivalence principle shown as a consequence of Poincaré invariance in [5]: the coupling of the field which describes gravity to matter and to itself must be governed by the same coupling constant λ. The second principle rests in the observation that conformal invariance is not a symmetry of matter in our world. A safe way to proceed is then to consider that matter fields are not affected by the Weyl transformations (7) (in the language of [17], matter fields are inert under Weyl transformations). Both principles imply that the resulting action of matter is obtained by replacing η_{ab} with the composite field $\hat{g}_{ab}$. Non-minimal couplings to $\hat{g}_{ab}$ are allowed by construction. Notice that no constraints are imposed on the matter sector.

3. A low-energy solution to the cosmological constant problem

When expressed in terms of the composite variable $\hat{g}_{ab}$, the nonlinear theory of gravity we are considering here has the form of general relativity plus matter, but it is a theory formulated in flat spacetime and the determinant of the metric is fixed by $\det(\hat{g}) = \det(\eta)$. On these grounds it can be understood as an extension of unimodular gravity which is invariant under general coordinate transformations. It is well-known that in unimodular gravity vacuum zero-point energies of matter do not gravitate. Here this is also true: the condition on the determinant implies that the corresponding (regulated) contribution in the effective action would have the form

$$\int \mathrm{d}\mathscr{V} E_{\mathrm{vac}}. \tag{8}$$

These energies act as a mere constant shift in the effective action, as we are used to in any special relativistic quantum field theory which does not contain gravity. Technically, the contributions of vacuum bubbles cancel out of correlation functions as a result of the linked-cluster theorem. This does not happen when considering general relativity (as an effective quantum field theory [18]) instead, leading to the cosmological constant problem [1, 3].

The mechanism behind this result is clearly the invariance of the theory under Weyl transformations of the gravitational field, whose infinitesimal version is given by (7).[2] This symmetry forbids radiative corrections which would otherwise couple to the determinant of the field g_{ab}. Is is not difficult to show this at one-loop level through *e.g.* the effective action scheme [19, 20, 21]. Given a theory of matter with an arbitrary combination of matter fields with different spin (0, 1/2 and 1), minimally coupled to $\hat{g}_{ab}$ as we have argued above, the heat kernel expansion permits to write the regulated effective action in terms of a cutoff κ so that one can take account of the necessary counterterms. In doing this we find three kinds of contributions. The first one is a zeroth-order contribution similar to (8). This contribution is irrelevant as we know from standard quantum field theories in flat spacetime such as electrodynamics [19]. The next contribution has the same functional form than (3), leading to a renormalization of the coupling constant λ. If we call λ_0 the bare coupling constant (whose square is proportional to the bare gravitational constant G_0) then one can read from the one-loop action the equation

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_0^2} + C_1\kappa^2 + C_2 \log\left(\frac{\kappa}{C_3}\right), \tag{9}$$

where C_1, C_2 and C_3 are constants with convenient physical dimensions and whose value depends on the particle content of the matter sector [20]. There exist other contributions which involve quadratic expresions in the "Ricci" tensor $R_{ab}(\hat{g})$ but we are going to ignore these, which also appear in general relativity and are expected to lead to higher-derivative deviations from the second-order field equations at high energies. Thus in essence we have only one physically relevant renormalization group equation, given by (9). The invariance of the theory under Weyl transformations implies, in contrast with general relativity, the absence of any renormalization group equation describing the running of the cosmological constant sector. Most importantly, it seems perfectly possible to realize this symmetry in a non-anomalous way at the semiclassical level: the background structure permits to construct Euclidean path integral measures for matter which are invariant both under transverse diffeomorphisms as well as Weyl transformations (with inert matter). These measures would be defined in terms of scalar products in which the corresponding evolution operators are self-adjoint [21]. The extra structure permits the

[2] Indeed, the extension of this symmetry to matter in the form of the usual conformal transformations was one of the first options considered (and discarded) in the literature to deal with the cosmological constant problem [1]. Previous formulations failed to notice however that, if matter fields are not affected under these transformations, one can ensure both radiative stability of the cosmological constant sector while having non-zero mass terms for matter.

construction of scalar products invariant under the internal symmetries. For example, for a real scalar field one would have simply

$$(\phi_1, \phi_2) = \int \mathrm{d}\mathscr{V} \phi_1 \phi_2. \tag{10}$$

The same symmetry arguments imply that the theory does not contain any bare cosmological constant, *i.e.* Weyl symmetry forbids the occurrence of such a constant in the action. However, this theory describe gravity in a way which is compatible with all the known experiments in gravitation, as the corresponding field equations contain solutions for arbitrary values of the cosmological constant. It is easy to see that the field equations are, by construction, traceless. In particular, in the gauge $\det(g) = \det(\eta)$ they reduce to the trace-free Einstein equations:

$$R_{ab} - \frac{1}{4} R g_{ab} = \lambda^2 \left(T_{ab} - \frac{1}{4} T g_{ab} \right). \tag{11}$$

As a consequence, any solution of the Einstein field equations for arbitrary values of the cosmological constant is imported to this theory. In principle one could expect additional solutions in which the Ricci scalar and the trace of the stress-energy momentum tensor are not tied up through a constant but a general function, but this is not the case.

This can be seen by means of the following argument. Given the fact that the canonical stress-energy tensor of gravity and matter is conserved under solutions [15], we recover the Einstein field equations with a phenomenological integration constant, unrelated to zero-point energies of matter, playing the role of a cosmological constant; this is shown in Sec. VI-A of [8]. The arguments above show that this effective cosmological constant is not renormalized by radiative corrections: its value is protected by local symmetries. The resulting field equations include potential energies in the matter sector as, even if gravity is not directly coupled to these terms by construction, they inevitably appear in the definition of the canonical stress-energy tensor. Notice that the conservation of the canonical stress-energy tensor is not an additional equation on its own, but a consequence of the translational symmetry of the theory and the field equations (11). This is in contrast with the situation in usual formulations of unimodular gravity (see the corresponding discussion in [8] and references therein).

Radiative corrections are intrinsically a feature of the semiclassical theory in which matter fields are quantized, so that the form of the matter action is fundamental to address any question concerning them. In Weyl transverse gravity, Weyl invariance dictates a fairly different coupling between gravity and matter fields with respect to the situation in general relativity. Although it should be clear that any argument trying to read the form of these contributions from the form of the classical equations of motion (maybe in some specific gauge) alone is misleading, there still exist (somewhat inexplicably) confusion in the literature in this respect. A well-known procedure to obtain the form of these radiative corrections leads to a definite answer which settles down the issue of radiative stability unambiguosly, as dictated by the form of the symmetries of the gravitational action.

In sumary, Weyl transverse gravity is a promising candidate to reconcile the classical and quantum-mechanical aspects of the cosmological constant problem. Any theory of emergent gravity which is described by means of this theory in the infrared regime would surpass this problem: on the one hand, Weyl invariance implies that there is no bare cosmological constant at the level of the action nor radiative corrections to it when semiclassical effects are taken into account; on the other hand, one recovers the same predictions than in classical general relativity at the purely classical level. As a result one can construct classical solutions to the gravitational field equations whose cosmological constant is guaranteed to be stable (indeed, unchanged) under radiative corrections due to the protection of the underlying symmetries. And, maybe surprisingly, all the necessary input to obtain this is deceptively simple: that gravity is mediated by gravitons.

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[1] C. P. Burgess. The Cosmological Constant Problem: Why it's hard to get Dark Energy from Micro-physics. *ArXiv e-prints*, September 2013.
[2] Steven Weinberg. The cosmological constant problem. *Rev. Mod. Phys.*, 61:1–23, Jan 1989.
[3] J. Martin. Everything you always wanted to know about the cosmological constant problem (but were afraid to ask). *Comptes Rendus Physique*, 13:566–665, July 2012.
[4] S. Carlip. Challenges for emergent gravity. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, (0):–, 2013.
[5] S. Weinberg. Photons and Gravitons in S-Matrix Theory: Derivation of Charge Conservation and Equality of Gravitational and Inertial Mass. *Physical Review*, 135:1049–1056, August 1964.
[6] C.W. Misner, K.S. Thorne, and J.A. Wheeler. *Gravitation*. Number pt. 3 in Gravitation. W. H. Freeman, 1973.
[7] E. Wigner. On unitary representations of the inhomogeneous lorentz group. *Annals of Mathematics*, 40(1):pp. 149–204, 1939.
[8] Carlos Barceló, Raúl Carballo-Rubio, and Luis J. Garay. Unimodular gravity and general relativity from graviton self-interactions. *Phys.Rev.*, D89:124019, 2014.
[9] E. Álvarez, D. Blas, J. Garriga, and E. Verdaguer. Transverse Fierz Pauli symmetry. *Nuclear Physics B*, 756:148–170, November 2006.
[10] T. Ortín. *Gravity and Strings*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2007.
[11] S. N. Gupta. Gravitation and Electromagnetism. *Physical Review*, 96:1683–1685, December 1954.
[12] S. Deser. Self-interaction and gauge invariance. *General Relativity and Gravitation*, 1:9–18, March 1970.
[13] L. M. Butcher, M. Hobson, and A. Lasenby. Bootstrapping gravity: A consistent approach to energy-momentum self-coupling. *Physical Review D*, 80(8):084014, October 2009.
[14] C. Barceló, R. Carballo-Rubio, and L. J. Garay. Absence of cosmological constant problem in special relativistic field theory of gravity. *ArXiv e-prints*, June 2014.
[15] N. Rosen. General relativity and flat space. i. *Phys. Rev.*, 57:147–150, Jan 1940.
[16] D. G. Boulware and S. Deser. Classical general relativity derived from quantum gravity. *Annals of Physics*, 89:193–240, January 1975.
[17] Enrique Alvarez and Roberto Vidal. Weyl transverse gravity (WTDiff) and the cosmological constant. *Phys.Rev.*, D81:084057, 2010.
[18] C. P. Burgess. Quantum Gravity in Everyday Life: General Relativity as an Effective Field Theory. *Living Reviews in Relativity*, 7:5, April 2004.
[19] D. V. Vassilevich. Heat kernel expansion: user's manual. *Phys. Rep.*, 388:279–360, December 2003.
[20] M. Visser. Sakharov's Induced Gravity. *Modern Physics Letters A*, 17:977–991, 2002.
[21] V. Mukhanov and S. Winitzki. *Introduction to Quantum Effects in Gravity*. Cambridge University Press, 2007.