Reply to “Comment on ‘Ratchet universality in the presence of thermal noise’”

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The Comment by Quintero *et al.* [preceding Comment, *Phys. Rev. E* **88**, 066101 (2013)] does not dispute the central result of our paper [Martínez and Chacón, *Phys. Rev. E* **87**, 062114 (2013)], which is a theory explaining the interplay between thermal noise and symmetry breaking in the ratchet transport of a Brownian particle moving on a periodic substrate subjected to a temporal biharmonic excitation $\gamma[\eta \sin(\omega t) + \alpha(1 - \eta) \sin(2\omega t + \varphi)]$. In the Comment, the authors claim, on the sole basis of their numerical simulations for the *particular* case $\alpha = 2$, that “there is no such universal force waveform and that the evidence obtained by the authors otherwise is due to their particular choice of parameters.” Here we demonstrate by means of theoretical arguments and additional numerical simulations that all the conclusions of our original article are preserved.

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The foregoing Comment by Quintero *et al.* [1] offers some criticisms on certain particular (numerical) aspects of our previous paper [2], in which we studied theoretically and numerically a universal model: a Brownian particle moving on a periodic substrate subjected to a biharmonic excitation

$$\dot{x} + \sin x = \sqrt{\sigma} \xi(t) + \gamma F_{\text{bihar}}(t),$$

$$F_{\text{bihar}}(t) \equiv \eta \sin(\omega t) + \alpha(1 - \eta) \sin(2\omega t + \varphi), \quad (1)$$

where γ is an amplitude factor; the parameters ($\eta \in [0, 1]$, $\alpha > 0$) and φ account for the relative amplitude and initial phase difference of the two harmonics, respectively; $\xi(t)$ is a Gaussian white noise with zero mean and $\langle \xi(t) \xi(t + s) \rangle = \delta(s)$; and $\sigma = 2k_b T$, with k_b and T the Boltzmann constant and temperature, respectively.

Quintero *et al.* state in the abstract of their Comment that “The authors claim that their simulations prove the existence of a universal waveform of the external force that optimally enhances directed transport, hence confirming the validity of a previous conjecture put forth by one of them in the limit of vanishing noise intensity.” We disagree with this statement. Note that the existence of such a universal waveform was clearly conjectured in Ref. [3] in the context of a criticality scenario: Optimal enhancement of directed ratchet transport (DRT) is achieved when maximally effective (i.e., critical) symmetry breaking occurs. The mathematical proof of the ratchet conjecture was subsequently completed in Ref. [4], where such a universal waveform was shown to be unique for both temporal and spatial biharmonic forces. This universal waveform is a direct consequence of the degree of symmetry breaking (DSB) mechanism: It is possible to consider a quantitative measure of the DSB on which the strength of directed transport by symmetry breaking must depend. Such a theory of ratchet universality has been applied to predict successfully the behavior of two kinds of soliton ratchets [5,6]. The theory demonstrated in our original article [2] uses, among other theoretical ideas, ratchet universality to explain the interplay between thermal noise and symmetry breaking in

the ratchet transport of Eq. (1). The authors of the Comment neither take into account nor cite Refs. [3,5,6].

All the criticisms made by Quintero *et al.* are based solely on numerical simulations of Eq. (1) for the *particular* case $\alpha = 2$. However, the theoretical discussion of the structural stability of the ratchet scenario under changes in the parameter α [see the text and Fig. 1(c) in Ref. [2]] is *essential* to understand the theory demonstrated in our original article. Indeed, the authors claim that the numerical results shown in Fig. 1 of their Comment allow one to reach their two main conclusions.

(a) “First of all, our results are not compatible with the existence of an optimal force waveform The apparent (approximate) confirmation of the prediction $\eta_{\text{opt}} = 4/5$ seems to arise from the specific choice of simulation parameters made by the authors.” As anticipated above, this conclusion arises from a misunderstanding on the part of the authors concerning the theory demonstrated in our original article [2]. Specifically, we demonstrated in Ref. [2] that the effect of thermal noise on the purely deterministic ratchet scenario can be understood as an effective noise-induced change of the potential barrier that allows one to understand the existence and behavior of the deviation $\Delta\eta(\alpha) \equiv \eta_{\text{opt}}(\alpha) - \eta_{\text{opt}}^{\sigma \rightarrow 0}(\alpha)$ as α is changed, while keeping constant the remaining parameters. Clearly, as mentioned in Ref. [2], the effect of noise on the DRT depends on the amplitude of the biharmonic excitation while keeping constant the remaining parameters. This means that the deviation $\Delta\eta(\alpha)$ will depend in turn on the amplitude factor γ , while our theory, which was shown to be numerically confirmed for the particular value $\gamma = 2$ just as an illustrative example, holds for other values of the amplitude factor. Indeed, Fig. 1 shows an additional illustrative example for $\gamma = 6$. One finds that the range where the DSB mechanism dominates over the thermal interwell activation mechanism is larger for $\gamma = 6$ than for $\gamma = 2$, as expected from our theory.

(b) “Second, although we cannot decrease γ below 0.8 without introducing too much uncertainty, the figure clearly illustrates that the trend of the value of η that optimizes v [for the average velocity] is toward the value $2/3$, which all theories predict in their range of validity, i.e., in the limit of

FIG. 1. (Color online) Value of η where the average velocity is maximum $\eta_{\text{opt}}^{\sigma>0}$ versus α [see Eq. (1)] for $\varphi = \varphi_{\text{opt}} \equiv \pi/2$, $\omega = 0.08\pi$, and $\sigma = 2$ and two values of the amplitude factor $\gamma = 2$ (squares) and $\gamma = 6$ (dots). Also plotted is the theoretical prediction for the purely deterministic case $\eta_{\text{opt}}(\alpha) \equiv 2\alpha/(1 + 2\alpha)$ (dashed line) and the function $\eta^*(\alpha) \equiv (4\alpha - 2)/(4\alpha - 1)$ (solid line) (cf. Ref. [2]).

weak external forces.” Note that this extrapolation argument invalidates itself: Directed ratchet transport is not properly defined in the limit of weak (with respect to the potential barrier) external forces in the presence of finite but arbitrary thermal noise, which is indeed confirmed in Fig. 1 of the Comment by the absence of numerical results for values of $\gamma \in]0, 0.8[$.

The rest of the Comment presents a few examples of the application of a numerical fitting method for the average velocity that is solely subjected to obvious constraints regarding the breaking of relevant symmetries in Eq. (1). Two observations concerning this method are in order. First, as is well known, numerical and experimental data providing the

dependence of the average velocity on relevant parameters, such as η and φ , in many ratchets fit generally into soft curves that exhibit few extrema. It is thus not so surprising that the authors’ fitting method yielded good fits having so many free adjustable parameters [see, for example, Fig. 3(b) in the Comment]. Note that a few parameters are enough to fit much more complex sets of data. A recent example with additional historical interest is discussed in Ref. [7]. Second, the authors’ fitting method is not a physical theory since it has no predictive power at all, precisely because of its dependence on many adjustable parameters. This is in sharp contrast to the theory of the ratchet universality that predicted and explained, for example, why the directed soliton current is dependent on the number of atoms in the DRT of bright solitons formed in a quasi-one-dimensional Bose-Einstein condensate [6], as well as the strong enhancement of DRT of topological solitons in Frenkel-Kontorova chains due to the introduction of phase disorder into the asymmetric periodic driving [5]. Finally, the authors again misunderstand the ratchet universality when commenting on their results shown in their Fig. 4 (final paragraph): The ratchet universality does not predict a sinusoidal dependence of the average velocity on the initial phase difference φ for general values of the amplitude factor. This is only valid for sufficiently small amplitudes (see Ref. [3]). Only the optimal values of φ are predicted to be universal. In the present case, these values are $\varphi_{\text{opt}} = \{\pi/2, 3\pi/2\}$ (see Ref. [3]), which are indeed confirmed by the numerical results shown in Fig. 4 of the Comment.

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