Cosmological Perturbations in LQC: Mukhanov-Sasaki Equations in Different Approaches

i-link 16 September 2014

Guillermo A. Mena Marugán

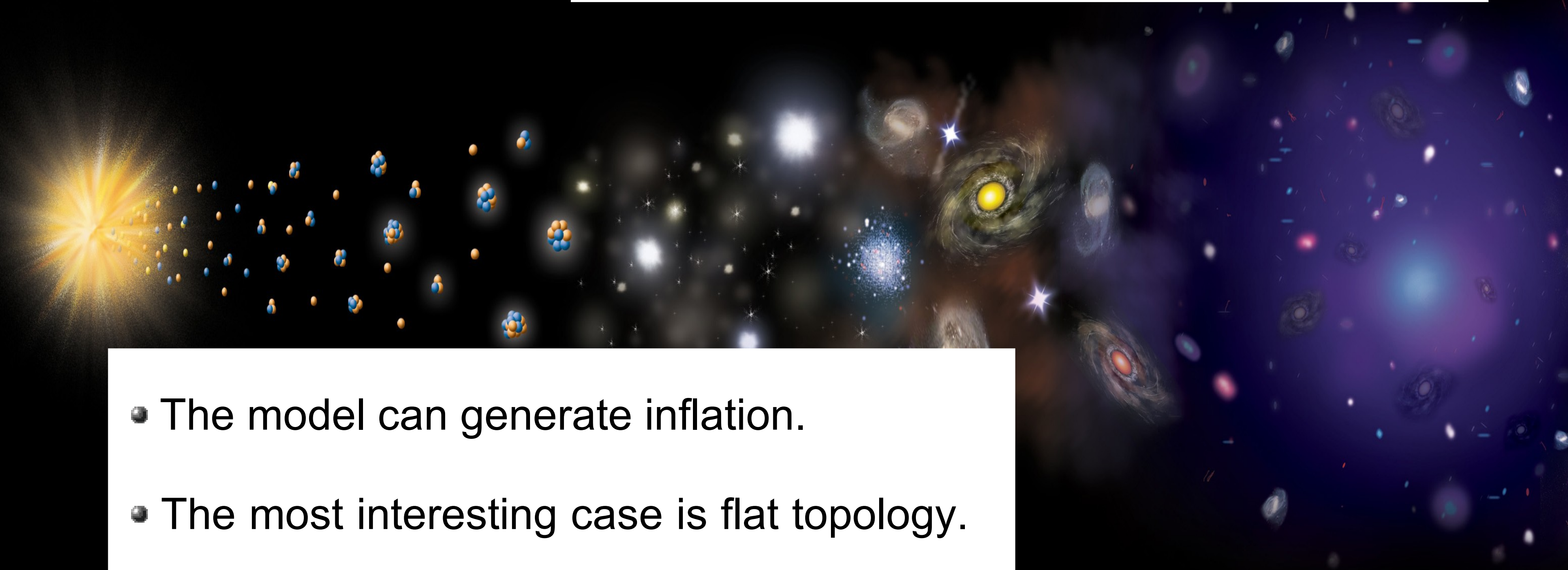
IEM-CSIC (Laura Castelló Gomar, Mikel Fernández-Méndez & Javier Olmedo)

75 AÑOS CSIC

Consejo Superior de Investigaciones Científicas

The model

- We consider **perturbed** FRW universes filled with a **massive** scalar field, in the context of LQC.



- The model can generate inflation.
- The most interesting case is flat topology.
- We assume **compact** spatial sections.

LQC approaches

- In LQC, two types of approaches have been considered:

☆ **Effective** equations from the closure of the constraint algebra.

☆ <u>Direct</u> quantization:

➔ **Hybrid** approach.

➔ ***Dressed metric***, with perturbations propagating on it as test fields.

Our strategy

- **Approximations**: As few as possible. Should be derived or at least checked for consistency.

- **LQC techniques**, with a quantum metric, and exploring potentially observable consequences.

- We want to include the **quantum nature** of spacetime, rather than treating perturbations as test fields in a generalized QFT.

Perturbations of **compact** FRW

Approximation: Truncation at **<u>quadratic</u>** perturbative order in the action.

- Uses the **modes** of the Laplace-Beltrami operator of the FRW spatial sections.
- Perturbations have **no zero modes**.
- Corrections to the action are **quadratic**.
- Not necessarily the same **truncation** order in the **metric.**
- The system is **symplectic** and **constrained**.

Hybrid approach

Approximation: Effects of (loop) quantum geometry are only accounted for in the background

- Loop quantum corrections on matter **d.o.f.** and perturbations are ignored.
- Succesfully applied in Gowdy cosmologies.

Mukhanov-Sasaki variables

- We use these **gauge invariants** for the perturbations.
- Then, the primordial power spectrum is easy to derive.
- Their field equations match criteria for the choice of a unique Fock quantization.
- Their use facilitates comparison with other approaches.
- Although we will fix the gauge, they can be part of a canonical set which includes the perturbative constraints.

Uniqueness of the Fock description

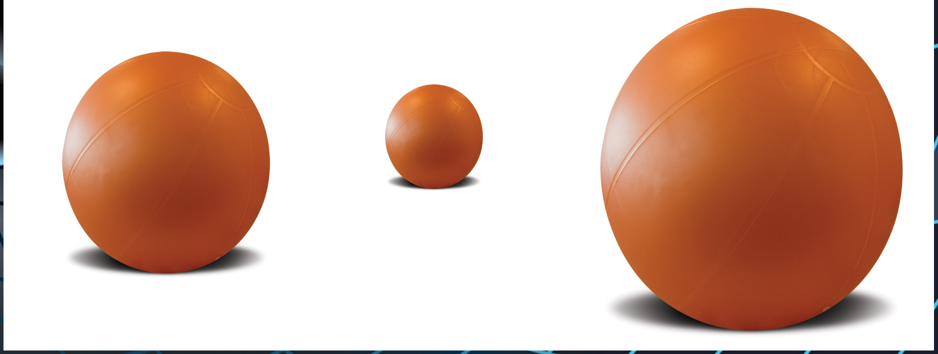
- The **ambiguity** in selecting a **Fock representation** in QFT is removed by:

  - appealing to *background* **symmetries**.
  - demanding the **UNITARITY** of the quantum evolution.

- There is additional ambiguity in the **separation of the background** and the matter field. This introduces time-dependent canonical field transformations.

- Our proposal selects a **UNIQUE canonical pair** and an **EQUIVALENCE CLASS** of invariant **Fock representations** for their CCR's.

Other works DO NOT incorporate the same field scaling. This may affect the quantum description, and the *effective* approach.



Loop Quantum FRW Cosmology

- Avoids the Big Bang.

Classical system: FRW

Massive scalar field coupled to a compact, flat FRW universe.

- **Geometry:** Ashtekar-Barbero variables.

$$A_a^i = c\,{}^0e_a^i(2\pi)^{-1}; \qquad E_i^a = p\sqrt{{}^0e}\,{}^0e_i^a(2\pi)^{-2}.$$

$$\{c, p\} = 8\pi G\gamma/3. \qquad V = |p|^{3/2}.$$

Scale factor and its momentum.

$$a^2 = e^{2\alpha} = |p|(2\pi\sigma)^{-2}; \qquad \pi_\alpha = -pc(\gamma 8\pi^3\sigma^2)^{-1}. \qquad \sigma^2 = G(6\pi^2)^{-1}.$$

- **Matter:**

$$\varphi = (2\pi)^{3/2}\sigma\phi; \qquad \pi_\varphi = (2\pi)^{-3/2}\sigma^{-1}\pi_\phi.$$

Classical system: Modes

- We expand inhomogeneities in a (real) **Fourier basis**: $\vec{n} \in \mathbb{Z}^3, \qquad n_1 \geq 0\,.$

$$Q_{\vec{n},+} = \frac{1}{2\pi^{3/2}} \cos \vec{n} \cdot \vec{\theta}\,, \qquad Q_{\vec{n},-} = \frac{1}{2\pi^{3/2}} \sin \vec{n} \cdot \vec{\theta}\,.$$

- The basis is **orthonormal**, and we exclude the zero mode in the expansions.

- These functions are eigenmodes of the Laplace-Beltrami operator of the standard flat metric on the three-torus, with eigenvalue $-\omega_n^2 = -\vec{n} \cdot \vec{n}\,.$

- We only consider **scalar perturbations**.

Classical system: Inhomogeneities

- Mode expansion of the inhomogeneities: metric and field.

$$h_{ij} = (\sigma e^{\alpha})^2 \left[{}^0h_{ij} + 2\epsilon (2\pi)^{3/2} \sum \left\{ a_{\vec{n},\pm}(t) Q_{\vec{n},\pm} {}^0h_{ij} + b_{\vec{n},\pm}(t) \left(\frac{3}{\omega_n^2} (Q_{\vec{n};\pm})_{,ij} + Q_{\vec{n},\pm} {}^0h_{ij} \right) \right\} \right],$$

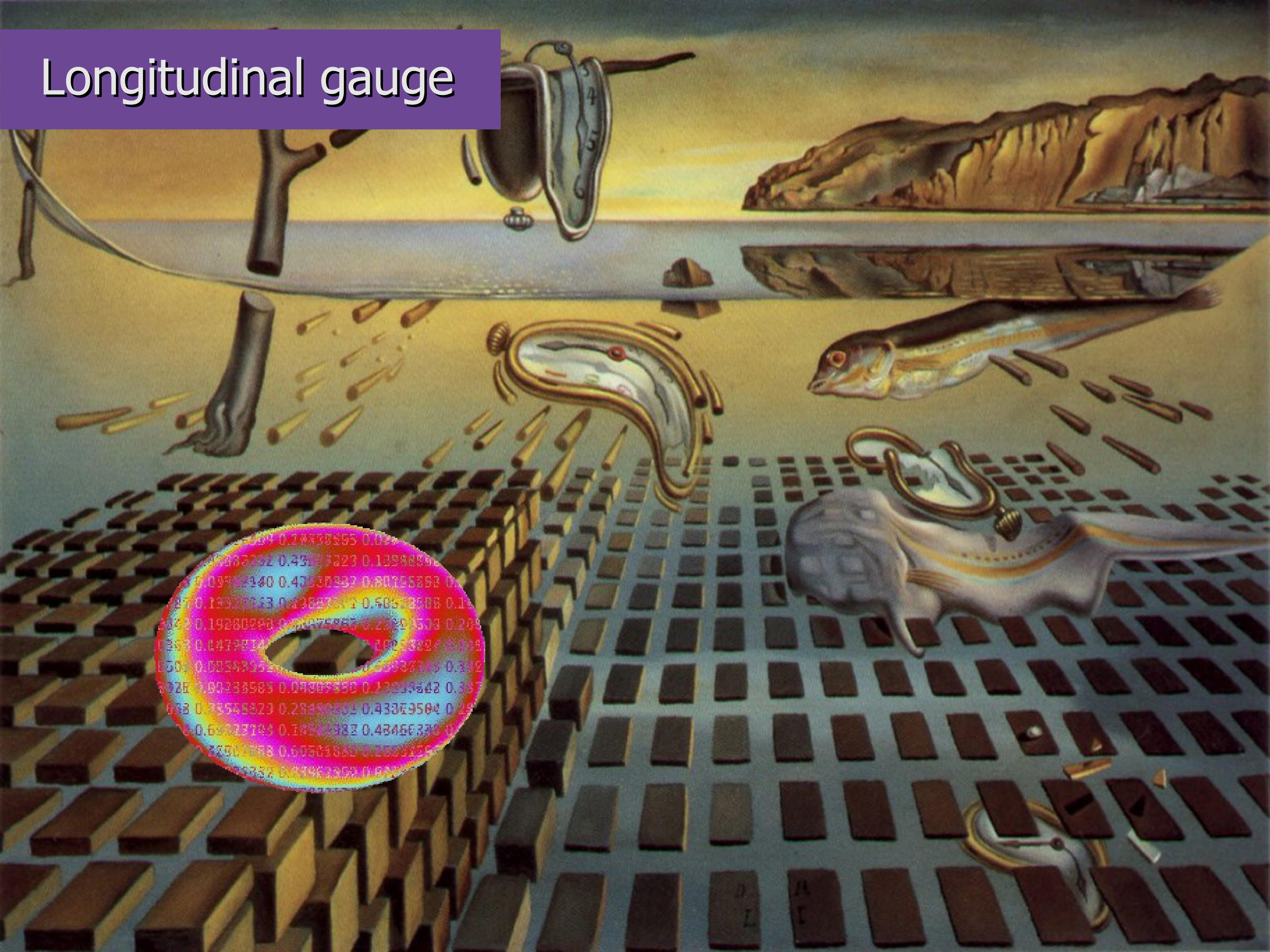
$$N = \sigma N_0(t) \left[1 + \epsilon (2\pi)^{3/2} \sum g_{\vec{n},\pm}(t) Q_{\vec{n},\pm} \right], \qquad N_i = \epsilon (2\pi)^{3/2} \sigma^2 e^{\alpha} \sum \frac{k_{\vec{n},\pm}(t)}{\omega_n^2} (Q_{\vec{n},\pm})_{;i},$$

$$\Phi = \frac{1}{\sigma} \left[\frac{\varphi(t)}{(2\pi)^{3/2}} + \epsilon \sum f_{\vec{n},\pm}(t) Q_{\vec{n},\pm} \right].$$

- Truncating at **quadratic** order in perturbations:

$$H = \frac{N_0 \sigma}{2} C_0 + \epsilon^2 \sum \left(N_0 H_2^{\vec{n},\pm} + N_0 g_{\vec{n},\pm} H_1^{\vec{n},\pm} + k_{\vec{n},\pm} \tilde{H}_1^{\vec{n},\pm} \right).$$

Longitudinal gauge



Longitudinal gauge

- We adopt a **longitudinal gauge**.
- After **REDUCTION**, the background variables are corrected with **quadratic perturbations** to form a **CANONICAL SET**.
- The remaining **Hamiltonian constraint** reads:

$$H=\frac{N_0\sigma}{2}C_0+\epsilon^2 N_0\sum H_2^{\vec{n},\pm},$$

$$H_2^{\vec{n},\pm}=\bar{E}_{\bar{f}\bar{f}}\bar{f}_{\vec{n},\pm}^2+\bar{E}_{\bar{f}\pi}\bar{f}_{\vec{n},\pm}\pi_{\bar{f}_{\vec{n},\pm}}+\bar{E}_{\pi\pi}\pi^2_{\bar{f}_{\vec{n},\pm}},$$

that is,quadratic in the ***RESCALED*** perturbative field variables.

Mukhanov-Sasaki

- The **Mukhanov-Sasaki gauge invariants** are related to the perturbative variables by a **linear transformation** in the gauge-fixed system:

$$v_{\vec{n},\pm} = A_n(\bar{q}_A, \pi_{\bar{q}_A})\,\bar{f}_{\vec{n},\pm} + B_n(\bar{q}_A, \pi_{\bar{q}_A})\,\pi_{\bar{f}_{\vec{n},\pm}},$$
$$\pi_{v_{\vec{n},\pm}} = C_n(\bar{q}_A, \pi_{\bar{q}_A})\,\bar{f}_{\vec{n},\pm} + D_n(\bar{q}_A, \pi_{\bar{q}_A})\,\pi_{\bar{f}_{\vec{n},\pm}},$$

$\bar{q}_A = \{\bar{\alpha}, \bar{\varphi}\}$ are the **canonical homogeneous variables** after correcting them with quadratic perturbations, and $\pi_{\bar{q}_A}$ their momenta.

- This change can be completed at our truncation order into a **canonical transformation** on the whole of the phase space:

$$\tilde{q}_A = \bar{q}_A + \frac{\epsilon^2}{2}\sum\left(\partial_{\pi_{\bar{q}_A}}\pi_{\bar{f}_{\vec{n},\pm}}\right)\bar{f}_{\vec{n},\pm} - \frac{\epsilon^2}{2}\sum\left(\partial_{\pi_{\bar{q}_A}}\bar{f}_{\vec{n},+}\right)\pi_{\bar{f}_{\vec{n},\pm}},$$
$$\pi_{\tilde{q}_A} = \pi_{\bar{q}_A} - \frac{\epsilon^2}{2}\sum\left(\partial_{\bar{q}_A}\pi_{\bar{f}_{\vec{n},\pm}}\right)\bar{f}_{\vec{n},\pm} + \frac{\epsilon^2}{2}\sum\left(\partial_{\bar{q}_A}\bar{f}_{\vec{n},\pm}\right)\pi_{\bar{f}_{\vec{n},\pm}}.$$

Mukhanov-Sasaki

- After this canonical transformation, the **new Hamiltonian constraint** (at our perturbative order) is:

$$H=\frac{N_0\sigma}{2}C_0(\tilde{q}_A,\pi_{\tilde{q}_A})+\epsilon^2 N_0\sum \tilde{H}_2^{\vec{n},\pm}(\tilde{q}_A,\pi_{\tilde{q}_A},v_{\vec{n},\pm},\pi_{v_{\vec{n},\pm}}),$$

$$\epsilon^2\sum_{\vec{n},\pm}\tilde{H}_2^{\vec{n},\pm}=\frac{\sigma}{2}\sum_A\left([\bar{q}_A-\tilde{q}_A]\partial_{\bar{q}_A}C_0+[\pi_{\bar{q}_A}-\pi_{\tilde{q}_A}]\partial_{\pi_{\bar{q}_A}}C_0\right)+\epsilon^2\sum_{\vec{n},\pm}H_2^{\vec{n},\pm}.$$

- The quadratic perturbative Hamiltonian is just the **Mukhanov-Sasaki Hamiltonian** in the rescaled variables.

$$4\pi e^{\tilde{\alpha}}\sum\tilde{H}_2^{\vec{n},\pm}=\pi_{v_{\vec{n},\pm}}^2+\left[4\pi^2\omega_n^2+e^{-4\tilde{\alpha}}\left(19\pi_{\tilde{\varphi}}^2-18\frac{\pi_{\tilde{\varphi}}^4}{\pi_{\tilde{\alpha}}^2}\right)+\sigma^2 m^2 e^{2\tilde{\alpha}}\left(1-2\tilde{\varphi}^2-12\tilde{\varphi}\frac{\pi_{\tilde{\varphi}}}{\pi_{\tilde{\alpha}}}\right)\right]v_{\vec{n},\pm}^2.$$

- It has **no crossed** configuration-momentum **term**.

Mukhanov-Sasaki

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$$\epsilon^2 \sum_{\vec{n},\pm} \tilde{H}_2^{\vec{n},\pm} = \frac{\sigma}{2} \sum_A \left([\bar{q}_A - \tilde{q}_A] \partial_{\bar{q}_A} C_0 + [\pi_{\bar{q}_A} - \pi_{\tilde{q}_A}] \partial_{\pi_{\bar{q}_A}} C_0 \right) + \epsilon^2 \sum_{\vec{n},\pm} H_2^{\vec{n},\pm}.$$

- The quadratic perturbative Hamiltonian is just the **Mukhanov-Sasaki Hamiltonian** in the rescaled variables.

$$4\pi e^{\tilde{\alpha}} \sum \tilde{H}_2^{\vec{n},\pm} = \pi_{v_{\vec{n},\pm}}^2 + \left[4\pi^2 \omega_n^2 + e^{-4\tilde{\alpha}} \left(19\pi_{\tilde{\varphi}}^2 - 18 \frac{\pi_{\tilde{\varphi}}^4}{\pi_{\tilde{\alpha}}^2} \right) + \sigma^2 m^2 e^{2\tilde{\alpha}} \left(1 - 2\tilde{\varphi}^2 - 12\tilde{\varphi} \frac{\pi_{\tilde{\varphi}}}{\pi_{\tilde{\alpha}}} \right) \right] v_{\vec{n},\pm}^2 .$$

- It has **no crossed** configuration-momentum **term**.

Quantization: Homogeneous sector

- We quantize the homogeneous sector with standard loop techniques.
- We can adopt a basis of **volume** eigenstates $\{|v\rangle ; v \in \mathbb{R}\}$, with $\hat{v} \propto |\hat{p}|^{3/2}$.
- The inner product is **discrete**: $\forall\, v_1, v_2 \in \mathbb{R},\ \langle v_1 | v_2 \rangle = \delta^{v_1}_{v_2}$.
- On straight edges, holonomy elements are linear in $N_{\bar{\mu}} := e^{i\bar{\mu}c/2}$.
- We use the so-called improved dynamics. Then

$$\hat{N}_{\bar{\mu}} |v\rangle := |v+1\rangle, \quad \hat{v}|v\rangle = v|v\rangle.$$

Quantization: Homogeneous sector

- The inverse volume is regularized as usual in LQC.
- We **decouple the zero-volume** state and change the constraint densitization

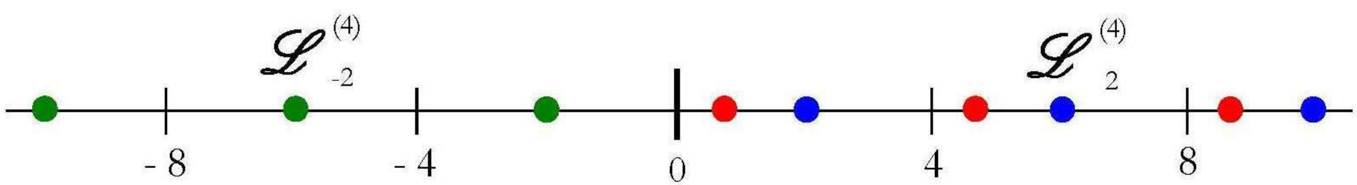
$$\hat{C}_0 = \left[\widehat{\frac{1}{V}}\right]^{1/2} \hat{C}_0 \left[\widehat{\frac{1}{V}}\right]^{1/2}.$$

$$\hat{C}_0 = -\frac{3}{4\pi G \gamma^2}\hat{\Omega}_0^2 + \hat{\pi}_\phi^2 + m^2 \hat{\phi}^2 \hat{V}^2.$$

- With **our proposal**, the gravitational part is a difference operator:

$$\widehat{\Omega}_0^2 |v\rangle = f_+(v)|v+4\rangle + f(v)|v\rangle + f_-(v)|v-4\rangle.$$

that acts on the **superselection** sectors $\mathscr{L}^{(4)}_{\pm\epsilon} := \{\pm(\epsilon + 4n),\ \ n \in \mathbb{N}\}, \epsilon \in (0,4].$



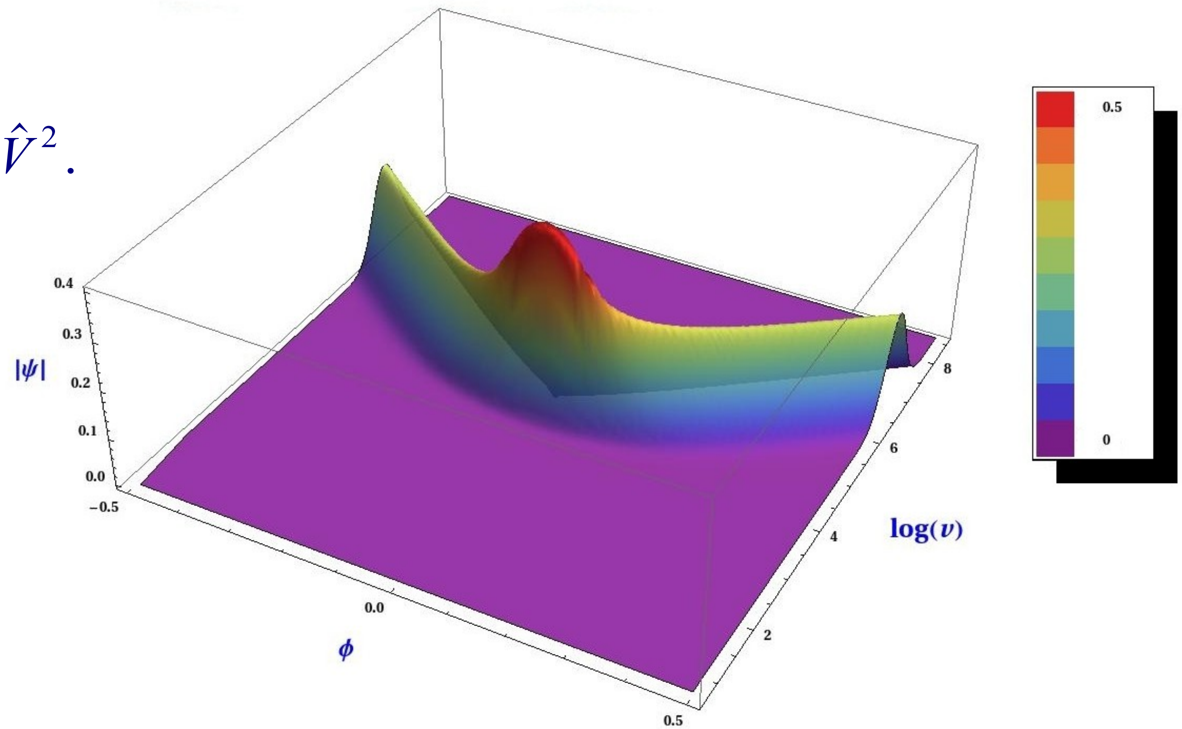
Quantization: Homogeneous sector

- $\Omega_0 = pc$ has been approximated with holonomies by $\Omega_0 \approx 2\pi G \gamma v \sin b$, with $\{b, v\} = 2.$

- States evolve in the scalar field *with the square root* of

$$\hat{H}_0^2 = \frac{3}{4\pi G \gamma^2} \hat{\Omega}_0^2 - m^2 \hat{\phi}^2 \hat{V}^2 .$$

(Or any alternate Hamiltonian for positive frequencies...)



Fock and hybrid quantizations

- We use annihilation and creation operators for the (***rescaled***) Mukhanov-Sasaki variables, constructed, e.g. with ***no mass***.

- We obtain a **Fock space** $\mathscr{F}$, with basis of n-*particle* states:

$$\left\{ \left| N \right\rangle = \left| N_{(1,0,0),\,+}, N_{(1,0,0),\,-}, \ldots \right\rangle ; \quad N_{\vec{n},\pm} \in \mathbb{N}, \ \sum N_{\vec{n},\pm} < \infty \right\}.$$

- The Hilbert space of the hybrid quantization is $H_{kin}^{FRW-LQC} \otimes H_{kin}^{matt} \otimes \mathscr{F}$.

- Any traslational invariant Fock representation in the same equivalence class would be acceptable. Restrictions may come from demands on **non-linear operators**.

Fock and hybrid quantizations

- We substitute π_ϕ^2 by H_0^2 in the quadratic perturbative contribution to the Hamiltonian.

- This perturbative contribution $\tilde{H}_2 = \sum \tilde{H}_2^{\vec{n},\pm}$ becomes linear in the homogeneous field momentum:

$$\tilde{H}_2^{\vec{n},\pm} \equiv \frac{\sigma}{2V} C_2^{\vec{n},\pm}; \qquad \boxed{C_2^{\vec{n},\pm} = -\Theta_e^{\vec{n},\pm} - \Theta_o^{\vec{n},\pm} \pi_\phi,}$$

Hybrid quantization

- We quantize the quadratic contribution of the perturbations adapting the **proposals of the homogeneous sector** and using a symmetric factor ordering:

  - We **symmetrize** products of the type $\hat{\phi}\hat{\pi}_{\phi}$.
  - We take a **symmetric geometric** factor ordering $V^k A \rightarrow \hat{V}^{k/2}\hat{A}\hat{V}^{k/2}$.
  - We adopt the **LQC** representation $(cp)^{2\mathrm{m}} \rightarrow \left[\hat{\Omega}_0^2\right]^m$.
  - In order to **preserve the FRW superselection sectors**, we adopt the prescription $(cp)^{2\mathrm{m}+1} \rightarrow \left[\hat{\Omega}_0^2\right]^{m/2}\hat{\Lambda}_0\left[\hat{\Omega}_0^2\right]^{m/2}$, where $\hat{\Lambda}_0$ is defined like $\hat{\Omega}_0$ but with double steps.

- The Hamiltonian constraint reads then $\hat{C}_0 - \sum \hat{\Theta}_e^{\vec{n},\pm} - \sum \left(\hat{\Theta}_o^{\vec{n},\pm}\hat{\pi}_{\phi}\right)_{sym} = 0.$

Born-Oppenheimer ansatz

- Consider states whose evolution in the inhomogeneities and FRW geometry **split**, with *positive frequency* in the homogeneous sector:

$$\Psi=\chi_0(V,\phi)\psi(N,\phi),\qquad \chi_0(V,\phi)=\boldsymbol{P}\left[\exp\left(i\int_{\phi_0}^{\phi}d\tilde{\phi}\ \hat{H}_0(\tilde{\phi})\right)\right]\chi_0(V).$$

The FRW state is peaked (*semiclassical*) and evolves unitarily.

- Disregard **nondiagonal** elements for the FRW geometry sector in the constraint and call:

$$\boldsymbol{d}_\phi\hat{O}=\partial_\phi\hat{O}-i\ [\hat{H}_0,\hat{O}].$$

Born-Oppenheimer ansatz

- The diagonal FRW-geometry part of the constraint gives:

$$-\partial_\phi^2\psi - i\left(2\langle \hat{H}_0\rangle_\chi - \langle\hat{\Theta}_o\rangle_\chi\right)\partial_\phi\psi = \left[\langle \hat{\Theta}_e + (\hat{\Theta}_o\hat{H}_0)_{sym}\rangle_\chi + i\left\langle \boldsymbol{d}_\phi \hat{H}_0 - \frac{1}{2}\boldsymbol{d}_\phi\hat{\Theta}_o\right\rangle_\chi\right]\psi .$$

- The term in cyan can be ignored if $\langle \hat{H}_0\rangle_\chi$ is **not negligible small**.

- Besides, if we can **neglect**: a) The second derivative of ψ,
  b) The total ϕ-derivatives.

$$-i\partial_\phi\psi = \frac{\langle \hat{\Theta}_e + (\hat{\Theta}_o\hat{H}_0)_{sym}\rangle_\chi}{2\langle \hat{H}_0\rangle_\chi}\psi .$$

Schrödinger-like equation; similar (but not so) to the dressed metric approach.

Born-Oppenheimer ansatz

- There are **restrictions** on the range of validiy.
- The extra terms are negligible if so are the ϕ -derivatives of

$$\langle \hat{H}_0 \rangle_\chi, \ \langle \hat{\Theta}_e \rangle_\chi, \ \langle \hat{\Theta}_o \rangle_\chi, \ \langle (\hat{H}_0 \hat{\Theta}_o)_{sym} \rangle_\chi .$$

- These derivatives contain contributions arising from $[\hat{\Omega}_0^2, \hat{V}]$.

  In the efecive regime,these are proportional to $\sin 2b$.
- These effects are also important in the closure of the constraint algebra.

Alternate factor ordering

- At the truncation order, the constraint $\boldsymbol{C}=\pi_\phi^2-H_0^2-\Theta_e-\Theta_o\pi_\phi$ can be written:

$$\boldsymbol{C}=\left[\pi_\phi+H_0+\frac{1}{2}(\Theta_e+\Theta_o\pi_\phi)H_0^{-1}\right]\left[\pi_\phi-H_0-\frac{1}{2}(\Theta_e+\Theta_o\pi_\phi)H_0^{-1}\right].$$

- Hence, for perturbed solutions of *homogeneous* positive frequency, with a **Born-Oppenheimer** ansatz and ignoring nondiagonal elements:

$$\boxed{-i\partial_\phi\psi=\frac{1}{2}\langle\hat{H}_0^{-1/2}\left(\hat{\Theta}_e+(\hat{\Theta}_o\hat{H}_0)_{sym}\right)\hat{H}_0^{-1/2}\rangle_\chi\psi}\;-\frac{i}{2}\langle\hat{H}_0^{-1/2}\boldsymbol{d}_\phi(\hat{\Theta}_o\hat{H}_0^{-1})\hat{H}_0^{1/2}\rangle_\chi\psi.$$

Alternate factor ordering

- This Schrödinger equation is like the one obtained in the **dressed metric** approach.
- The **difference** between the two **factor orderings** is a commutator.

Effective Mukhanov-Sasaki equations

- Starting from the **Born-Oppenheimer** form of the constraint and assuming a direct **effective** counterpart for the **inhomogeneities**:

$$d^2_{\eta_\chi} v_{\vec{n},\pm} = -v_{\vec{n},\pm}\left[4\pi^2\omega_n^2 + \langle \hat{\theta}_{e,(v)} + \hat{\theta}_{o,(v)} \rangle_\chi\right],$$

$$\langle \hat{\theta}_{e,(v)} + \hat{\theta}_{o,(v)} \rangle_\chi v^2_{\vec{n},\pm} = -\frac{\langle 2\hat{\Theta}_e + 2(\hat{\Theta}_o \hat{H}_0)_{sym} - i\, \boldsymbol{d}_\phi \hat{\Theta}_o \rangle_\chi}{2\langle [1\hat{/}V]^{-2/3} \rangle_\chi} - 4\pi^2\omega_n^2 v^2_{\vec{n},\pm} - \pi^2_{v_{\vec{n},\pm}}.$$

where we have defined the **state-dependent conformal time**

$$d\eta_\chi = \langle [1\hat{/}V]^{-2/3} \rangle_\chi (dt/V).$$

- The effective equations are of harmonic **oscillator** type, with no dissipative term, and **hyperbolic in the ultraviolet** regime.

Effective Mukhanov-Sasaki equations

- In the alternate factor ordering, with the same assumptions:

$$d^2_{\eta_\chi^{dress}} v_{\vec{n},\pm} = -v_{\vec{n},\pm}\left[4\pi^2\omega_n^2 + \langle \hat{\theta}^{dress}_{e,(v)} + \hat{\theta}^{dress}_{o,(v)} \rangle_\chi\right],$$

$$\langle \hat{\theta}^{dress}_{e,(v)} + \hat{\theta}^{dress}_{o,(v)} \rangle_\chi v^2_{\vec{n},\pm} = -\frac{\langle 2\hat{H}_0^{-1/2}[\hat{\Theta}_e + (\hat{\Theta}_o \hat{H}_0)_{sym}]\hat{H}_0^{-1/2} - i\hat{H}_0^{-1/2}\boldsymbol{d}_\phi(\hat{\Theta}_o\hat{H}_0^{-1})\hat{H}_0^{1/2}\rangle_\chi}{2\langle \hat{H}_0^{-1/2}[1/\hat{V}]^{-2/3}\hat{H}_0^{-1/2}\rangle_\chi}$$
$$-4\pi^2\omega_n^2 v^2_{\vec{n},\pm} - \pi^2_{v_{\vec{n},\pm}}.$$

where we have defined the **state-dependent conformal time**

$$d\eta_\chi^{dress} = \langle \hat{H}_0^{-1/2}[1/\hat{V}]^{-2/3}\hat{H}_0^{-1/2}\rangle_\chi \langle \hat{H}_0 \rangle_\chi (dt/V).$$

- There is a **change** in the derivative contribution to the potential (usually negligible), and the FRW state is replaced with $\hat{H}_0^{-1/2}\chi$.

Conclusions

- We have considered the **hybrid quantization** of a FRW universe with a massive scalar field perturbed at **quadratic** order in the action.
- The system is a **constrained symplectic manifold**. **Backreaction** is included at the considered perturbative order.
- The model has been described in terms of **Mukhanov-Sasaki** variables.
- A **Born-Oppenheimer** ansatz leads to a Schrödinger equation for the inhomogeneities. We have discussed the range of validity.
- An alternate factor ordering gives similar results to the **dressed metric**.
- We have derived the effective **Mukhanov-Sasaki equations**. The ultraviolet regime is **hyperbolic**.