Effective Dynamics of Perturbations in Loop Quantum Cosmology

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Introducción

Introduction

In previous chapters,

we constructed a (scalarly) perturbed FLRW model with

- a massive scalar field as matter content
- (flat) compact spatial sections

This model is very interesting

because it can allows us to study inflation.

Now we have begun to study the effective dynamics of the model, following works for the Gowdy model, in which it was found that the inhomogeneities are amplified in the bouncing regime.

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(Classical) Cosmological Perturbations

3+1 decomposition

Let $(\mathcal{M}, g)$ be a globally hyperbolic spacetime.

Let t be a global time function.

t foliates the spacetime in spacelike hypersurfaces Σ_t.

We define

$h_{\alpha\beta} \rightsquigarrow$ metric induced on Σ_t by $g_{\alpha\beta}$

$N^\alpha \rightsquigarrow$ shift vector

$N \rightsquigarrow$ lapse function

The line element can be written as

$$ds^2 = -(N^2 - N_a N^a)dt^2 + 2N_a dt dx^a + h_{ab} dx^a dx^b.$$

$a, b, \ldots = 1, 2, 3 \rightsquigarrow$ spatial indices

Truncation of the classical system

ADM variables: FLRW (with a scalar field) + inhomogeneities

$$\Phi(t,x) = \frac{1}{l_0^{3/2}\sigma}\left[\varphi(t) + \delta\varphi(t,x)\right],$$
$$h_{ab}(t,x) = \sigma^2 e^{2\alpha(t)}\left[{}^0h_{ab}(x) + \epsilon_{ab}(t,x)\right],$$
$$N(t,x) = \sigma\left[N_0(t) + \delta N_0(t,x)\right],$$
$$N_a(t,x) = \delta N_a(t,x).$$

$$\sigma^2 = \frac{4\pi G}{3l_0^2}$$
$$l_0^3 = \int d^3x\sqrt{{}^0h}$$

${}^0h_{ab} \rightsquigarrow$ fiducial metric on the 3-torus

The inhomogeneities can be Fourier expanded, e.g.

$$\delta\varphi(t,x) = \textstyle\sum_n f_n(t)\tilde{Q}^n(x)$$

($\tilde{Q}_n$ are real eigenfunctions of the Laplace-Beltrami operator, ${}^0\Delta\tilde{Q}_n = -\omega_n^2\tilde{Q}_n$).

We truncate the action at quadratic order

in the coefficients of the expansions.

Constraints. Gauge fixing

The Hamiltonian is a linear combination of constraints:

- The corrected Hamiltonian constraint $C_0 + \sum C^n_{|2}$
  (which appears with the homogeneous lapse).
- Linear constraints $C^n_{|1}$ and $C^n_{\_1}$
  (with the perturbations of the lapse and the shift, resp.).

We fix the linear constraints classically.

In particular, consider the longitudinal gauge, in which

$$h_{ab} \propto {}^0h_{ab} \text{ and } N_a = 0.$$

After the reduction, one is left with the homogeneous α and φ and the field-like $\delta\varphi$, which do not have canonical Dirac brackets.

Nonetheless, we can find a new set of canonical variables.

Choice of the field parametrization

Classically, the parametrization of the inhomogeneities is irrelevant. However,

different parametrizations lead to inequivalent quantum theories.

Fortunately, we can invoke some results of uniqueness of quantum field theory in curved spacetimes. ▶ Laura's talk

In particular, in the classical background, the requirements of

- a translation-invariant complex structure (symmetric vacuum)
- unitarily implementable field dynamics

suffice to select

- a scaling of the field, $e^{\alpha}\delta\varphi$, and its conjugate momentum
- a class of unitarily equivalent Fock representations for them.

Reparametrization of the system

We introduce a set of canonical variables including the scaled field and its preferred momentum:

$$\bar{f}_n = e^{\alpha} f_n, \quad \bar{\pi}_{\bar{f}_n} = e^{-\alpha}(1 + F_n)\pi_{f_n} + G_n f_n$$

$O(\omega_n^{-2})$ background functions

while the homogeneous variables get 2nd-order corrections (backreaction).

In these variables, $C^n_{|2}$ adopts a Klein-Gordon-like form with background-dependent mass and $O(\omega_n^{-2})$ corrections.

A representative of the class of preferred Fock representations can be constructed from the annihilation-like variables

$$a_{\bar{f}_n} = \frac{1}{\sqrt{2\omega_n}}(\omega_n \bar{f}_n + i\pi_{\bar{f}_n}).$$


Quantization

Hybrid Quantization

▶Bea's talk

We want to adopt

- a polymer representation of the homogeneous gravitational d.o.f.
- a Schrödinger representation for the homogeneous field
- a standard Fock quantization for its field-like perturbation

In this approximation,

only the background incorporates the effects of quantum geometry,

but an infinite number of d.o.f. can be treated.

The kinematical Hilbert space of the theory

is constructed as the product

$$\mathcal{H}_{\rm kin}^{\rm tot} = \mathcal{H}_{\rm kin}^{\rm LQC} \otimes \mathcal{H}_{\rm kin}^{\varphi} \otimes \mathcal{F}.$$

$L^2(\mathbb{R}_{\rm B}, d\mu_{\rm B})$

$L^2(\mathbb{R}, d\varphi)$ Fock space

How do we choose the Fock representation for the inhomogeneities?

Homogeneous sector: Ashtekar variables

In a homogeneous and isotropic universe,

the Ashtekar-Barbero connection and the densitized triad

can be parametrized by two variables, c and p, satisfying

$$\{c, p\} = \frac{8\pi G\gamma}{3}, \quad |p| = l_0^2\sigma^2 e^{2\bar{\alpha}}, \quad pc = -\gamma l_0^3 \sigma^2 \bar{\pi}_{\bar{\alpha}}.$$

In terms of these variables,

the classical Hamiltonian constraint of the homogeneous system is

$$C_0 = \frac{1}{|p|^{3/2}} \left(-\frac{6}{\gamma^2} c^2 p^2 + 8\pi G (\pi_\phi^2 + m^2 |p|^3 \phi^2) \right),$$

Holonomy-flux algebra

However, the fundamental variables for quantization

are not the connection and the triad, but

- Holonomies of the connection along straight edges of length $l_0\bar{\mu}(p)$,

  parametrized by the functions $N_{\bar{\mu}} = e^{i\bar{\mu}c/2}$.

  The improved dynamics scheme has been adopted: $l_0\bar{\mu} = l_0\sqrt{\Delta/p}$,

  where Δ is an imput from Loop Quantum Gravity:

  the minimum non-zero eigenvalue of the area operator.

- Fluxes of the densitized triad (proportional to p).

Fundamental algebra: $\{N_{\bar{\mu}}, p\} = \dfrac{4\pi i G\gamma\bar{\mu}}{3} N_{\bar{\mu}}.$

Representation

Mimicking the representation employed in LQG, the holonomy-flux algebra is represented in $\mathcal{H}_{\text{kin}}^{\text{LQC}} = L^2(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Bohr}})$.

The momentum representation is more frequently employed:

- Orthonormal basis: $\{|v\rangle \mid v \in \mathbb{R}\}$, $\langle v|v'\rangle = \delta_{vv'}$.
- Fundamental operators: $\hat{N}_{\bar{\mu}}|v\rangle = |v+1\rangle, \quad \hat{p}|v\rangle = p(v)|v\rangle$.

As this representation is not continuous, there is no operator for c.

The Hamiltonian constraint must be regularized.

This is done by following the programme of Loop Quantum Gravity

Regularization

The term cp can be expressed in terms of a holonomy around a closed squared loop in the limit of vanishing area of the loop.

Now, instead of a vanishing area, we take a loop with the minimum one, Δ.

Thus, we obtain $(cp)^2 \to \hat{\Omega}_0^2$, where

$$\hat{\Omega}_0 = \frac{|\hat{p}|^{3/4}}{4i\sqrt{\Delta}}\left[\widehat{\mathrm{sgn}(p)}\left(\hat{N}_{2\bar{\mu}} - \hat{N}_{-2\bar{\mu}}\right) + \left(\hat{N}_{2\bar{\mu}} - \hat{N}_{-2\bar{\mu}}\right)\widehat{\mathrm{sgn}(p)}\right]|\hat{p}|^{3/4}.$$

In addition, inverse powers of p are regularized expressing them in terms of Poisson brackets of the fundamental operators.

Then, the brackets are promoted to commutators. The result is

$$\widehat{\left[\frac{1}{|p|^{1/2}}\right]} = \frac{3}{4\pi\gamma G\hbar\sqrt{\Delta}}\widehat{\mathrm{sgn}(p)}\sqrt{|\hat{p}|}\left(\hat{N}_{-\bar{\mu}}\sqrt{|\hat{p}|}\hat{N}_{\bar{\mu}} - \hat{N}_{\bar{\mu}}\sqrt{|\hat{p}|}\hat{N}_{-\bar{\mu}}\right).$$

Second-order constraint

The 2nd-order Hamiltonian has the structure

$$C^n_{|2} \propto \frac{1}{2} e^{-\alpha} \big(E^n_{\pi\pi} \bar{\pi}^2_{\bar{f}_n} + 2 E^n_{f\pi} f_n \bar{\pi}_{\bar{f}_n} + E^n_{ff} \bar{f}^2_n \big),$$

where the E-coefficients are functions of the homogeneous variables.

The prescription we follow to quantize it is:

- Normal ordering for annihilation and creation operators.
- Symmetrizations: $\phi \pi_\phi \rightsquigarrow \frac{1}{2}(\hat{\phi}\hat{\pi}_\phi + \hat{\pi}_\phi \hat{\phi})$, $AV^k \rightsquigarrow \hat{V}^{k/2} \hat{A} \hat{V}^{k/2}$
  - $(cp)^{2k} \rightsquigarrow \hat{\Omega}_0^{2k}$
  - $(cp)^{2k+1} \rightsquigarrow |\hat{\Omega}_0|^k \hat{\Lambda}_0 |\hat{\Omega}_0|^k$

$$\hat{\Lambda}_0 = \frac{1}{8i\sqrt{\Delta}} \hat{V}^{1/2} \big[\widehat{\mathrm{sgn}(v)} \big(\hat{N}_{4\bar{\mu}} - \hat{N}_{-4\bar{\mu}} \big) + \big(\hat{N}_{4\bar{\mu}} - \hat{N}_{-4\bar{\mu}} \big) \widehat{\mathrm{sgn}(v)} \big] \hat{V}^{1/2}.$$

In this way, the superselection sectors are preserved.

Effective Dynamics

Derivation of the effective dynamics

Now we have a quantum model, but it is very intricate.

As a first approach we studied its effective dynamics in the massless case. ▶ Dani's talk

In simple models, the peaks of certain semiclassical states follow simple trajectories which obey the effective constraint obtained by

$$\hat{p} \to p$$
$$\hat{N}_{\bar{\mu}} \to N_{\bar{\mu}}$$

There are two types of corrections:

- Regularization of $\widehat{|p|^{-1/2}} \to$ inverse-triad corrections
- Regularization of $cp \to$ holonomy corrections

This algorithm has proven useful in more involved systems (of course, one should check its validity!)

Implementation

We introduce holonomy corrections in our model by making

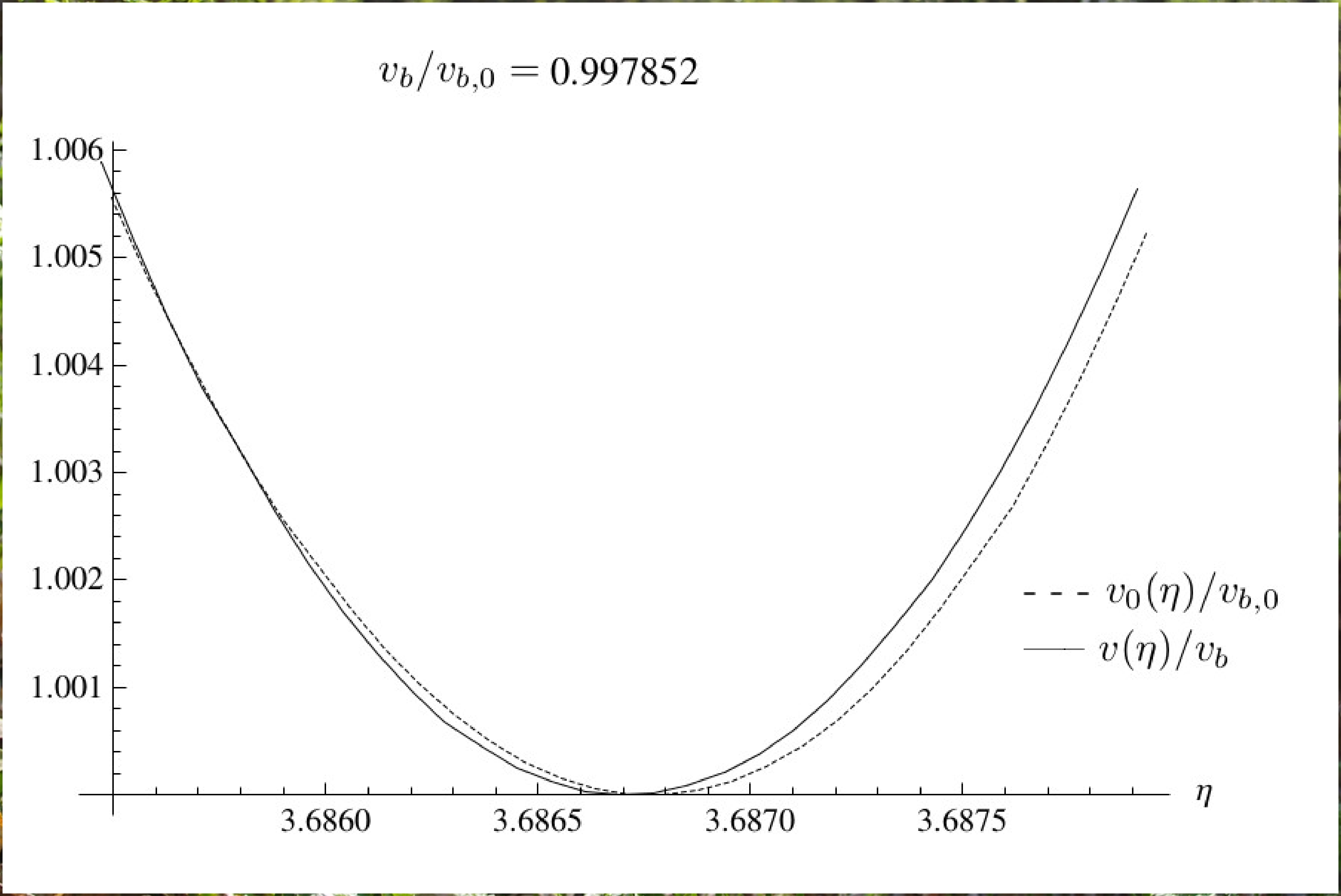
$$(pc)^2 \to \Omega^2 = \left(p\frac{\sin(\bar{\mu}c)}{\bar{\mu}}\right)^2$$
$$pc \to \Lambda = p\frac{\sin(2\bar{\mu}c)}{2\bar{\mu}}$$

This implementation of the effective dynamics, with two steps, has qualitative as well as quantitative consequences, e.g. the bounce in the volume does not coincide with the max. density.

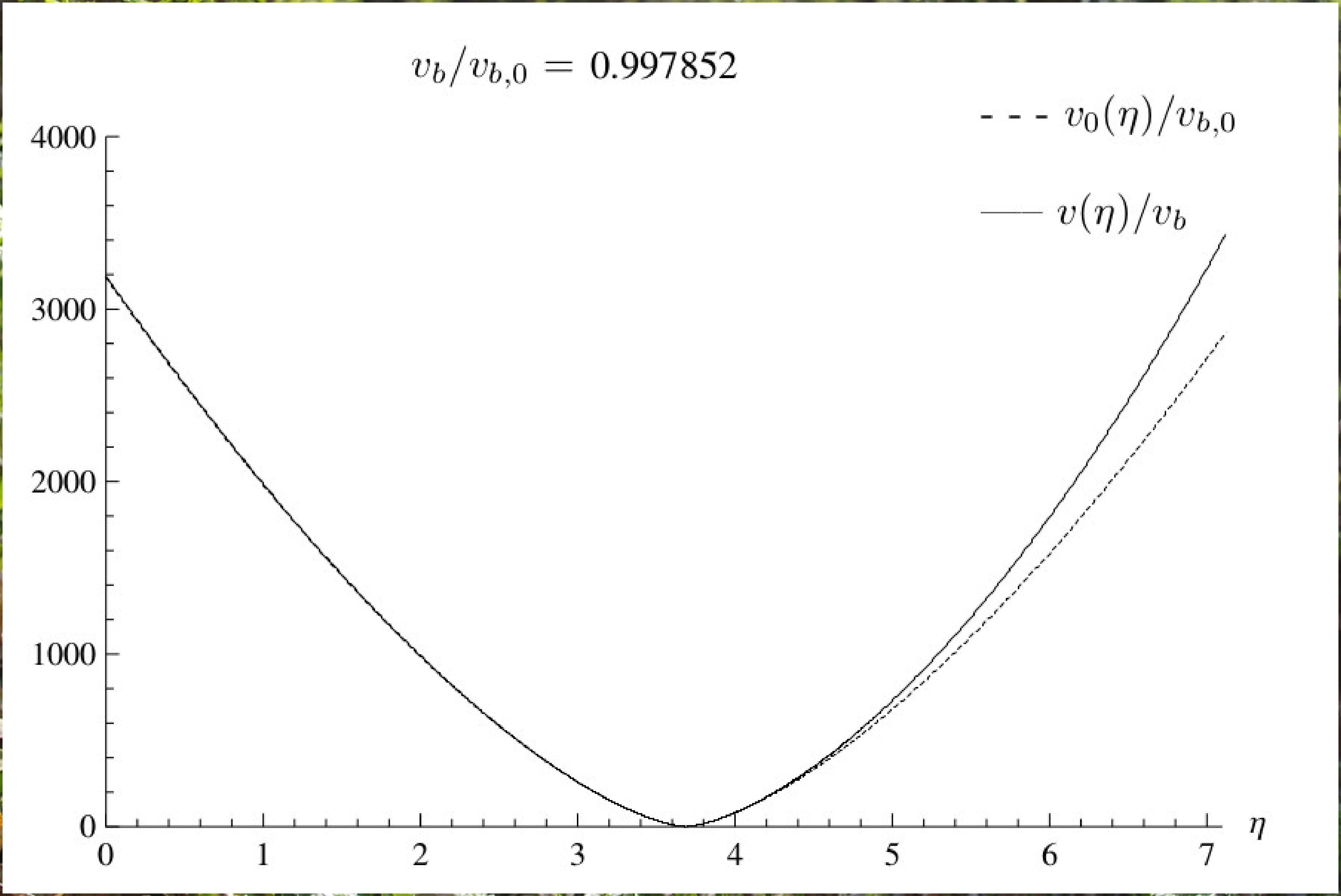
Nonetheless, we would expect other effects of the backreaction to appear in other prescriptions as well.

E.g., the energy transfer between background and perturbations.

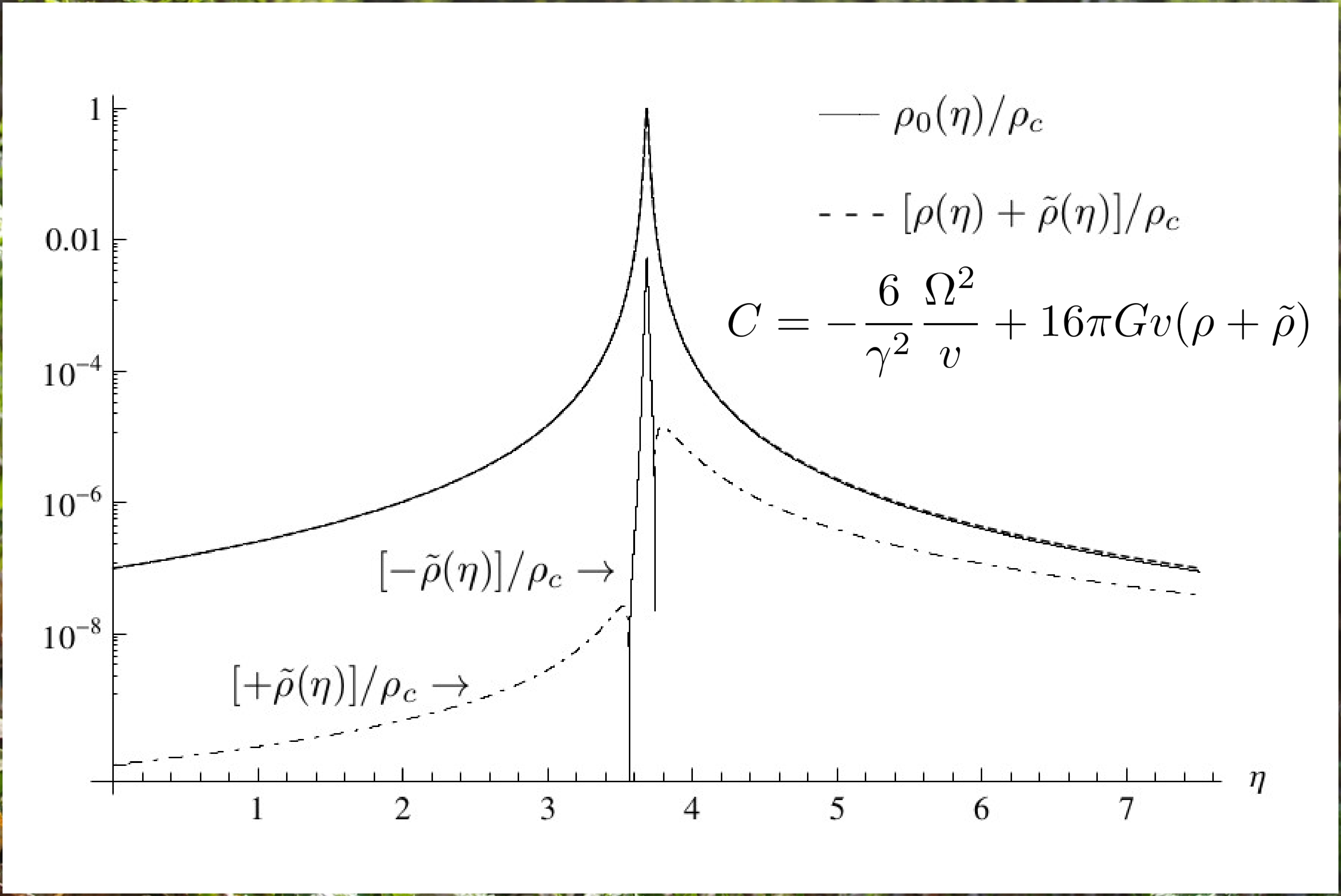
Effects of the backreaction (I)



Effects of the backreaction (II)



Effects of the backreaction (III)



Perturbation amplification (I)

We studied the evolution of the system (at η_0).

setting initial conditions well before the bounce

and evolving them until long after the bounce.

Initial conditions for the inhomogeneities:

- Gaussian distribution for the amplitude
- Homogeneous distribution for the phase α

(idea: mimicking a vaccum state)

Statistically, the perturbations are amplified through the bounce.

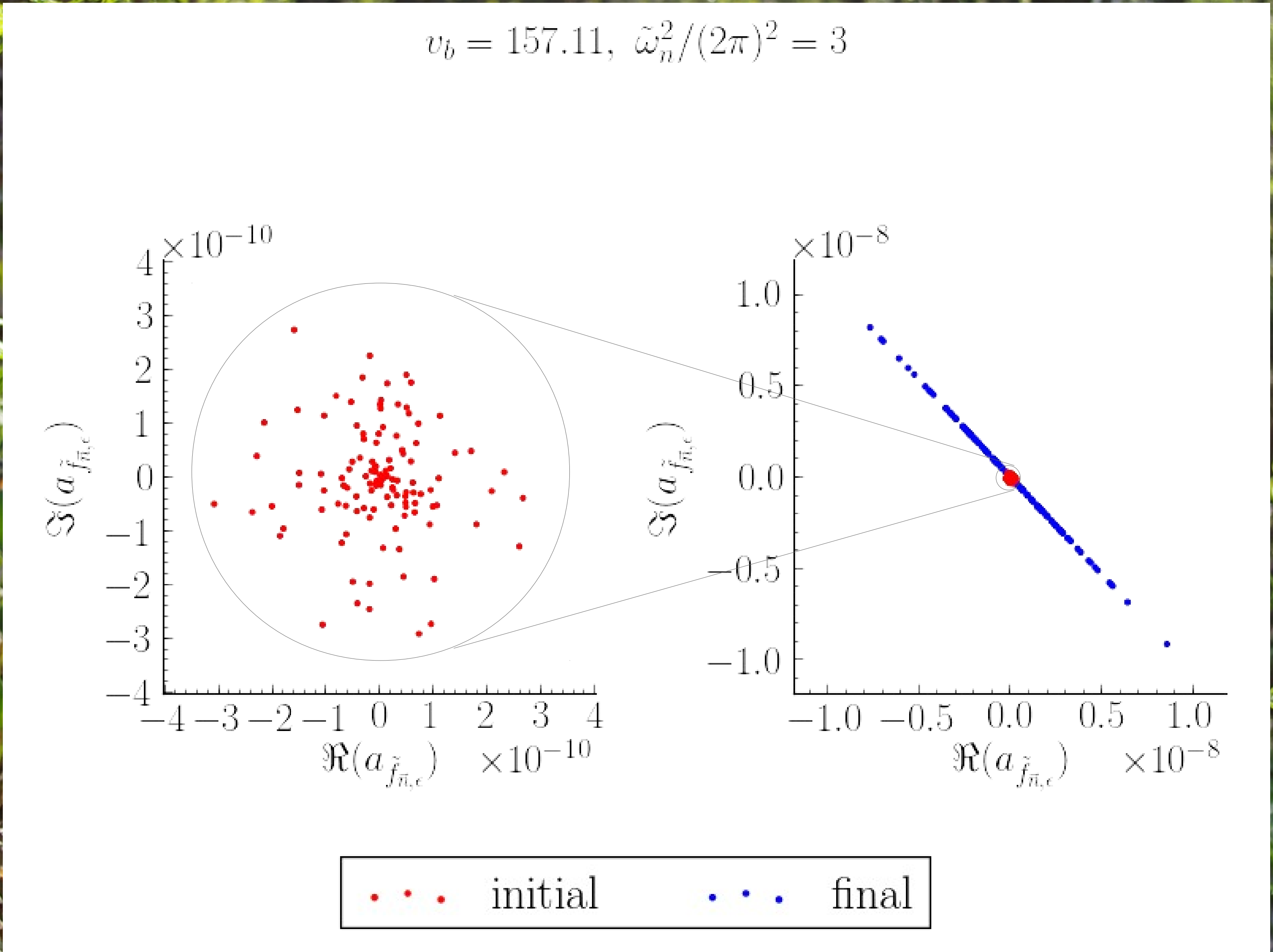
The average amplification is modulated by the frequency.

Besides, there is an effect of alignment of the phases.

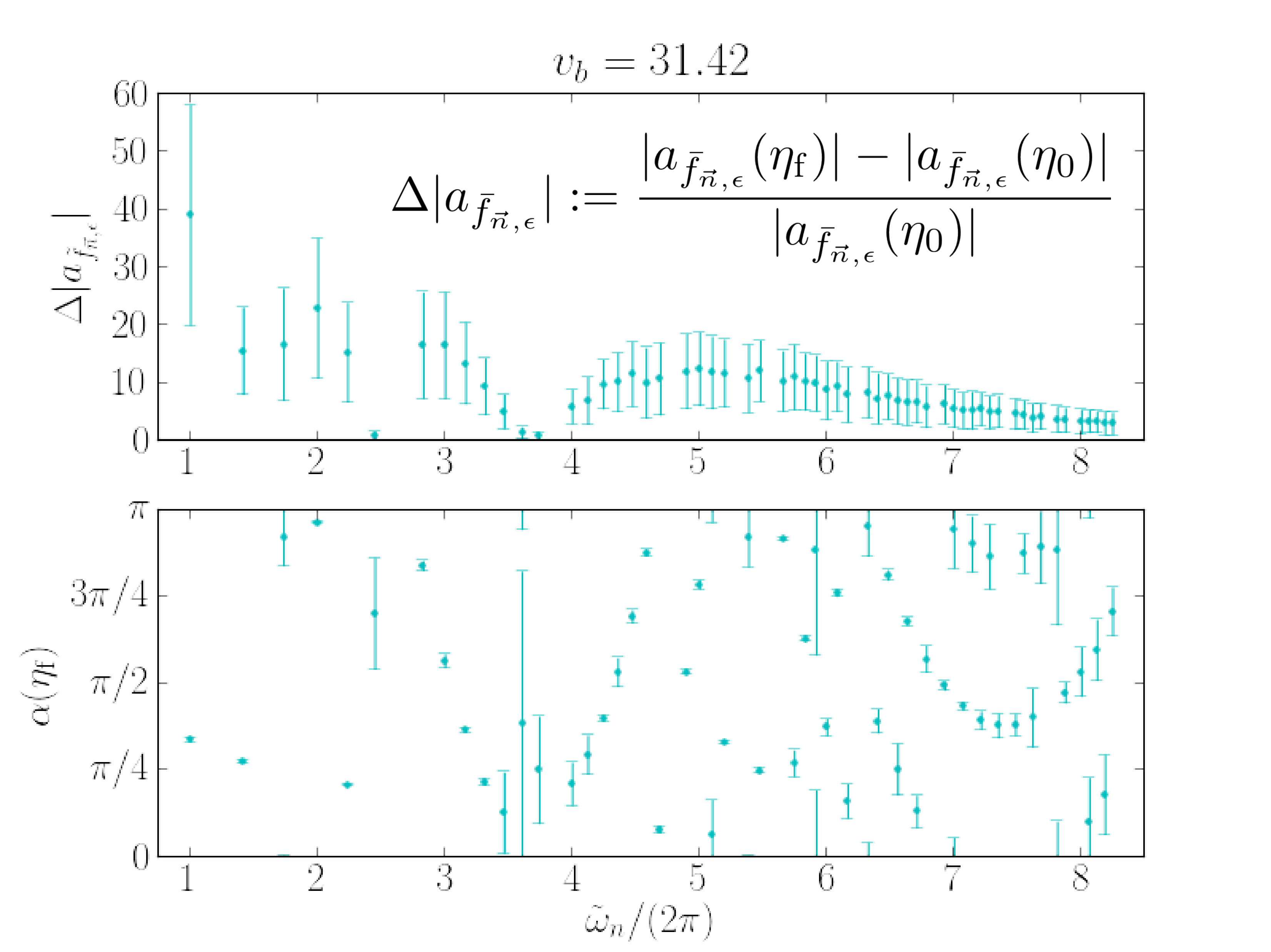
The following figures were obtained neglecting the backreaction

and choosing $\eta_{\mathrm{f}} - \eta_{\mathrm{bounce}} = \eta_{\mathrm{bounce}} - \eta_0$

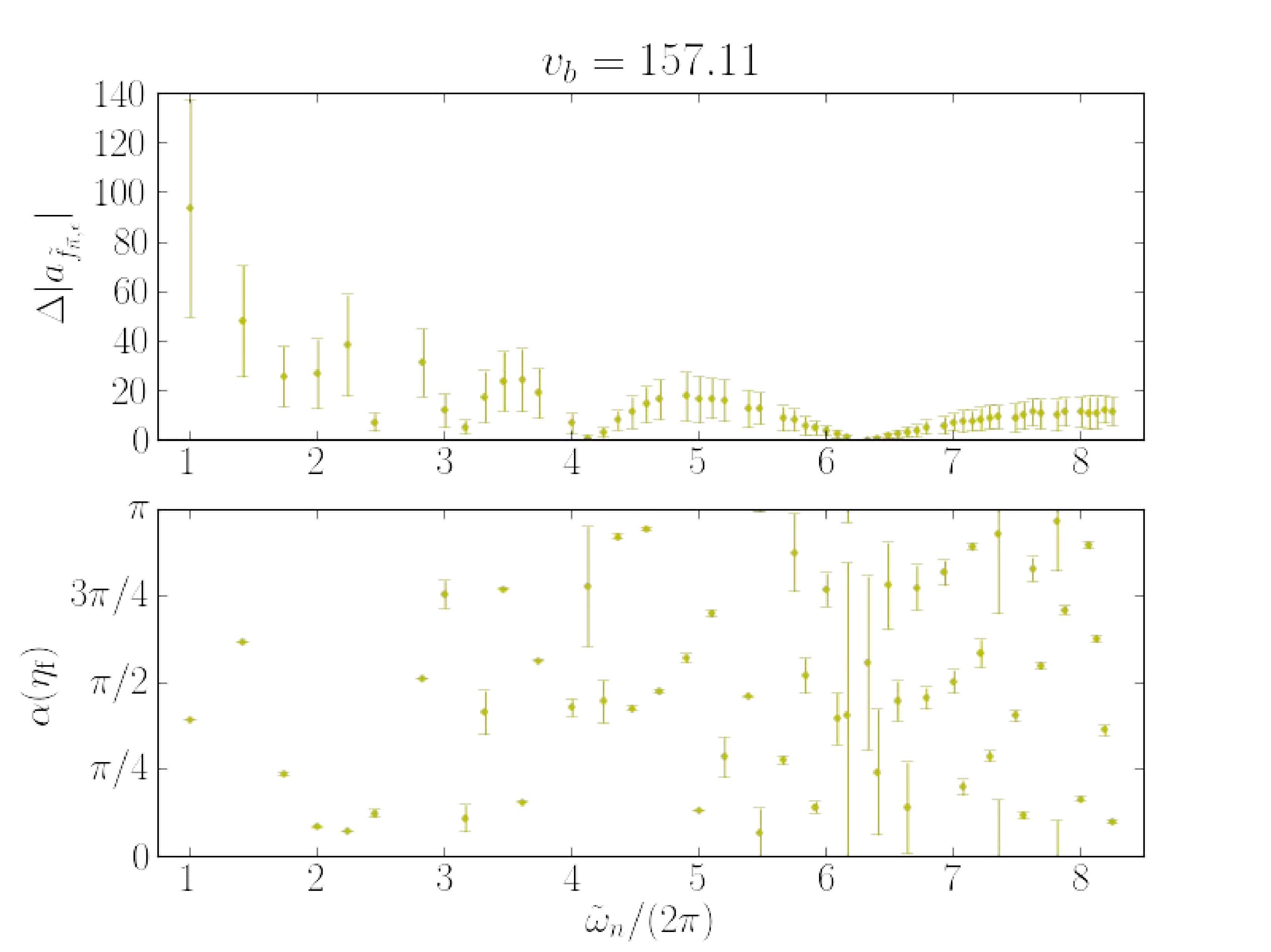
Perturbation amplification (II)



Modulation of the amplification (I)



Modulation of the amplification (II)



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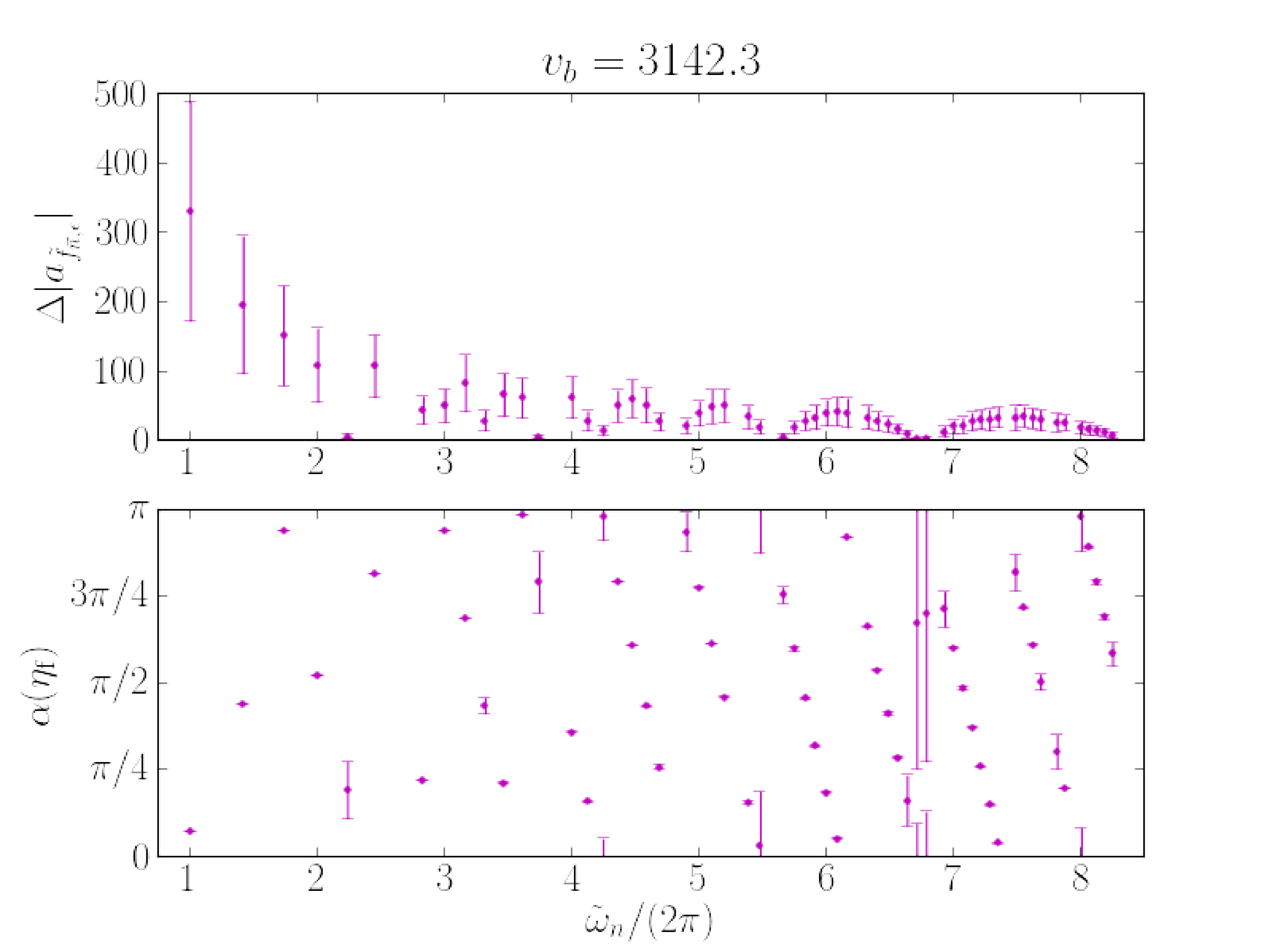

Conclusions

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We have studied the plausible effective dynamics
of the hybrid quantization of the perturbed FLRW model.

Results:

- The perturbations are boosted in the bounce.
  The average amplification oscillates with the frequency.
  The ultraviolet modes are not amplified singnificantly.
- There is a parallel effect of alignment of the phases.
- There is an energy transfer between the background
  and the perturbations.