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# Mimicking Matter in the Hybrid Quantum Gowdy Models

- LQG as a GR quantum theory $\longrightarrow$ LQC
- Flat FRW model leads to a Big Bounce.
- Simplest inhomogeneous Universe $\longrightarrow$ $T^3$ Gowdy model with linear polarization.
- Two spatial Killing vectors; solutions with LRS.
- More physical scenario: Minimally coupled massless scalar field. Non zero modes vary in the same direction as gravitational waves.
- Hybrid quantization and approximations.
- Family of states that could mimick exotic matter content in a flat FRW Universe.

# Hybrid Quantization of the Gowdy model

- Loop quantization of the homogeneous sector (Bianchi I) minimally coupled to a massless scalar field Φ is combined with a Fock quantization of gravitational and matter fields varying in one angular direction.

$$\widehat{C}_G = \widehat{C}_{FRW} - \frac{\pi G \hbar^2}{8}\left(\widehat{\Omega}\widehat{\Theta} + \widehat{\Theta}\widehat{\Omega}\right) + \frac{2\pi G \hbar^2}{\beta}\widehat{e^{2\Lambda}}\,\widehat{H}_0 + \frac{\pi G \hbar^2 \beta}{4}\widehat{e^{-2\Lambda}}\,\widehat{D}\,\widehat{\Omega}^2\,\widehat{D}\,\widehat{H}_I$$

Where

$$\widehat{C}_{FRW} \equiv \frac{-3\pi G \hbar^2}{8}\widehat{\Omega}^2 - \frac{\hbar^2 \partial^2_{\phi}}{2} \qquad \beta \equiv \left[G\hbar/\left(16\pi^2 \gamma^2 \Delta\right)\right]^{1/3}$$

$$\widehat{H}_0 \equiv \sum_{\alpha \in \xi,\, \varphi} \sum_{m \in \mathbb{Z}-[0]} |m|\, \hat{a}^{(\alpha)\dagger}_{m}\, \hat{a}^{(\alpha)}_{m}$$

$$\widehat{H}_0 \equiv \sum_{\alpha \in \xi,\, \varphi} \sum_{m \in \mathbb{Z}-[0]} \frac{1}{2|m|}\left(2\hat{a}^{(\alpha)\dagger}_{m}\hat{a}^{(\alpha)}_{m} + \hat{a}^{(\alpha)\dagger}_{m}\hat{a}^{(\alpha)\dagger}_{-m} + \hat{a}^{(\alpha)}_{m}\hat{a}^{(\alpha)}_{-m}\right)$$

# Approximations: Spectrum of the Flat FRW operator

- Eigenfunctions $e^{\varepsilon}_{\rho}(v)$ of the geometry, with eigenvalue $\rho^2$, are exponentially suppresed for $v \leq \rho/2$, specially for $\rho >> 10$.

  Oscillatory behavior at large values of v.

- $\widehat{D}$ comes from the regularization of the inverse of the volume in LQC.

  It acts by multiplication on $|v\rangle$ states. Eigenvalue $D(v) \rightarrow 1$ when $v >> 1$, and $D(v)=0$ when $v=0$.

- We can restrict ourselves to states with $\rho >> 10$, $v >> 1$.

- We can expand the homogeneous and isotropic part of the states in the $|e^{\varepsilon}_{\rho}\rangle$ basis.

# Approximations: anisotropy and interaction terms

- Consider smooth anisotropy wavefunctions f(Λ) that for variations in Λ smaller than $q_\varepsilon$ satisfy $f(\Lambda+\Lambda_0)\simeq f(\Lambda)+\Lambda_0\partial_\Lambda f(\Lambda)$
- Then the anisotropy term approximately factorizes:

  $\hat{\Omega}\hat{\Theta}+\hat{\Theta}\hat{\Omega}\simeq 2\hat{\Omega}'\hat{\Theta}'$, acting on states $\left|e^{\varepsilon}_{\rho}\right\rangle\otimes\left|f(\Lambda)\right\rangle$, where $\hat{\Omega}'$ is analogous to $\hat{\Omega}$ if v >> 1, and $\hat{\Theta}'$ is the discretization of $-4\mathrm{i}\,\partial_\Lambda$ at the scale $q_\varepsilon \geq \log\left(1+\frac{4}{\rho^*}\right)$, for ρ ≥ ρ*.
- We consider Gaussian profiles peaked in Λ and its momentum:

$$|\psi_{\bar{\Lambda}}\rangle\simeq\sum_{\Lambda\in\mathscr{L}^{q_\varepsilon}_{\Lambda^*}}\frac{\sqrt{\sigma_s}}{\sqrt[4]{\pi}}\,\mathrm{e}^{\frac{\sigma_s^2}{2q_\varepsilon^2}(\Lambda-\bar{\Lambda})^2}\,|\Lambda\rangle \quad \text{with} \quad \sigma_s\ll\pi/2$$

then $\langle\psi_{\bar{\Lambda}}|\hat{\Theta}'|\psi_{\bar{\Lambda}}\rangle=0$, and $\langle\psi_{\bar{\Lambda}}|\widehat{\mathrm{e}^{-2\Lambda}}|\psi_{\bar{\Lambda}}\rangle$<< 1 if $\bar{\Lambda}$ >> $q_\varepsilon^2/\sigma_s^2$

# Approximate solutions to the Gowdy model

- One arrives at the approximate Hamiltonian constraint:

$$\widehat{C}_{app} = \frac{-3\pi G \hbar^2}{8}\widehat{\Omega}^2 - \frac{\hbar^2 \partial_\phi^2}{2} + \frac{2\pi G \hbar^2}{\beta} e^{\widehat{2\Lambda}} \widehat{H}_0$$

- Solving $\langle \phi, v, \Lambda, n^\xi, n^\varphi | \widehat{C}_{app} | \Psi \rangle = 0$ for ρ, solutions take the form:

$$\Psi(\phi, v, \Lambda, n^\xi, n^\varphi) = \int_{-\infty}^{\infty} dp_\phi \, \Psi(p_\phi, \Lambda, n^\xi, n^\varphi) e_{\rho}^{\varepsilon}(v) e_{p_\phi}(\phi)$$

Where $\Psi(p_\phi, \Lambda, n^\xi, n^\varphi) = \Psi(p_\phi, n^\xi, n^\varphi)\psi_{\overline{\Lambda}}(\Lambda)$, and ρ satisfies:

$$\rho^2 = \frac{4}{3\pi G \hbar^2} p_\phi^2 + \frac{16}{3\beta} e^{2\Lambda} H_0(n^\xi, n^\varphi)$$

- We can focus our attention on states with $\Lambda = \overline{\Lambda}$, considering that our anisotropy profiles are strongly peaked there.

# Peak depending on constants of motion

- Friedmann equation for flat FRW: $3\frac{\dot{a}^2}{a^2}=8\pi G\omega$

  Comparing with the classical constraint for flat FRW minimally coupled to a massless ϕ:

$$\omega=\frac{p_\phi^2}{2a^6}=(8\pi^2G^2\hbar^2\Delta)^{-1}\frac{p_\phi^2}{v^2} \qquad \rho^2=\frac{\dot{a}^2}{a^2}\frac{a^6}{\pi^2G^2\hbar^2}$$

- Setting $\bar{\Lambda}=\bar{\Lambda}(p_\phi^2,H_0)$ we regard the solution of $\hat{C}_{app}$ as a corrected flat Friedmann equation, for small content in inhomogeneities:

$$\rho^2=\frac{4}{3\pi G\hbar^2}p_\phi^2+\frac{16}{3\beta}e^{2\bar{\Lambda}(p_\phi^2,H_0)}H_0(n^\xi,n^\varphi)\equiv F(p_\phi^2,H_0)\longrightarrow p_\phi^2=F^{-1}(\rho^2,H_0)$$

If $H_0\ll 1$ and $\omega_0\equiv\frac{3\pi G\hbar^2}{8a^6}\rho^2=\frac{3}{8\pi G}(\frac{\dot{a}^2}{a^2})$:

$$\omega=\omega_0+(8\pi^2G^2\hbar^2\Delta)^{-1}\frac{1}{v^2}\sum_{n=1}^{\infty}[\frac{1}{n!}\frac{\partial^n F^{-1}(\rho^2,H_0)}{\partial H_0^n}]_{H_0=0}$$

# Peak depending on the volume

- More physically interesting scenario $\longrightarrow$ Allowing the peak to depend on the volume so it can behave as new matter content.
- Setting $\bar{\Lambda}=\bar{\Lambda}(v, p_{\phi}^{2}, H_0)$, makes $\psi_{\bar{\Lambda}}(\Lambda) \longrightarrow \psi_{\bar{\Lambda}}(\Lambda, v, p_{\phi}^{2}, H_0)$.
- Are the approximations still valid?

- On the one hand, for anisotropy states $|f(v,\Lambda)\rangle$ with same properties as the previous ones in their $\Lambda$ dependence:

  $\hat{\Omega}\hat{\Theta}+\hat{\Theta}\hat{\Omega}\simeq 2\hat{\Omega}'\hat{\Theta}'$ still holds, but states no longer separate.

  $\langle\psi_{\bar{\Lambda}}|2\hat{\Omega}'\hat{\Theta}'|\psi_{\bar{\Lambda}}\rangle\neq 0$ now, but $\langle\psi_{\bar{\Lambda}}|\hat{\Theta}'|\psi_{\bar{\Lambda}}\rangle=0$ holds as well.

  Therefore, if $\hat{\Theta}'$ 's dispersion is sufficiently small, we may disregard this term compared to $\hat{\Omega}^2$.

# Peak depending on the volume: Interaction term

- On the other hand, acting on the anisotropy states gives:

$$\widehat{e^{-2\Lambda}}\widehat{\Omega}^2|\psi_{\overline{\Lambda}}\rangle = \sum_{v\in\mathscr{L}_{\varepsilon}^{+}} \sum_{\Lambda\in\mathscr{L}_{\Lambda_*}^{q_{\varepsilon}}} \psi_{\overline{\Lambda}}(\Lambda, v) e^{-2\Lambda}\widehat{\Omega}^2|\Lambda, v\rangle$$

  Where D(v) $\longrightarrow$ 1 holds in the regime v >> 1.

- $\widehat{\Omega}^2$ basically produces shifts of step 4 in the volume.
- Besides, $\langle\psi_{\overline{\Lambda}}|\widehat{e^{-2\Lambda}}|\psi_{\overline{\Lambda}}\rangle \ll 1$ if $\overline{\Lambda}(v) \gg q_{\varepsilon}^2/\sigma_s^2$ stands.

  We can focus our attention on v dependences such that $\overline{\Lambda}(v)$ increases as the volume grows larger. Then, we may disregard the interaction term compared to $\widehat{\Omega}^2$.

- This kind of dependence in our Friedmann equation would mean that the density corrections would decrease slower than the free scalar field contribution $\longrightarrow$ Behave as new matter.
- Further analysis of this term may provide more information on the details of the dependence.