## Colored bosons on top FBA and angular cross section for $t\bar{t}$ production

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(Dated: April 16, 2013)

## Abstract

With full data set that corresponds to an integrated luminosity of 9.4 fb<sup>-1</sup>, CDF has updated the top quark forward-backward asymmetry (FBA) as functions of rapidity difference  $|\Delta y|$  and  $t\bar{t}$  invariant mass  $M_{t\bar{t}}$ . Beside the sustained inconsistency between experiments and standard model (SM) predictions at large  $|\Delta y|$  and  $M_{t\bar{t}}$ , an unexpected large first Legendre moment with  $a_1 = 0.39 \pm 0.108$  is found. In order to solve the large top FBA, we study the contributions of color triplet scalar and color octet vector boson. We find that the top FBA at  $|\Delta y| > 1$  ( $M_{t\bar{t}} > 450$  GeV) in triplet and octet model could be enhanced to be around 40% (30%) and 26% (20%), whereas the first Legendre moment is  $a_1^{\mathbf{Di}} = 0.38$  and  $a_1^{\mathbf{Axi}} = 0.23$ , respectively.

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It is believed that the standard model (SM) is just an effective theory of a fundamental one yet to be discovered, even it is consistent with most experimental data. For pursuing more basic elements which are made of our universe, the search of new physics beyond SM is inevitable. If such new physics exists, it can be probed either directly at collider or indirectly through precise measurements. The recent measurements at Tevatron on the forward-backward asymmetry (FBA) in the top-quark pair production at  $\sqrt{s} = 1.96$  TeV may provide the clue for the existence of new physics. The FBA of top pair system is defined by

$$A_{FB} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)},\tag{1}$$

where  $\Delta y = y_t - y_{\bar{t}}$ ,  $y_{t(\bar{t})}$  is the rapidity of top (anti-top) quark and N is the number of events for  $\Delta y \ge 0$ .

With full Tevatron data set which corresponds to an integrated luminosity of 9.4 fb<sup>-1</sup>, CDF Collaboration measures the top-quark FBA with lepton+jets topology at parton level to be [1]

$$A_{FB} = 0.164 \pm 0.047 \,. \tag{2}$$

The result is roughly in agreement with early CDF and D0 data [2, 3]. CDF also reports the linear mass  $(A_{FB}(M_{t\bar{t}}))$  and rapidity  $(A_{FB}(|\Delta y|))$  dependences and the associated slope as  $(15.2 \pm 5.0) \times 10^{-4}$  and  $(28.6 \pm 8.5) \times 10^{-2}$ , respectively. The former is  $1.3\sigma$  deviation from the SM prediction and the latter is  $2.1\sigma$ . Additionally, CDF also analyzes the angular differential cross section for  $t\bar{t}$  production and observes an unexpected large first Legendre moment, where it is associated with top FBA [4].

In the SM, since the top-quark pair production is dominated by the strong interaction QCD contribution, due to C-parity invariance, a vanishing FBA at the leading order (LO) is expected. However, a nonvanishing FBA can be induced at the next-to-leading order (NLO) [5] and beyond [6–12]. The SM predictions have been improved and the resultant range could be from around 6% to around 10% [12, 13], however, by comparing with the central value of Eq. (2), the inconsistency between SM and data does not disappear. Although the anomaly of FBA has derived many studies of new physics in the literature [14–25], based on the new measurements and updated data, we investigate the issue by introducing new u-channel and s-channel effects. For illustration, we study the color triplet [14] and color octet models [15, 16, 26], where the former is a representative of u-channel and the latter is a s-channel.

In order to study the angular cross section for  $t\bar{t}$  production and the unexpected large top FBA as functions of  $|\Delta y|$  and  $M_{t\bar{t}}$ , here we consider two extensions of the SM. One is to introduce a color triplet scalar, called diquark, to the SM. Although there are many possible representations of diquark in SM gauge symmetry, for simplicity, we only focus on the representation (3, 1, -3/4). The other is color octet gauge boson which could be arisen from an extended  $SU(3)_R \times SU(3)_L$  gauge symmetry, called chiral color gauge model [27, 28]. In such model, the SM QCD could be taken as an unbroken symmetry of the extended one. Since the couplings of the new color gauge boson to quarks have the axial vector currents, hereafter, we call the massive color octet gauge boson as axigluon.

Now we briefly introduce the relevant pieces for the new models. Firstly, we discuss the color triplet model. The SM gauge invariant interactions of color triplet diquark with quarks are written by

$$-\mathcal{L}_{\mathbf{3}} = f_{ij}^{\mathbf{3}} \bar{u}_{i\alpha} P_L u_{j\beta}^c \bar{K}_{\gamma}^{\alpha\beta} H_{\mathbf{3}}^{\gamma\dagger} + h.c. , \qquad (3)$$

where the indices *i* and *j* are the quark flavors,  $f_{ij}^3 = -f_{ji}^3$ ,  $(\alpha, \beta, \gamma)$  stand for the color indices and  $P_{L(R)} = (1 \mp \gamma_5)/2$  is the helicity projection. The antisymmetric tensors in color space are defined as  $\bar{K}^{\alpha\beta}_{\gamma} = (K^{\gamma}_{\alpha\beta})^{\dagger}$  and the Ks are given by [29]

$$K_{1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad K_{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad K_{3} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (4)$$

where the antisymmetric tensors satisfy  $Tr(K^a\bar{K}_b) = \delta^a_b$  and  $K^a_{\alpha\beta}\bar{K}^{\rho\sigma}_a = \frac{1}{2}(\delta^{\sigma}_{\alpha}\delta^{\rho}_{\beta} - \delta^{\rho}_{\alpha}\delta^{\sigma}_{\beta})$ . As a result, the process  $u\bar{u} \to t\bar{t}$  could proceed through the following interactions

$$-\mathcal{L}_{\mathbf{3}} = 2f_{ut}^{\mathbf{3}}\bar{u}_{\alpha}P_{L}t_{\beta}^{c}\bar{K}_{\gamma}^{\alpha\beta}H_{\mathbf{3}}^{\gamma\dagger} + h.c.$$

$$\tag{5}$$

For color octet gauge boson of  $SU(3)_R \times SU(3)_L$ , we write the interactions of the axigluon with quarks as

$$\mathcal{L}_A = g_V \bar{q} \gamma_\mu \mathbf{Z}_1 T^b q G_A^{b\mu} + g_A \bar{q} \gamma_\mu \gamma_5 \mathbf{Z}_2 T^b q G_A^{b\mu} , \qquad (6)$$

where the flavor and color indices are suppressed,  $q^T = (u, c, t)$ ,  $g_{V,A}$  are the gauge couplings of axigluon to the first two generation quarks,  $T^b$  are the Gell-Mann matrices which are normalized by  $Tr(T^bT^c) = \delta^{ac}/2$  and  $\mathbf{Z}_i$  is a 3 × 3 diagonalized matrix with  $\operatorname{diag}(Z_i) = (1, 1)$  1,  $\zeta_i$ ). Here  $\zeta_{1(2)} = g_V^t/g_V(g_A^t/g_A)$ .  $g_{V(A)}^t$  denotes the gauge coupling of the third-generation quark and its value depends on a specific model, e.g.  $\zeta_{1(2)} = 1(-1)$  in Ref. [15]. Although flavor changing neutral currents (FCNCs) at tree level could be induced by  $g_{V(A)}^t \neq g_{V(A)}$ and have an interesting influence on low energy physics [30], since the small effects do not have a significant contributions to the studying phenomena, we will not further discuss the FCNC effects. In our numerical analysis, the parametrisation of free parameters is the same as that in Ref. [15]. Therefore, the gauge coupling of SM QCD is given by  $g_s = g \sin \phi \cos \phi$ ,  $\sin \phi = g_R/g$ ,  $\cos \phi = g_L/g$ ,  $g = \sqrt{g_R^2 + g_L^2}$  is the combination of new gauge couplings,  $g_V^t = g_V = -\frac{1}{2}g \cos 2\phi$  and  $g_A^t = -g_A = g/2$ . If we take the value of  $g_s$  as input, the mixing angle  $\phi$  and mass of axigluon are the main free parameters.

Before presenting the scattering amplitude squares for  $q\bar{q} \rightarrow t\bar{t}$  which are mediated by gluon and the colored bosons, let us first define a convenient notation for the following calculations. The momenta of the incoming quark and anti-quark, outgoing top and outgoing anti-top quarks are denoted by  $p_q$ ,  $p_{\bar{q}}$  (q=u,d),  $p_t$  and  $p_{\bar{t}}$  respectively such that  $p_q + p_{\bar{q}} =$  $p_t + p_{\bar{t}}$ . The momentum can be written as:

$$p_{q,\bar{q}} = \frac{\sqrt{\hat{s}}}{2} (1, 0, 0, \pm 1)$$

$$p_{t,\bar{t}} = \frac{\sqrt{\hat{s}}}{2} (1, \pm\beta\sin\theta, 0, \pm\beta\cos\theta)$$
(7)

where  $\beta^2 = 1 - 4m_t^2/\hat{s}$  and  $\theta$  is the scattering angle in the center-of-mass frame of the  $t\bar{t}$ . Neglecting the light quark masses of the incoming partons, the Mandelstam variables are defined as follows:

$$\hat{s} = (p_q + p_{\bar{q}})^2 = (p_t + p_{\bar{t}})^2 ,$$
  

$$\hat{t} = (p_q - p_t)^2 = (p_{\bar{q}} - p_{\bar{t}})^2 = m_t^2 - \frac{\hat{s}}{2} (1 - \beta \cos \theta) ,$$
  

$$\hat{u} = (p_q - p_{\bar{t}})^2 = (p_{\bar{q}} - p_t)^2 = m_t^2 - \frac{\hat{s}}{2} (1 + \beta \cos \theta) .$$
(8)

Accordingly, the averaged amplitude square for QCD gluon and color triplet can be derived by [14]

$$\overline{\sum |\mathcal{M}_{SM+H_3}|^2} = \frac{4\pi^2 \alpha_s^2}{N_c^2} (N_c^2 - 1) \left( 1 + \beta^2 \cos^2 \theta + \frac{4m_t^2}{\hat{s}} \right) + \frac{\pi \alpha_s}{N_c^2} \left( \frac{N_c^2 - 1}{2} \right) \frac{f_{ut}'^2 \hat{s}}{\hat{u} - m_{H_3}^2} \left( (1 + \beta \cos \theta)^2 + \frac{4m_t^2}{\hat{s}} \right) + \frac{1}{8N_c^2} N_c (N_c - 1) \frac{f_{ut}'^4 \hat{s}^2}{(\hat{u} - m_{H_3}^2)^2} (1 + \beta \cos \theta)^2 , \qquad (9)$$

where we have already summed over final state color and averaged over the initial spin and color,  $m_{H_3}$  is the mass of diquark,  $f'_{ut} = 2f_{ut}$  and  $N_c = 3$ . We note that since the propagator in the *u*-channel diagram depends on the scattering angle  $\theta$ , the FBA may arise not only from  $\cos \theta$  terms in Eq. (9), but also from the constant terms and the  $\cos^2 \theta$  terms. Although the interference between SM and diquark is negative when  $\hat{u} - m_{H_3}^2 < 0$ , however the pure diquark contribution is positive and dominates in the considered  $M_{t\bar{t}}$  range.

The averaged amplitude square for QCD gluon and axigluon contributions can be obtained as

$$\overline{\sum \left|\mathcal{M}_{SM+G_{A}}\right|^{2}} = \frac{N_{c}^{2}-1}{4N_{c}^{2}} \left\{ 16\pi^{2}\alpha_{s}^{2} \left(1+\beta^{2}\cos\theta^{2}+\frac{4m_{t}^{2}}{\hat{s}}\right) + \frac{8\pi\alpha_{s}\hat{s}(\hat{s}-m_{A}^{2})}{(\hat{s}-m_{A}^{2})^{2}+m_{A}^{2}\Gamma_{A}^{2}} \left[g_{V}g_{V}^{t}\left(1+\beta^{2}\cos^{2}\theta+\frac{4m_{t}^{2}}{\hat{s}}\right)+2g_{V}g_{A}^{t}\beta\cos\theta\right] + \frac{\hat{s}^{2}}{(\hat{s}-m_{A}^{2})^{2}+m_{A}^{2}\Gamma_{A}^{2}} \left[\left(g_{V}^{2}+g_{A}^{2}\right)\left((g_{V}^{t})^{2}\left(1+\beta^{2}\cos^{2}\theta+\frac{4m_{t}^{2}}{\hat{s}}\right)+(g_{A}^{t})^{2}\left(1+\beta^{2}\cos^{2}\theta-\frac{4m_{t}^{2}}{\hat{s}}\right)\right)+8g_{V}g_{V}^{t}g_{A}g_{A}^{t}\beta\cos\theta\right]\right\}.$$
(10)

Clearly, the FBA is only from the linear  $\cos \theta$  terms and associated with axial-vector coupling. Unlike diquark model, with  $g_V g_A^t$ ,  $g_V g_V^t g_A g_A^t < 0$ , the interference between SM and axigluon is positive when  $\hat{s} - m_A^2 < 0$ , whereas axigluon contribution is negative. Therefore, both contributions to the FBA are in competition.

Beside the experimental limit on the mass of new particles, the direct strict constraint on the free parameters is from the  $t\bar{t}$  production cross section. Based on 8.8 fb<sup>-1</sup> of Tevatron data, the recent combination of CDF and D0 results for  $t\bar{t}$  cross section is [31]

$$\sigma(p\bar{p} \to t\bar{t})^{\exp} = 7.65 \pm 0.20 \pm 0.36 \text{ pb}.$$
 (11)

In order to study new physics on  $t\bar{t}$  production cross section and top FBA, we implement the diquark and axigluon matrix elements, shown in Eqs. (9) and (10), in Pythia8175 [32] as semi-internal  $2 \rightarrow 2$  processes. Thus,  $t\bar{t}$  pair production in the two models could be dealt with as a normal internal Pythia process. For estimating the SM NLO contributions, we use POWHEG-BOX-1.0 [33].

All estimations in our analysis are performed at parton level and we do not take into account the effects from parton showering, hadronizations and detector conditions. The taken inputs are the experimental value given by Eq. (11) within  $2\sigma$  errors,  $m_t = 172.5$  GeV and  $\alpha_s = 0.1095$  at  $m_Z$ . The renormalization and factorisation scales are fixed at  $\mu_R = \mu_F = m_t$ . The event samples are generated using CTEQ6L1 [34] set parton distribution functions for diquark and axigluon models, whereas CTEQ6M [34] is used for SM NLO.

Before we discuss the contributions of new physics to top FBA, we first study the allowed parameter space, where the main constraint is from the  $t\bar{t}$  production cross section given in Eq. (11). For color triplet, since the free parameters are  $f_{ut}$  and  $m_{H_3}$ , we display the scatters of  $\sigma(p\bar{p} \to t\bar{t})$  with respect to  $f'_{ut} = 2f_{ut}$  and  $m_{H_3}$  in left panel of Fig. 1. Similarly, for axigluon model, the scatters of  $\sigma(p\bar{p} \to t\bar{t})$  with respect to mixing angle  $\phi$  and  $m_A$  are shown in right panel of Fig. 1.



FIG. 1: Scatter plots of  $t\bar{t}$  production cross section for color triplet diquark (left panel) and axigluon (right panel) within  $1\sigma$  (green) and  $2\sigma$ (red) of  $\sigma_{t\bar{t}}^{exp}$  and  $A_{FB}$ , respectively.

With the allowed parameters shown in Fig. 1, we now discuss the influence of color triplet and octet bosons on the FBA. In order to display the dependence of free parameters, we will fix the masses of new bosons and chosen the allowed values for the couplings. Consequently, with  $m_{H_3} = 665$  GeV, the top FBA as a function  $|\Delta y|$  for diquark is shown in left panel of Fig. 2. The dashed, dotted and dott-dashed lines stand for  $f'_{ut} = (2.5, 2.6, 2.7)$ , where the corresponding  $t\bar{t}$  production rates are  $\sigma(p\bar{p} \to t\bar{t}) = (7.29, 7.70, 8.19)$  pb. For axigluon model, we present the results in right panel of Fig. 2. For escaping the limit from the search of new resonance, we take  $m_A = 1525$  GeV. The dashed, dotted and dash-dotted lines denote  $\phi = (25^0, 30^0, 35^0)$ , where the corresponding  $t\bar{t}$  cross sections are  $\sigma(p\bar{p} \to t\bar{t}) =$ (7.35, 7.72, 7.94) pb. For comparisons, we also show the SM NLO prediction in the plots by solid line. By the two plots, we see clearly both models could enhance the top FBA and match the CDF data.



FIG. 2: Top FBA as a function of  $|\Delta y|$ , where the dashed, dotted and dash-dotted lines stand for  $f'_{ut} = (2.5, 2.6, 2.7)$  in diquark model and  $\phi = (25^0, 30^0, 35^0)$  in axigluon model, respectively. The solid line is the SM NLO.

Using the same taken values of parameters for  $|\Delta y|$  dependence, we present the FBA as a function of  $M_{t\bar{t}}$  in Fig. 3. The FBA in both models is enhanced at  $M_{t\bar{t}} > 450$  GeV and fits well to current CDF data within  $1\sigma$  errors. For further comparing our results with data



FIG. 3: The legend is the same as Fig. 2 but for function of  $M_{t\bar{t}}$ .

and SM NLO, we show the values of FBA with the chosen ranges of  $|\Delta y|$  and  $M_{t\bar{t}}$  in Tables I and II. Beside  $M_{t\bar{t}} \leq 750$  GeV which is presented in CDF paper [1], we display the FBA up to 1400 GeV in Fig. 4, where we have integrated the FBA over the width of 7 bins in [350, 1400] GeV. It is found that when  $M_{t\bar{t}} < 1100$  GeV, the two models induce a positive asymmetry. When  $M_{t\bar{t}} > 1100$  GeV, the asymmetry induced by the diquark is positive and

TABLE I: The asymmetry as a function of  $|\Delta y|$  in color triplet and octet models. The data are quoted from [1].  $m_{H_3} = 665$  GeV,  $f'_{ut} = 2.7$ ,  $m_A = 1525$  GeV,  $\phi = 30^0$  are used for numerical calculations.

$ \Delta y $	Data	SM@NLC	) $H_3$	$G_A$
0.0 - 0.5	$0.048 \pm 0.034 \pm 0.022$	0.025	0.060	0.042
0.5 - 1.0	$0.180 \pm 0.057 \pm 0.046$	0.071	0.194	0.126
1.0 - 1.5	$0.356 \pm 0.080 \pm 0.036$	0.113	0.352	0.219
$\geq 1.5$	$0.477 \pm 0.132 \pm 0.074$	0.171	0.562	0.370
< 1.0	$0.101 \pm 0.040 \pm 0.029$	0.042	0.118	0.077
$\geq 1.0$	$0.392 \pm 0.093 \pm 0.043$	0.131	0.432	0.266

TABLE II: The legend is the same as Table I but for  $M_{t\bar{t}}$  dependence.

$M_{t\bar{t}}$	Data	SM@NLO	$H_3$	$G_A$
< 450	$0.084 \pm 0.046 \pm 0.026$	0.048	0.075	0.064
450 - 550	$0.255 \pm 0.062 \pm 0.034$	0.085	0.245	156
550 - 650	$0.370 \pm 0.084 \pm 0.087$	0.115	0.358	0.263
$\geq 650$	$0.493 \pm 0.158 \pm 0.110$	0.170	0.414	0.398
< 450	$0.084 \pm 0.046 \pm 0.030$	0.048	0.075	0.064
$\geq 450$	$0.295 \pm 0.058 \pm 0.033$	0.099	0.313	0.205

grows with  $M_{t\bar{t}}$ , whereas that induced by the axigluon is negative and falls with  $M_{t\bar{t}}$  [15]. Thus we can use the different behavior at large  $M_{t\bar{t}}$  to rule out one of the two models.

CDF recently reports the measurements of the Legendre moments for differential cross section of  $t\bar{t}$  production with respect to the scattering angle  $\theta$  of top-quark in the  $t\bar{t}$  center of mass. The moment is defined by

$$a_{\ell}' = \frac{2\ell + 1}{2} \int_{-1}^{1} d\cos\theta \frac{d\sigma(\cos\theta)}{d\cos\theta} P_{\ell}(\cos\theta) ,$$
  
$$= \frac{2\ell + 1}{2} \sigma < P_{\ell}(\cos\theta) >$$
(12)

with  $\ell$  being the degree of Legendre polynomial. In order to compute the Legendre moments, we normalize  $a'_0$  to be unity. Thus, the Legendre moments  $a_\ell$  can be estimated from a sample



FIG. 4: Differential asymmetry  $A_{FB}$  as a function of  $M_{t\bar{t}}$  integrated over 7 bins chosen in [350, 1400] GeV.

of  $N_{ev}$  events as

$$a_{\ell} = (2\ell + 1) < P_{\ell}(\cos \theta) >$$
  
=  $\frac{2\ell + 1}{N_{ev}} \sum_{i=1}^{N_{ev}} P_{\ell}(\cos \theta_i).$  (13)

It is found the first Legendre moment is  $a_1 = 0.39 \pm 0.108$  [4] and the result is in disagreement with SM NLO of  $a_1^{\text{SM}} = 0.15^{+0.066}_{-0.033}$  [4, 11]. In our calculations, the first Legendre moment of diquark and axigluon model at the preference point respectively is

$$a_1^{\mathbf{Di}} = 0.38,$$
  
 $a_1^{\mathbf{Axi}} = 0.23.$  (14)

Clearly, diquark fits well CDF result. The first eight Legendre moments  $a_{\ell}$ ,  $\ell=1-8$ , are shown in the left panel of Fig 5. For understanding the new physics contributions, we calculate the normalised differential cross section as a function of  $\cos \theta$  and present the results in right panel of Fig. 5, where the solid, dashed and dotted lines represent the SM NLO, diquark with  $m_{H_3} = 665$  GeV and  $f'_{ut} = 2.7$  and axigluon with  $m_A = 1525$  GeV and  $\phi = 30^0$ , respectively. In order to compare with CDF data, in the calculations we divide [-1, 1] into 10 bins and integrate the angular differential cross section over the width of 10-bin in  $\cos \theta$ . The right plot in Fig. 5 shows that the diquark mode is in better agreement with the data



FIG. 5: Left panel: Legendre moments estimated for diquark and axigluon models compared to SM NLO and CDF results. Right panel: normalised angular differential cross section for  $p\bar{p} \rightarrow t\bar{t}$ as a function of  $\cos \theta$ , where the differential cross section has been integrated over 10 bins chosen in [-1, 1].

than axilguon model. Note that, the difference in the calculations of FBA between  $t\bar{t}$  frame and partonic center-of-mass frame is numerically negligible.

In summary, we have studied the top FBA in diquark and axigluon models, where the former is a representative of a u-channel and the latter is a s-channel. According to our analysis, both models could enhance the FBA and fit well in  $|\Delta y|$  and  $M_{t\bar{t}}$  distributions. We also show that the top FBA induced by s-channel will decrease from positive to negative at  $M_{t\bar{t}} > 1100$  GeV, while the u-channel is still growing slightly. We also give the first eight Legendre moments in diquark and axigluon models and find that diquark could explain the large  $a_1$  obtained by CDF.

## Acknowledgments

We would like to thank Dr. Y.C. Chen of IOP Academia Sinica for useful discussions. This work is supported by the National Science Council of R.O.C. under Grant #: NSC-100-2112-M-006-014-MY3 (CHC). The work of R.B was supported by the Spanish Consejo Superior de Investigaciones Cientificas (CSIC) and by a part from National Center for Theoretical Sciences (NCTS) where this work was initiated. The work of M.E was supported by LIA International Laboratory for Collider Physics between IN2P3/CNRS, France, the Moroccan CNRST and KTH Sweden; LAPP, Université Savoie, IN2P3/CNRS.

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