Polarization dynamics of a multimode vertical-cavity surface-emitting laser subject to orthogonal optical injection

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We have studied the effect of orthogonal optical injection on a multitransverse-mode vertical-cavity surface-emitting laser (VCSEL). The injection beam comes from a similar VCSEL. Our results reveal that, when the receiver operates with several strong transverse modes of parallel polarization, optical injection can induce polarization switching in all of them while only one mode is locked to the external frequency. The induced switching can occur for a very weak (a few microwatts) injected beam. Periodic oscillations and low-frequency irregular pulsations are observed as well. © 2012 Optical Society of America

1. INTRODUCTION

Optical injection on vertical-cavity surface-emitting lasers (VCSELs) has been an active research area since 1990's [1–19]. Optical injection can induce frequency locking and rich nonlinear phenomena including enhanced relaxation oscillation [1], bistability [2], nondegenerate four-wave mixing [5], subharmonic generation [1,3,10], and chaotic dynamics [6,10]. With optical injection, it is possible to reduce chirp and nonlinearity [7] and to achieve all-optical inverting via frequency locking [8]. Depending on the polarization of the injected signal relative to the polarization of receiver, two configurations have been used in the study: parallel injection and orthogonal injection. Parallel injection means that polarization of the injected signal is parallel to the polarization of the receiver; orthogonal injection refers to the case in which the polarization of the injection is perpendicular to that of the receiver. However, polarization of a VCSEL is not always stable. It may vary with the operating temperature and bias current of the VCSEL [20]. Besides, higher-order transverse modes can start oscillation when the bias current is increased, and adjacent transverse modes have parallel or orthogonal polarizations [21]. The complexity in polarization and transverse mode operation increases richness of dynamics when optical injection is applied to a VCSEL. In recent years, single-frequency optical injection has been studied in two-transverse-mode VCSELs both theoretically and experimentally [4,5,8,9,11,15,18]. It has been observed that optical injection can induce polarization switching (PS) [11], mode switching [8,11], and bistability [18] via frequency locking between one frequency of the VCSEL and the injected beam. Nondegenerate four-wave mixing [5], antiphase dynamics and chaotic oscillation [4,15] are obtained outside the locking regime. Very recently, we studied experimentally two-frequency orthogonal optical injection on a multimode

VCSEL. We obtained complete polarization switching when the two dominant modes are locked to the corresponding modes of the injected light [19]. The interests in effects of optical injection on VCSELs arise from both potential applications and intellectual curiosity for mechanisms underlying the phenomena. For example, control of polarization is an important issue in telecommunication applications because coupling of a VCSEL to an optical fiber may result in different loss for the two polarizing modes and lead to an increase of the intensity noise at the fiber output [10]. On the other hand, a well-controlled polarization switching and polarization bistability may be useful for all-optical signal processing functionalities [8,18]. From the point of view of fundamental sciences, the interplay of polarization and transverse modes in optically injected VCSELs still needs investigating.

In this paper we are reporting our experimental investigation on the polarization dynamics of a multitransverse-mode VCSEL subject to a single-frequency orthogonal optical injection. Our work differs from the previous ones in the following aspects. First, our VCSEL operates with more than two transverse modes and demonstrates unique behaviors under optical injection. In the previous studies [5], it was shown that transverse modes of parallel polarization behave independently of each other for both parallel and orthogonal optical injection. On contrary, we have observed simultaneous polarization switching of several X-polarized transverse modes when one of the modes is frequency-locked to the injected beam even when the injection power is only a few microwatts. This opens up the possibility of controlling the polarization of the VCSEL in the multitransverse-mode regime. Second, while most experiments use a tunable semiconductor laser as the transmitter, our transmitter is a VCSEL that is very similar to the receiver: they are from the same manufacturer, of the same model number, and came in same package. Given the recent efforts in studying VCSEL-by-VCSEL optically

injection-locked systems [14], it is necessary to understand polarization dynamics in such a system.

2. EXPERIMENTAL SETUP AND CHARACTERISTICS OF THE RECEIVER

In our experiment, a proton-implanted VCSEL, V_1 (receiver), emitting at 847 nm receives a single-frequency injected signal provided by another VCSEL, V_2 (transmitter), of the same model, as shown in Fig. 1. The temperatures of the two VCSELs are stabilized at 24.01 °C and 32.02 °C separately by temperature controllers of same model (Thorlabs TEC2000). Their bias currents are controlled by current drivers (Thorlabs LDC200C) with accuracy 0.001 mA. The dominant polarization of both VCSELs is parallel to the optical table, which is termed X polarization. The polarization perpendicular to the optical table is termed Y polarization. The X polarization of the transmitter is selected by a polarizing beam splitter (PBS) and sent through an optical isolator (ISO). Since the isolator makes the polarization rotated by 45°, a half-wave plate (HWP) is placed behind the isolator to rotate the polarization of the transmitted light by another 45°, becoming perpendicular to the optical table. Now the polarization of the injected light is orthogonal to the dominant polarization of the receiver. The injection is sent into the receiver by mirror M_2 and a nonpolarizing plate beamsplitter M_1 . We obtain the optimal alignment when the injection power necessary to induce polarization switching is minimized at the boundary of the PS region.

For the purpose of observation and measurement, the output of V_1 is split at M_1 : the transmitted part is sent to a Fabry–Perot (F–P) spectrum analyzer (FSR 150 GHz) and a charge coupled device (CCD) camera, the reflected light is sent to a one-meter spectrometer (Jobin Yvon 1000m) and a fast detector (Newport 1580B, 12 GHz) which can be connected to an RF spectrum analyzer (Agilent EXA N9010A, 9 kHz to 26.5 GHz) or a digital oscilloscope (Tektronix DPO 7254, 2.5 GHz). With the aid of another PBS and a half-wave plate, we can detect spectrum and dynamics of each polarization of V_1 as well as its power. The power of each polarization is measured with a power meter. We can also set up

Fig. 1. Experimental setup, where PBS stands for polarizing beamsplitter, BS for nonpolarizing beamsplitter, HWP for half-wave plate, ISO for optical isolator, FC for fiber coupler, and PD for fast photo-detector. M_1 is a plate beamsplitter, M_2 and M_3 are mirrors.

two fast detectors to observe temporal behaviors of the X and Y polarizations simultaneously. Half of the beam from the transmitter V_2 can be sent to the spectrometer or to the F–P spectrum analyzer and the CCD camera. In order to send the light of V_2 to the F–P spectrum analyzer, we use a mirror, M_3 , installed on a translational stage. M_3 and the PBS that reflects the output of V_2 are carefully aligned to get the frequency structure of V_2 at the F–P spectrum analyzer. The frequency (wavelength) detuning is obtained from the F–P spectrum analyzer (spectrometer). The neutral density filter (NDF) is used to adjust the injection power. The power of the injected signal is measured in front of the collimating lens of V_1 .

As shown in Fig. 2(a), the solitary V_1 is dominantly polarized in the X direction and no polarization switch occurs. From threshold to 3.33 mA, the VCSEL operates in fundamental mode (TEM_{00} mode). The second mode, a higher-order transverse mode, starts lasing from 3.33 mA. Its beam profile indicates that it can be described as a TEM_{01} mode. From 3.65 mA to 4.75 mA, the VCSEL operates with three transverse modes. The third mode is a TEM_{10} mode. The fourth transverse mode can be described as a TEM_{02} mode. It is on from 4.75 mA and is essentially Y polarized. In our study, we focused on the bias current between 4.8 mA and 5.6 mA, where the receiver operates with four transverse modes.

The polarization feature of V_1 and the spatial profile of each transverse mode are shown in Fig. 2(b). The fundamental mode and the TEM_{10} mode are X polarized; however, each has a nonlasing Y mode with a higher frequency. The fourth mode (TEM_{02}) is Y polarized. The TEM_{01} mode has two polarized states: an elliptically polarized state and a nonlasing Y polarized state. The frequency of the Y polarization is 8 GHz higher. As illustrated, the X component of the elliptical polarization is much stronger than its Y component, therefore

Fig. 2. (Color online) Polarization-resolved (a) L–I curves and (b) optical spectra of the solitary V_1 . The bias current of V_1 is 5.219 mA. (Blue: X polarization; pink: Y polarization.)

this polarization can be regarded as X polarization essentially. The frequency difference is 45 GHz between the TEM_{00} and TEM_{10} mode and 60 GHz between the TEM_{00} and essentially X -polarized TEM_{01} mode. The frequency of the Y -polarized TEM_{02} mode is 124 GHz higher than the TEM_{00} mode.

3. EXPERIMENTAL RESULTS

In our experiment, the bias current of V_2 , I_2 , is 3.100 mA. The output from V_2 is single frequency with the beam profile of the fundamental mode. We tune I_1 , the bias current of the receiver, to change the frequency detuning, $\Delta\nu$, between V_1 and V_2 . We define $\Delta\nu$ as the frequency difference between the injected light, ν_2 , and the nonlasing Y -polarized TEM_{01} mode of the solitary V_1 , ν_{1Y} . The frequency detuning is read from the optical spectra obtained from the F-P spectrum analyzer. The reading uncertainty is ~ 0.6 GHz. Another control parameter is the injection power, P_{inj} . The injection power is measured in front of the collimating lens of V_1 . Its value is adjusted by using a neutral density filter (NDF). The maximum injection power is $19.6 \mu\text{W}$.

Figure 3 demonstrates polarization switching in the total output power when the injection power is varied and frequency detuning is relatively small. In Fig. 3(a), dominantly X -polarized total power switches to Y -polarized power when P_{inj} is greater than $10 \mu\text{W}$. The corresponding frequency detuning is $\Delta\nu = -2.0$ GHz. When $\Delta\nu = -1.4$ GHz, polarization switching occurs for even lower injection power. This is similar to [10,11,17], in which a minimum injection power induces PS for appropriate detuning. However, the power of Y (X) polarization is lower (higher) than that in Fig. 3(a) by approximately $15 \mu\text{W}$. This is because the transverse modes do not always have PS together. As shown in Fig. 4,

Fig. 3. (Color online) Polarization-resolved total power versus P_{inj} for (a) $\Delta\nu = -2.0$ GHz and (b) $\Delta\nu = -1.4$ GHz. (Blue diamond: X polarization; pink square: Y polarization.)

Fig. 4. (Color online) (a) X -polarized and (b) Y -polarized modal intensities versus frequency detuning, where the injection power is $19.6 \mu\text{W}$.

though the three X -polarized modes start to drop simultaneously, they do not reach their lowest intensity together. Among the three modes, the TEM_{01} mode undergoes polarization switching in a detuning range of ~ 3 GHz. The other two, the X -polarized fundamental mode and TEM_{10} mode, stay at their lowest intensity for a very narrow range of detuning and then begin to increase. Therefore, though the total power still undergoes polarization switching in Fig. 3(b), the switching is not as complete as in Fig. 3(a). The modal intensity also shows that the influence of the injection on the fourth transverse mode is minimal: its Y -polarized state only has a slight decrease and the X polarization increases slightly when the detuning is changed from -3 GHz to 0 GHz.

The polarized optical spectra of V_1 in the PS region are given in Fig. 5. It is obvious that frequency locking is achieved between the Y -polarized TEM_{01} mode and the injected beam. Frequencies of the Y -polarized fundamental mode and TEM_{10} mode are ~ 7 GHz higher than their X -polarized counterparts, indicating that their nonlasing Y -polarizations start oscillation now. Their frequencies, however, are far from that of the injection. Thus it is reasonable to say that the underlying physical mechanism is not frequency locking for the fundamental mode and TEM_{10} mode. Note that the Y (X) polarization of the TEM_{02} mode has a slight decrease (increase). This agrees with what is shown in Fig. 4.

As a comparison, we give the polarized optical spectra of V_1 when the injected signal has frequency locking with the fundamental mode and the TEM_{10} mode, respectively. Figures 6(a) and 6(b) show injection-induced polarization switching in the TEM_{00} mode for I_1 to be 3.408 mA. For this bias current, the TEM_{01} mode has started oscillation but is

Fig. 5. (Color online) Polarization-resolved optical spectra of V_1 for $\Delta\nu$ to be -2 GHz and P_{inj} to be $19.5 \mu\text{W}$. (a) X polarization (blue: solitary V_1 ; green: V_1 with injection) and (b) Y polarization (blue: solitary V_1 ; green: V_1 with injection). The pink, dashed curve represents injected signal in both polarized spectra.

much weaker than the fundamental mode. The frequency detuning in Figs. 6(a) and 6(b) is -67 GHz. It is obvious that the fundamental mode becomes Y -polarized and frequency locking is achieved between the injected signal and the Y -polarized fundamental mode. The TEM_{01} (the weak peak in Fig. 6(a)) is not affected by the injection. This is somewhat similar to the behavior of the TEM_{02} mode in Fig. 5, in which the three strong X -polarized modes undergo polarization switching whereas the TEM_{02} mode only has a slight change in its intensity. When the injection gets close to the TEM_{10} mode, it causes polarization switching in the X -polarized TEM_{10} mode but does not affect polarization of the other modes, as shown in Figs. 6(c) and 6(d) for detuning to be -16 GHz.

When the frequency detuning and injection power are varied separately, the output of the receiver demonstrates polarization switching and various instabilities. The dynamics of V_1 is mapped in Fig. 7. Experimentally, we consider that the mode switches from X to Y polarization when its X -polarized modal intensity drops to one-tenth of its original. Similarly, the mode is considered switching back when its X -polarized modal intensity rises to one-tenth of its original. The modal polarization switching, however, is complex. As shown in Fig. 4, the TEM_{10} mode switches twice as detuning is varied from ~ -3 GHz to 1 GHz. Therefore we define a modal polarization switching regime instead of illustrating switching/switching back boundary for each mode. The modal polarization

Fig. 6. (Color online) Polarization-resolved optical spectra of V_1 for (a, b) $I_1 = 3.408$ mA, $\Delta\nu = -67$ GHz and (c, d) $I_1 = 4.799$ mA, $\Delta\nu = -16$ GHz. P_{inj} is $19.5 \mu\text{W}$. (a) and (c) give X polarization (blue: solitary V_1 ; green: V_1 with injection; pink: injected signal). (b) and (d) are Y polarization (blue: solitary V_1 ; green: V_1 with injection; pink dashed: injected signal). An inset is included in (a) to illustrate with clarity the TEM_{01} mode of the solitary V_1 .

switching refers to the regime where there are one or more transverse modes switching from X -polarized state to Y -polarized state. As for the slight discrepancy between Fig. 4 and Fig. 7 (in Fig. 4, modal switching regime is from ~ -3 GHz to ~ 0.5 GHz for maximum injection power; in Fig. 7, the

Fig. 7. (Color online) Dynamics map of V_1 in the parameter plane of detuning versus injection power, in which regions of modal polarization switching and three types of instabilities are illustrated, respectively.

modal switching regime is from ~ -2 GHz to ~ 1.5 GHz for maximum injection power), it is probably caused by uncertainties in reading and alignment of the F-P spectrum analyzer. The overall dynamics, however, repeats the same pattern as the detuning is tuned from negative value to positive. In the modal PS regime, when all the X -polarized modes have PS, the output is stable. Otherwise, a low-frequency (<0.2 GHz) shoulder is observed in the RF power spectrum of the receiver, as shown in Fig. 8(a). The corresponding time series manifest antiphase, irregular fluctuations [Fig. 8(b)]. This is because modal polarization switch and switching back do not always occur simultaneously for all transverse modes in the modal polarization regime. When some modes switch whereas others do not, polarization competition will happen and results in irregular antiphase fluctuations of relatively low frequencies.

We analyzed the RF power spectrum of V_1 and categorized the instabilities into three types. In the analysis, we take a distinguishable peak in the power spectrum into account when the magnitude of the peak is equal or greater than 3 dBm. Among the three types of instabilities, type 1 and type 2 are also observed when the VCSEL is subject to a two-frequency optical injection [19]. When the detuning is greater than -10 GHz, a sharp peak is observed in the power spectrum and its frequency changes with the detuning. We term this type of oscillation as type 1 instability. As shown in Fig. 9(a), there is a sharp peak of ~ 0.8 GHz in the RF power spectrum of V_1 for detuning to be -7.4 GHz. For this specific detuning, the frequency difference between the injection and the elliptically polarized TEM_{01} mode is 0.6 GHz. These two frequencies are the same by taking the reading uncertainty into account. On the contrary, type 1 instability is not observed when the external signal is injected to the TEM_{00} or TEM_{10} mode of the receiver because these two modes are X -polarized in solitary operation. Therefore the oscillation is probably the beating of the injected beam and the y -component of the TEM_{01} mode. The second type of instability is represented by a peak ~ 1 to 2 GHz in the power spectrum and a low-frequency enhancement of the power spectrum of the Y -polarization, as shown in Fig. 9(b). It occurs immediately before a mode switches from X to Y polarization and the bandwidth of the frequency peak in the power spectrum is wider than that in type 1 instability. The origin of this instability is likely due to noise-induced mode hopping that typically accompanies polarization switching. Polarization mode hopping in quantum-well VCSELs is only observed when the laser is biased in the vicinity of the current of an abrupt PS

Fig. 8. (Color online) (a) Power spectrum of the Y polarization and (b) polarization-resolved time series (blue: X ; pink: Y) for $\Delta\nu = -0.2$ GHz and $P_{inj} = 19.5 \mu W$. The receiver is in the modal PS regime. The time series of X polarization is shifted downward for visual convenience.

[22–25]. Type 2 instability only occurs when the parameters are close to the PS boundary, as shown in Fig. 7. Also the RF spectrum of Fig. 9(b) shows an enhancement of low-frequency component similar to that observed in VCSELs showing polarization mode hopping [24]. It also occurs as the signal from the transmitter is injected on the TEM_{10} mode. The third type of instability includes a low-frequency shoulder (<1 GHz) and a sharp peak at ~ 9 GHz [Fig. 9(c)]. The low-frequency shoulder represents polarization competition between X and Y polarizations and is wider than that in type 2 instability. The magnitude of the 9 GHz peak is affected by injection power. When the injection power is decreased but not too weak, the magnitude of the peak increases, as shown in Fig. 9(d). Since the low-frequency shoulder is wide and there is a ~ 9 GHz peak in the power spectrum, it may indicate a transition to chaos via undamped relaxation oscillation. However, we did not obtain such a frequency of relaxation oscillation experimentally. Therefore the origin of this instability needs further investigation. Our theoretical modeling and numerical simulation are under way.

We also investigated whether there is any bistability for the observed PS since detuning-induced PS is typically accompanied by hysteresis regions in VCSELs [26]. When the frequency detuning is decreased, we observed a very narrow region of hysteresis. However, since the width of the region is similar to the level of reading uncertainty, we do not think that this is solid evidence of bistability. This may be related to the low power of the injected signal.

4. DISCUSSION AND CONCLUSION

It was reported that, when the VCSEL operates with two transverse modes—a fundamental mode and a first-order

Fig. 9. (Color online) Power spectrum of the Y polarization of V_1 . (a) Type 1 instability for $\Delta\nu = -7.4$ GHz, (b) type 2 instability for $\Delta\nu = -2$ GHz, (c) type 3 instability for $\Delta\nu = 0.4$ GHz. The injection power is $19.5 \mu\text{W}$. The inset shows the low-frequency shoulder in the power spectrum. (d) Type 3 instability for $P_{\text{inj}} = 8.6 \mu\text{W}$.

mode—with parallel polarization, the behaviors of the two modes are independent for both parallel and orthogonal optical injection. That is, optical injection only affects one mode

while the other mode is unperturbed [5]. In our study, this modal independence was observed in Fig. 6 when the external signal is injected to the fundamental mode and the TEM_{10} mode, respectively. However, a strong modal correlation evidenced by PS in three modes was demonstrated when the TEM_{01} mode received the injection. The fourth mode, orthogonally polarized to the first three, is quite independent of the PS that occurs in the other modes. This implies that both spatial profile overlapping and modal strength play roles in the observed PS. When the external beam is injected into the TEM_{01} mode, the bias current of V_1 is more than two times the threshold. For this relatively high current, all the X -polarized transverse modes are strong and the TEM_{01} mode is the strongest one among them, as shown in Fig. 5. Therefore, it is possible that the spatial coupling between the TEM_{01} mode and the other two modes (the TEM_{00} and TEM_{10} modes) are significantly stronger than their couplings for lower bias currents (Fig. 6) and this stronger coupling leads to the PS in all the three modes. The fourth transverse mode is much weaker than the first three modes, so its behavior is more independent. Our results also show that PS can be achieved with a very low injection power (a few microwatts), which is lower than most injected power used in previous studies [5,11,17]. This indicates a possible way to control the polarization of a multimode VCSEL via a weak signal.

The dynamics underlying the transition to polarization switching has been discussed for single [12,13] and multimode [10,11] VCSELs subject to orthogonal optical injection. Physics and bifurcations have been studied by doing a detailed analysis of the changes in optical spectra observed when changing the injected power for a fixed value of the frequency detuning. A better insight into the dynamics obtained in our experiment can be gained by comparing with results for multimode VCSELs [10,11]. Switching from the X to the Y -polarized fundamental transverse mode is also obtained in [11] for low values of the injected power when the frequency of the optical injection is near to that of the high-order transverse mode. The transition to locked high-order transverse mode observed in [11] is not obtained in our experiment, possibly due to the limited injected power in our setup. However, a direct comparison with [10, 11] has its limitations because in our free-running VCSEL three different modes have appreciable power whereas in [10, 11] only the fundamental mode of the solitary VCSEL is lasing. This difference suggests that an analysis of the dynamics, similar to that performed in [10–13], would be desirable in our system for a better understanding of the physics and bifurcations. But this analysis is beyond the scope of this paper.

In conclusion, we have studied experimentally the effect of a single-frequency orthogonal optical injection on a VCSEL operating with four transverse modes. Our results reveal that when the strongest mode, the TEM_{01} mode, is frequency-locked to the injection, polarization switching is achieved in the TEM_{01} mode as well as the other strong modes. This leads to polarization switching in the total output power of the receiver. The results imply that spatial overlapping and modal intensities play roles in the modal polarization switching in the fundamental mode and the TEM_{10} mode. We have identified three types of instabilities outside the modal polarization switching regime and proposed physical mechanisms for some of them.

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