Neutron skins in spherical and deformed nuclei from Skyrme HF

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Outline

Motivation :

- New phenomena in nuclei far away from the stability valley
- Neutron-skin formation in n-rich nuclei
- Density dependence of the symmetry energy
- EOS of asymmetric nuclear matter

Theoretical approach :

- Deformed HF+BCS formalism with Skyrme forces (SLy4)

Results :

- Proton, neutron and charge rms radii (Ni, Kr and Sn isotopes)
- Neutron skin thickness from various definitions
- Neutron skin thickness in deformed nuclei

Skyrme Hartree-Fock

Schroedinger equation

$$\left[-\vec{\nabla} \cdot \frac{\hbar^2}{2m^*(\vec{r})} \vec{\nabla} + U_q(\vec{r}) + \vec{W}(\vec{r}) \cdot (-i)(\vec{\nabla} \times \vec{\sigma}) \right] \Phi_i = e_i \Phi_i$$

$$m^*(\vec{r}), \quad U_q(\vec{r}), \quad \vec{W}(\vec{r})$$

Algebraic combinations of Skyrme parameters, densities & derivatives

$$\rho(\vec{r}) = \sum_i |\Phi_i(\vec{r})|^2$$

Axially deformed Skyrme Hartree-Fock

$$\Phi_i(\vec{R}, \sigma, q) = \chi_{q_i}(q) \left[\Phi_i^+(r, z) e^{i\Lambda^- \varphi} \chi_{+1/2}(\sigma) + \Phi_i^-(r, z) e^{i\Lambda^+ \varphi} \chi_{-1/2}(\sigma) \right]$$

Expansion into eigenfunctions of deformed harmonic oscillator

$$\Phi_i(\vec{R}, \sigma, q) = \chi_{q_i} \sum_{\alpha} C_{\alpha}^i \phi_{\alpha}(\vec{R}, \sigma) \quad \alpha = \{n_r, n_z, \Lambda, \Sigma\}$$

$$\rho(r, z) = \sum_i \left[|\Phi_i^+(r, z)|^2 + |\Phi_i^-(r, z)|^2 \right]$$

Pairing correlations in BCS approximation

Pairing interaction G

$$H = \sum_{k>0} \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \sum_{kk'>0} a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}'} a_{k'}$$

BCS eqs.

Number eq.

$$2 \sum_i v_i^2 = N$$

Gap eq.

$$\Delta = G \sum_{k>0} u_k v_k$$

$$v_i^2 = \frac{1}{2} \left[1 - \frac{e_i - \lambda}{E_i} \right]; \quad E_i = \sqrt{(e_i - \lambda)^2 + \Delta^2};$$

$$\rho(r, z) = \sum_i 2v_i^2 \left[\left| \Phi_i^+(r, z) \right|^2 + \left| \Phi_i^-(r, z) \right|^2 \right]$$

Densities, radii, and form factors

Multipole decomposition

$$\rho(r, z) = \sum_{\lambda} \rho_{\lambda}(R) P_{\lambda}(\cos \theta) = \rho_0(R) + \rho_2(R) P_2(\cos \theta) + \dots$$

$$\rho_{\lambda}(R) = \frac{2\lambda + 1}{2} \int_{-1}^{+1} P_{\lambda}(\cos \theta) \rho(R \cos \theta, R \sin \theta) d(\cos \theta)$$

Normalization

$$\int \rho(\vec{R}) d\vec{R} = X; \quad 4\pi \int \rho_0(R) R^2 dR = X; \quad X = Z, N$$

Mean square radii

$$\langle r_{p,n}^2 \rangle = \frac{\int R^2 \rho_{p,n}(\vec{R}) d\vec{R}}{\int \rho_{p,n}(\vec{R}) d\vec{R}}$$

Formfactors

$$F_{p,n}(\vec{q}) = \frac{\int \rho_{p,n}(\vec{R}) e^{i\vec{q} \cdot \vec{R}} d\vec{R}}{\int \rho_{p,n}(\vec{R}) d\vec{R}}$$

Helm model

$$\rho_{hs}(r; R_d) = \frac{3X}{4\pi R_d^3} \Theta(R_d - r)$$

$$\rho_G(r, \sigma) = (2\pi\sigma^2)^{-3/2} e^{-r^2/2\sigma^2}$$

$$\rho_{Helm}(\vec{r}) = \int \rho_{hs}(\vec{r}') \rho_G(\vec{r} - \vec{r}') d\vec{r}'$$

F.T.

$$\begin{aligned} F_{Helm}(q) &= F_{hs}(q, R_d) F_G(q, \sigma) \\ &= \frac{3}{qR_d} j_1(qR_d) e^{-\sigma^2 q^2/2} \end{aligned}$$

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$$\sigma^2 = \frac{2}{q_{\max}^2} \ln \frac{3 j_1(q_{\max} R_d)}{R_d q_{\max} F(q_{\max})}$$

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Isotopic chains

Charge rms radii

Charge rms radii of tin isotopes

$$\langle r_{ch}^2 \rangle = \langle r_p^2 \rangle + \langle r_{ch}^2 \rangle_p + (N/Z) \langle r_{ch}^2 \rangle_n + r_{CM}^2 + r_{SO}^2$$

Proton and neutron rms radii

$$\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

Hadron scattering

Antiproton atoms

GDR

SDR

Neutron-skin definitions (based on radii)

$$R_{hs} = \sqrt{5/3} \langle r^2 \rangle^{1/2}$$

$$R_d = 4.49341 / q_1$$

$$R_{Helm} = \sqrt{5/3} R_{rms}^{Helm} = \sqrt{R_d^2 + 5\sigma^2}$$

$$\Delta R_X = R_{X,n} - R_{X,p}$$

Neutron-skin definitions (based on density tails)

$$R_2 : \frac{\rho_n(R_2)}{\rho_n(0)} = 0.01$$

$$R_{1,a} : \frac{\rho_n(R_{1,a})}{\rho_p(R_{1,a})} = 4$$

$$R_{1,b} : \frac{\rho_p(R_{1,b})}{\rho_p(0)} = 0.01$$

$$\Delta R = R_2 - R_1$$

$R_{1,a}$

$R_{1,b}$

R_2

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HF+BCS proton and neutron densities $\rho_0(R)$

Ni isotopes

HF+BCS proton and neutron densities $\rho_0(R)$

Ni isotopes

Kr isotopes

HF+BCS proton and neutron densities $\rho_0(R)$

Ni isotopes

Kr isotopes

Sn isotopes

Neutron skin thickness

$$\Delta R_d = R_d [n - p]$$

$$\Delta R_{Helm} = \sqrt{R_d^2 + 5\sigma^2} [n - p]$$

$$\Delta R_{hs} = \sqrt{5/3} \langle r^2 \rangle^{1/2} [n - p]$$

$$\Delta R = R_2 - R_{1,a}$$

$$\Delta R = R_2 - R_{1,b}$$

Neutron skin thickness

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$$\Delta R = R_2 - R_{1,a}$$

$$\Delta R = R_2 - R_{1,b}$$

Densities in deformed nuclei

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Conclusions

Theoretical approach based on deformed Skyrme (SLy4) HF+BCS

- Three isotopic chains (Ni, Kr, Sn) chains
- Good agreement with experiment (charge radii)
- Calculate proton and neutron density distributions (and radii)

Explore

- Different definitions of neutron-skin thickness
- Skin formation in isotopic chains
- Neutron-skin along different directions in deformed nuclei

Find

- Strong dependence of the neutron-skin thickness on the various definitions. Linear increase with N
- While the radial extensions of the densities in deformed nuclei (prolate and oblate) change with the direction, the neutron-skin thickness remains practically constant along the different directions

Multipolar densities in deformed nuclei

$$\rho(r, z) = \sum_{\lambda} \rho_{\lambda}(R) P_{\lambda}(\cos \theta) = \rho_0(R) + \rho_2(R) P_2(\cos \theta) + \dots$$

Appendix