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Phase-tunable colossal magnetothermal resistance in ferromagnetic Josephson valves

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We propose a heat valve based on the interplay between *thermal* transport and proximity-induced *exchange* splitting in Josephson tunnel junctions. We demonstrate that the junction electron heat conductance strongly depends on the relative alignment of the exchange fields induced in the superconductors. *Colossal* magnetothermal resistance ratios as large as $\sim 10^7\%$ are predicted to occur under proper temperature and phase conditions, as well as suitable ferromagnet-superconductor combinations. Moreover, the quantum phase tailoring, intrinsic to the Josephson coupling, offers an additional degree of freedom for the control of the heat conductance. Our predictions for the phase-coherent and spin-dependent tuning of the thermal flux can provide a useful tool for heat management at the nanoscale. © 2013 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4800578>]

The study of heat transport and dynamics in meso-¹ and nanoscopic² solid-state systems is a research field that has attracted much attention in recent years because of the impressive progress achieved in nanoscience and nanofabrication techniques. At such scale heat may play a significant role in determining the properties of the devices, and therefore it is of particular interest to control and manipulate^{3,4} the thermal flux as well to understand the origin of dissipative phenomena. Prototypical cases in which the understanding of heat transport is crucial are, for instance, the fine temperature control in ultrasensitive cryogenic radiation detectors,¹ general cooling applications at the nanoscale,¹ and the emerging field of *coherent* caloritronic circuitry where the quantum phase allows for enhanced operation.^{5–11}

It has been known for a few decades that phase-dependent thermal transport through weakly coupled superconducting condensates is in principle possible.^{12–16} However, only recently the first Josephson heat interferometer was demonstrated.^{17–20} The experiment of Ref. 19 proves that, in addition to the Josephson charge supercurrent, phase coherence extends to dissipative observables such as the thermal current. This heat interferometer represents a prototypical building block to implement future coherent caloritronic circuits like, for instance, heat transistors and thermal splitters.

In this letter, we put forward the concept of a ferromagnetic Josephson junction acting as a thermal valve. In particular, we address the interplay between thermal transport and proximity-induced *exchange* splitting in a Josephson tunnel weak-link consisting of two superconducting electrodes with an internal exchange splitting.²¹ The latter is induced from nearby-contacted ferromagnetic layers [see Fig. 1(a)]. We show that the junction electron thermal conductance strongly

depends on the relative alignment of the exchange fields induced in the superconductors. As a result, *colossal* magnetothermal resistance (MTR) ratios as large as $\sim 10^7\%$ are predicted to occur for suitable exchange fields and proper temperature conditions. Moreover, the quantum phase tailoring, characteristic for the Josephson effect, adds a further degree of freedom for enhanced heat conductance control.

Our system is schematized in Fig. 1(a). It consists of two equal ferromagnet-superconductor bilayers (FS_{L,R}) tunnel-coupled through an insulating barrier (I) and implementing a Josephson junction. The FS_L and FS_R bilayers are in thermal steady-state and reside at different temperatures T_L and T_R , respectively. For definiteness, we assume $T_L \geq T_R$ so that the structure is temperature-biased only, while there is no voltage drop across the Josephson junction. t_S (t_F) labels the S (F) layer thickness while φ denotes the macroscopic quantum phase difference over the junction. Furthermore, the z -axis is the one parallel to the magnetization (exchange field) of the left F layer (h_L), which is kept fixed, whereas the one in the right ferromagnet (h_R) is misaligned by an angle α [see Fig. 1(b)]. Experimentally this can be achieved either by using ferromagnetic films with different coercive fields or by pinning the magnetization in the left electrode through an exchange-bias with an additional magnetic layer.²² h_R can therefore be freely rotated by applying an in-plane magnetic field as low as a few tens of Oe.^{23,24}

We first derive an expression for the electronic contribution to the heat current ($\dot{Q}$) flowing through the Josephson junction. If $T_L \neq T_R$, there is a finite heat current flowing through the junction which is given by

$$\dot{Q} = \frac{1}{2e^2 R_N} \int_{-\infty}^{+\infty} d\epsilon \epsilon \text{Tr}[\hat{N}_L \hat{N}_R - \hat{M}_L \hat{M}_R \cos \varphi] [F_R - F_L], \quad (1)$$

where $F_{L,R} = \tanh[\epsilon/(2k_B T_{L,R})]$ is the electronic distribution function, R_N is the junction normal-state resistance, k_B is the

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FIG. 1. (a) A schematical view of the FSISF Josephson heat valve discussed in the text. (b) The exchange fields ($\mathbf{h}_{L,R}$) in the F layers are confined to the z - y plane, and are misaligned by an angle α . (c) Temperature dependence of the self-consistently calculated superconducting order parameter Δ for different values of the exchange field h . Δ_0 is the zero-temperature, zero-exchange field order parameter and T_c is the superconducting critical temperature.

Boltzmann constant, and e is the electron charge. In the following analysis, we neglect the phonon heat current as well as the heat exchanged between electrons and lattice phonons.^{12,17,19} With our convention, $\dot{Q} > 0$ represents the thermal current flowing out the left superconducting electrode when $T_L > T_R$. The two contributions to the heat current stem from the normal, $\hat{N}_j = (\hat{g}_j^R - \hat{g}_j^A)/2$, and phase-coherent (anomalous), $\hat{M}_j = (\hat{f}_j^R - \hat{f}_j^A)/2$, parts of the quasiparticle spectral function.^{12,15} Here, $\hat{g}^{R(A)}$ and $\hat{f}^{R(A)}$ are the normal and anomalous retarded (advanced) Green's functions (GFs) in electrode $j = L, R$. Equation (1) is the generalization of the Maki-Griffin heat current equation¹² for the case of spin-dependent density of states (DoS). In particular, we obtain the oscillatory behavior of the heat current as a function of the superconducting phase difference φ predicted for the first time in Ref. 12, and recently demonstrated in Ref. 19. We emphasize that Eq. (1) also accounts for the entropy production rate in the junction ($\dot{S}$). By neglecting coupling with the phonon bath and with the electromagnetic environment, the entropy production rate can be written as $\dot{S} = -\dot{Q}(\frac{1}{T_L} - \frac{1}{T_R})$ which is always positive for $T_L \neq T_R$. This explicitly shows that the entropy is increasing, in agreement with the second principle of thermodynamics. We stress that a pure temperature bias across the junction is a crucial condition to preserve phase dependence in thermal transport. Indeed, any dc voltage drop occurring across the Josephson weak-link would make φ time-dependent and, therefore, the φ -dependent component of $\dot{Q}$ in Eq. (1) would not contribute to steady-state dc heat transport.^{12,14,19}

Instead of analyzing the heat current which depends on an arbitrarily large temperature difference across the junction, we shall focus on the behavior of the electron thermal conductance (κ) which is defined for small temperature differences as $\kappa = \dot{Q}/\delta T$

$$\kappa = -\frac{1}{2e^2 R_N} \int_{-\infty}^{+\infty} d\epsilon \epsilon \left(\frac{\partial F}{\partial T} \right) \text{Tr}[\hat{N}_L \hat{N}_R - \hat{M}_L \hat{M}_R \cos \varphi], \quad (2)$$

where $\delta T = T_L - T_R$, and $(\partial F/\partial T) = -\epsilon/[2k_B T^2 \cosh^2(\epsilon/2k_B T)]$. By deriving the second equality, we have assumed that $\delta T \ll T = (T_R + T_L)/2$. Equations (1) and (2) are rather general, and allow to compute the heat current and the thermal conductance for an arbitrary tunneling junction provided that values of the GFs on both sides of the interfaces are known.

With the help of Eq. (2), we can determine the heat conductance for the junction sketched in Fig. 1(a). We assume that $|\mathbf{h}_L| = |\mathbf{h}_R| = h$, and that the S/F interface is highly transmissive so that both the superconductor and the ferromagnet are strongly affected by proximity effect.^{25,26} At the same time, in order to preserve superconductivity in the leads, we assume that the F layers are thin enough. In particular, if t_S is smaller than the superconducting coherence length, and t_F is smaller than the length of the condensate penetration into the ferromagnet, the latter induces a homogeneous effective exchange field (h) in S through proximity effect which modifies the superconducting gap (Δ). h and the effective gap in S (Δ) are given by $h/h_0 = \nu_F t_F (\nu_S t_S + \nu_F t_F)^{-1}$ and $\Delta/\Delta_0 = \nu_S t_S (\nu_S t_S + \nu_F t_F)^{-1}$, respectively. Here, h_0 is the original exchange field existing in the ferromagnetic layer and ν_S (ν_F) is the normal-state DoS at the Fermi energy in S (F). If $\nu_S = \nu_F$ and for $t_F \ll t_S$ it follows that $\Delta \approx \Delta_0$ while $h/h_0 \approx t_F/t_S \ll 1$.²⁷ We focus first on the case that the magnetizations of the F layers in Fig. 1 are either parallel (P, $\alpha = 0$) or antiparallel (AP, $\alpha = \pi$) to each other. Thus, GFs $\hat{g}^{R(A)}$ and $\hat{f}^{R(A)}$ entering Eqs. (1) and (2) are 2×2 diagonal matrices in spin space with diagonal elements given by²⁷ $g_{\pm}^R = (\epsilon \pm h + i\Gamma)$ $f_{\pm}^R/\Delta(T) = (\epsilon \pm h + i\Gamma)/\sqrt{(\epsilon \pm h + i\Gamma)^2 - \Delta^2(h, T)}$. In particular, Δ has to be determined self-consistently.²¹ The temperature dependence of the order parameter for different values of h is shown in Fig. 1(c). The parameter Γ accounts for the inelastic scattering energy rate within the relaxation time approximation.²⁸⁻³⁰ Similar expressions hold for the advanced GFs by replacing $i\Gamma$ by $-i\Gamma$. The real part of the functions $g_{\pm}^R$ gives the modified DoS in the superconductors which is spin-dependent due to the finite exchange field in the F layers.

We propose an experiment in which one can switch the misalignment angle between the P and AP configurations, and determine the MTR ratio defined as

$$\text{MTR} = \frac{\kappa_P - \kappa_{AP}}{\kappa_{AP}}, \quad (3)$$

where $\kappa_{P(AP)}$ are the heat conductances for the P and AP cases which are obtained from Eq. (2).

In Fig. 2, we show the behavior of the MTR as a function of temperature and the superconducting phase difference. All panels show an overall huge MTR ($\sim 10^5 - 10^7\%$) ratio within a broad range of parameters. We demonstrate in this way that by switching between the P and AP configuration one realizes an almost *perfect* heat valve effect as the thermal conductance in the AP configuration is practically negligible with respect to that in the P one. This colossal MTR is one of the key results of the present letter. Figures 2(a) and 2(b) show that the heat valve effect is maximized at certain finite temperature (i.e., for $T/T_c \sim 0.1$) and for sufficiently large exchange fields. Here, T_c is the superconducting critical temperature. It is worth emphasizing that due to the $\cos \varphi$ interference term in Eq. (2) the MTR ratio can be additionally largely tuned by the phase difference between the superconductors. Such a phase-tunable thermal transport mechanism originates from the Josephson effect and is unique to weakly coupled superconductors.¹² In the lower panels of Fig. 2, the MTR dependence on φ is displayed. The minimum value of the MTR is achieved for zero phase difference, whereas it reaches its maximum value for $\varphi = \pi$. We also emphasize that the phase-coherent term in Eq. (2) does not describe pure tunneling of Cooper pairs.^{12,13} Furthermore, we point out that while the P configuration

maximizes the heat current, the DC Josephson effect is maximized by the AP one.²⁷

The obtained colossal MTR ratio can be understood by inspection of Eq. (2). If we assume for simplicity that $\varphi = \pi/2$ [see Fig. 2(b)], then only the normal term $\text{Tr}[\hat{N}_L \hat{N}_R]$ in Eq. (2) contributes to the spectral conductance. The heat current (and hence the thermal conductance) is due to quasiparticle transmission from the hot to the cold electrode. For a given energy, the number of states available for the heat transport is given by the product of the DoS on both sides of the tunnel barrier. As discussed above, the effective exchange field in the S/F electrodes leads to a spin-dependent DoS. The latter is of BCS-like shape with spin-dependent gaps at $\Delta_{\pm} = \Delta \pm h$, equivalent to a Zeeman-split superconductor in a magnetic field.^{21,31} In the P configuration, the DoS of the left and right electrode coincide for both spin-up and spin-down, and therefore quasiparticles with energies around $\epsilon \sim \Delta_{\pm}$ contribute most to the heat conductance. The situation is different in the AP configuration, where the DoS for each spin-channel is shifted on both sides of the barrier by an amount $2h$. The main contribution to κ_{AP} comes from quasiparticles with energies $\epsilon \sim \Delta_+$ and therefore the spectral function $\text{Tr}[\hat{N}_L \hat{N}_R]$ in the AP configuration is approximately a factor $\sim \sqrt{\Gamma/\Delta}$ smaller than in the P one. Moreover, in both the P and AP cases the spectral contribution is weighted by the function $\epsilon \partial F / \partial T(\epsilon)$. The latter decays as $e^{-\epsilon/2T}$ for $\epsilon > 2T$ and hence the main contribution to κ_{AP} (from $\epsilon \sim \Delta_+$) has an additional exponentially small factor $e^{-h/T}$ with respect to the main contribution to κ_P (from $\epsilon \sim \Delta_-$). This explains the smallness of κ_{AP} and the huge MTR ratio obtained for large values of h .

As discussed above, the maximum MTR ratio is reached for a certain finite temperature. According to Figs. 2(a) and 2(b) further increase of T leads to a decrease of the MTR,

FIG. 2. (a) Magnetothermal resistance ratio MTR vs temperature T calculated for a few values of the exchange field h at $\varphi = 0$. (b) MTR ratio vs T calculated for the same values of h as in panel (a) at $\varphi = \pi/2$. (c) MTR ratio vs φ calculated for several values of the exchange field at $T = 0.1T_c$. (d) MTR ratio vs φ calculated at $T = 0.5T_c$ for the same values of h as in panel (c).

which can be explained, on the one hand, by the suppression of the energy gap $\Delta(T)$ and on the other hand by the fact that, increasing T , the contribution from quasiparticles with energies larger than $\Delta_{\pm}$ becomes more and more important leading to a smaller difference between κ_{AP} and κ_P . Notice that for $0 \leq \varphi < \pi/2$ the condensate part of the spectral function in Eq. (2) ($\text{Tr}[\hat{M}_L \hat{M}_R]$) gives a negative contribution to the heat conductance. This explains the lower values of MTR for small phase difference shown in Figs. 2(c) and 2(d).

For an arbitrary angle α between the magnetizations of the left and right electrode [see Fig. 1(b)], one can show straightforwardly that $\kappa_{\alpha} = \kappa_P \cos^2(\alpha/2) + \kappa_{AP} \sin^2(\alpha/2)$. We define MTR_{α} as $\text{MTR}_{\alpha} = (\kappa_{\alpha} - \kappa_{AP})/\kappa_{AP}$. In Figs. 3(a) and 3(b), we show the temperature dependence of MTR_{α} for different values of α at $\varphi = 0$. All curves show similar behavior, and again very large values for the MTR can be achieved with a proper choice of the parameters. According to Figs. 3(a) and 3(b), the effect is maximized for $\alpha = 0$, i.e., when the junction is switched between the P and AP configurations. Figures 3(c) and 3(d) show the impact of the inelastic parameter Γ on the MTR. The overall tendency is that by increasing Γ the MTR ratio is reduced, as the “normal” character of transport is strengthened leading to a suppression of the MTR. The latter, indeed, originates from the presence of the superconducting gap. Moreover, the MTR ratio reaches its maximum at higher temperature by increasing Γ [see Fig. 3(c)].

In the light of a realistic implementation, ferromagnetic insulators (FIs) such as Eu chalcogenides barriers^{32–34} combined with a conventional superconductor (e.g., aluminum) are probably the most suitable candidates for the S/F bilayers. It has been shown recently²³ that the interface proximity effect in a FI/S structure leads to an effective exchange field in S. For an S layer thinner than the superconducting coherence length this exchange field leads to a spin-split of

the BCS DoS, as the one considered in our calculations. For example, for a typical Al-based Josephson junction with a critical current of the order of 100 nA these effect should be already observable at a few hundreds of mK. Furthermore, a proper phase bias can be realized by inserting the Josephson thermal valve in a suitable dc superconducting quantum interference device (i.e., a dc SQUID), along the lines of Ref. 19, or by driving a dissipationless supercurrent through the junction. In the case of a metallic F layer, the results presented above are valid for highly transparent S/F interfaces. Nevertheless, even in the case of a finite interface resistance R_b they are qualitatively valid. In such a case the superconductor still exhibits a spin-split DoS, however with an additional damping factor proportional to R_b^{-1} . The latter will suppress the MTR ratio similarly as it does a finite Γ .

Concerning potential applications, the present thermal valve might be used whenever a precise control and mastering of the temperature are required, for instance, for on-chip heat management as a *switchable heat sink*. This setup can be useful as well to tune the operation temperature of sensitive radiation detectors.^{1,35} In the context of quantum computing architectures,³⁶ the Josephson thermal valve can also be used to influence the behavior and the dynamics of two-level quantum systems through temperature manipulation. Similarly, the relation between the Josephson critical supercurrent and the temperature can be exploited for designing tunable thermal Josephson weak-links of different kinds.^{1,37–39}

In conclusion, we have investigated thermal transport through a heat valve consisting of a Josephson junction between two S/F bilayers as electrodes. In particular, we predict that the electron heat conductance depends strongly on the relative alignment of the magnetizations of the F layers. Under specific conditions of temperature bias and phase difference across the junction, one can obtain a colossal

FIG. 3. (a) MTR_{α} ratio vs T calculated for several values of the misalignment angle α at $h = 0.1\Delta_0$ and $\varphi = 0$. (b) MTR_{α} ratio vs T calculated for the same α values as in panel (a) at $h = 0.5\Delta_0$ and $\varphi = 0$. (c) MTR ratio vs T calculated for a few values of Γ at $h = 0.5\Delta_0$ and $\varphi = 0$. (d) MTR ratio vs φ calculated for the same Γ values as in panel (c) at $h = 0.5\Delta_0$ and $T = 0.1T_c$.

magnetothermal resistance ratio as high as several orders of magnitude. The spin-dependent and phase-tunable mechanisms of heat flux control discussed here will likely prove useful for thermal management at the nanoscale, and for the development of coherent spin caloritronic nanocircuits.^{40,41}

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- ¹F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, *Rev. Mod. Phys.* **78**, 217 (2006).
- ²Y. Dubi and M. Di Ventra, *Rev. Mod. Phys.* **83**, 131 (2011).
- ³O.-P. Saira, M. Meschke, F. Giazotto, A. M. Savin, M. Möttönen, and J. P. Pekola, *Phys. Rev. Lett.* **99**, 027203 (2007).
- ⁴J. P. Pekola, F. Giazotto, and O.-P. Saira, *Phys. Rev. Lett.* **98**, 037201 (2007).
- ⁵M. Meschke, W. Guichard, and J. P. Pekola, *Nature* **444**, 187 (2006).
- ⁶E. V. Bezuglyi and V. Vinokur, *Phys. Rev. Lett.* **91**, 137002 (2003).
- ⁷J. Eom, C.-J. Chien, and V. Chandrasekhar, *Phys. Rev. Lett.* **81**, 437 (1998).
- ⁸V. Chandrasekhar, *Supercond. Sci. Technol.* **22**, 083001 (2009).
- ⁹V. V. Ryazanov and V. V. Schmidt, *Solid State Commun.* **42**, 733 (1982).
- ¹⁰G. I. Panaitov, V. V. Ryazanov, and V. V. Schmidt, *Phys. Lett.* **100**, 301 (1984).
- ¹¹P. Virtanen and T. T. Heikkilä, *Appl. Phys. A* **89**, 625 (2007).
- ¹²K. Maki and A. Griffin, *Phys. Rev. Lett.* **15**, 921 (1965).
- ¹³G. D. Guttman, B. Nathanson, E. Ben-Jacob, and D. J. Bergman, *Phys. Rev. B* **55**, 3849 (1997).
- ¹⁴G. D. Guttman, E. Ben-Jacob, and D. J. Bergman, *Phys. Rev. B* **57**, 2717 (1998).
- ¹⁵E. Zhao, T. Löftwander, and J. A. Sauls, *Phys. Rev. Lett.* **91**, 077003 (2003).
- ¹⁶E. Zhao, T. Löftwander, and J. A. Sauls, *Phys. Rev. B* **69**, 134503 (2004).
- ¹⁷F. Giazotto and M. J. Martínez-Pérez, *Appl. Phys. Lett.* **101**, 102601 (2012).
- ¹⁸M. J. Martínez-Pérez and F. Giazotto, *Appl. Phys. Lett.* **102**, 092602 (2013).
- ¹⁹F. Giazotto and M. J. Martínez-Pérez, *Nature* **492**, 401 (2012).
- ²⁰R. W. Simmonds, *Nature* **492**, 358 (2012).
- ²¹F. Giazotto and F. Taddei, *Phys. Rev. B* **77**, 132501 (2008).
- ²²J. Nogués and I. K. Schuller, *J. Magn. Magn. Mater.* **192**, 203 (1999).
- ²³B. Li, N. Roschewsky, B. A. Assaf, M. Eich, M. Epstein-Martin, D. Heiman, M. Münzenberg, and J. S. Moodera, *Phys. Rev. Lett.* **110**, 097001 (2013).
- ²⁴K. Senapati, M. G. Balmire, and Z. H. Barber, *Nature Mater.* **10**, 849 (2011).
- ²⁵A. I. Buzdin, *Rev. Mod. Phys.* **77**, 935 (2005).
- ²⁶F. S. Bergeret, K. B. Efetov, and A. Volkov, *Rev. Mod. Phys.* **77**, 1321 (2005).
- ²⁷F. S. Bergeret, A. F. Volkov, and K. B. Efetov, *Phys. Rev. Lett.* **86**, 3140 (2001).
- ²⁸R. C. Dynes, J. P. Garno, G. B. Hertel, and T. P. Orlando, *Phys. Rev. Lett.* **53**, 2437 (1984).
- ²⁹J. P. Pekola, T. T. Heikkilä, A. M. Savin, J. T. Flyktman, F. Giazotto, and F. W. J. Hekking, *Phys. Rev. Lett.* **92**, 056804 (2004).
- ³⁰Unless differently stated, throughout our analysis we set a realistic value for Γ of $10^{-4}\Delta_0$.
- ³¹R. Meservey and P. M. Tedrow, *Phys. Rep.* **238**, 173 (1994).
- ³²X. Hao, J. Moodera, and R. Meservey, *Phys. Rev. B* **42**, 8235 (1990).
- ³³T. Santos, J. Moodera, K. Raman, E. Negusse, J. Holroyd, J. Dvorak, M. Liberati, Y. Idzerda, and E. Arenholz, *Phys. Rev. Lett.* **101**, 147201 (2008).
- ³⁴G.-X. Miao, M. Müller, and J. S. Moodera, *Phys. Rev. Lett.* **102**, 076601 (2009).
- ³⁵F. Giazotto, T. T. Heikkilä, G. P. Pepe, P. Helisto, A. Luukanen, and J. P. Pekola, *Appl. Phys. Lett.* **92**, 162507 (2008).
- ³⁶M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2002).
- ³⁷F. Giazotto and J. P. Pekola, *J. Appl. Phys.* **97**, 023908 (2005).
- ³⁸S. Tirelli, A. M. Savin, C. Pascual Garcia, J. P. Pekola, F. Beltram, and F. Giazotto, *Phys. Rev. Lett.* **101**, 077004 (2008).
- ³⁹A. M. Savin, J. P. Pekola, J. T. Flyktman, A. Anthore, and F. Giazotto, *Appl. Phys. Lett.* **84**, 4179 (2004).
- ⁴⁰G. E. W. Bauer, A. H. MacDonald, and S. Maekawa, *Solid State Commun.* **150**, 459 (2010).
- ⁴¹G. E. W. Bauer, E. Saitoh, and B. J. van Wees, *Nature Mater.* **11**, 391 (2012).