Analytical simulation of edge waves observed around the Balearic Islands

P. L.-F. Liu
School of Civil and Environmental Engineering, Cornell University, USA

S. Monserrat and M. Marcos
Grup d’Oceanografia Interdisciplinar/Oceanografia Física, (GOIFis), IMEDEA (CSIC-UIB) and Departament de Física, Universitat de les Illes Balears, Spain

Received 21 February 2002; accepted 1 July 2002; published 13 September 2002.

[1] We present sea level spectra computed from bottom pressure records at two locations around the Balearic Islands, Western Mediterranean. In these spectra, several sea level oscillation peaks are observed at both sides of Mallorca Island. Moreover, the relative magnitudes of the first two peaks switch during the “rissaga” events. To understand the appearance of these modes and their changes during ‘rissaga’, a simple analytical model is presented for atmospheric pressure induced edge waves on a circular shelf with a circular island in the middle. The analytical solutions indicate that indeed the peaks in the observed spectra can be identified as the fundamental modes of the edge waves. The analytical solutions also confirm that the magnitude of the second peak becomes higher than the first peak only when the phase speed of the atmospheric pressure waves become almost the same as that of the long ocean wave on the Mallorca shelf. INDEX TERMS: 4203 Oceanography: General: Analytical modeling; 4594 Oceanography: Physical: General: Marginal and semienclosed seas; 4560 Oceanography: Physical: Surface waves and tides (1255). Citation: Liu, P. L.-F., S. Monserrat, and M. Marcos, Analytical simulation of edge waves observed around the Balearic Islands, Geophys. Res. Lett., 29(17), 1847, doi:10.1029/2002GL015555, 2002.

1. Introduction

[2] An extensive field experiment was carried out in the Ciutadella region, on the west coast of Menorca (Balearic Islands in the Western Mediterranean), during the summer of 1997 (LAST-97) (see Figure 1). A set of sea level and atmospheric pressure gauges were deployed and high-quality simultaneous sea level data on the shelf and inside several inlets were obtained, including measurements in Palma Bay, which is located on the south side of Mallorca Island (see Figure 1). The aim of this field experiment was to gain understanding of the rissaga phenomena, which generate large sea level oscillations, up to 3-meter trough-to-crest wave heights with a period of about 10 minutes in Ciutadella Inlet. These oscillations have been linked to the passage of atmospheric pressure disturbances, generating long ocean waves on the continental shelf surrounding the islands that, in turn, force the resonance response in the inlets [Monserrat et al., 1991; Gomis et al., 1993; Rabino-vich and Monserrat, 1998]. In this paper we will not discuss the inlet oscillations and resonance phenomenon. Instead we shall focus our attention on the waves measured on the shelf of Mallorca.

2. Observations of Shelf Modes

[3] During the LAST-97 field experiment, from June to September 1997, four sea level recorders (bottom pressure gauges MW1, MW2, MW3 and MW4) were deployed on the shelf in front of Ciutadella. As shown in Figure 1, two more pressure gauges were located inside the Ciutadella Inlet, one near the middle (M_0) and the other close to the end of the inlet (M_2). The last pressure gauge was installed near the middle of the neighboring inlet of Platja Gran (M_1). A tide gauge has been permanently recording the sea level fluctuations inside Palma Bay (M_3). All bottom pressure gauges recorded continuously for 30 seconds intervals and stored the data with a sampling interval \( \Delta t = 1 \) min. The spectra of one of the instruments located on the shelf near Ciutadella Inlet (MW4) and of one in Palma Bay (M_3) are shown in Figure 2a. The spectral contents have been computed for a background period of 4 days (selected when the wave heights inside the inlets are only few centimeters) by using a Kaiser-Bessel window of 512 points with half window overlapping, resulting in 42 degrees of freedom.

[4] Monserrat et al. [1998] and Liu et al. [2002] have studied the spectra on the shelf near Ciutadella and have revealed the presence of some low frequency peaks that are generally referred to as ‘modes related to the shelf’. These peaks (51, 32, 24 and 14 min) appear to be present in all the instruments on the shelf and inside the nearby inlets. Figure 2a shows that some of these peaks (in particular, 32 and 24 min) are also observed in Palma Bay. Since Palma Bay is located at the other side of Mallorca Island (see Figure 1), these low frequency peaks might not be a local feature of Ciutadella region, but could be associated with the characteristics of the shelf surrounding the islands. The presence of this pair of peaks is not coincidental, since it appears in all different periods of data recording.

[5] The spectra, measured by the same instruments shown in Figure 2a during a 4-day period covering a rissaga event are presented in Figure 2b. The spectra are obviously more energetic in both locations. However, the interesting
point is that the 24 min peaks are now much higher than those of 32 min peaks in both places.

[7] In summary, the following observations are repeatable for other selected periods for analysis. Namely, the pair of peaks (32 and 24 min) is always present on the shelf near Palma and Ciutadella. The first peak (32 min) is more energetic than the second peak (24 min) for all background periods selected, but the 24 min peak becomes more energetic during a rissaga event.

[8] The primary difference between a background period and a rissaga period is the presence of a train of atmospheric pressure traveling over the islands during the rissaga event. The characteristics of these atmospheric waves are well documented [Monserrat and Thorpe, 1992; Garcia et al., 1996]. They are non-dispersive waves, travelling from the SW to the NE and with a phase speed always ranging between 20 and 30 m/s. This phase speed approximately coincides with the long waves speed on the shelf between Mallorca and Menorca, and this pressure wave condition has been suggested as a key factor in the rissaga generation [Marcos et al., 2002].

[9] As shown in Figure 1, Mallorca sits on a shelf, which drops off very sharply at the 100 m water depth contour. The distance between the coastline and the 100 m contour is roughly 10–15 km around the island, except to the northwest where the shelf extends to Menorca. To understand and to provide an interpretation of the field observations reported above, we will develop a simple analytical model for waves generated on the shelf by the propagating atmospheric pressure waves. In order to obtain analytical solutions both the shelf and the island are assumed to be circular. Although the geometry of the island and shelf is overly simplified, we believe that the simple model will capture the essential characteristics of the field observations.

3. Analytical Solutions for Forced Long Waves on a Circular Shelf with a Circular Island

[10] Suppose that there is an island of radius $b$ surrounded by a circular shelf of radius $a$ and depth $h_1$, beyond that is deep water of depth $h_2$. The governing equation for the long wave responses due to a moving atmospheric pressure can be expressed as (e.g., Greenspan [1956])

$$\nabla \cdot (gh\nabla \eta) - \frac{\partial^2 \eta}{\partial t^2} = -\frac{1}{\rho} (h^2 \nabla^2 p_a + \nabla h \cdot \nabla p_a)$$  \hspace{1cm} (1)

in which $\eta(x, y, t)$ is the free surface displacement and $p_a$ the moving atmospheric pressure that can be represented by a propagation model, i.e.,

$$p_a = P_0 e^{i(kr - wt)}$$  \hspace{1cm} (2)

[11] Following the approach taken by Longuet-Higgins [1967], the wave field is divided into two regions: In region 1 where $b > r > a$ and $h = h_1$, and in region 2 where $r \geq a$ and $h = h_2$. Since the water depth is a constant in each region ($j = 1, 2$), the governing equation (1), can be simplified and the solution of the simplified equation should also be a simple harmonic function in time with a frequency $\omega$. Thus, we assume that the following solution form:

$$\eta = \xi_j e^{-i\omega t}, \quad j = 1, 2$$  \hspace{1cm} (3)

Figure 2. Sea level spectra for Ciutadella shelf (solid) and Palma Bay (dashed) (a) for a 4-day period of background oscillations and (b) for a 4-day period including a rissaga event. The periods (in minutes) of some significant peaks are indicated. (See Figure 1 for the positions of the instruments).
in which \( \xi_j \) satisfies the equation

\[
g_{r} \partial^2 \xi_j + \omega^2 \xi_j = \frac{h\rho P_0}{\rho} \nabla^2 (e^{i\kappa r})
\]

\[
= \frac{h\rho P_0}{\rho} \sum_{n=0}^{\infty} \varepsilon_n \nu^n k^2 J_n(kr) \cos n\theta
\]

where \((r, \theta)\) are the cylindrical coordinates, \(\varepsilon_n = 2\), for \(n = 1, 2, 3, \ldots\) and \(\varepsilon_0 = 1\) for \(n = 0\), \(J_n\) is the Bessel function of the first kind, of order \(n\). The solutions of the above equation take the following form:

\[
\xi_1 = \sum_{n=0}^{\infty} \varepsilon_n \nu^n \cos n\theta \left[ \frac{P_0}{\rho(c^2 - gh_1)} J_n(kr) + A_n J_n(kr) + B_n Y_n(kr) \right]
\]

\[
\xi_2 = \sum_{n=0}^{\infty} \varepsilon_n \nu^n \cos n\theta \left[ \frac{P_0}{\rho(c^2 - gh_2)} J_n(kr) + C_n H_n^{(1)}(kr) \right]
\]

in which \(c = \omega/k\) is the phase speed of the atmospheric pressure disturbance, \(Y_n\) is the Bessel function of second kind of order \(n\) and \(H_n^{(1)}\) denotes the Hankel function of the first kind of order \(n\). We remark here that the solution in region 2 has already satisfied the radiation boundary condition at infinity. The phase speed of long waves in the different regions is \(\sqrt{gh_j} = \omega/k, j = 1, 2\). It is obvious that the particular solution is singular when the phase speed of the atmospheric wave is the same as that of the long ocean wave, implying that the energy transfer from atmosphere to ocean is most efficient at this phase speed and the wave amplitude could grow in the \(r\)-direction. \(A_n, B_n, C_n\) are to be determined from the matching conditions.

[12] Solutions in region 1 and 2 are matched at the edge of the shelf, \(r = a\), by requiring that

\[
\xi_1 = \xi_2 \quad \text{at } r = a
\]

Furthermore, no-flux boundary condition along the perimeter of the island, \(r = b\), is required, i.e.,

\[
\frac{d\xi_1}{dr} = 0 \quad \text{at } r = b
\]

[13] Substituting (5) and (6) into (7) and (8) yields three equations for \(A_n, B_n, C_n\). Thus,

\[
A_n J_n(k_1 a) + B_n Y_n(k_1 a) - C_n H_n(k_2 a) = F_1
\]

\[
A_n J_n(k_1 a) + B_n Y_n(k_2 a) - C_n \frac{1}{\sqrt{\beta}} H_n^{(1)}(k_2 a) = F_2
\]

\[
A_n J_n(k_1 b) + B_n Y_n(k_2 b) = F_3
\]

in which \(H_n \equiv H_n^{(1)}\) and \(\beta \equiv h_1/h_2\) have been used and the prime denotes the differentiation with respect to the argument of the function. The right hand side terms of the above equations are known and given as

\[
F_1 = \frac{P_0}{\rho} \left( \frac{1}{c^2 - gh_2} - \frac{1}{c^2 - gh_1} \right) J_n(ka)
\]

\[
F_2 = \frac{P_0}{\rho} \left( \frac{1}{c^2 - gh_2} - \frac{1}{c^2 - gh_1} \right) J_n'(kb)
\]

\[
F_3 = -\frac{P_0}{\rho} \left( 1 - \frac{1}{\sqrt{\beta}} \right) J_n'(kb)
\]

[14] The system of linear algebraic equations, (9)–(11), can be readily solved. The analytical solution for \(A_n\) can be expressed as

\[
A_n = \frac{F_1}{\sqrt{\beta}} \frac{H_n'(ka)}{Y_n'(kb)} - \frac{F_2}{\sqrt{\beta}} \frac{J_n'(ka)}{Y_n'(kb)} \Delta_n^{-1} \frac{Y_n'(ka)}{H_n'(ka)}
\]

where

\[
\Delta_n = \frac{J_n(k_1 a)H_n'(ka)}{\sqrt{\beta}} - \frac{J_n'(k_1 a)H_n(ka)}{\sqrt{\beta}} - \frac{J_n'(k_1 b)Y_n'(ka)}{\sqrt{\beta}} - \frac{Y_n'(k_1 a)H_n'(ka)}{\sqrt{\beta}} - \frac{Y_n'(k_1 a)H_n(ka)}{\sqrt{\beta}}
\]

Once \(A_n\) is known, \(B_n, C_n\) can be found from (9)–(11).

[15] We remark here that in the absence of any forcing, i.e., \(P_0 = 0\), free waves exist at the discrete modes when \(\Delta_n = 0\). As pointed out by Longuet-Higgins [1967], the roots of \(\Delta_n = 0\) cannot be real, since there must be energy leakages into infinity; hence the free modes must decay in time. Because we are mostly interested in the shelf wave responses that are more or less uniform around the perimeter of the circular island, we will focus our attention only on the \(n = 0\) modes so that the free surface responses are independent of \(\theta\). In Table 1 we have presented the dimensionless frequencies and damping rates for the first four free wave modes with \(n = 0\). The following parameter values are used: \(\alpha = a/b = 4/3\) and \(\alpha = h_1/h_2 = 1/8\), which are the representative geometrical values for the Mallorca Island and surrounding shelf. The dimensionless frequency and the damping rate are defined as:

\[
k_1 a \equiv \omega a/\sqrt{gh_1} = \omega_1 + i\omega_2
\]

<table>
<thead>
<tr>
<th>(k_1 a)</th>
<th>(\omega_1)</th>
<th>(\omega_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.74</td>
<td>-126.47</td>
<td></td>
</tr>
<tr>
<td>5.21</td>
<td>-124.48</td>
<td></td>
</tr>
<tr>
<td>6.35</td>
<td>-113.7</td>
<td></td>
</tr>
<tr>
<td>8.16</td>
<td>-125.95</td>
<td></td>
</tr>
</tbody>
</table>

If the radius of the shelf is chosen as 50 km and the water depth on the shelf is 65 m, the wave period, \(T\) can be expressed in terms of \(\omega_1\) as \(T = 207/\omega_1\) (min). Thus, the first four free wave modes correspond to \(T = 76\) min, 40 min, 33 min, and 25 min, respectively. The e-folding times for these
times are in the order of magnitude of 10 min, except for
$T = 33$ min mode for which the e-folding time is about 3 hr.
Therefore, the 33 min mode is the expected dominant free
wave mode for $n = 0$.

[16] Due to the atmospheric pressure forcing, the shelf
wave responses depend on an additional parameter, namely
$\gamma = c/\sqrt{gh_1} = k/k_1$, where $k$ is the wave number of
the atmospheric pressure. In Figure 3, the normalized wave
amplitude along the island perimeter, $\xi_1(r = b; n = 0) ggh_1(\gamma - 1)/P_0$, is plotted against the dimensionless fre-
ness $k_1a$ with $\gamma$ as an additional parameter. Four different
response curves for $\gamma = 0.10, 0.50, 0.98$ and 1.50 are shown
for the range of $0 < k_1a < 15$. It is clear that the envelopes of
these response curves have peaks at around the dominating
free wave mode $k_1a \approx 6.35$, or $T \approx 33$ min. As the $\gamma$ value
becomes small ($\gamma = 0.10$ and 0.50 in Figure 3), due to the
non-dispersive characteristics in atmospheric pressure
waves, the range of $ka$ values is much larger than that of
$k_1a$, i.e., $ka = k_1a/\gamma$. Therefore, the frequency of oscillations
in the response curve is much higher for the smaller $\gamma$ value.
On the other hand, as $\gamma$ value becomes closer to unity (e.g.,
$\gamma = 0.98$ in Figure 3), the first three peaks in the response
curves occur at dimensionless frequencies $k_1a = 5.1, 7.7, 11.4$. The corresponding periods are 41 min, 27 min, and 18
min. It is interesting to point out that the maximum response
for the forced waves with $n = 0$ occurs at a shorter wave
period, $T = 27$ min than that for the free wave mode, $T = 33$ min.
This seems to be consistent with the field observations
shown in Figure 2. As the $\gamma$ value increases, (e.g., $\gamma = 1.50$ in
Figure 3), the peak frequencies shift upwards, $T = 32$ min,
17 min, etc. and the lowest frequency response becomes the
largest. These features do not agree with observations during
the rissaga events. We reiterate here that the normalized
wave amplitudes shown in Figure 3 have been multiplied by
a factor, $[\gamma^2 - 1]$. Therefore the dimensional wave amplitude
is much larger when $\gamma$ is near unity.

[17] In summary, based on the simple analytical model
for a circular island on a circular shelf, we can make the
following conclusions.

- Using the representative geometric parameters for
Malorca Island and its surrounding shelf, there is only one
possible dominant free wave mode that oscillates uni-
formly around the island. The order of magnitude of the
wave period is about 30 min.
- For the atmospheric pressure forced waves, the frequen-
cies of the peak responses shift according to the relative speed
of the atmospheric pressure waves to the celerity of the water
waves. Again, using the representative parameters for
Malorca Island, only when these two phase speeds are close
each other the frequency of oscillations in the response
curve agree with observations and the second peak becomes
higher than the first peak.
- The theoretical results agree with most of observations
shown in Figure 2, despite of the simplifications made in
shelf geometry and bathymetry. By changing slightly the
values of the parameters ($\alpha$ and $\beta$) does not alter the
essential features and conclusions made above.

[18] It is proposed that, based on the observations and the
analytical results, the two wave modes observed near
Ciutadella (32 min and 24 min) could be related to the
shelf surrounding the islands rather than to local bathymet-
ric characteristics in Ciutadella region.

---

**Figure 3.** The normalized wave amplitude along the
island perimeter, $\xi_1(r = b; n = 0) ggh_1(\gamma - 1)/P_0$, vs. the
dimensionless frequency $k_1a$ with $\gamma$ as an additional
parameter.

---

**References**

Garcia, M., D. Gomis, and S. Monserrat, Pressure-forced seiches
of large amplitude in inlets of the Balearic Islands, 2. Observational

Greenspan, H. P., The generation of edge waves by moving pressure

Gomis, D., S. Monserrat, and J. Tintori, Pressure-forced seiches of large
amplitude in inlets of the Balearic Islands, *J. Geophys. Res.*, 98, 14,437–

Liu, P. L.-F., S. Monserrat, M. Marcos, and A. B. Rabinovich, Coupling
between two inlets: Observation and modeling, *J. Geophys. Res.*, sub-
mitted, 2002.

Longuet-Higgins, M. S., On the trapping of wave energy round islands,

Marcos, M., S. Monserrat, R. Medina, and C. Vidal, Influence of the atmo-
spheric wave velocity in the coastal amplification of meteotsunamis,
*Underwater ground failures on tsunami generation, modelling, risk and

Monserrat, S., A. Ibbetson, and A. J. Thorpe, Atmospheric gravity waves and

Monserrat, S., and A. J. Thorpe, Gravity-wave observations using an array
of microbarographs in the Balearic Islands, *Q. J. Roy. Meteor. Soc.*, 118,

Monserrat, S., A. B. Rabinovich, and B. Casas, On the reconstruction of the
transfer function for atmospherically generated seiches, *Geophysical Res.

Rabinovich, A. B., and S. Monserrat, Generation of meteorological tsunami
(large amplitude seiches) near the Balearic and Kuril islands, *Natural

---

P. L.-F. Liu, School of Civil and Environmental Engineering, Hollister Hall,
Cornell University, Ithaca, New York 14853, USA. (pl13@cornell.edu)

S. Monserrat and M. Marcos, Departament de Fisica, Universitat de les
Illes Balears, E-07071 Palma de Mallorca, Balears, Spain. (s.monserrat@uib.es)