Hierarchical Temporal Multi-Layer Decentralised MPC Strategy for Drinking Water Networks: Application to the Barcelona Case Study

V. Puig, C. Ocampo-Martinez and S. Montes de Oca

***Abstract*— In this paper, a hierarchical temporal multi-layer decentralised model predictive control (HTML-DMPC) approach for drinking water networks (DWN) is proposed. The upper temporal layer works with a daily scale and is in charge of achieving the global objectives, which correspond with the optimal selection of the sources and the path to the reservoirs. On the other hand, the lower temporal layer is in charge of manipulating the set-point of the actuators to satisfy the local objectives, i.e., the minimisation of the energy needed for pumping water to the reservoirs. The system decomposition is based on graph partitioning theory, which considers the graph representation of the DWN topology. The obtained system decomposition allows to establish a hierarchical flow of information between the MPC controllers. Hence, the proposed DMPC strategy results in a hierarchical-like scheme. Results obtained when used selected simulation scenarios show the effectiveness of the proposed control strategy in terms of system modularity, reduced computational burden and, at the same time, the admissible loss of performance in contrast to a centralised MPC (CMPC) strategy.**

I. Introduction

Drinking water management is a subject of increasing interest because of its social, economic and environmental impact. The most important issues include the sustainable use of limited resources and the reliability of service to consumers with adequate quality and pressure levels. Expected climatic change over the current century is predicted to manifest itself regionally through changes in water availability. Such changes will have direct consequences through impacts on the availability and quality of water in the water cycle. Optimal management strategies are required to reduce the vulnerability of urban water systems (such as drinking water networks — DWN —) to climatic variability and change.

The leading control technique for the management of large-scale systems such as DWNs is model predictive control (MPC) [1], [2], [3]. The success of MPC is due to its ability to handle several dynamically- coupled, manipulated and controlled variables (up to several hundreds) and constraints on them. Indeed, constraints handling (e.g., related to physical limits) are almost impossible to tackle by traditional frameworks based on $\mathscr{H}_2$ or $\mathscr{H}_\infty$ optimisation. Since MPC directly embodies technical specifications (model, performance, limits) into the control algorithm, no a-posteriori patches (like antiwindup) are required to take into account limitations on the system variables. Traditional MPC procedures assume that all available information is centralised. In fact, a global dynamical model of the system must be available for control design. Moreover, all measurements must be collected in one location to estimate all states and compute all control actions.

When considering large-scale systems, these assumptions usually fail to hold, either because gathering all measurements in one location is not feasible, or because a centralised high-performance computing unit is not available. This fact can be collected as the absence of scalability. Subsequently, a model change would require re-tuning the centralised controller. It is obvious that the cost of setting up and maintaining the monolithic solution of the control problem is prohibitive. A way of circumventing these issues might be by looking into *decentralised* MPC (DMPC) or *distributed* MPC techniques, where networked local MPC controllers are in charge of the control of part of the entire system. Thus, DMPC has became in one of the hottest topics in control during the early 21st century, opening the door to the research towards solving new open issues and related problems of the strategy. Few works have been recently published in this area; see, e.g., [4], [5], [6], [7], [8], among others. Classical theory of observability, robust feasibility, partitioning, among others, should be reviewed in order to establish new paradigms for DMPC and large-scale systems.

The main contribution of this paper consists in presenting the application of a hierachical temporal multi-layer DMPC (HTML-DMPC) approach to the Barcelona DWN. The aim is to show that this approach reduces the computational burden with respect to the centralised counterpart and reduces the level of suboptimality of the system performance with respect to a pure hierarchical DMPC approach presented in [9]. Moreover, important features such as the system modularity are presented in a decentralised scheme. The advantage of this hierarchical-like DMPC approach is the simplicity of its implementation given the absence of negotiations among controllers. To apply the proposed DMPC approach, the network is decomposed into subsystems using a novel automatic decomposition algorithm reported in [9], which is based on graph partitioning.

The paper is structured as follows: Section II describes the case study considered in the paper. Section III introduces the HTML-DMPC and the hierarchical-like DMPC strategy applied to the case study, both seen as the techniques used for each temporal layer. Section IV discusses the main results derived from the application of the proposed control strategy. Finally, conclusions and directions for further research are reported in Section V.

This work has been funded by the Spanish Ministry of Science and Technology through the WATMAN project, Ref. DPI2009-13744.

V. Puig, C. Ocampo-Martinez and S Montes de Oca are with Advanced Control Systems Group (SAC) at Institut de Robòtica i Informàtica Industrial (CSIC-UPC), Llorens i Artigas, 4-6, 08028 Barcelona, Spain. e-mail: {vpuig,cocampo}@iri.upc.edu

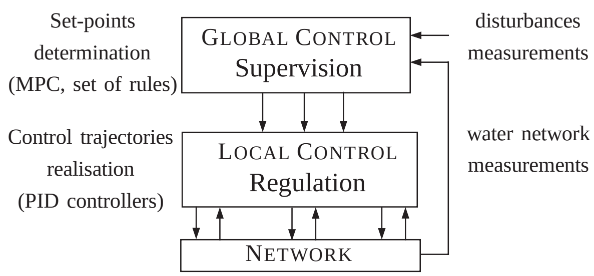

Fig. 1. Hierarchical structure for a water transport network

II. CASE-STUDY DESCRIPTION

A. System Description

The Barcelona DWN, managed by Aguas de Barcelona, S.A. (AGBAR), not only supplies drinking water to Barcelona city but also to the metropolitan area. The sources of water are the Ter and Llobregat rivers, which are regulated at their head by some dams with an overall capacity of 600 cubic hectometres. Currently, there are four drinking water treatment plants (WTP) and several underground sources (wells) that can provide water through pumping stations. Those different water sources currently provide a flow of around 7 m^3/s. The water flow from each source is limited, what implies different water prices depending on water treatments and legal extraction canons.

The Barcelona DWN is structurally organised in two layers. The upper layer, named as *transport network*, links the water treatment plants with the reservoirs distributed all over the city. The lower layer, named *distribution network* is sectorised in subnetworks. Each subnetwork links a reservoir with each consumer. This paper is focused on the transport network. Thus, each subnetwork of the distribution network is modelled as a demand sector. The demand of each sector is characterised by a demand pattern, which can be predicted by using a time-series model [10]. The control system of the transport network is also organised in two layers (see Figure 1). The upper layer is in charge of the global control of the network, establishing the set-points of the regulatory controllers at the lower layer. Regulatory controllers are of PID type, while the supervisory layer controller is of MPC type. Regulatory controllers hide the network non-linear behaviour to the supervisory controller. This fact allows the MPC supervisory controller to use a control-oriented linear model.

B. System Management Criteria

AGBAR has provided the management policies for the Barcelona DWN, given their knowledge of the system. These management criteria are briefly explained below. More details can be found in [11].

1) Minimising water production and transport costs: The main economic costs associated with drinking water production (treatment) are due to chemicals, legal canons, and electricity costs. The corresponding performance index to be minimised is expressed as

$$f_1(t) = (\alpha_1 + \alpha_2(t))\, u(t), \qquad (1)$$

where u denotes the manipulated flows through the system actuators, α_1 corresponds to a known vector related to the economic cost of the water according to the source (treatment plant, dwell, etc.), and $\alpha_2(t)$ is associated with the economic cost of the flow through certain actuators (pumps only) and their control cost (pumping), and varies with time since pumping efforts have different values according to the time of the day (electricity costs). Variable t denotes the discrete time.

2) Safety storage term: The satisfaction of water demands should be fulfilled at every time instant with some degree of safely given by water availability. A quadratic expression for this concept is written as

$$f_2(t) = (x(t) - \beta\ x^{\max})^T (x(t) - \beta\ x^{\max}), \qquad (2)$$

where x denotes the water volumes at network tanks and β is a term which determines the safety volume to be considered for the control law computation.

3) Smoothness of control actions: To smooth out the control action of MPC, the expression

$$f_3(t) = \Delta u(t)^T \Delta u(t) \qquad (3)$$

penalises variations $\Delta u(t) = u(t) - u(t-1)$ of the control signal between consecutive sampling intervals.

C. Control-oriented Modelling

Control-oriented modelling principles for DWNs have been widely presented in the literature, see [1], [11]. In order to obtain a control-oriented model of the DWN, the constitutive network elements as well as their basic relationships should be discussed. The reader is referred to the aforementioned references for further details of DWN modelling and specific insights related to the case study of this paper.

In general terms and according to [11], [9], among others, the the control-oriented flow-based model of a DWN in discrete-time state-space form can be written as

$$x(t+1) = Ax(t) + \Gamma\, \upsilon(t), \qquad (4a)$$

$$E_1\, \upsilon(t) = E_2, \qquad (4b)$$

where $x \in \mathbb{R}^n$ is the state vector corresponding to the water volumes of the n tanks, $\Gamma = [B \quad B_p]$, and $\upsilon(t) = [u(t)^T \quad d(t)^T]^T$. In turn, $u \in \mathbb{R}^m$ represents the vector of manipulated flows through the m actuators (pumps and valves), and $d \in \mathbb{R}^p$ corresponds to the vector of the p water demands (sectors of consume). Moreover, A, B, B_p, E_1, and E_2 are matrices of suitable dimensions dictated by the network topology. This model is complemented by the main physical constraints of the DWN given by the variables related to the tank volumes and manipulated flows. In the case of tank volumes, the physical constraint related to the range of volume capacities for the i-th tank is expressed as

$$x_i^{\min} \leq x_i(t) \leq x_i^{\max}, \quad \forall t, \qquad (5)$$

where $x_i^{\min}$ and $x_i^{\max}$ denote the minimum and maximum volume capacity, respectively, given in m^3. On the other hand, physical constraints related to manipulated flows through the system actuators are expressed as

$$u_i^{\min} \leq u_i(t) \leq u_i^{\max}, \quad \forall t, \tag{6}$$

where $u_i^{\min}$ and $u_i^{\max}$ denote the minimum and the maximum flow capacity, respectively, given in m^3/s.

The Barcelona DWN model (4) contains a total amount of 67 tanks and 121 actuators, these latter divided in 46 pumps and 75 valves. Moreover, the network has 88 demand sectors and 16 water nodes. Both the demand episodes and the network calibration/simulation setup are provided by AGBAR. Figure 6 (further below) depicts the considered network.

III. HTML-DMPC Approach

A. Description of the approach

Figure 2 presents two control levels. The first control level is denoted as *Supervisory Control Level*, establishing the set-points of the regulatory controllers at the lower control level (*Regulatory Control Level*). Regulatory controllers are of PI or PID type, while the controllers of the supervisory control level are of MPC type.

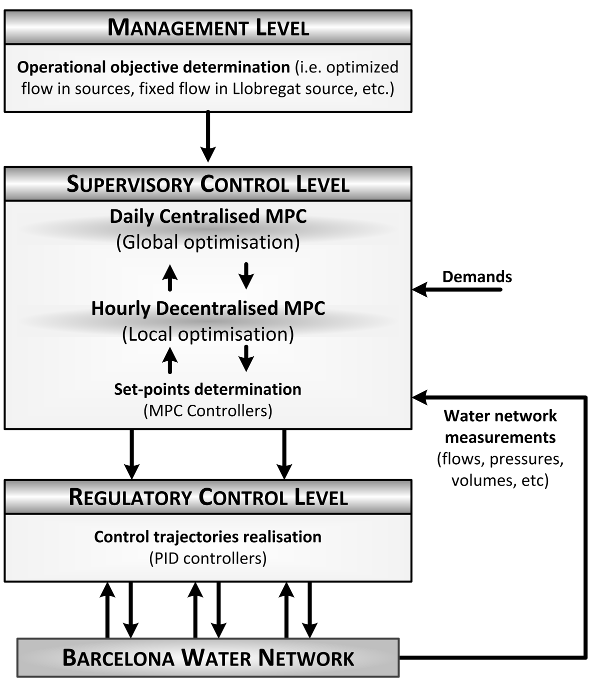

Fig. 2. HTML-DMPC structure

The *Supervisory Control Level* of the hierarchical structure is also divided in two control layers that are characterized by operating at different time scales:

- *Daily Centralized MPC Control*: This centralised (or global) optimisation is carried out at daily time scale to coordinate the subsystems.
- *Hourly Decentralized MPC Control*: This decentralized (or local) optimisation for each subsystem operates at hourly time scale.

The motivation for the HTML-DMPC approach comes from the results obtained just using a hierarchical DMPC approach [9]. Analysing these results, it was noticed an increment of the total costs of operation when using the hierarchical DMPC strategy with respect to a CMPC strategy. This loss of performance can be explained because the DMPC strategy does not take into account in a proper way the water costs related to external water sources since it is a global objective. On the other hand, DMPC controllers are mainly focused on the reduction of pumping costs (local objective) within each subsystem. By contrast, the information of water costs is properly managed for the CMPC controller by optimising it but at the price of *moving* more water inside the network. This leads to an increment in the electric costs (the water transportation cost) when CMPC controller is used. Therefore, in order to enforce the global objective, the economical unitary cost of the shared variables that act as sources is calculated by the daily optimisation in order to fulfill the global objective. The daily optimisation determines this price by finding the optimal paths from all water sources taking into account the flow capacity and unitary cost in each point of the network.

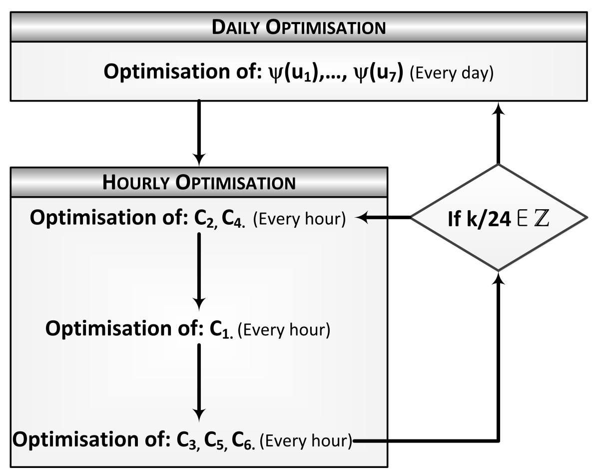

Fig. 3. Daily and Hourly optimisation of HTML-DMPC

Figure 3 summarizes the interplay between the daily and hourly optimisations in the HTML-DMPC approach. At the daily level, the unitary costs of the shared links between subsystems, denoted by $\psi(u_j)$ for $j = 1, \ldots, m$, are determined. These costs are used by the DMPC controllers at the hourly optimisation level. In the figure, notation C_i for $i \in \{1, \ldots, 6\}$ corresponds to the MPC controllers fo each partition. The way of designing them and the corresponding approach is explaned in Section III.C.

B. Daily layer

The sector demands of a DWN presents a 24-hour periodic behaviour are shown in Figure 4.

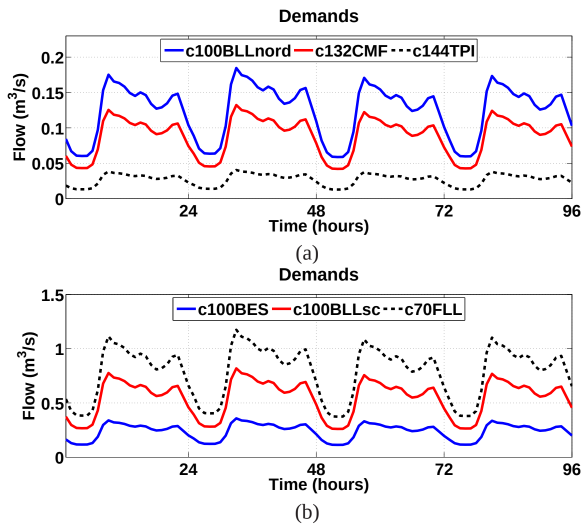

Fig. 4. Demands evolution of the Barcelona DWN

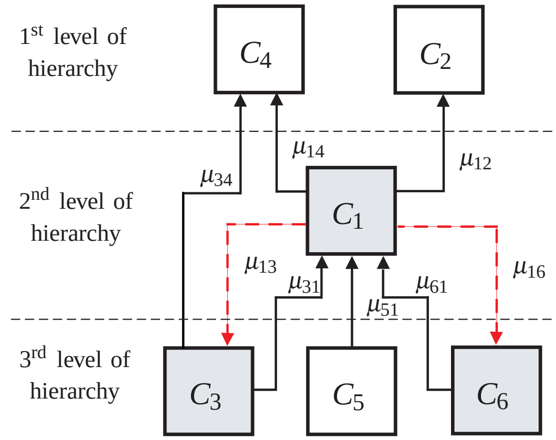

Fig. 5. Hierarchy of MPC controllers C_i. Their solution sequence is top-down

Thus, when looking at the tank volume evolution serving a demand sector using a daily sampling rate, it can be noted that volumes are almost the same. The reason is that the water volume presents the same repetitive evolution than the demands and, at the end of the day, the volume reaches its minimum (safety volume). For this reason, when modelling the network at daily level, it can be assumed that volumes do not change, i.e., the tank evolution can be reduced to

$$x(t+1) = x(t). \tag{7}$$

Therefore, tanks behave as nodes and the DWN can be represented by a static model. Regarding the control objectives at daily level, the only operational goal (of the three goals considered in Section II-B in hourly scale) that makes sense is the economic cost. It is quite important to highlight that, even this layer considers an optimisation problem coming from a centralised MPC, the static nature of the model makes the problem tractable in the sense of computational burden. Therefore, the optimisation problem of this layer does not correspond with the CMPc problem.

C. Hourly layer

Using the Barcelona DWN decomposition obtained by means of the partitioning algorithm presented in [9], a DMPC strategy can be implemented in order to manage the whole network. This DMPC strategy considers

- the dynamic system model (4) split in subsystems;
- the physical constraints (5)-(6) for each subsystem;
- a demand forecasting algorithm (taken from [11], [10]); and
- a multi-objective cost function, expressed by using (1), (2), and (3) as

$$J = \gamma_1 \sum_{i=0}^{H_u-1} f_1(t+i|t) + \gamma_2 \sum_{i=1}^{H_p} f_2(t+i|t) + \gamma_3 \sum_{i=0}^{H_u-1} f_3(t+i|t), \tag{8}$$

where H_p and H_u correspond to the prediction and control horizons, respectively, index t represents the current time instant while index i represents the predicted time along H_p. In this paper, the prediction horizon is related to the 24-hours demand seasonality. Moreover, $H_u = H_p$, following the criterion of the DWN management company. γ_i denote the weighting factors used for giving priority to each control objective [12].

For completeness reasons, a brief description of the DMPC strategy proposed in [9] is presented in this paper. The authors encourage the reader to get through this reference for the full comprehension of the approach. The proposed DMPC methodology is based on the consideration of a hierarchical-like topology given by the existence of bidirectional flow of information between DMPC controllers (see Figure 5). This scheme is therefore different to those proposed by [13], where the *pure* hierarchical control scheme determines a sequence of information distribution among the subsystems, where top-down communication is available from upper to lower level of the hierarchy and the unidirectionality of the information flow between controllers is stated.

Denoting C_i as the MPC controller related to the subsystem S_i (for $i \in \{1,\dots,6\}$), and μ_{ij} as the set of control actions u (manipulated flows) going from S_i to S_j (for $j \in \{1,\dots,6\}$, $i \neq j$)[1], the solution sequence of the described hierarchical-like control problem for the complete Barcelona DWN at each iteration $t \in \mathbb{Z}_{\geq 1}$ is the following:

- C_4 computes the control actions of S_4 and sets μ_{14} and μ_{34}.
- In parallel, C_2 computes the control actions of S_2 and the set μ_{12}.
- C_1 computes the control actions of S_1 and sets μ_{31}, μ_{51}, and μ_{61}. Sets μ_{12}, μ_{13}, μ_{14}, and μ_{16} are considered as

[1]Notice that μ_{ij} not only contains values of each component at time t but also all values over H_u, i.e., if $\mu_{ij} = \{u_a, u_b, \dots\}$.

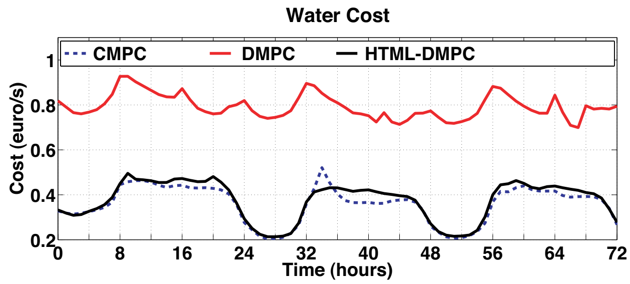

Fig. 7. Water cost of the three MPC strategies

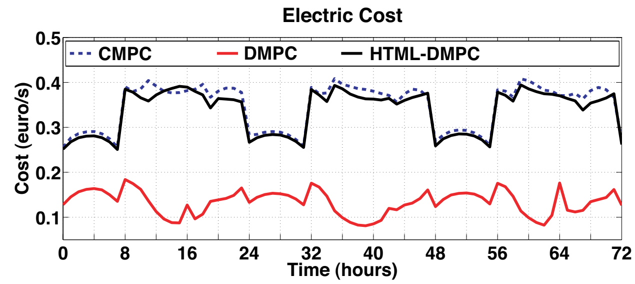

Fig. 8. Electric cost of the three MPC strategies

sets of *virtual demands*[2] within the controller C_1.

- C_5 computes the control actions of S_5 considering μ_{51} as a set of virtual demands.
- C_3 computes the control actions of S_3 considering μ_{31} and μ_{34} as sets of virtual demands. C_3 also computes the set μ_{13} to be used as a set of virtual demands for C_1 at iteration $t+1$.
- C_6 computes the control actions of S_6 considering μ_{61} as a set of virtual demands. C_6 also computes μ_{16} to be virtual demands for C_1 at iteration $t+1$.

IV. Results

This section presents the results of the application of the proposed HTML-DMPC approach using the partitioned Barcelona DWN according to [9].

The results obtained by using the proposed control strategy are compared with those obtained employing a CMPC approach and DMPC strategy without the multi-layer scheme proposed in [9]. The results are obtained for 72 hours (July 24-26, 2007). The weights of the cost function (8) are $\gamma_1 = 100$, $\gamma_2 = 10$ and $\gamma_3 = 0.005$, which represents the weights associated to the normalized functions (1)-(3). The tuning of these parameters has been chosen in a way that highest priority objective is the economic cost, which should be minimized while maintaining adequate levels of safety volume and control action smoothness. The hourly layer uses the same control and prediction horizons $H_p = H_u = 24$.

Table I summarizes the obtained control results in terms of economic cost. For each MPC approach, the water, electric and total cost is detailed. Figure 8 presents the evolution electric cost along the three days. Finally, comparing the total costs, the HTML-DMPC presents similar results than the CMPC and the total cost of DMPC approach is higher of about 30% with respect to the CMPC.

TABLE I
Performance comparisons

Index	CMPC	DMPC	HTML-DMPC
Water Cost	93.01	205.55	97.11
Electric Cost	90.31	34.58	87.53
Total Cost	183.33	240.13	184.65

[2]See [9] for all the detailed definitions and concepts related to the DMPC strategy.

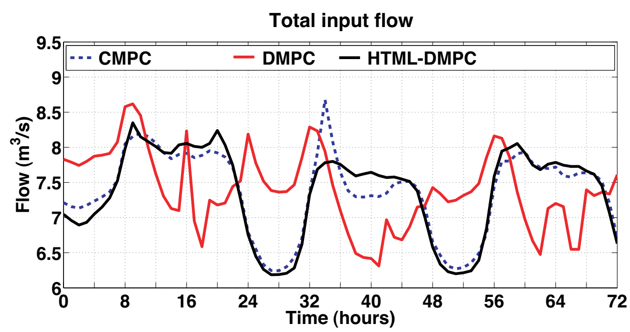

Fig. 9. Total flow of Barcelona DWN

Figure 9 shows the total water inflow of the Barcelona DWN. This inflow for the CMPC approach and HTML-DMPC approach presents similar behaviour. In contrast, the HTML-DMPC takes the water with a greater flow from Abrera and Ter sources. This is the reason of the high water cost of DMPC presented in Table I. These results confirms the need for the daily CMPC optimisation.

Figure 10 shows the volume in one of the key tanks and the behaviour of the volume follows the demand evolution. It can be noticed that the HTML-DMPC approach presents a behaviour closer to the CMPC than the DMPC from [9].

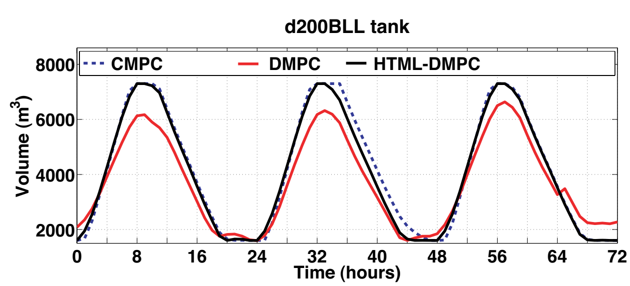

Fig. 10. Volume behaviour of d200BLL tank

Finally, Figure 11 presents the evolution of one the key pumps. The behaviour of this pump follows the demand evolution. This element is located in a critical area of the Barcelona DWN because it is one of the main routes of the Llobregat source. Notice that although the similar behaviour of the flow with respect to the considered approach, the global performance (costs in water at source and electricity)

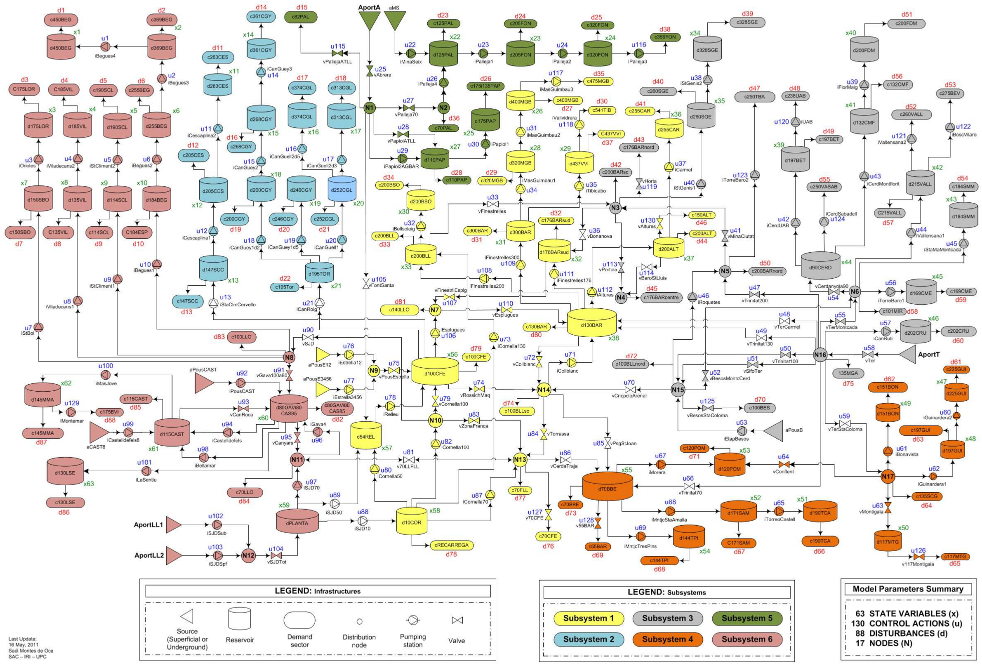

Fig. 6. Partition of the Barcelona DWN

is the desired one.

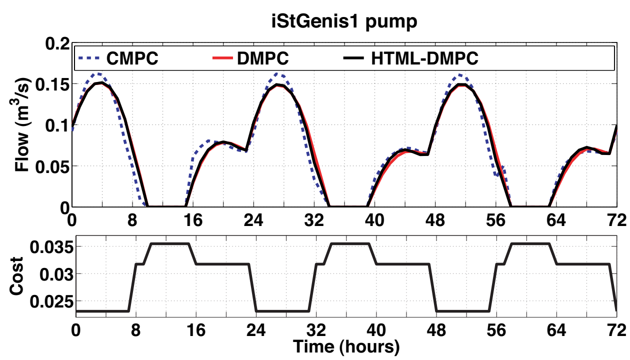

Fig. 11. Flow Behavior of iSJD70 pump of Barcelona DWN

V. Conclusions

This paper has proposed a hierarchical temporal multi-layer DMPC approach for DWN. The upper temporal layer works with a daily time scale and it is in charge of achieving the global control (optimal water source selection). On the other hand, the lower temporal layer is in charge of manipulating the set-point of the actuators to satisfy the local objectives (electric cost minimisation). The system decomposition is based on graph partitioning theory. Results obtained in selected simulation scenarios has shown the effectiveness of the control strategy in terms of system modularity, reduced computational burden and, at the same time, the very loss of performance in contrast to a CMPC strategy and a hierarchical-like DMPC strategy previously presented by the authors. Future work is focused on the formalisation of the proposed approach in terms of feasibility, robustness and stability, and its generalisation to large-scale systems.

References

[1] M. Brdys and B. Ulanicki, *Operational Control of Water Systems: Structures, algorithms and applications*. UK: Prentice Hall International, 1994.

[2] M. Marinaki and M. Papageorgiou, *Optimal Real-time Control of Sewer Networks*. Secaucus, NJ (USA): Springer, 2005.

[3] P. V. Overloop, *Model Predictive Control on Open Water Systems*. Delft, The Netherlands: Delft University Press, 2006.

[4] T. Keviczky, F. Borrelli, and G. Balas, "Decentralized receding horizon control for large scale dynamically decoupled systems," *Automatica*, vol. 42, no. 12, pp. 2105–2115, December 2006.

[5] J. B. Rawlings and B. T. Stewart, "Coordinating multiple optimization-based controllers: New opportunities and challanges," *Journal of Process Control*, vol. 18, no. 9, pp. 839–845, October 2008.

[6] R. Negenborn, B. De Schutter, and J. Hellendoorn, "Multi-agent model predictive control for transportation networks: Serial vs. parallel schemes," *Engineering Applications of Artificial Intelligence*, vol. 21, no. 3, pp. 353–366, April 2008.

[7] A. N. Venkat, I. A. Hiskens, J. B. Rawlings, and S. J. Wright, "Distributed MPC strategies with application to power system automatic generation control," *IEEE Tran on Control Systems Technology*, vol. 16, no. 6, pp. 1192–1206, November 2008.

[8] R. Scattolini, "Architectures for distributed and hierarchical Model Predictive Control: A review," *Journal of Process Control*, vol. 19, no. 5, pp. 723–731, May 2009.

[9] C. Ocampo-Martinez, S. Bovo, and V. Puig, "Partitioning approach oriented to the decentralised predictive control of large-scale systems," *Journal of Process Control*, vol. 21, no. 5, pp. 775–786, 2011.

[10] J. Quevedo, V. Puig, G. Cembrano, and J. Blanch, "Validation and reconstruction of flow meter data in the Barcelona water distribution network," *Control Engineering Practice*, vol. 11, no. 6, pp. 640–651, June 2010.

[11] C. Ocampo-Martinez, V. Puig, G. Cembrano, R. Creus, and M. Minoves, "Improving water management efficiency by using optimization-based control strategies: the Barcelona case study," *Water Science & Technology: Water supply*, vol. 9, no. 5, pp. 565–575, 2009.

[12] J. Maciejowski, *Predictive Control with Constraints*. Great Britain: Prentice Hall, 2002.

[13] D. Šiljak, *Decentralized control of complex systems*. Academic Press, 1991.