Misner-brane cosmology

Pedro F. González-Díaz.
Isaac Newton Institute, 20 Clarkson Road, Cambridge, CB3 0EH, UK
and
Instituto de Matemáticas y Física Fundamental
Consejo Superior de Investigaciones Científicas
Serrano 121, 28006 Madrid, SPAIN

June 1, 2000

Abstract

A brane universe derived from the Randall-Sundrum models is considered in which an additional Misner-like periodicity is introduced in the extra direction. This model solves the ambiguity in the choice of the brane world by identifying the branes with opposite tensions, in such a way that if one enters the brane with positive tension, one finds oneself emerging from the brane with negative tension, without having experienced any tension. We show that the cosmological evolution resulting from this model matches that of the standard Friedmann scenario, at least in the radiation dominated era, and that there exist closed timelike curves only in the bulk, but not in the branes which are chronologically protected from causality violations by quantum-mechanically stable chronology horizons.

In his public lecture *Space and time warps* Stephen Hawking has recently pointed out [1] that string theory might now be offering a new room for possible violation of the chronology protection conjecture [2], coming out from some special mixing between our four flat directions of space and time and the extra highly curved or warped directions considered in string theories, so traveling superluminarly or back in time cannot be ruled out yet. The present letter describes a string-theory inspired cosmological scenario where the above issue can be addressed properly. More precisely, we shall consider a brane world derived from the Randall-Sundrum models (originally intended to solve the hierarchy problem [3,4]) by introducing a periodicity on the extra direction which solves any ambiguity in the choice of the brane world and induces the emergence of nonchronal regions.

Let us start with a five-dimensional spacetime with the fifth dimension, ω, compactified on S^1, with $-\omega_c \leq \omega \leq \omega_c$, and satisfying the orbifold symmetry $\omega \leftrightarrow -\omega$. On the fifth direction there are two domain walls, with the brane at $\omega = 0$ having positive tension and that at $\omega = \omega_c$ having negative tension. A first model assumed [3] that we live in the negative-tension brane where the mass scales are severely suppressed, but had the serious problem of a repulsive gravity [5]. A second model assumed [4] that we are living in the positive-tension brane, while the other brane was moved off to infinity. This avoids any repulsive character of gravity, but unfortunately leads to field equations on the brane at $\omega = 0$ which are nonlinear in the source terms [6]. In spite of the several attempts made to reconcile this non coventional behaviour with the standard Friedmann scenario based on inserting a cosmological constant in the brane universe [7-10], within the spirit of the Randall-Sundrum approach [3,4], it appears clear that such a situation is rather unconfortable in a number of respects, not less of which is the fact that in any classical gravitational theory where isotropy and homogeneity are assumed, one should expect standard cosmology to hold.

In order to represent the universe we live in, our approach chooses neither of the two branes on ω individually, but both of them simultaneously; that is to say, we shall provide the fifth direction with a periodic character, in such a way that the branes at $\omega = 0$ and $\omega = \omega_c$ are identified with each other, so if one enters the brane at $\omega = 0$, one finds oneself emerging from the brane at $\omega = \omega_c$, without having experienced any tension. If we then set the brane at $\omega = 0$ into motion toward the brane at $\omega = \omega_c$ with a given speed v, in units of the speed of light, our space would resemble five-dimensional Misner space, the differences being in the spatial topology and in the definition of time and the closed-up extra direction which would also contract at a rate v. Then, time dilation between the two branes would inexorably lead to the creation of a nonchronal region which will start forming at the future of a given chronology horizon. We shall first consider the metric of the five-dimensional spacetime in terms of Gaussian coordinates centered e.g. on the brane at $\omega = 0$. If we assume the three spatial sections on the branes to be flat, then such a metric can be written in the form [11]

$$ds^2 = c^2(\omega,t)\left(d\omega^2 - dt^2\right) + a^2(\omega,t)\sum_{j=2}^{4} dx_j^2, \tag{1}$$

where if we impose the orbifold condition $\omega \leftrightarrow -\omega$, the scale factors c and a are given by

$$c^2(\omega,t) = \frac{\dot{f}(u)\dot{g}(v)}{[f(u)+g(v)]^{\frac{2}{3}}}, \quad a^2(\omega,t) = [f(u)+g(v)]^{\frac{2}{3}}\,, \tag{2}$$

with $u = t - |\omega|$ and $v = t + |\omega|$ the retarded and advanced coordinates satisfying the orbifold symmetry, where we have absorbed some length constants into the definition of t and ω, and the overhead dot denotes derivative with respect to time t. If no further symmetries are introduced then $f(u)$ and $g(v)$ are arbitrary functions of u and v, respectively [11] . However, taking metric (1) to also satisfy the (Misner) symmetry [12,13]

$$\begin{gathered}(t,\omega,x_2,x_3,x_4) \leftrightarrow \\ (t\cosh(n\omega_c) + \omega\sinh(n\omega_c), t\sinh(n\omega_c) + \omega\cosh(n\omega_c), \\ x_2, x_3, x_4)\,,\end{gathered} \tag{3}$$

where n is any integer number, makes the functions $f(u)$ and $g(v)$ no longer arbitrary. Invariance of metric (1) under symmetry (3) can be achieved if we choose for $f(u)$ and $g(v)$ e.g. the simple expressions

$$f(u) = \ln u, \quad g(v) = \ln v. \tag{4}$$

Imposing symmetry (3) together with the choice for the scale factors given by expressions (4) fixes the topology of the five-manifold to correspond to the identification of the domain walls at $\omega = 0$ and at $\omega = \omega_c$ with each other, so that if one enters one of these branes then one finds oneself emerging from the other. Although other possible, perhaps more complicate choices for the functions $f(u)$ and $g(v)$ could also be done in order to achieve fulfillment of the above symmetry, in what follows we shall restrict ourselves to use Eqs. (4) to define our brane universe model satisfying the Misner like symmetry (3), and denote it as the Misner-brane universe. We shall show that the choice (4) actually corresponds to an early universe which is radiation dominated.

The periodicity property on the extra direction can best be explicited by introducing the coordinate transformation

$$\omega = T\sinh(W), \quad t = T\cosh(W), \tag{5}$$

with which metric (1) becomes

$$ds^2 = \frac{\left(\frac{\dot{T}^2}{T^2} - \dot{W}^2\right)}{\ln^{\frac{2}{3}} T^2}\left(T^2 dW^2 - dT^2\right) + \ln^{\frac{2}{3}} T^2 \sum_{j=2}^{4} dx_j^2. \tag{6}$$

Although now metric (6) and the new coordinate $T = \sqrt{t^2-\omega^2}$ (which is timelike, provided that $\ln T \geq \text{Const} \pm W$) are both invariant under symmetry (3), the new extra coordinate W transforms as

$$W \equiv \frac{1}{2}\ln\left(\frac{t+|\omega|}{t-|\omega|}\right) \leftrightarrow W + n\omega_c \tag{7}$$

under that symmetry. On the two identified branes making up the Misner-brane universe, we can describe the four-dimensional spacetime by a metric which can be obtained by slicing the five-dimensional spacetime given by metric (6), along surfaces of constant W, i.e.

$$ds^2 = -\frac{\dot{T}^2}{T^2 \ln^{\frac{2}{3}} T^2} dT^2 + \ln^{\frac{2}{3}} T^2 \sum_{j=2}^{4} dx_j^2. \qquad (8)$$

The energy-momentum tensor of this brane universe will now have the form:

$$T_i^k = \frac{\delta(\omega - n\omega_c)}{c_b} \mathrm{diag}\left(-\rho, p, p, p, 0\right), \quad n = 0, 1, 2, 3..., \qquad (9)$$

where $c_b \equiv (t, \omega = n\omega_c)$. This tensor should be derived using the Israel's jump conditions [14] that follow from the Einstein equations. Using the conditions computed by Binétruy, Deffayet and Langlois [6] and the metric (6) we then [11] obtain for the energy density and pressure of our Misner-brane universe:

$$\rho = -\frac{4T\dot{W}}{\kappa_{(5)}^2 \ln^{\frac{2}{3}} T^2 \left(|\dot{T}^2 - T^2\dot{W}^2|\right)^{\frac{1}{2}}} \qquad (10)$$

$$p = \frac{2T\dot{T}^2 \ln^{\frac{2}{3}} T^2}{\kappa_{(5)}^2 \left(|\dot{T}^2 - T^2\dot{W}^2|\right)^{\frac{5}{2}}} \frac{d}{dt}\left(\frac{T\dot{W}}{\dot{T}}\right) - \frac{1}{3}\rho. \qquad (11)$$

Thus, both the energy density ρ and the pressure p, defined by expressions (10) and (11), respectively, identically vanish on the sections W=const. Therefore, taking the jump of the component (ω, ω) of the Einstein equations with the orbifold symmetry [6], one gets on the identified branes

$$\frac{\dot{a}_b^2}{a_b^2} + \frac{\ddot{a}_b}{a_b} = \frac{\dot{a}_b \dot{c}_b}{a_b c_b}, \qquad (12)$$

where $a_b \equiv a(t, \omega = n\omega_c)$, with $n = 0, 1, 2, 3...$, is the scale factor in our Misner-brane universe.

The breakdown of arbitrariness of functions $f(u)$ and $g(v)$ imposed by symmetry (3) prevents the quantity c_b to be a constant normalizable to unity, so the right-hand-side of Eq. (12) can be expressed in terms of coordinates T, W as:

$$\frac{\dot{a}_b \dot{c}_b}{a_b c_b} = -\frac{1 + \frac{1}{3\ln T}}{3T^2 \cosh^2 W \ln T}. \qquad (13)$$

A simple dimensional analysis (performed after restoring the constants absorbed in the definitions of t and ω in Eqs. (2)) on the right-hand-side of Eq. (13) indicates that if this side is taken to play the role of the source term of the corresponding Friedmann equation, then it must be either quadratic in the energy density if we use $\kappa_{(5)}^2 = M_{(5)}^{-3}$ (with $M_{(5)}$ the five-dimensional reduced Planck mass) as the gravitational coupling, or linear in the energy density and pressure if we use $\kappa_{(4)}^2 = 8\pi G_N = M_{(4)}^{-2}$ (with $M_{(4)}$

the usual four-dimensional reduced Planck mass) as the gravitational coupling. Since $\kappa_{(4)}^2$ should be the gravitational coupling that enters the (Friedmann-) description of our observable four-dimensional universe, we must choose the quantity in the right-hand-side of Eq. (13) to represent the combination $-\kappa_{(4)}^2(\rho_b + 3p_b)/6$ which should be associated with the geometrical left-hand-side part of Eq. (12) of the corresponding Friedmann equation, when the term proportional to the bulk energy-momentum tensor $T_{\omega\omega}$ is dropped by taking the bulk to be empty (see later on). We have then,

$$\rho_b + 3p_b = \frac{2\left(1 + \frac{1}{3\ln T}\right)}{\kappa_{(4)}^2 T^2 \cosh^2 W \ln T}. \tag{14}$$

The four-dimensional metric (8) can be expressed as that of a homogeneous and isotropic universe with flat spatial geometry, $ds^2 = -d\eta^2 + a(\eta)_b^2 \sum_{j=2}^4 dx_j^2$, if we take for the cosmological time $\eta = 3a(\eta)_b^2/(4\cosh W) = 3\ln^{2/3} T^2/(4\cosh W)$. In this case, the scale factor $a(\eta)_b$ corresponds to that of a radiation dominated flat universe, with $\cosh W = const$ expressing conservation of rest energy, and $p_b = \rho_b/3$ at small η. For small η, it follows then from Eq. (14)

$$\rho_b \equiv \rho_b(T,\eta) \simeq \frac{4}{3\kappa_{(4)}^2 T^2 \cosh^2(W) a(\eta)_b^6},$$

or

$$\rho_b(\eta) = a(\eta)_b^2 T^2 \cosh^2 W \rho_b(T,\eta) \simeq \frac{3}{32\pi G_N \eta^2},$$

when expressed in terms of the cosmological time η only.

Having thus shown that the Misner-brane cosmology based on ansatz (4) matches the standard cosmological evolution in the radiation dominated era, we turn now to investigate the nonchronal character of the spacetimes described by metric (6). Nonchronal regions in such spacetimes can most easily be uncovered if we re-define the coordinates entering this metric, such that $Y = W - \ln T$ and $\Theta = T^2$. In terms of the new coordinates, the line element (6) reads:

$$ds^2 = -\frac{\left(\dot{Y}^2 + \frac{\dot{Y}\dot{\Theta}}{\Theta}\right)}{\ln^{\frac{2}{3}}\Theta}\left(\Theta dY^2 + dY d\Theta\right) + \ln^{\frac{2}{3}}\Theta \sum_{j=2}^{4} dx_j^2. \tag{15}$$

This metric is real only for $\Theta > 0$ in which case Y is always timelike if $\dot{Y} > 0$. One will therefore [13] have closed timelike curves (CTC's) only in the bulk, provided $\Theta > 0$, $\dot{Y} > 0$. There will never be CTC's in any of the branes, that is the observable universe.

Singularities of metrics (6), (8) and (15) will appear at $T = 0$ and $T = 1$. The first one corresponds to $\omega = t = 0$, and the second one to $\eta = 0$, the initial singularity at $Y = W$, $t^2 = 1 + \omega^2$, in a radiation dominated universe. We note that the source term $-\kappa_{(4)}^2(\rho_b + 3p_b)/6$ given by Eq. (14) also diverges at these singularities. The geodesic incompleteness at $T = 1$ can be removed in the five-dimensional space, by extending metric (6) with coordinates defined e.g by $X = \int dW/\ln^{\frac{1}{3}} T^2 - 3\ln^{\frac{2}{3}} T^2/4$,

$Z = \int dW/\ln^{\frac{1}{3}} T^2 + 3\ln^{\frac{2}{3}} T^2/4$. Instead of metric (6), we obtain then

$$ds^2 = \frac{2}{3}(Z-X)\left\{\exp\left[\sqrt{\frac{8}{27}}(Z-X)^{\frac{3}{2}}\right]\dot{X}\dot{Z}dXdZ + \sum_{j=2}^{4} dx_j^2\right\}, \qquad (16)$$

where one can check that whereas the singularity at $T = 0$ still remains, the metric is now regular at $T = 1$. Since replacing W for Y in Eqs. (6) and (15) simultaneously leads to the condition $Y = -\frac{1}{2}\ln T + const.$, and hence, by the definition of Y, $Y = const$ and $W = const$ at $T = 1$, one can choose the singularities at $T = 1$ to correspond to the brane positions along ω, and interpret such singularities as chronology horizons in the five-space. So, CTC's will only appear in the bulk on nonchronal regions defined by $0 < T < 1$, $n\omega_c < W < (n+1)\omega_c$, with $n = 0, 1, 2, 3, ...$ The resulting scenario can be regarded to be a typical example of the kind of models alluded by Hawking [1] on how curved or warped extra dimensions would induce the existence of nonchronal regions in higher dimensional cosmological spacetimes inspired in string theory. In the present case, the extra direction is mixed up with our four dimensions in such a way that, although CTC's are allowed to occur in the bulk, any violation of the chronology protection conjecture is fully prevented in the observable universe by the big-bang singularity itself.

For any equation of state the combination $\rho_b + 3p_b$ given in Eq. (14) is divergent at the geodesic incompletenesses at $T = 0$ and $T = 1$. The classical divergence at the chronology horizons is of course removed in the extended coordinate frame X, Z. However, as it happens in wormholes [15] and other topological generalizations [16-18] of the Misner space, if one considers a quantum field propagating in our spacetime, then the renormalized stress-energy tensor $\langle T_{\mu\nu}\rangle_{ren}$ would diverge at the chronology horizons [19]. The existence of this semiclassical instability would support a chronology protection conjecture also against the existence of our universe model. Two situations have been however considered where that conjecture is violated. Both of them use an Euclidean continuation and lead to a vanishing renormalized stress-energy tensor everywhere, even on the chronology horizons. In what follows, we briefly review them, as adapted to our present problem. In order to convert metric (16) into a positive definite metric, it is covenient to use new coordinates p, q, defined by $X = p - q$, $Z = p + q$, or $T^2 = \exp\left[(4q/3)^{3/2}\right]$, $W^2 = 4p^2q/3$. A positive definite metric is then obtained by the continuation $p = i\xi$ which, in turn, implies $W = i\Omega$. Furthermore, using Eqs. (5) we can also see that this rotation converts the extra direction ω in pure imaginary and keeps t and T real, while making the first two of these three quantities periodic and leaving T unchanged. Two ansätze can then be used to fix the value of P_Ω, the period of Ω in the Euclidean sector. On the one hand, from $\exp(W) \to \exp(i\Omega)$ we obtain $P_\Omega = 2\pi$, a result that allows us to introduce a self-consistent Li-Gott vacuum [20], and hence obtain $\langle T_{\mu\nu}\rangle_{ren} = 0$ everywhere. On the other hand, if we take $\exp(p) \to \exp(i\xi)$, then we get $P_\Omega = 2\pi\ln^{1/3} T^2$. In this case, for an automorphic scalar field $\phi(\gamma X, \alpha)$, where γ represents symmetry (3), α is the automorphic parameter, $0 < \alpha < 1/2$, and $X = t, \omega, x^2, x^3, x^4$, following the analysis carried out in [21,22], one can derive solutions of the field equation $\Box\phi = \Box\bar{\phi} = 0$ by demanding t-independence for the mode-frequency. This amounts [22] to a quantum condition on time T which, in this case, reads $\ln T^2 = (n+\alpha)^3 \ln T_0^2$, where T_0 is a small constant time. The use of this condition in the Hadamard function leads to a value for $\langle T_{\mu\nu}\rangle_{ren}$ which is again vanishing everywhere [22]. This not only solves the problem of the semiclassical instability, but can also regularize expression (16) at $T = 0$ and $T = 1$:

$$\rho_b + 2p_b = \frac{2T_0^{-2(n+\alpha)^3}\left(1 + \frac{1}{3(n+\alpha)^3 \ln T_0}\right)}{\kappa_{(4)}^2 \cosh^2(W)(n+\alpha)^3 \ln T_0}, \tag{17}$$

which can never diverge if we choose the constant T_0 such that $\ln T_0 \neq 0$.

At first sight, it could seem that Misner symmetry describes simple and familiar spacetimes. Specifically one would believe this by showing that Misner symmetry converts the five-dimensional metric (1) into merely a reparametrization of the Kasner-type solution [23]. However, the simple transformation $Q = \ln T$ converts metric (1) into

$$ds^2 = (2Q)^{-2/3}\dot{u}\dot{v}\left(-dQ^2 + dW^2\right) + (2Q)^{2/3}\sum_{j=2}^{4} dx_j^2,$$

which differs from a Kasner-type metric by the factor $\dot{u}\dot{v} = 1 - |\dot{\omega}|^2$ in the first term of the right-and-side. This factor cannot generally be unity in the five-dimensional manifold. On surfaces of constant $W = W_0$, according to Eqs. (5), we have $\dot{\omega} = \tanh W_0$, so $\dot{u}\dot{v} = \cosh^{-2} W_0$ which can only be unity for $W_0 = 0$ that is on the brane at $\omega = 0$. However, besides identifying the two branes according to Eq. (7), the Misner approach also requires that the closed up direction ω contracts at a given nonzero rate $d\omega_c/d\eta = -v_0$ [24]. This in turn means that once the branes are set in motion toward one another at the rate v_0, symmetry (7) should imply that for constant W_0,

$$\frac{dW_0(0)}{d\eta} = 0 \leftrightarrow \frac{dW_0(\eta)}{d\eta} - nv_0 = 0,$$

so that $dW_0(\eta)/d\eta \neq 0$ if $n \neq 0$. In this case, we have $\triangle W_0(\eta) = n\int_0^\eta d\omega_c = n \triangle_\eta \omega_c$, and hence $W_0(\eta) = W_0(0) + n \triangle_\eta \omega_c = n \triangle_\eta \omega_c > 0$, provided that we initially set $W_0 \equiv W_0(0) = 0$. It follows that $\dot{u}\dot{v}$ can only be unity on the brane at $\omega = 0$ when $n = 0$ (i.e. at the very moment when the brane universe was created and started to evolve. We note that if we substract the zero-point contribution $\alpha \ln^{1/3} T_0^2$, the quantization of T discussed above amounts to the relation $\eta \propto n^2$ and, therefore, initial moment at $\eta = 0$ means $n = 0$), taking on smaller-than-unity values thereafter, to finally vanish as $\eta, n \to \infty$. Thus, one cannot generally consider metric (1) or metrics (6) and (8) to be reparametrizations of the Kasner solution neither in five nor in four dimensions, except at the very moment when brane at $\omega = 0$ starts being filled with radiation, but not later even on this brane.

We note that in the case that Kasner metric would exactly describe our spacetime (as it actually happens at the classical time origin, $T = 1$, $n = 0$), Misner identification reduces to simply identifying the plane $W = 0$ with $W = n\omega_c$, that is identifying W on a constant circle, which does not include CTC's. This picture dramatically changes nevertheless once n and η become no longer zero, so that $\dot{u}\dot{v} = \cosh^{-2} W_0 < 1$ and the metric cannot be expressed as a reparametrization of the Kasner metric. In that case, there would appear a past apparent singularity [actually, a past event (chronology) horizon] at $T = 1$ for observers at later times $\eta, n \neq 0$, which is extendible to encompass nonchronal regions containing CTC's, as showed before by using the extended metric (16). Indeed, the particular value of T-coordinate $T = 1$ measures a quantum transition at which physical domain walls (three-branes) with energy density ρ_b created themselves, through a process which can be simply represented by the conversion of the inextendible physical singularity of Kasner metric [23] at $T = 1$, $n = 0$ into the coordinate singularity of the Misner-brane metric at $T = 1$, relative to observers placed at later times $\eta, n \neq 0$, which is continuable into a nonchronal region on the bulk space.

On the other hand, since the energy density ρ and pressure p on any of the two candidate branes vanish, one might also think that, related to the previous point, we are actually dealing with a world with no branes, but made up enterely of empty space. The conversion of the field-equation term (13) in a stress-energy tensor would then simply imply violation of momentum-energy conservation. However, the existence of an event (chronology) horizon which is classically placed at $T = 1$ for the five-dimensional spacetime amounts to a process of quantum thermal radiation from vacuum, similar to those happening in black holes or de Sitter space [25,26], which observers at later times $\eta, n > 0$ on the branes would detect to occur at a temperature $\beta \propto \ln^{-1/3} T^2$, when we choose for the period of Ω (which corresponds to the Euclidean continuation of the *timelike* coordinate W on hypersurfaces of constant T) $P_\Omega = 2\pi \ln^{1/3} T^2 \propto a$. Thus, for such observers, the branes would be filled with radiation having an energy density proportional to $\ln^{-4/3} T^2 \propto \eta^{-2} = \rho_b$ and temperature $\propto \eta^{-1/2}$, i.e. just what one should expect for a radiation dominated universe and we have in fact obtained from Eq. (14). Observers on the branes at times corresponding to $T > 1$, $n \neq 0$ would thus interpret all the radiating energy in the four-dimensional Misner-brane universe to come from quantum-mechanical particle creation near an event horizon at $T = 1$.

Moreover, in order to keep the whole two-brane system tensionless relative to a *hypothetical* observer who is able to pass through it by tunneling along the fifth dimension (so that when the observer enters the brane at $\omega = 0$ she finds herself emerging from the brane at $\omega = \omega_c$, without having experienced any tension), one *must* take the tension $V_{\omega=0} = \rho_b > 0$ and the tension $V_{\omega=\omega_c} = -\rho_b$, and therefore the total tension experienced by the hypothetical observer, $V = V_{\omega=0} + V_{\omega=\omega_c}$ will vanish. Given the form of the energy density ρ_b, this necessarily implies that current observers should live on just one of the branes (e.g. at $\omega = 0$) and cannot travel through the fifth direction to get in the other brane (so current observers are subjected to chronology protection [2]), and that, relative to the hypothetical observer who is able to make that traveling, the brane which she emerges from (e.g. at $\omega = \omega_c$) must then be endowed with an antigravity regime with $G_N < 0$ [5], provided she first entered the brane with $G_N > 0$ (e.g. at $\omega = 0$).

Chronology protection conjecture states [2] that the laws of physics prevent the existence of CTC's and possible time machines constructed out of them, at least in a semiclassical approximation where the quantum fields propagate in a classical background spacetime. As formulated in this way, this conjecture is violated in our model and, indeed, in all nonchronal spacetime models admitting similar Euclidean continuations [20-22]. The results of the present letter imply, nevertheless, that, although the laws of physics actually allow CTC's and time machines to occur, they place them outside our observable universe, in such a way that such constructs can neither be directly observed, nor break causality. It is in this sense that a chronology protection must be understood in the present work.

To sum up, we have considered a brane universe in which the two domain walls on the fifth direction of the Randall-Sundrum approach are identified by using a Misner-like symmetry, resulting in a cosmological evolution which matches that of the standard Friedmann scenario at early times. This universe model has CTC's in the bulk, but not in the branes which are chronologically protected by quantum-mechanically stable chronology horizons, so providing the chronology protection conjecture with a new, less demanding interpretation.

Acknowledgements

For helpful comments and a careful reading of the manuscript, the author thanks C. Sigüenza. This research was supported by DGICYT under Research Projects Nos. PB97-1218 and PB98-0520.

References

1 S.W. Hawking, *Space and Time Warps*, public lecture, http://www.hawking.org.uk/lectures/lindex.html

2 S.W. Hawking, Phys. Rev. D46(1992)603.

3 L. Randall and R. Sundrum, Phys. Rev. Lett. 83(1999)3370.

4 L. Randall and R. Sundrum, Phys. Rev. Lett. 83(1999)4690.

5 T. Shiromizu, K. Maeda and M. Sasaki, *Einstein Equations on the 3-Brane World*, gr-qc/9910076.

6 P. Binétruy, C. Deffayet and D. Langlois, Nucl. Phys. B565(2000)269.

7 C. Csáki, M. Graesser, C. Kolda and J. Terning, Phys. Lett. B462(1999)34.

8 J.M. Cline, C. Grosjean and G. Servant, Phys. Rev. Lett. 83(1999)4245.

9 P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B477(2000)285.

10 P.F. González-Díaz, Phys. Lett. B481(2000)353.

11 D.N. Vollick, *Cosmology on a three-brane*, hep-th/9911181.

12 C.W. Misner, in *Relativity and Cosmology*, edited by J. Ehlers (American Mathematical Society, Providence, RI, USA, 1967).

13 S.W. Hawking and G.F.R. Ellis, *The large scale structure of space-time* (Cambridge University Press, Cambridge, UK, 1973).

14 W. Israel, Nuovo Cimento B44(1966)1; B48(1966)463.

15 M.S. Morris, K.S. Thorne and U. Yortsever, Phys. Rev. Lett. 61(1988)1446.

16 J.R. Gott, Phys. Rev. Lett. 66(1991)1126.

17 P.F. González-Díaz, Phys. Rev. D54(1996)6122.

18 P.F. González-Díaz and L.J. Garay, Phys. Rev. D59(1999)064026.

19 S.-W. Kim and K.S. Thorne, Phys. Rev. D43(1991)3929.

20 Li-Xin-li and J.R. Gott, Phys. Rev. Lett. 80(1998)2980.

21 S.V. Sushkov, Class. Quantum Grav. 14(1997)523.

22 P.F. González-Díaz, Phys. Rev. D58(1998)124011.

23 R.M. Wald, *General Relativity* (The University of Chicago Press, Chicago, USA, 1984).

24 K.S. Thorne, in *Directions in General Relativity*, Proceedings of the 1993 International Symposium, Maryland, in Honor of C. Misner (Cambridge University Press, Cambridge, UK, 1993), Vol. 1.

25 S.W. Hawking, Commun. Math. Phys. 43(1975)199.

26 G.W. Gibbons and S.W. Hawking, Phys. Rev. D15(1977)2738.