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Linking Starobinsky-type inflation in no-scale supergravity to MSSM

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Abstract. A novel realization of the Starobinsky inflationary model within a moderate extension of the Minimal Supersymmetric Standard Model (MSSM) is presented. The proposed superpotential is uniquely determined by applying a continuous $R$ and a $Z_2$ discrete symmetry, whereas the Kähler potential is associated with a no-scale-type SU(54, 1)/SU(54) × U(1)$_R$ × $Z_2$ Kähler manifold. The inflaton is identified with a Higgs-like modulus whose the vacuum expectation value controls the gravitational strength. Thanks to a strong enough coupling (with a parameter $c_T$ involved) between the inflaton and the Ricci scalar curvature, inflation can be attained even for subplanckian values of the inflaton with $c_T \geq 76$ and the corresponding effective theory being valid up to the Planck scale. The inflationary observables turn out to be in agreement with the current data and the inflaton mass is predicted to be $3 \cdot 10^{13}$ GeV. At the cost of a relatively small superpotential coupling constant, the model offers also a resolution of the $\mu$ problem of MSSM for $c_T \leq 4500$ and gravitino heavier than about $10^4$ GeV. Supplementing MSSM by three right-handed neutrinos we show that spontaneously arising couplings between the inflaton and the particle content of MSSM not only ensure a sufficiently low reheating temperature but also support a scenario of non-thermal leptogenesis consistently with the neutrino oscillation parameters.

Keywords: particle physics - cosmology connection, supersymmetry and cosmology, cosmology of theories beyond the SM, inflation

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1 Introduction

After the announcement of the recent PLANCK results [1, 2], inflation based on the potential of the Starobinsky model [3–5] has gained a lot of momentum [6–20] since it predicts [3–5, 21] a (scalar) spectral index very close to the one favored by the fitting of the observations by the standard power-law cosmological model with cold dark matter (CDM) and a cosmological constant (ΛCDM). In particular, it has been shown that Starobinsky-type inflation can be realized within extensions of the Standard Model (SM) [22, 23] or Minimal SUSY SM (MSSM) [24, 25]. However, the realization of this type of inflation within Supergravity (SUGRA) is not unique. Different super- and Kähler potentials are proposed [7–9] which result to the same scalar potential. Prominent, however, is the idea [6, 7] of implementing...
this type of inflation using a Kähler potential, $K$, corresponding to a $\text{SU}(N,1)/\text{SU}(N) \times \text{U}(1)$ Kähler manifold inspired by the no-scale models [26–28]. Such a symmetry fixes beautifully the form of $K$ up to an holomorphic function $f_K$ which exclusively depends on a modulus-like field and plays the role of a varying gravitational coupling. The stabilization of the non-inflaton accompanying field can to be conveniently arranged by higher order terms in $K$. In this context, a variety of models are proposed in which inflaton can be identified with either a matter-like [6, 7, 24, 25] or a modulus-like [7, 8] inflaton. The former option seems to offer a more suitable framework [24, 25] for connecting the inflationary physics with a low-energy theory, such as the MSSM endowed with right handed neutrinos, $N^c_i$, since the non-inflaton modulus is involved in the no-scale mechanism of soft SUSY breaking (SSB). On the other hand, the inflationary superpotential, $W_{\text{MI}}$, is arbitrarily chosen and not protected by any symmetry. Given that, the inflaton takes transplanckian values during inflation, higher order corrections — e.g., by non-renormalizable terms in $W_{\text{MI}}$ — with not carefully tuned coefficients may easily invalidate or strongly affect [10, 29–31] the predictions of an otherwise successful inflationary scenario.

It would be interesting, therefore, to investigate if the shortcoming above can be avoided in the presence of a strong enough coupling of the inflaton to gravity [32–36], as done [37–44] in the models of non-minimal Inflation (nMI). This idea can be implemented keeping the no-scale structure of $K$, since the involved $f_K$ can be an analytic function, selected conveniently. In view of the fact that $f_K$ depends only on a modulus-like field, we here focus on this kind of inflaton — contrary to refs. [24, 25]. As a consequence, the direct connection of the inflationary model with the mechanism of the SSB is lost. Note, in passing, that despite their attractive features, the no-scale models [24, 25] of SSB enface difficulties — e.g., viable SUSY spectra are obtained only when the boundary conditions for the SSB terms are imposed beyond the Grand Unified Theory (GUT) scale and so the low energy observables depend on the specific GUT.

Focusing on a modulus-like inflaton, the link to MSSM can be established through the adopted $W_{\text{MI}}$. Its form in our work is fixed by imposing a continuous $R$ symmetry, which reduces to the well-known $R$-parity of MSSM, and a $Z_2$ discrete symmetry. As a consequence, $W_{\text{MI}}$ resembles the one used in the widely employed models [45–47] of standard F-term Hybrid Inflation (FHI) — with singlet waterfall field though. As a bonus, a dynamical generation of the reduced Planck scale arises in Jordan Frame (JF) through the vacuum expectation value (v.e.v) of the inflaton. Therefore the inflaton acquires a higgs-character as in the theories of induced gravity [48–54]. To produce an inflationary plateau with the selected $W_{\text{MI}}$, $f_K$ is to be taken quadratic, in accordance with the adopted symmetries. This is to be contrasted with the so-called modified Cecotti model [7–10, 55, 56] where the inflaton appears linearly in the super- and Kähler potentials. The inclusion of two extra parameters compared to the original model — cf. refs. [7, 8, 10] — allows us to attain inflationary solutions for subplanckian values of the inflaton with the successful inflationary predictions of the model being remained intact. As a bonus, the ultraviolet (UV) cut-off scale [11, 57–63] of the theory can be identified with the Planck scale and so, concerns regarding the naturalness of the model can be safely eluded.

Our inflationary model — let name it for short no-scale modular inflation (nSMI) — has ramifications to other fundamental open problems of the MSSM and post-inflationary cosmological evolution. As a consequence of the adopted $U(1)_R$ symmetry, the generation [47, 64–67] of the mixing term between the two electroweak Higgses is explained via the v.e.v of the non-inflaton accompanying field, provided that a coupling constant in $W_{\text{MI}}$ is rather
suppressed and the masses of the gravitino ($\tilde{G}$) lie in the multi-TeV region — as dictated in many versions [68–75] of MSSM after the recent LHC [76, 77] results on the Higgs boson mass. Finally, the observed [78] baryon asymmetry of the universe (BAU) can be explained via spontaneous [79, 80] non-thermal leptogenesis (nTL) [81–83] — consistently with the $\tilde{G}$ constraint [84–91] and the data [92, 93] on the neutrino oscillation parameters.

The basic ingredients — particle content and structure of the super- and Kähler potentials — of our model are presented in section 2. In section 3 we describe the inflationary potential, derive the inflationary observables and confront them with observations. Section 4 is devoted to the resolution of the $\mu$ problem of MSSM. In section 5 we outline the scenario of nTL, exhibit the relevant imposed constraints and describe our predictions for neutrino masses. Our conclusions are summarized in section 6. Throughout the text, the subscript of type, $\chi$ denotes derivation with respect to (w.r.t.) the field $\chi$ (e.g., $\chi \chi = \partial^2/\partial \chi^2$) and charge conjugation is denoted by a star.

### 2 Model description

We focus on a moderated extension of MSSM with three $N^c_i$'s augmented by two superfields, a matter-like $S$ and a modulus-like $T$, which are singlets under $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$. Besides the local symmetry of MSSM, $G_{SM}$, the model possesses also the baryon number symmetry $U(1)_B$, a nonanomalous $R$ symmetry $U(1)_R$ and a discrete $Z_2$. Note that global continuous symmetries can effectively arise [94] in many compactified string theories. The charge assignments under the global symmetries of the various matter and Higgs superfields are listed in table 1. We below present the structure of the superpotential (section 2.1) and the Kähler potential (section 2.2) of our model.

#### 2.1 The superpotential

The superpotential of our model naturally splits into two parts:

$$ W = W_{MSSM} + W_{MI}, $$

where $W_{MSSM}$ is the part of $W$ which contains the usual terms — except for the $\mu$ term — of MSSM, supplemented by Yukawa interactions among the left-handed leptons and $N^c_i$:

$$ W_{MSSM} = h_{ij} e_i^c L_j H_d + h_{ij} d_i^c Q_j H_u + h_{ij} u_i^c Q_j H_u + h_{ij} N_i^c L_j H_u. $$

Here the $i$th generation SU(2)$_L$ doublet left-handed quark and lepton superfields are denoted by $Q_i$ and $L_i$ respectively, whereas the SU(2)$_L$ singlet antiquark [antilepton] superfields by $u_i^c$ and $d_i^c$ [$e_i^c$ and $N_j^c$] respectively. The electroweak Higgs superfields which couple to the up [down] quark superfields are denoted by $H_u$ [$H_d$].

On the other hand, $W_{MI}$ is the part of $W$ which is relevant for nSMI, the generation of the $\mu$ term of MSSM and the Majorana masses for $N^c_i$'s. It takes the form

$$ W_{MI} = \lambda S (T^2 - M^2/2) + \lambda_\mu S H_u H_d + \frac{1}{2} M_i N_i^c N_i^c + \lambda_{ij} N_i^c T^2 N_j^c N_j^c / 2m_P, $$

where $m_P = 2.44 \cdot 10^{18}$ GeV is the reduced Planck mass. The imposed $U(1)_R$ symmetry ensures the linearity of $W_{MI}$ w.r.t. $S$. This fact allows us to isolate easily via its derivative the contribution of the inflaton $T$ into the F-term SUGRA scalar potential, placing $S$ at the origin. The imposed $Z_2$ prohibits the existence of the term $ST$ which, although does
not drastically modifies our proposal, it complicates the determination of SUSY vacuum and the inflationary dynamics. On the other hand, the imposed symmetries do not forbid non-renormalizable terms of the form $T^{2n+2}$ where $n \geq 1$ is an integer. For this reason we are obliged to restrict ourselves to subplanckian values of $T$.

The second term in the right-hand side (r.h.s.) of eq. (2.1c) provides the $\mu$ term of MSSM along the lines of refs. [47, 64–67] — see section 4. The third term is the Majorana mass term for the $N_c$’s and we assume that it overshadows (for sufficiently low $\lambda_{ijN_c}$’s) the last non-renormalizable term which is neglected henceforth. Here we work in the so-called right-handed neutrino basis, where $M_iN_c$ is diagonal, real and positive. These masses together with the Dirac neutrino masses in eq. (2.1b) lead to the light neutrino masses via the seesaw mechanism. The same term is important for the decay [79, 80] of the inflaton after the end of nSMI to $\tilde{N}_c$, whose subsequent decay can activate nTL. As a result of the imposed $Z_2$, a term of the form $TN_c^2$ is prohibited and so the decay of $T$ into $N_c$ is processed by suppressed SUGRA-induced interactions [79], guaranteeing thereby a sufficiently low reheat temperature compatible with the $\tilde{G}$ constraint and successful nTL — see section 5.1.

In the limit where $m_P$ tends to infinity, we can obtain the SUSY limit, $V_{SUSY}$, of the SUGRA potential — see section 3.1 —, which corresponds to $W_{MI}$ in eq. (2.1c). This is

$$V_{SUSY} = |\lambda T^2 + \lambda_{ij}H_uH_d - \lambda M^2/2|^2 + 4\lambda^2 |ST|^2 + M_{iN_c}^2|\tilde{N}_c|^2 + \lambda_M^2(|H_u|^2 + |H_d|^2)|S|^2,$$

where the complex scalar components of the superfields $T$ and $S$ are denoted by the same symbol. From the potential in eq. (2.2a), we find that the SUSY vacuum lies at

$$\langle H_u \rangle = \langle H_d \rangle = \langle \tilde{N}_c \rangle = 0, \quad \langle S \rangle \approx 0 \text{ and } \sqrt{2}\langle |T| \rangle = M.$$

Contrary to the Cecotti model [7, 8, 55, 56] our modulus $T$ can take values $M \leq m_P$ at the SUSY vacuum. Also, $\langle T \rangle$ breaks spontaneously the imposed $Z_2$ and so, it can comfortably decay via SUGRA-inspired decay channels — see section 5.1 — reheating the universe and rendering [80] spontaneous nTL possible. No domain walls are produced due to the spontaneous breaking of $Z_2$ at the SUSY vacuum, since this is broken already during nSMI.

With the addition of SSB terms, as required in a realistic model, the position of the vacuum shifts [47, 64–67] to non-zero $\langle S \rangle$ and an effective $\mu$ term is generated from the second term in the r.h.s. of eq. (2.1c) — see section 4. Let us emphasize that SSB effects explicitly break $U(1)_R$ to the $Z_2^R$ matter parity, under which all the matter (quark and lepton) superfields change sign. Combining $Z_2^R$ with the $Z_2^f$ fermion parity, under which all fermions change sign, yields the well-known $R$-parity. Recall that this residual symmetry prevents the rapid proton decay, guarantees the stability of the lightest SUSY particle (LSP) and therefore it provides a well-motivated CDM candidate. Needless to say, finally, that such a connection of the Starobinsky-type inflation with this vital for MSSM $R$-symmetry can not be established within the modified Cecotti model [8, 9, 55, 56], since no symmetry can

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Table 1. The global charges of the superfields of our model.
prohibit a quadratic term for the modulus-like field in conjunction with the tadpole term in $W_{\text{MI}}$.

### 2.2 The Kähler potential

According to the general discussion of [26, 27], the Kähler manifold which corresponds to a Kähler potential of the form

$$K = -3m_P^2 \ln \left( f_K(T) + f_K^*(T^*) - \frac{\Phi_A\Phi_A^*}{3m_P^2} + k_{S\Phi_A}\frac{|S|^2|\Phi_A|^2}{3m_P^4} \ldots \right), \quad (2.3a)$$

with $f_K$ being an holomorphic function of $T$, exhibits a $\text{SU}(N_1)/\text{SU}(N) \times \text{U}(1)_R \times \mathbb{Z}_2$ global symmetry. Here $N_1 - 1 = 53$ is the number of scalar components of $S$, $N_i^c$ and the MSSM superfields which are collectively denoted as

$$\Phi^A = \tilde{e}_i^c, \tilde{u}_i^c, \tilde{d}_i^c, \tilde{N}_i^c, \tilde{L}_i, \tilde{Q}_i, H_u, H_d \text{ and } S. \quad (2.3b)$$

Note that summation over the repeated (small or capital) Greek indices is implied. The third term in the r.h.s. of eq. (2.3a) — with coefficients $k_{S\Phi_A}$ being taken, for simplicity, real — is included since it has an impact on the scalar mass spectrum along the inflationary track — see section 3.1. In particular, the term with coefficient $k_{SS} = k_S \simeq 1$ assists us to avoid the tachyonic instabilities encountered in similar models [7–9, 32–36] — see section 3.1. The ellipsis represents higher order terms which are irrelevant for the inflationary dynamics since they do not mix the inflaton $T$ with the matter fields. This is, in practice, a great simplification compared to the models of nMI — cf. ref. [44]. Contrary to other realizations of the Starobinsky model — cf. refs. [7–9] —, we choose $f_K$ to be quadratic and not linear with respect to $T$, i.e.,

$$f_K(T) = c_T T^2 / m_P^2 \quad (2.3c)$$

in accordance with the imposed $\mathbb{Z}_2$ symmetry which forbids a linear term — the coefficient $c_T$ is taken real too. As in the case of eq. (2.1c), non-renormalizable terms of the form $T^{2n+2}$, with integer $n \geq 1$, are allowed but we can safely ignore them restricting ourselves to $T \leq m_P$.

The interpretation of the adopted $K$ in eq. (2.3a) can be given in the “physical” frame by writing the JF action for the scalar fields $\Phi^\alpha = \Phi^A, T$. To extract it, we start with the corresponding EF action within SUGRA [32–34, 38, 44] which can be written as

$$S = \int d^4x \sqrt{-\hat{g}} \left( -\frac{1}{2} m_P^2 \hat{\mathcal{R}} + K_{\alpha\beta} \hat{\Phi}^\alpha \hat{\Phi}^{\ast \beta} - \hat{V}_{\text{MI0}} + \ldots \right), \quad (2.4a)$$

where $K_{\alpha\beta} = \hat{K}_{\hat{\Phi}^\alpha \hat{\Phi}^{\ast \beta}}$ with $\hat{K}_{\hat{\beta}\hat{\alpha}} K_{\hat{\alpha}\hat{\gamma}} = \delta_{\hat{\beta}}^{\hat{\gamma}}$, $\hat{g}$ is the determinant of the EF metric $\hat{g}_{\mu\nu}$, $\hat{\mathcal{R}}$ is the EF Ricci scalar curvature, $\hat{V}_{\text{MI0}}$ is defined in section 3.1, the dot denotes derivation w.r.t. the JF cosmic time and the ellipsis represents terms irrelevant for our analysis. Performing then a suitable conformal transformation, along the lines of refs. [38, 44] we end up with the following action in the JF

$$S = \int d^4x \sqrt{-g} \left( -\frac{m_P^2}{2} \left( -\frac{\Omega}{3} \right) \mathcal{R} + m_P^2 \Omega_{\alpha\beta} \hat{\Phi}^\alpha \hat{\Phi}^{\ast \beta} - V_{\text{SUSY}} + \ldots \right), \quad (2.4b)$$
The only surviving term of the frame function can be found from the relation:

\[- \frac{\Omega}{3} = e^{-K/3m^2_P} = f_K(T) + f_K^*(T^*) - \frac{\Phi_A \Phi^A}{3m^2_P} + k_S \Phi_A \left| \frac{S^2 |\Phi_A|^2}{3m^2_P} \right| + \cdots \]  

(2.4c)

The last result reveals that $T$ has no kinetic term, since $\Omega_{TT^*} = 0$. This is a crucial difference between the Starobinsky-type models and those [44] of nMI, with interest consequences [11] to the derivation of the ultraviolet cutoff scale of the theory — see section 3.2. Furthermore, given that $(\Phi^4) \simeq 0$, recovering the conventional Einstein gravity at the SUSY vacuum, eq. (2.2b), dictates:

\[ f_K((T)) + f_K^*((T^*)) = 1 \quad \Rightarrow \quad M = m_P/\sqrt{c_T}. \]  

(5.5)

Given that the analysis of inflation in both frames yields equivalent results [21, 95–97], we below—see section 3.1 and 3.2—carry out the derivation of the inflationary observables exclusively in the EF.

3 The inflationary scenario

In this section we outline the salient features of our inflationary scenario (section 3.1) and then, we present its predictions in section 3.4, calculating a number of observable quantities introduced in section 3.2. We also provide a detailed analysis of the UV behavior of the model in section 3.3.

3.1 The inflationary potential

The EF F-term (tree level) SUGRA scalar potential, $\tilde{V}_{M0}$, of our model — see eq. (2.4a) — is obtained from $W_{\text{MI}}$ in eq. (2.1c) and $K$ in eq. (2.3b) by applying the standard formula:

\[ \tilde{V}_{M0} = e^{K/m_P^2} \left( K^{\alpha\beta} F_\alpha F_\beta - \frac{3|W_{\text{MI}}|^2}{m^2_P} \right), \]  

(3.1)

where $F_\alpha = W_{\text{MI}}\Phi^\alpha + K\Phi^\alpha W_{\text{MI}}/m^2_P$. Setting the fields $\Phi^\alpha = S, N_c, H_u$ and $H_d$ at the origin the only surviving term of $\tilde{V}_{M0}$ is

\[ \tilde{V}_{M0} = e^{K/m_P^2} K^{SS} W_{\text{MI},S} W_{\text{MI},S^*} = \frac{\lambda^2 |2T^2 - M^2|^2}{4(f_K + f_K^*)^2}. \]  

(3.2)

It is obvious from the result above that a form of $f_K$ as the one proposed in eq. (2.3c) can flatten $\tilde{V}_{M0}$ sufficiently so that it can drive nSMI. Employing the dimensionless variables

\[ x_\phi = |\phi|/m_P, \quad f_T = 1 - c_T x^2_\phi \quad \text{and} \quad x_M = M/m_P \quad \text{with} \quad \phi = |T|/\sqrt{2} \]  

(3.3)

and setting $\arg T = 0$, $\tilde{V}_{M0}$ and the corresponding Hubble parameter $\tilde{H}_{\text{MI}}$ read

\[ \tilde{V}_{M0} = \frac{\lambda^2 m^4_P x^2_\phi - x^2_M}{4c_T^2 x^2_\phi} = \frac{\lambda^2 m^4_P f_T^2}{4c_T^2 x^2_\phi} \quad \text{and} \quad \tilde{H}_{\text{MI}} = \frac{\tilde{V}^{1/2}_{M0}}{\sqrt{3m_P}} \simeq \frac{\lambda m_P}{2\sqrt{3}c_T^2}, \]  

(3.4)

where we put $x_M = 1/\sqrt{c_T}$—by virtue of eq. (5.5)—in the final expressions.
Expanding $T$ and $\Phi^\alpha$ in real and imaginary parts as follows

$$T = \frac{\phi}{\sqrt{2}} e^{i\theta/m} \quad \text{and} \quad X^\alpha = \frac{x^\alpha + i\bar{x}^\alpha}{\sqrt{2}} \quad \text{with} \quad X^\alpha = S, H_u, H_d, \tilde{N}_i^c$$  \hspace{1cm} (3.5)

we can check the stability of the inflationary direction

$$\theta = x^\alpha = \bar{x}^\alpha = 0 \quad \text{where} \quad x^\alpha = s, h_u, h_d, \tilde{v}_i^\alpha,$$  \hspace{1cm} (3.6)

w.r.t. the fluctuations of the various fields. In particular, we examine the validity of the extremum and minimum conditions, i.e.,

$$\left. \frac{\partial \tilde{V}_{M10}}{\partial \chi^\alpha} \right|_{eq.\ (3.6)} = 0 \quad \text{and} \quad \tilde{m}_{\chi^\alpha}^2 > 0 \quad \text{with} \quad \chi^\alpha = \theta, x^\alpha, \bar{x}^\alpha.$$

(3.7a)

Here $\tilde{m}_{\chi^\alpha}^2$ are the eigenvalues of the mass matrix with elements

$$\tilde{M}_{\alpha\beta}^{2\text{g}} = \left. \frac{\partial^2 \tilde{V}_{M10}}{\partial \chi^\alpha \partial \chi^\beta} \right|_{eq.\ (3.6)} \quad \text{with} \quad \chi^\alpha = \theta, x^\alpha, \bar{x}^\alpha$$  \hspace{1cm} (3.7b)

and hat denotes the EF canonically normalized fields. Taking into account that along the configuration of eq. (3.6) $K_{\alpha\beta}$ defined below eq. (2.4a) takes the form

$$(K_{\alpha\beta}) = \text{diag} \left( \begin{array}{cccc} 6/\bar{x}_\phi, & 1/c_T x_\phi^2, & \ldots, & 1/c_T x_\phi^2 \end{array} \right)$$  \hspace{1cm} (3.8)

— here we take into account that $H_u$ and $H_d$ are SU(2)$_L$ doublets —, the kinetic terms of the various scalars in eq. (2.4a) can be brought into the following form

$$K_{\alpha\beta} \Phi^\alpha \Phi^{*\beta} = \frac{1}{2} \left( i \dot{\phi}^2 + \dot{\theta}^2 \right) + \frac{1}{2} \left( \ddot{x}^\alpha + \ddot{\bar{x}}^\alpha \right) \quad \text{and} \quad \ddot{\bar{x}}^\alpha = \ddot{x}^\alpha / \sqrt{c_T x_\phi}.$$  \hspace{1cm} (3.9a)

where the hatted fields are defined as follows

$$d\hat{\phi}/d\phi = J = \sqrt{6}/x_\phi, \quad \hat{\theta} = \sqrt{6}\theta, \quad \hat{x}^\alpha = x^\alpha / \sqrt{c_T x_\phi} \quad \text{and} \quad \ddot{\bar{x}}^\alpha = \ddot{x}^\alpha / \sqrt{c_T x_\phi}.$$  \hspace{1cm} (3.9b)

Upon diagonalization of the relevant sub-matrices of $\tilde{M}_{\alpha\beta}^{2\text{g}}$, eq. (3.7b), we construct the scalar mass spectrum of the theory along the direction in eq. (3.6). Our results are summarized in table 2, assuming $k_{SH_u} \approx k_{SH_d} = k_{SH}$ in order to avoid very lengthy formulas for the masses of $\tilde{h}_\pm$ and $\tilde{h}_\pm$. The various unspecified there eigenvalues are defined as follows:

$$\tilde{h}_\pm = (h_u \pm h_d)/\sqrt{2}, \quad \tilde{h}_\pm = (\bar{h}_u \pm \bar{h}_d)/\sqrt{2} \quad \text{and} \quad \tilde{\psi}_\pm = (\tilde{\psi}_T \pm \tilde{\psi}_S)/\sqrt{2},$$  \hspace{1cm} (3.10a)

where the spinors $\tilde{\psi}_T, \tilde{\psi}_S$ and $N_i^c$ associated with the superfields $S, T$ and $N_i^c$ are related to the normalized ones in table 2 as follows:

$$\tilde{\psi}_S = \sqrt{6}\tilde{\psi}_S/x_\phi, \quad \tilde{\psi}_T = \tilde{\psi}_T/\sqrt{c_T x_\phi} \quad \text{and} \quad \tilde{N}_i^c = N_i^c/\sqrt{c_T x_\phi}.$$  \hspace{1cm} (3.10b)

We also use the shorthand notation:

$$f_{SH} = 2 + 3k_{SH}c_T x_\phi^2 \quad \text{and} \quad f_{S\tilde{N}_i^c} = 2 + 3k_{S\tilde{N}_i^c}c_T x_\phi^2.$$  \hspace{1cm} (3.11)
Note that, due to the large effective masses that the $\chi$'s in eq. (3.7b) acquire during nSMI, they enter a phase of oscillations about $\chi = 0$ with reducing amplitude. As a consequence — see eq. (3.9b) —, $\dot{\chi} \approx \chi/\sqrt{f_K}$ since the quantity $f_K/(2f_K^{1/2})$, involved in relating $\dot{\chi}$ to $\chi$, turns out to be negligibly small compared with $\chi/\sqrt{f_K}$ — cf. ref. [43]. Moreover, we have numerically verified that the various masses remain greater than $\dot{H}_{MI}$ during the last 50 e-foldings of nSMI, and so any inflationary perturbations of the fields other than the inflaton are safely eliminated — see also section 3.4.

From table 2 it is evident that $k_S \gtrsim 1$ assists us to achieve $\hat{m}_\psi^2 > 0$ — in accordance with the results of [7–9]. On the other hand, given that $f_T \leq 0$, $\hat{m}_\psi^2 > 0$ requires

$$\lambda f_T f_{SH} + 6 \lambda \mu c_T^2 x_\phi^2 < 0 \quad \Rightarrow \quad \lambda_{\mu} < -\frac{\lambda f_T f_{SH}}{6c_T^2 x_\phi^2} \simeq \frac{\lambda}{3c_T} + \frac{1}{2} \lambda k_{SH} x_\phi^2 \simeq 2 \cdot 10^{-5} - 10^{-6}, \quad (3.12)$$

as $k_{SH}$ decreases from 3 to 0.5. Here we have made use of eqs. (3.16a) and (3.20b) — see section 3.2. We do not consider such a condition on $\lambda_{\mu}$ as unnatural, given that $h_{UV}$ in eq. (2.1b) is of the same order of magnitude too — cf. ref. [98]. In table 2 we also present the masses squared of chiral fermions along the trajectory of eq. (3.6), which can be served for the calculation of the one-loop radiative corrections. Employing the well-known Coleman-Weinberg formula [99], we find that the one-loop corrected inflationary potential is

$$\hat{V}_{MI} = \hat{V}_{MI0} + \frac{1}{64\pi^2} \left( \hat{m}_\theta^4 \ln \frac{\hat{m}_\theta^4}{\Lambda^4} + 2 \hat{m}_s^4 \ln \frac{\hat{m}_s^4}{\Lambda^4} + 4 \hat{m}_{h_+}^4 \ln \frac{\hat{m}_{h_+}^4}{\Lambda^4} + 4 \hat{m}_{h_-}^4 \ln \frac{\hat{m}_{h_-}^4}{\Lambda^4} - 2 \hat{m}_\psi^4 \ln \frac{\hat{m}_\psi^4}{\Lambda^4} \right),$$

where $\Lambda$ is a renormalization group (RG) mass scale. As we numerically verify the one-loop corrections have no impact on our results. The absence of gauge interactions and of a direct renormalizable coupling between $T$ and $N_5^c$ assists to that direction — cf. refs. [44, 100]. Based on $\hat{V}_{MI}$, we can proceed to the analysis of nSMI in the EF, employing the standard slow-roll approximation [101–104]. It can be shown [48–51] that the results calculated this way are the same as if we had calculated them using the non-minimally coupled scalar field in the JF.

<table>
<thead>
<tr>
<th>Fields</th>
<th>Eingestates</th>
<th>Masses Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 real scalar</td>
<td>$\hat{\theta}$</td>
<td>$\hat{m}<em>\theta^2 = \lambda^2 m_P^2 (f_T + 2c_T^2 x</em>\phi^2) / 3c_T^4 x_\phi^4 \simeq 4\dot{H}_{MI}^2$</td>
</tr>
<tr>
<td>2 real scalars</td>
<td>$\hat{s}$, $\hat{\bar{s}}$</td>
<td>$\hat{m}<em>s^2 = \lambda^2 m_P^2 (1 + c_T^2 x</em>\phi^2 (2 - c_T^2 x_\phi^2 + 6k_{SH} f_{SH}^2)) / 6c_T^4 x_\phi^4$</td>
</tr>
<tr>
<td>4 real scalars</td>
<td>$\hat{h}<em>+, \hat{\bar{h}}</em>+$</td>
<td>$\hat{m}<em>{h</em>+}^2 = \lambda m_P^2 f_T (\lambda f_T f_{SH} - 6\lambda \mu c_T^2 x_\phi^2) / 12c_T^4 x_\phi^4$</td>
</tr>
<tr>
<td>4 real scalars</td>
<td>$\hat{h}<em>-, \hat{\bar{h}}</em>-$</td>
<td>$\hat{m}<em>{h</em>-}^2 = \lambda m_P^2 f_T (\lambda f_T f_{SH} + 6\lambda \mu c_T^2 x_\phi^2) / 12c_T^4 x_\phi^4$</td>
</tr>
<tr>
<td>6 real scalars</td>
<td>$\hat{c}_i, \hat{\bar{c}}_i$</td>
<td>$\hat{m}<em>{c_i}^2 = (\lambda^2 m_P^2 f_T^2 f</em>{SH} N_i + 12 M_{\lambda N_5}^2 c_T^2 x_\phi^2) / 12c_T^4 x_\phi^4$</td>
</tr>
<tr>
<td>2 Weyl spinors</td>
<td>$\hat{\psi}_\pm$</td>
<td>$\hat{m}<em>{\psi</em>\pm}^2 \simeq \lambda^2 m_P^2 / 3c_T^4 x_\phi^4$</td>
</tr>
<tr>
<td>3 Weyl spinors</td>
<td>$\hat{N}_i^c$</td>
<td>$\hat{m}<em>{N_i^c}^2 = M</em>{\lambda N_5}^2 / c_T x_\phi^2$</td>
</tr>
</tbody>
</table>

Table 2. The mass spectrum of our model along the inflationary trajectory of eq. (3.6).
3.2 The inflationary observables — requirements

A successful inflationary scenario has to be compatible with a number of observational requirements which are outlined in the following.

3.2.1 The number of e-foldings

The number of e-foldings \( \hat{N}_* \), that the scale \( k_\ast = 0.05/\text{Mpc} \) suffers during nSMI has to be adequate to resolve the horizon and flatness problems of the standard Big Bang cosmology. Assuming that nSMI is followed in turn by a decaying-particle, radiation and matter domination and employing standard methods [37], we can easily derive the required \( \hat{N}_* \) for our model, with the result:

\[
\hat{N}_* \approx 19.4 + 2 \ln \frac{\hat{V}_{\text{MI}}(\phi_\ast)^{1/4}}{1 \text{ GeV}} - \frac{4}{3} \ln \frac{\hat{V}_{\text{MI}}(\phi_T)^{1/4}}{1 \text{ GeV}} + \frac{1}{3} \ln \frac{T_{\text{rh}}}{1 \text{ GeV}} + \frac{1}{2} \ln \frac{f_K(\phi_\ast)}{f_K(\phi_T)^{1/3}},
\]

(3.14)

where \( \phi_\ast \) [\( \hat{\phi}_\ast \)] is the value of \( \phi \) [\( \hat{\phi} \)] when \( k_\ast \) crosses the inflationary horizon. Also \( \phi_T \) [\( \hat{\phi}_T \)] is the value of \( \phi \) [\( \hat{\phi} \)] at the end of nSMI determined, in the slow-roll approximation, by the condition:

\[
\max \{\hat{\epsilon}(\phi_T), |\hat{\eta}(\phi_T)|\} = 1,
\]

(3.15a)

where the slow-roll parameters read

\[
\hat{\epsilon} = \frac{m_P^2}{2} \left( \frac{\hat{V}_{\text{MI}, \hat{\phi}}}{\hat{V}_{\text{MI}}} \right)^2 = \frac{m_P^2}{2J^2} \left( \frac{\hat{V}_{\text{MI}, \phi}}{V_{\text{MI}}} \right)^2 \approx \frac{4}{3f_T^2},
\]

(3.15b)

and

\[
\hat{\eta} = m_P^2 \frac{\hat{V}_{\text{MI}, \hat{\phi}}}{V_{\text{MI}}} = \frac{m_P^2}{J^2} \left( \frac{\hat{V}_{\text{MI}, \phi}}{V_{\text{MI}}} - \frac{\hat{V}_{\text{MI}, \phi}}{V_{\text{MI}}} J_{\phi} \right) \approx \frac{4(1 + f_T)}{3f_T^2}.
\]

(3.15c)

The termination of nSMI is triggered by the violation of the \( \epsilon \) criterion at a value of \( \phi \) equal to \( \phi_T \), which is calculated to be

\[
\hat{\epsilon}(\phi_T) = 1 \Rightarrow \phi_T = m_P \left( 1 + 2/\sqrt{3} \right)/c_T
\]

(3.16a)

since the violation of the \( \eta \) criterion occurs at \( \phi = \hat{\phi}_f \) such that

\[
\hat{\eta}(\hat{\phi}_f) = 1 \Rightarrow \hat{\phi}_f = m_P \left( 5/3c_T \right)^{1/2} < \phi_T.
\]

(3.16b)

On the other hand, \( \hat{N}_* \) can be calculated via the relation

\[
\hat{N}_* = \frac{1}{m_P^2} \int_{\hat{\phi}_f}^{\phi_\ast} d\hat{\phi} \frac{\hat{V}_{\text{MI}}}{V_{\text{MI}, \hat{\phi}}} = \frac{1}{m_P^2} \int_{\phi_T}^{\phi_\ast} d\phi J^2 \frac{\hat{V}_{\text{MI}}}{V_{\text{MI}, \phi}}.
\]

(3.17)

Given that \( \phi_T \ll \phi_\ast \), we can find a relation between \( \phi_\ast \) and \( \hat{N}_* \) as follows

\[
\hat{N}_* \approx \frac{3c_T}{4m_P^2} \left( \phi_\ast^2 - \hat{\phi}_f^2 \right) \Rightarrow \phi_\ast \approx 2m_P \sqrt{\hat{N}_*/3c_T}.
\]

(3.18a)

Obviously, nSMI with subplanckian \( \phi \)'s can be achieved if

\[
\phi_\ast \leq m_P \Rightarrow c_T \geq 4\hat{N}_*/3 \approx 76
\]

(3.18b)

for \( \hat{N}_* \approx 52 \). Therefore we need relatively large \( c_T \)'s.
3.2.2 The curvature perturbation

The amplitude $A_s$ of the power spectrum of the curvature perturbation generated by $\phi$ at the pivot scale $k_*$ is to be confronted with the data [1,2], i.e.

$$A_s^{1/2} = \frac{1}{2\sqrt{3} \pi m_P^2} \frac{\hat{V}_{\text{MI}}(\hat{\phi}_*)^{3/2}}{|\hat{V}_{\text{MI},\phi}(\hat{\phi}_*)|} = \frac{1}{2\pi m_P^2} \sqrt{\hat{V}_{\text{MI}}(\phi_*)} \approx 4.685 \cdot 10^{-5}. \quad (3.19)$$

Since the scalars listed in table 2 are massive enough during nSMI, the curvature perturbations generated by $\phi$ are solely responsible for $A_s$. Substituting eqs. (3.15b) and (3.18a) into the relation above, we obtain

$$\sqrt{A_s} = \frac{\lambda m_P^2 f_T(\phi_*)^2}{8\sqrt{2} c_T^2 \phi_*^2} \Rightarrow \lambda \simeq 6\pi \sqrt{2A_sc_T}/\hat{N}_s. \quad (3.20a)$$

Combining the last equality with eq. (3.19), we find that $\lambda$ is to be proportional to $c_T$, for almost constant $\hat{N}_s$. Indeed, we obtain

$$\lambda \simeq 3.97 \cdot 10^{-4} \pi c_T/\hat{N}_s \Rightarrow c_T \simeq 41637 \lambda \text{ for } \hat{N}_s \simeq 52. \quad (3.20b)$$

3.2.3 The other observables

The (scalar) spectral index $n_s$, its running $a_s$, and the scalar-to-tensor ratio $r$ must be consistent with the fitting [1,2] of the observational data, i.e.,

(a) $n_s = 0.96 \pm 0.014$, (b) $-0.0314 \leq a_s \leq 0.0046$ and (c) $r < 0.11 \quad (3.21)$

at 95% confidence level (c.l.). The observable quantities above can be estimated through the relations:

$$n_s = 1 - 6\epsilon_* + 2\eta_* \simeq 1 - 2/\hat{N}_s - 9/2\hat{N}_s^2, \quad (3.22a)$$

$$a_s = \frac{2}{3} \left(4\eta_*^2 - (n_s - 1)^2\right) - 2\xi_* \simeq -2\hat{\xi}_* \simeq -2/\hat{N}_s^3 + 3/2\hat{N}_s^3, \quad (3.22b)$$

$$r = 16\epsilon_* \simeq 12/\hat{N}_s^2. \quad (3.22c)$$

where $\hat{\xi} = m_P^4 \hat{V}_{\text{MI},\phi} \hat{V}_{MI,\phi^2} / \hat{V}_{MI}^2 = m_P \sqrt{2} \eta_* / J + 2\eta c$. The variables with subscript * are evaluated at $\phi = \phi_*$ and eqs. (3.15b) and (3.15c) have been employed.

3.3 The effective cut-off scale

As anticipated in eq. (3.18b), the realization of nSMI with subplanckian $\phi$’s requires relatively large $c_T$’s. This fact may [57–60] jeopardize the validity of the classical approximation, on which the analysis of the inflationary behavior is based. To see if this problem — which is rather questionable [32–34, 61–63] though — insists here, we have to extract the UV cut-off scale, $\Lambda_{\text{UV}}$, of the effective theory.

We first determine $\Lambda_{\text{UV}}$ analyzing the small-field behavior of the model in EF along the lines of ref. [11]. The EF action $S$ in eq. (2.4a) along the path of eq. (3.6) is written as

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} m_P^2 \hat{K} + \frac{1}{2} J^2 \phi^2 - \hat{V}_{\text{MI0}} + \cdots \right). \quad (3.23a)$$
Given the form of $J$ in eq. (3.9b) an expansion of the kinetic term in eq. (3.23a) about zero is not doable. Therefore we expand it about $\langle \phi \rangle = m_P / \sqrt{c_T}$ — see eqs. (2.2b) and (2.5) — and we find

$$J^2 \dot{\phi}^2 = 6c_T \left(1 - \frac{2 \sqrt{c_T} \delta \phi}{m_P} + \frac{3c_T \delta \phi^2}{m_P^2} - \frac{4c_T \sqrt{c_T} \delta \phi^3}{m_P^3} + \frac{5c_T^2 \delta \phi^4}{m_P^4} - \cdots \right) \dot{\phi}^2,$$  
(3.23b)

where $\delta \phi = (\phi - \langle \phi \rangle)$. Since there is no canonically normalized leading kinetic term, we define the canonically normalized inflaton at the SUSY vacuum $\hat{\delta} \phi = \sqrt{6c_T} \delta \phi$ — see also section 5.1 — and we reexpress eq. (3.23b) in terms of $\hat{\delta} \phi$, with result

$$J^2 \dot{\phi}^2 = \left(1 - \sqrt{\frac{2}{3}} \frac{\delta \phi}{m_P} + \frac{1}{2m_P^2} \frac{\delta \phi^2}{\sqrt{3}} + \frac{5}{36m_P^4} \delta \phi^4 - \cdots \right) \dot{\phi}^2.$$  
(3.23c)

On the other hand, $\hat{V}_{M\text{IO}}$ in eq. (3.4) can be expanded also in terms of $\hat{\delta} \phi$ as follows

$$\hat{V}_{M\text{IO}} = \frac{\lambda^2 m_P^2}{6c_T} \delta \phi^2 \left(1 - \sqrt{\frac{3}{2}} \frac{\delta \phi}{m_P} + \frac{25}{24m_P^3} \delta \phi^2 - \cdots \right).$$  
(3.23d)

From the derived expressions in eqs. (3.23c) and (3.23d) we conclude that $\Lambda_{UV} = m_P$ and therefore our model is valid up to $m_P$ as the original Starobinsky model [11].

The resulting $\Lambda_{UV}$ represents essentially the unitarity-violation scale [57–60] of the $\delta \phi - \delta \phi$ scattering process via $s$-channel graviton, $h^{\mu \nu}$, exchange in the JF. The relevant vertex is $c_T \delta \phi^2 \Box h / m_P$ — with $h = h^\mu_\mu$ — can be derived from the first term in the r.h.s. of eq. (2.4b) expanding the JF metric $g_{\mu \nu}$ about the flat spacetime metric $\eta_{\mu \nu}$ and the inflaton $\phi$ about its v.e.v as follows:

$$g_{\mu \nu} \simeq \eta_{\mu \nu} + h_{\mu \nu} / m_P \quad \text{and} \quad \phi = \langle \phi \rangle + \delta \phi.$$  
(3.24)

Retaining only the terms with two derivatives of the excitations, the part of the lagrangian corresponding to the two first terms in the r.h.s. of eq. (2.4b) takes the form

$$\delta \mathcal{L} = - \frac{\langle f_K \rangle}{4} F_{EH} (h^{\mu \nu}) + \left( m_P \langle f_K, \phi \rangle + \frac{c_T \delta \phi}{2m_P} \right) (\Box h - \partial_\mu \partial_\nu h^{\mu \nu}) \delta \phi + \cdots$$

$$= - \frac{1}{8} F_{EH} (\bar{h}^{\mu \nu}) + \left( \partial_\mu \delta \phi \partial_\mu \delta \phi \right) + \frac{c_T}{2 \sqrt{2} m_P} \frac{\sqrt{\langle f_K \rangle}}{\langle f_K \rangle} \delta \phi^2 \Box h + \cdots,$$  
(3.25a)

where the function $F_{EH}$, related to the linearized Einstein-Hilbert part of the lagrangian, reads

$$F_{EH} (h^{\mu \nu}) = h^{\mu \nu} \Box h_{\mu \nu} - h \Box h + 2 \partial_\mu h^{\mu \nu} \partial^\nu h_{\mu \nu} - 2 \partial_\mu h^{\mu \nu} \partial_\nu h$$  
(3.25b)

and the JF canonically normalized fields $\bar{h}_{\mu \nu}$ and $\delta \phi$ are defined by the relations

$$\bar{\delta} \phi = \sqrt{\frac{\langle f_K \rangle}{\langle f_k \rangle}} \delta \phi \quad \text{and} \quad \bar{h}_{\mu \nu} = \sqrt{\langle f_K \rangle} h_{\mu \nu} + \frac{m_P \langle f_K, \phi \rangle}{\sqrt{\langle f_K \rangle}} \eta_{\mu \nu} \delta \phi \quad \text{with} \quad f_K = 3m_P^2 f_{K, \phi}.$$  
(3.25c)

The interaction originating from the last term in the r.h.s. of eq. (3.25a) gives rise to a scattering amplitude which is written in terms of the center-of-mass energy $E$ as follows

$$A \sim \left( \frac{E}{\Lambda_{UV}} \right)^2 \quad \text{with} \quad \Lambda_{UV} = \frac{m_P}{3 \sqrt{2} c_T \sqrt{\langle f_K \rangle}} = m_P,$$  
(3.26)
where \( \langle f_K \rangle = 1/2 \) and \( \langle \tilde{f}_K \rangle = 3c_T \) and \( \Lambda_{\text{UV}} \) is identified as the UV cut-off scale in the JF, since \( A \) remains within the validity of the perturbation theory provided that \( E < \Lambda_{\text{UV}} \).

Although the expansions in eqs. (3.23d) and (3.25a) are obtained for \( \phi \simeq \langle \phi \rangle \) and are not valid [61–63] during nSMI, we consider \( \Lambda_{\text{UV}} \) as the overall UV cut-off scale of the model since reheating is an unavoidable stage of the inflationary dynamics [11]. Therefore, the validity of the effective theory implies [57–60]

\[
\hat{V}_{\text{MI}}(\phi)_{1/4} \ll \Lambda_{\text{UV}} \quad \text{with} \quad \Lambda_{\text{UV}} = m_P,
\]

which is much less restrictive than the corresponding condition applied in the models of nMI with quartic scalar potential, where \( \Lambda_{\text{UV}} \) turns out to be equal to \( m_P \) divided by the strength of the non-minimal coupling to gravity — cf. refs. [11, 38–40, 44].

### 3.4 Numerical results

As can be easily seen from the relevant expressions above, the inflationary dynamics of our model depends on the following parameters:

\[
\lambda, c_T, \lambda_\mu, k_{SS} = k_S, k_{SH}, k_{S^N}, M_{iN^c} \quad \text{and} \quad T_{rh}.
\]

Recall that \( M \) is related to \( c_T \) via eq. (2.5). Our results are essentially independent of \( \lambda_\mu \) and \( k \)'s, provided that we choose them so as \( \hat{m}_{\delta \phi}^2 \) and \( \hat{m}_{\delta \phi}^2 \) in table 2 are positive for every allowed \( \lambda \). We therefore set \( \lambda_\mu = 10^{-6} \), \( k_S = k_{S^N} = 1 \) and \( k_{SH} = 1.5 \) throughout our calculation. Moreover we take into account the contribution to \( \hat{V}_\text{MI} \), eq. (3.13), only from the heaviest \( N_i^c \) which is taken to be \( M_{3N^c} = 10^{14} \text{ GeV} \) — cf. section 5.5. We also choose \( \Lambda \simeq 10^{13} \text{ GeV} \) so as the one-loop corrections in eq. (3.13) vanish at the SUSY vacuum, eqs. (2.2b) and (2.5).

Finally \( T_{rh} \) can be calculated self-consistently in our model as a function of the inflaton mass, \( \hat{m}_{\delta \phi} \) and the strength of the various inflaton decays — see section 5.1. However, since the inflationary predictions depend very weakly on \( T_{rh} \) — see eq. (3.14) — we prefer to take here a constant \( T_{rh} = 6 \cdot 10^8 \text{ GeV} \) as suggested by our results on post-inflationary evolution — see section 5.5. Upon substitution of \( \hat{V}_\text{MI} \) from eq. (3.13) in eqs. (3.15a), (3.17) and (3.19) we extract the inflationary observables as functions of \( c_T, \lambda \) and \( \phi_* \). The two latter parameters can be determined by enforcing the fulfilment of eq. (3.14) and (3.19), for every chosen \( c_T \).

Our numerical findings are quite close to the analytic ones listed in section 3.2 for the sake of presentation.

The importance of the two extra variables \( (M \) and \( c_T \) — in eqs. (2.1c), (2.3a) and (2.3c) — compared to the Cecotti model [7–10] in reducing \( \phi_* \) below \( m_P \) can be easily inferred from figure 1. We there depict \( \hat{V}^{1/4}_\text{MI} \) as a function of \( \phi \) (both normalized to \( m_P \)) for \( \lambda = 2.26 \cdot 10^{-5} \) and \( c_T = 1 \) or \( \lambda = 0.0017 \) and \( c_T = 76 \) or \( \lambda = 0.1 \) and \( c_T = 4500 \) — the last value saturates an upper bound on \( c_T \) derived in section 4. Note that for \( c_T = 1 \) (or \( x_M = 1 \)) our result matches that of the original Starobinsky model [6, 95–97] — with the mass scale appearing in that model being replaced by \( \lambda m_P \simeq 2.2 \cdot 10^{13} \text{ GeV} \). Increasing \( c_T, \lambda \) increases too, whereas \( \phi_* \) and \( M \) decrease and for \( c_T \geq 76, \phi_* \) becomes subplanckian. On the other hand, we have to clarify that the corresponding values of the inflaton in the EF remain transplanckian, since integrating the first equation in eq. (3.9b) and using eqs. (3.18a) and (3.16a) we find:

\[
\phi = \hat{\phi}_c + \sqrt{6}m_P \ln (\phi/M) \Rightarrow \begin{cases} 
\hat{\phi}_c \simeq \hat{\phi}_c \sqrt{6}m_P \ln (\hat{N}_s/3)^{1/2} \\
\hat{\phi}_c \simeq \hat{\phi}_c \sqrt{6}m_P \ln (1 + 2/\sqrt{3})^{1/2}.
\end{cases}
\]

(3.28)
Figure 1. The inflationary scale $\hat{V}_{MI}^{1/4}$ as a function of $\phi$ for $\lambda = 2.26 \cdot 10^{-5}$ and $M = m_P$ ($c_T = 1$) or $\lambda = 1.7 \cdot 10^{-3}$ and $M/m_P = 0.115$ ($c_T = 76$) or $\lambda = 0.1$ and $M/m_P = 0.015$ ($c_T = 4500$). The values corresponding to $\phi_*$ and $\phi_\ell$ are also depicted.

where $\hat{\phi}_c$ is a constant of integration. E.g., setting $\hat{\phi}_c = 0$, we obtain $\hat{\phi}_* = 5.3m_P$ and $\hat{\phi}_\ell = 0.94m_P$ for any $c_T$ — with constant $N_*$. We do not consider this result as an upset of our proposal, since the inflaton field defined in the JF enters $W_{MI}$ and $K$. Therefore, possible corrections from non-renormalizable terms, which may be avoided for subplanckian values of inflaton, are applied in this frame, which is mostly considered as the physical frame.

From figure 1 we also infer that $\hat{V}_{MI}^{1/4}/m_P$ remains almost constant during nSMI. Indeed, if we plug eqs. (3.18a) and (3.20b) into eq. (3.4), we obtain

$$\hat{V}_{MI0}^{1/4}/m_P \simeq \left(3\pi\sqrt{2A_{\ell}/\hat{N}_*}\right)^{1/2} \simeq 0.0033 \ll 1.$$  \hspace{1cm} (3.29)

This result is more explicitly displayed in figure 2 too, where we draw the allowed values of $c_T$ (solid line), $10^5x_M$ (dashed line) and $10^3\hat{V}_{MI}(\phi_*)^{1/4}/m_P$ (dotted line) [$\phi_\ell$ (solid line) and $\phi_*$ (dashed line)] versus $\lambda$ (a) [(b)]. The lower bound of the depicted lines comes from the saturation of eq. (3.18b) whereas the upper bound originates from the perturbative bound on $\lambda$, $\lambda \leq \sqrt{4\pi} \simeq 3.54$. In figure 2a we see that eq. (3.27) is readily satisfied along the various curves and we can verify our analytic estimation in eq. (3.20b). Moreover, the variation of $\phi_\ell$ and $\phi_*$ as a function of $\lambda$ — drawn in figure 2b — is consistent with eqs. (3.16a) and (3.18a).

The overall allowed parameter space of our model is

$$76 \lesssim c_T \lesssim 1.5 \cdot 10^5, \quad 0.11 \gtrsim x_M \gtrsim 0.002 \quad \text{and} \quad 1.7 \cdot 10^{-3} \lesssim \lambda \lesssim 3.54 \quad \text{for} \quad \hat{N}_* \simeq 52.$$

(3.30a)

Letting $\lambda$ or $c_T$ vary within its allowed region in eq. (3.30a), we obtain

$$0.961 \lesssim n_s \lesssim 0.963, \quad -7.4 \lesssim a_s/10^{-4} \lesssim -6.7 \quad \text{and} \quad 4.2 \gtrsim r/10^{-3} \gtrsim 3.8,$$

(3.30b)

whereas the masses of the various scalars in table 2 remain well above $\hat{H}_{MI}$ both during and after nSMI for the selected $k_S, \lambda_\nu$ and $M_{3N\nu}$. E.g., for $\phi = \phi_*$ and $c_T = 150$, we obtain

$$\left(\hat{m}_\phi^2, \hat{m}_\nu^2, \hat{m}_{\text{h}^0}, \hat{m}_{\text{h}^+}, \hat{m}_{3\nu}^2\right)/\hat{H}_{MI}^2 \simeq (4,905,342,342,282).$$

(3.30c)
Figure 2. The allowed by eqs. (3.14), (3.19) and (3.27) values of $c_T$ (solid line), $10^3x_M$ (dashed line) and $10^3V_{MI}(\phi_*)^{1/4}/V_{UV}$ (dotted line) [$\phi_*$ (solid line) and $\phi_*$ (dashed line)] versus $\lambda$ (a) [(b)] for $k_S = 1$, $\lambda_\mu = 10^{-6}$, $M_{3Nc} = 10^{14}$ GeV and $T_{rh} = 6 \cdot 10^8$ GeV.

Clearly, the predicted $a_s$ and $r$ lie within the allowed ranges given in eq. (3.21b) and eq. (3.21c) respectively, whereas $n_s$ turns out to be impressively close to its central observationally favored value — see eq. (3.21a). Therefore, the inclusion of extra parameters, compared to the Cecotti model [7–10], does not affect the successful predictions on the inflationary observables.

4 The $R$ symmetry and the $\mu$ problem of MSSM

A byproduct of the $R$ symmetry associated with our model is that it assists us to understand the origin of $\mu$ term of MSSM. To see how this works, we write the part of the scalar potential which includes the SSB terms corresponding to $W_{MI}$ in eq. (2.1c). We have

$$V_{\text{soft}} = \lambda A_\lambda S T^2 + \lambda_\mu A_\mu S H_u H_d + B_{iNc} M_{iNc} N_i^c N_i^c - a_S S \lambda M^2 + \text{h.c.} + m_\alpha^2 |\Phi_\alpha|^2,$$

(4.1)

where $m_\alpha$, $A_\lambda$, $A_\mu$, $B_{iNc}$ and $a_S$ are SSB mass parameters. Rotating $S$ in the real axis by an appropriate $R$-transformation, choosing conveniently the phases of $A_\lambda$ and $a_S$ so as the total low energy potential $V_{\text{tot}} = V_{\text{SUSY}} + V_{\text{soft}}$ to be minimized — see eq. (2.2a) — and substituting in $V_{\text{soft}}$ the SUSY v.e.vs of $T, H_u, H_d$ and $N_i^c$ from eq. (2.2b) we get

$$\langle V_{\text{tot}}(S) \rangle = 2\lambda^2 M^2 S^2 - \lambda (|A_\lambda| + |a_S|) M^2 S,$$

(4.2a)

where we take into account that $m_S \ll M$. The minimization condition for the total potential in eq. (4.2a) w.r.t. $S$ leads to a non vanishing $\langle S \rangle$ as follows:

$$\frac{\partial}{\partial S} \langle V_{\text{tot}}(S) \rangle = 0 \Rightarrow \langle S \rangle \simeq (|A_\lambda| + |a_S|)/2\lambda.$$

(4.2b)

The generated $\mu$ term from the second term in the r.h.s. of eq. (2.1c) is

$$\mu = \lambda_\mu \langle S \rangle \simeq \lambda_\mu (|A_\lambda| + |a_S|)/2\lambda \sim \lambda_\mu m_{3/2}/\lambda.$$

(4.2c)
where $m_{3/2}$ is the $\tilde{G}$ mass. Due to eq. (3.12) the generation of the correct size of $\mu \geq 10^2$ GeV, as required by the radiative electroweak symmetry breaking, entails rather large $m_{3/2}$'s. Taking into account that relatively large $m_{3/2}$'s are necessitated in various versions [68–75] of MSSM in order for the mass $m_h$ of the lighter CP-even higgs boson, $h$, to be consistent with the latest LHC data [76, 77], we conservatively impose the bound $m_{3/2} \leq 10^6$ GeV. Combining the bounds above on $\mu$ and $m_{3/2}$ with those in eqs. (3.12) and (3.30a), we end up with the overall allowed regions of our model:

$$10^{-7} \lesssim \lambda_\mu \lesssim 10^{-5} \quad \text{and} \quad 1.7 \cdot 10^{-3} \lesssim \lambda \lesssim 0.1 \quad \Rightarrow \quad 76 \lesssim cT \lesssim 4500, \quad (4.3)$$

where $k_{SH} = 1.5$, we use eq. (3.20b) and we assume that the renormalization of the quantities above is negligible. Obviously the proposed resolution of the $\mu$ problem of MSSM relies on the existence of non-zero $A_\lambda$ and/or $a_S$ and the viability of the radiative electroweak symmetry breaking with $\mu \ll m_{3/2}$. These issues depend on the adopted model of SSB. We single out the following cases:

(i) If we wish to be fully consistent the no-scale structure of $K$ and suppose that the modulus, $z$, which is responsible for the SSB, is contained (somehow) in the logarithm of eq. (2.3a), $K$ is of the “sequestered-sector” form [105–107] and has the property that it generates no tree-level SSB scalar masses for the visible-sector fields and vanishing trilinear coupling constants. In this case the anomaly-mediated SSB [64–67, 105–107] is the dominant mechanism for obtaining $A_\lambda \neq 0$ and/or $a_S \neq 0$. Since the involved superfields $T$ and $S$ are $G_{SM}$ singlets, we expect $A_\lambda = 0$. However, according to the superconformal formalism, $M^2$ can be rescaled as $M^2 \varphi^2$ (where $\varphi$ is a superconformal compensator) and, in the presence of SSB, a non vanishing $a_S = 2m_{3/2}$ comes out. From the derived [68, 69] sparticle spectra in this kind of models the situation $\mu \ll m_{3/2}$ can be easily accommodated with $m_{3/2} \sim (10^5–10^6)$ GeV.

(ii) If we decide to deviate from the no-scale form of $K$ in eq. (2.3a), we can suppose that $z$ is not contained in the logarithm, and has an almost canonical Kähler potential [12, 13, 105–107]. In a such circumstance, both $A_\lambda$ and $a_S$ are expected to be non-zero, as in the gravity-mediated SSB [108], giving rise again to $\langle S \rangle \neq 0$. The hierarchy $\mu \ll m_{3/2}$ can be established in the so-called focus-point region [70–72] of the parameter space of MSSM, and more especially in portions of this region with low tuning and LSP abundance lower than the expectations [78].

In both cases above, our superpotential in eq. (2.1a) has to be extended by a SSB sector which should ensure the successful stabilization of $z$ — cf. refs. [12, 13, 105–107, 109–111]. We expect that these terms do not disturb the inflationary dynamics. Alternatively, the $\mu$ problem can be resolved [112] by imposing a Peccei-Quinn symmetry which is broken spontaneously at an intermediate scale by the v.e.vs of two $G_{SM}$ singlets which enter the superpotential via non-renormalizable terms. This scheme, already adopted, e.g., in refs. [39, 40, 113], can be applied as first realized in [112] in the case (ii) above and somehow modified in the case (i).

Let us clarify, finally, that the due hierarchy in eq. (3.12) between $\lambda_\mu$ and $\lambda$, is the inverse to that imposed in the models [47] of FHI, where $S$ plays the role of inflaton and $T$, $H_u$ and $H_d$ are confined at zero — playing the role of the waterfall fields. This is because, at the end of FHI, the mass squared of $T$ becomes negative for $S < M/\sqrt{2}$ and the mass matrix squared of the scalars $H_u - H_d$ develop a negative eigenvalue for $S < M/\sqrt{2}\lambda_\mu$. 

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Consequently, the correct cosmological scenario can be attained if we ensure that, at the end of FHI, $T$ acquires its v.e.v, while $H_u$ and $H_d$ remain equal to zero. To this end we demand \[ 47 \] $\lambda > \lambda_{\mu}$ so as the tachyonic instability in the $T$ direction occurs first, and $T$ start evolving towards its v.e.v, whereas $H_u$ and $H_d$ continue to be confined to zero. In our case, though, $|T|$ is the inflaton while $S$ and the $H_u - H_d$ system are safely stabilized at the origin both during and after the end of nSMI. Therefore, $|T|$ is led at its vacuum whereas $H_u$ and $H_d$ take their non-vanishing v.e.vs during the electroweak phase transition triggered by radiative corrections.

5 Non-thermal leptogenesis and neutrino masses

We below specify how our inflationary scenario makes a transition to the radiation dominated era (section 5.1) and give an explanation of the observed BAU (section 5.2) consistently with the $\tilde{G}$ constraint and the low energy neutrino data (section 5.3). Our results are summarized in section 5.5.

5.1 The inflaton decay

When nSMI is over, the inflaton continues to roll down towards the SUSY vacuum, eq. (2.2b). Soon after, it settles into a phase of damped oscillations around the minimum of $\hat{V}_{\text{MI0}}$ — note that $\theta$ is stabilized during and after nSMI at the origin and so, it does not participate neither into inflationary nor to post-inflationary dynamics. The (canonically normalized) inflaton, $\hat{\delta} \phi = \sqrt{6} \frac{\hat{V}_{\text{MI0}, \hat{\phi} \phi}}{m_P} = \lambda \frac{\hat{V}_{\text{MI0}, \hat{\phi} \phi}}{\sqrt{2} m_P} \simeq 3 \cdot 10^{13}$ GeV, (5.1)

where we make use of eq. (3.20b) in the last step. Since eq. (3.8) implies $<x_\phi> = 1/\sqrt{c_T}$ — see eqs. (3.3), (2.2b) and (2.5) —, the EF canonically normalized fields $\Phi^A$ in eq. (2.3b) are not distinguished from the JF ones at the SUSY vacuum.

The decay of $\hat{\delta} \phi$ is processed through the following decay channels:

5.1.1 Decay channel into $N^c_i$’s

The lagrangian which describes these decay channels arises from the part of the SUGRA langrangian \[ 108 \] containing two fermions. In particular,

$$\mathcal{L}_{\hat{\delta} \phi \rightarrow N^c_i} = -\frac{1}{2} e^{K/2m_P^2} W_{,N^c_i N^c_i} N^c_i N^c_i + \text{h.c.} = \frac{3}{2} \frac{M}{m_P} c_T^{1/2} \hat{\delta} \phi \ N^c_i N^c_i + \cdots$$

where an expansion around $\langle \phi \rangle$ is performed in order to extract the result above. We observe that although there is not direct coupling between $T$ and $N_i^c$ in $W_{\text{MI}}$ — recall that we assume that the third term in the r.h.s. of eq. (2.1c) prevails over the last one —, an adequately efficient decay channel arises, which gives rise to the following decay width

$$\hat{\Gamma}_{\hat{\delta} \phi \rightarrow N^c_i} = \frac{1}{16 \pi \lambda_{N^c_i}^2 \hat{m}_{\delta \phi}} \left( 1 - 4 M_{N^c_i}^2 / \hat{m}_{\delta \phi}^2 \right)^{3/2},$$

where we take into account that $\hat{\delta} \phi$ decays to identical particles.
5.1.2 Decay channel into $H_u$ and $H_d$

The lagrangian term which describes the relevant interaction comes from the F-term SUGRA scalar potential in eq. (3.1). Namely, we obtain

$$
\mathcal{L}_{\delta\phi\rightarrow H_uH_d} = -\frac{1}{2} e^{K/m_{\delta\phi}^2} K^{SS*} |W_S|^2 = -\frac{1}{2} \lambda \nu (\phi^2 - M^2) H_u H_d^* + \cdots
$$

$$
= -\lambda_H \delta\phi H_u^* H_d^* + \cdots \text{ with } \lambda_H = \lambda_\nu / \sqrt{2}.
$$

This interaction gives rise to the following decay width

$$
\hat{\Gamma}_{\delta\phi\rightarrow H} = \frac{2}{8\pi} \lambda_H \delta\phi,
$$

where we take into account that $H_u$ and $H_d$ are SU(2)$_L$ doublets. Eq. (3.12) facilitates the reduction of $\hat{\Gamma}_{\delta\phi\rightarrow H}$ to a level which allows for the decay mode into $N_i^c$’s playing its important role for nTL.

5.1.3 Three-particle decay channels

Focusing on the same part of the SUGRA langrangian [108] as in the paragraph 5.1.1, for a typical trilinear superpotential term of the form $W_y = y XYZ$ — cf. eq. (2.1b) —, where $y$ is a Yukawa coupling constant, we obtain the interactions described by

$$
\mathcal{L}_{\delta\phi\rightarrow XYZ} = -\frac{1}{2} e^{K/2m^{2}_\delta\phi} (W_{y,YZ}\psi_Y\psi_Z + W_{y,YX}\psi_Y\psi_X + W_{x,YZ}\psi_Y\psi_Y) + \text{h.c.}
$$

$$
= \lambda_y \frac{\delta\phi}{m_p} (X\psi_Y\psi_Z + Y\psi_Y\psi_z + Z\psi_Y\psi_Y) + \text{h.c.} \text{ with } \lambda_y = \sqrt{3/2}(y/2),
$$

where $\psi_X$, $\psi_Y$ and $\psi_Z$ are the chiral fermions associated with the superfields $X$, $Y$ and $Z$ whose the scalar components are denoted with the superfield symbol. Working in the large tan $\beta$ regime which yields similar $y$‘s for the 3rd generation, we conclude that the interaction above gives rise to the following 3-body decay width

$$
\hat{\Gamma}_{\delta\phi\rightarrow XYZ} = \frac{14n_f}{512\pi^3} \lambda_y^2 \delta\phi^2 m_p^2,
$$

where for the third generation we take $y \simeq (0.4-0.6)$, computed at the $\delta\phi$ scale, and $n_f = 14$ [$n_f = 16$] for $\delta\phi < M_{3N_i}^c$ [$\delta\phi > M_{3N_i}^c$] — summation is taken over SU(3)$_c$ and SU(2)$_L$ indices.

Since the decay width of the produced $N_i^c$ is much larger than $\hat{\Gamma}_{\delta\phi}$ the reheating temperature, $T_{rh}$, is exclusively determined by the inflaton decay and is given by [114]

$$
T_{rh} = \left( \frac{72}{5\pi^2 g_*} \right)^{1/4} \sqrt{\hat{\Gamma}_{\delta\phi} m_p} \text{ with } \hat{\Gamma}_{\delta\phi} = \hat{\Gamma}_{\delta\phi\rightarrow N_i^c} + \hat{\Gamma}_{\delta\phi\rightarrow H} + \hat{\Gamma}_{\delta\phi\rightarrow XYZ},
$$

where $g_* \approx 228.75$ counts the effective number of relativistic degrees of freedom of the MSSM spectrum at the temperature $T \simeq T_{rh}$. Let us clarify here that in our models there is no decay of a scalaron as in the original (non-SUSY) [3–5, 22, 23] Starobinsky inflation and some [14–20] of its SUGRA realizations; thus, $T_{rh}$ in our case is slightly lower than that obtained there. Indeed, spontaneous decay of the inflaton to scalars takes place only via three-body interactions which are suppressed compared to the two-body decays of scalaron. On the other hand, we here get also $\hat{\Gamma}_{\delta\phi\rightarrow H}$ in eq. (5.3b), due to explicit coupling of $\delta\phi$ into $H_u$ and $H_d$, which can be kept at the same level with $\hat{\Gamma}_{\delta\phi\rightarrow XYZ}$ due to the rather low $\lambda_\nu$‘s required here — see eq. (3.12).
5.2 Lepton-number and gravitino abundances

The mechanism of nTL [81–83] can be activated by the out-of-equilibrium decay of the $N_i^c$'s produced by the $\delta \phi$ decay, via the interactions in eq. (5.2a). If $T_{rh} \ll M_{N_i^c}$, the out-of-equilibrium condition [115, 116] is automatically satisfied. Namely, $N_i^c$ decay into (fermionic and bosonic components of) $H_u$ and $L_i$ via the tree-level couplings derived from the last term in the r.h.s. of eq. (2.1b). The resulting — see section 5.3 — lepton-number asymmetry $\varepsilon_i$ (per $N_i^c$ decay) after reheating can be partially converted via sphaleron effects into baryon-number asymmetry. In particular, the $B$ yield can be computed as

$$ (a) \quad Y_B = -0.35 Y_L \quad \text{with} \quad (b) \quad Y_L = \frac{5}{4} \frac{T_{rh}}{\hat{m}_{\delta \phi}} \sum_{i=1}^{3} \frac{\hat{\Gamma}_{\delta \phi \rightarrow N_i^c}}{\hat{\Gamma}_{\delta \phi}} \varepsilon_i, \quad (5.6) $$

The numerical factor in the r.h.s. of eq. (5.6a) comes from the sphaleron effects, whereas the one $(5/4)$ in the r.h.s. of eq. (5.6b) is due to the slightly different calculation [114] of $T_{rh}$ — cf. refs. [115, 116].

The required for successful nTL $T_{rh}$ must be compatible with constraints on the $\tilde{G}$ abundance, $Y_{\tilde{G}}$, at the onset of nucleosynthesis (BBN). This is estimated to be [86–91]:

$$ Y_{\tilde{G}} \simeq 1.9 \cdot 10^{-22} T_{rh} / \text{GeV}, \quad (5.7) $$

where we assume that $\tilde{G}$ is much heavier than the gauginos of MSSM. Let us note that non-thermal $\tilde{G}$ production within SUGRA is [79] also possible but strongly dependent on the mechanism of SSB. It can be easily suppressed [80, 117, 118] when a tiny mixing arises between the inflaton and the field responsible for SSB provided that the mass of the latter is much lower than the inflationary scale. Therefore, we here prefer to adopt the conservative estimation of $Y_{\tilde{G}}$ in eq. (5.7).

Both eqs. (5.6) and (5.7) calculate the correct values of the $B$ and $\tilde{G}$ abundances provided that no entropy production occurs for $T < T_{rh}$. This fact can be achieved if the Polonyi-like field $z$ decays early enough without provoking a late episode of secondary reheating. In both cases of section 4, $z$ is expected to be displaced from its true minimum to lower values due to large mass that it acquires during nSMI. In the course of the decaying-inflaton period which follows nSMI, $z$ adiabatically tracks an instantaneous minimum [119–121] until the Hubble parameter becomes of the order of its mass. Successively it starts to oscillate about the true SUSY breaking minimum and may or may not dominate the Universe, depending on the initial amplitude of the coherent oscillations. The domination may be eluded in a very promising scenario [12, 13, 109–111] which can be constructed assuming that $z$ is strongly stabilized through a large enough coupling in a higher order term of Kähler potential, similar to that used for the stabilization of $S$ — see eq. (2.3a). A subsequent difficulty is the possible over-abundance of the LSPs which are produced by the $z$ decay. From that perspective, it seems that the case (ii) — cf. refs. [109–111, 119–121] — is more tolerable than the case (i) — see refs. [122, 123].

5.3 Lepton-number asymmetry and neutrino masses

As mentioned in section 5.2, the decay of $N_i^c$ emerging from the $\delta \phi$ decay, can generate [124–126] a lepton asymmetry $\varepsilon_i$ caused by the interference between the tree and one-loop decay diagrams, provided that a CP-violation occurs in $h_{ijN}$'s — see eq. (2.1b). The produced $\varepsilon_i$
can be expressed in terms of the Dirac mass matrix of $\nu_i$, $m_D$, defined in the $N_i^c$-basis, as follows:

$$
\varepsilon_i = \sum_{j \neq i} \frac{\text{Im} \left( (m_D^\dagger m_D)^2 \right)_{ij}}{8\pi (H_u)^2 (m_D^\dagger m_D)_{ii}} \left( F_S (x_{ij}) + F_V (x_{ij}) \right),
$$

(5.8a)

where $x_{ij} := M_{jN^c}/M_{iN^c}$, $(H_u) \simeq 174$ GeV, for large $\tan \beta$ and the functions $F_{V,S}$ read \[124–126\]

$$
F_V (x) = -x \ln (1 + x^{-2}) \quad \text{and} \quad F_S (x) = \frac{-2x}{x^2 - 1}.
$$

(5.8b)

Also $m_D$ is the Dirac mass matrix of $\nu_i$'s and $m_D^\dagger m_D$ in eq. (5.8a) can be written as follows:

$$
m_D^\dagger m_D = U^c d_D^\dagger d_D U^c.
$$

(5.8c)

where $U^c$ are the $3 \times 3$ unitary matrix which relates $N_i^c$ in the $N_i^c$-basis with the corresponding in the weak basis. With the help of the seesaw formula, $m_D$ and $M_{iN^c}$ involved in eq. (5.8a) can be related to the light-neutrino mass matrix $m_{\nu}$. Working in the $N_i^c$-basis, we have

$$
m_\nu = -m_D d_{N^c}^{-1} m_D^T
$$

with $M_{1N^c} \leq M_{2N^c} \leq M_{3N^c}$ real and positive. Based on the analysis of \[43, 128\], we find $m_{\nu}$ via

$$
m_\nu = U_{\nu}^c d_{\nu} U_{\nu}^\dagger
$$

where $d_{\nu} = \text{diag} (m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$

(5.10)

with $m_{\nu_1}$, $m_{\nu_2}$ and $m_{\nu_3}$ being the real and positive light neutrino mass eigenvalues. These can be found assuming normal [inverted] ordered (NO [IO]) $m_{\nu}$'s and using a reference neutrino mass and the observed \[92, 93\] low energy neutrino mass-squared differences. Also $U_{\nu}$ is the PMNS matrix which is a function of the mixing angles $\theta_{ij}$ and the CP-violating Majorana ($\varphi_1$ and $\varphi_2$) and Dirac ($\delta$) phases. Taking also $m_{id}$ as input parameters we can construct the complex symmetric matrix

$$
W = -d_D^{-1} m_\nu d_D^{-1}
$$

(5.11)

from which we can extract $d_{N^c}$ as follows \[43, 128\]:

$$
d_{N^c}^{-2} = U^c W W^\dagger U^c.
$$

(5.12)

Acting this way — see section 5.5 —, we can determine the elements of $U^c$ and the $M_{iN^c}$'s, compute $m_D^\dagger m_D$ through eq. (5.8c) and finally obtain the $\varepsilon_i$'s via eq. (5.8a).

5.4 Post-inflationary requirements

The success of our post-inflationary scenario can be judged, if, in addition to the constraints of section 3.2, it is consistent with the following requirements:

5.4.1 The bounds on $M_{1N^c}$

We impose the following bounds on $M_{1N^c}$:

(a) $M_{1N^c} \gtrsim 10 T_{th}$ and (b) $\tilde{m}_{\delta \phi} \gtrsim 2 M_{1N^c}$.

(5.13)

The first inequality is applied to avoid any erasure of the produced $Y_L$ due to $\nu^c_i$ mediated inverse decays and $\Delta L = 1$ scatterings \[128\]. The second bound ensures that the decay of $\delta \phi$ into a pair of $N_i^c$'s is kinematically allowed for at least one species of the $N_i^c$'s.
5.4.2 Constraints from neutrino physics

We take as inputs the best-fit values [92] — see also [93] — on the neutrino mass-squared differences, $\Delta m_{21}^2 = 7.62 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = (2.55 [2.43]) \times 10^{-3} \text{ eV}^2$, on the mixing angles, $\sin^2 \theta_{12} = 0.32$, $\sin^2 \theta_{13} = 0.0246 [0.025]$, and $\sin^2 \theta_{23} = 0.613 [0.6]$ and the Dirac phase $\delta = 0.8 \pi [0.03 \pi]$ for NO [IO] $m_{\nu}$’s. Moreover, the sum of $m_{\nu}$’s is bounded from above by the current data [1, 78], as follows

$$\sum_i m_{\nu} \leq 0.28 \text{ eV at 95\% c.l.}$$ (5.14)

5.4.3 The observational results on $Y_B$

We take [1, 78]:

$$Y_B \simeq (8.55 \pm 0.217) \times 10^{-11} \text{ at 95\% c.l.}$$ (5.15)

5.4.4 The bounds on $Y_{3/2}$

Successful BBN entails [88–91]:

$$Y_{3/2} \lesssim \begin{cases} 10^{-14} & \text{for } m_{3/2} \simeq 0.69 \text{ TeV}, \\ 10^{-13} & \text{for } m_{3/2} \simeq 10.6 \text{ TeV}, \\ 10^{-12} & \text{for } m_{3/2} \simeq 13.5 \text{ TeV}. \end{cases}$$ (5.16)

Here we consider the conservative case where $\tilde{G}$ decays with a tiny hadronic branching ratio.

5.5 Numerical results

As shown in section 5.1, nSMI predicts a constant value of $\tilde{m}_{\delta \phi}$. Consequently, $T_{th}$ and $Y_B$ — see eqs. (5.5) and (5.6) — are largely independent of the precise value of $c_T$ and $\lambda$ in the range of eq. (3.30a) — contrary to the case of FHI [45, 46, 113]. Just for definiteness we specify that throughout this section we take $c_T = 150$ which corresponds to $\lambda = 0.0034$, $n_b = 0.963$ and $\tilde{m}_{\delta \phi} = 3 \times 10^{13} \text{ GeV}$. On the other hand, $T_{th}$ and $Y_B$ depend on $\lambda_{\mu}$, $y$ and the masses of the $N_i$’s into which $\delta \phi$ decays. Throughout our computation we take $y = 0.5$, which is a typical value encountered [98] into various MSSM settings with large $\tan \beta$, and so the corresponding decay width via eq. (5.4b) is confined to $\tilde{\Gamma}_{\delta \phi \rightarrow XYZ} = 0.45 \text{ GeV}$. Note that varying $y$ in its plausible [98] range $(0.4–0.6)$, $\tilde{\Gamma}_{\delta \phi \rightarrow XYZ}$ ranges from 0.28 to 0.64 GeV causing minor changes to our results.

Following the bottom-up approach described in section 5.3, we find the $M_{iN_i}$’s by using as inputs the $m_{iD}$’s, a reference mass of the $\nu_i$’s — $m_{1\nu}$ for NO $m_{i\nu}$’s, or $m_{3\nu}$ for IO $m_{i\nu}$’s —, the two Majorana phases $\varphi_1$ and $\varphi_2$ of the PMNS matrix, and the best-fit values, mentioned in section 5.4, for the low energy parameters of neutrino physics. In our numerical code, we also estimate, following [127], the RG evolved values of the latter parameters at the scale of nTL, $\Lambda_L = \tilde{m}_{\delta \phi}$, by considering the MSSM with tan $\beta \simeq 50$ as an effective theory between $\Lambda_L$ and the SSB scale, $M_{\text{SUSY}} = 1.5 \text{ TeV}$. We evaluate the $M_{iN_i}$’s at $\Lambda_L$, and we neglect any possible running of the $m_{iD}$’s and $M_{iN_i}$’s. Therefore, we present their values at $\Lambda_L$.

Fixing $\lambda_{\mu}$ at an intermediate value in its allowed region — see eq. (4.3) — $\lambda_{\mu} = 10^{-6}$ which results, via eq. (5.3b) in $\tilde{\Gamma}_{\delta \phi \rightarrow H} = 1.3 \text{ GeV}$ we can get a first picture for the parameters which yield $Y_B$ and $Y_{3/2}$ compatible with eqs. (5.15) and (5.16), respectively in table 3. We consider strongly NO (cases A and B), almost degenerate (cases C, D and E) and strongly IO (cases F and G) $m_{i\nu}$’s. In all cases the current limit of eq. (5.14) is safely met — in the case
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<tr>
<td><strong>Leptogenesis-Scale Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$m_{1D}/0.1 \text{ GeV}$</td>
<td>16</td>
</tr>
<tr>
<td>$m_{2D}/\text{GeV}$</td>
<td>40</td>
</tr>
<tr>
<td>$m_{3D}/10 \text{ GeV}$</td>
<td>10</td>
</tr>
<tr>
<td>$M_{1N_c}/10^{11} \text{ GeV}$</td>
<td>12.3</td>
</tr>
<tr>
<td>$M_{2N_c}/10^{12} \text{ GeV}$</td>
<td>22.2</td>
</tr>
<tr>
<td>$M_{3N_c}/10^{14} \text{ GeV}$</td>
<td>25</td>
</tr>
<tr>
<td><strong>Open Decay Channels of the Inflaton, $\delta\phi$, Into $N_{i}^c$</strong></td>
<td></td>
</tr>
<tr>
<td>$\delta\phi \rightarrow N_{i}^c$</td>
<td>$N_{1}^c$</td>
</tr>
<tr>
<td>$\Gamma_{\delta\phi \rightarrow N_{i}^c}$ (%)</td>
<td>3</td>
</tr>
<tr>
<td><strong>Resulting $B$-Yield</strong></td>
<td></td>
</tr>
<tr>
<td>$10^{11}Y_B$</td>
<td>8.54</td>
</tr>
<tr>
<td><strong>Resulting $T_{th}$ and $\tilde{G}$-Yield</strong></td>
<td></td>
</tr>
<tr>
<td>$T_{th}/10^8 \text{ GeV}$</td>
<td>5.9</td>
</tr>
<tr>
<td>$10^{13}Y_{3/2}$</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 3. Parameters yielding the correct BAU for various neutrino mass schemes, $k_{SH} = 1.5$, $\lambda_\mu = 10^{-6}$ and $y = 0.5$. Shown also are the branching ratios of the $\delta\phi$ decay into $N_{i}^c$ with $i = 2$ except for the case A where $i = 1$. Recall that these results are independent of the variables $\lambda, c_T, k_S$ and $k_{S\tilde{N}_c}$. 


D this limit is almost saturated. We observe that with NO or IO \( m_{\nu} \)'s, the resulting \( M_{iN'} \)'s are also hierarchical. With degenerate \( m_{\nu} \)'s, the resulting \( M_{\nu} \)'s are closer to one another. Consequently, in the latter case more \( \delta \phi \)-decay channels are available, whereas for the case A only a single decay channel is open. In all other cases — even in the case C where the decay channel \( \delta \phi \rightarrow N_3^cN_3^c \) is kinematically permitted —, the dominant contributions to \( Y_B \) arise from \( \varepsilon_2 \). Therefore, the branching ratios, which are also presented in table 3, \( \tilde{\Gamma}_{\delta \phi \rightarrow N_i^c} \slash \tilde{\Gamma}_{\delta \phi} \) with \( i = 1 \) for the case A and \( i = 2 \) for the other cases are crucial for the calculation \( Y_B \) from eq. (5.6). We notice that these ratios introduce a considerable reduction in the derivation of \( Y_B \), given that \( \Gamma_{\delta \phi \rightarrow n_i^c} \lesssim \Gamma_{\delta \phi \rightarrow X} \lesssim \Gamma_{\delta \phi \rightarrow H} \). This reduction can be eluded if we adopt — as in refs. [39, 40, 45, 46, 113] — the resolution of the \( \mu \) problem proposed in [112] since then the decay mode in eq. (5.3a) disappears. In table 3 shown also are the values of \( \Gamma_{\delta \phi} \) — as in refs. [39, 40, 128] — the resolution of the \( \mu \) problem of MSSM, as explained in section 4.

Since we do not consider any particular GUT here, the \( m_{iD} \)'s are free parameters. For the sake of comparison, however, we mention that the simplest realization of a SUSY Left-Right [Pati-Salam] GUT predicts [113, 129] \( h_{iN} = h_{iE} \ [m_{iD} = m_{iU}] \), where \( m_{iU} \) are the masses of the up-type quarks and we ignore any possible mixing between generations. Taking into account the SUSY threshold corrections [98] in the context of MSSM with universal gaugino masses and \( \tan \beta \approx 50 \) — favored by the recent LHC results [76, 77] — these predictions are translated as follows:

\[
(m^0_{1D}, m^0_{2D}, m^0_{3D}) \simeq \begin{cases} 
(0.023, 4.9, 100) \text{ GeV} & \text{for a Left-Right GUT,} \\
(0.0005, 0.24, 100) \text{ GeV} & \text{for a Pati-Salam GUT.} 
\end{cases} \tag{5.17}
\]

Comparing these values with those listed in table 3, we remark that our model is not compatible with any GUT-inspired pattern of large hierarchy between the \( m_{iD} \)'s, especially in the two lighter generations, since \( m_{1D} \gg m^0_{2D} \) and \( m_{2D} > m^0_{3D} \). On the other hand, in the cases A, B, D, E and F we are able to place \( m_{3D} \approx m^0_{3D} \). This arrangement can be understand if we take into account that \( m_{1D} \) and \( m_{2D} \) separately influences the derivation of \( M_{1N'} \) and \( M_{2N'} \) respectively — see, e.g., refs. [39, 40, 128]. Consequently, the displayed \( m_{2D} \sim 10 \text{ GeV} \) assists us to obtain the \( \varepsilon_2 \)'s required by eq. (5.15) — note that in the case A \( m_{2D} \approx 40 \text{ GeV} \) kinematically blocks the channel \( \delta \phi \rightarrow N_3^cN_3^c \). On the other hand, \( m_{1D} \gtrsim 0.5 \text{ GeV} \) is necessitated in order to obtain the observationally favored \( \varepsilon_1 \) in the case A and fulfill eq. (5.13a) in the other cases. Note that the phases \( \varphi_1 \) and \( \varphi_2 \) in table 3 are selected in each case, so that the required \( m_{iD} \) and \( M_{iN'} \), which dominate the \( Y_B \) calculation, and the resulting \( T_{th} \) are almost minimized.

In order to extend the conclusions inferred from table 3 to the case of a variable \( \lambda_\mu \), we can examine how the central value of \( Y_B \) in eq. (5.15) can be achieved by varying \( m_{2D} \) as a function of \( \lambda_\mu \). The resulting contours in the \( \kappa - m_{2D} \) plane are presented in figure 3 — since the range of \( Y_B \) in eq. (5.15) is very narrow, the 95% c.l. width of these contours is negligible. The convention adopted for these lines is also described in the figure. In particular, we use solid, dashed, or dot-dashed line for \( m_{\nu}, m_{1D}, m_{3D}, \varphi_1, \) and \( \varphi_2 \) corresponding to the cases B, D, or F of table 3 respectively. Since increasing \( \lambda_\mu \), the resulting \( T_{th} \) is expected to get larger than that shown in table 3 — see eqs. (5.3b) and (5.5) — we select for the plot in figure 3 one case from every low-energy mass scheme of \( m_{\nu} \)'s with \( M_{iN'} \) large enough, such that eq. (5.13a) is comfortably satisfied for every \( \lambda_\mu \) within the range of eq. (4.3) with \( k_{SH} = 1.5 \). This equation sets, actually, the limits on the contours depicted in figure 3. For
\( \lambda_\mu \gtrsim 6 \cdot 10^{-7} \) we get \( \hat{\Gamma}_{\delta\phi \rightarrow H} > \hat{\Gamma}_{\delta\phi \rightarrow XYZ} \) and so, increasing \( \lambda_\mu \) the branching fraction in eq. (5.6b) drops and larger \( m_{2D} \)'s are required to obtain \( Y_B \) compatible with eq. (5.15). On the other hand, for \( \lambda_\mu \lesssim 6 \cdot 10^{-7} \), \( \hat{\Gamma}_{\delta\phi \rightarrow XYZ} \) gets larger than \( \hat{\Gamma}_{\delta\phi \rightarrow H} \) and so, the branching fraction in eq. (5.6b) remains almost constant and no sizable variation of \( m_{2D} \) is required. At the upper termination points of the contours, we obtain \( T_{rh} \approx 5 \cdot 10^9 \) GeV or \( \tilde{G} \approx 9.4 \cdot 10^{-13} \). The constraint of eq. (5.16), therefore, will cut any possible extension of the curves would be available for possible larger \( \lambda_\mu \)'s. Along the depicted contours, the resulting \( M_{2N_c} \)'s vary in the range \((1.4–4) \cdot 10^{12} \) GeV whereas \( M_{1N_c} \) and \( M_{3N_c} \) remain close to their values presented in the corresponding cases of table 3.

In conclusion, nTL is a realistic possibility within our model, thanks to the spontaneously arising couplings in SUGRA, even without direct couplings of the inflaton to \( N_{c_i} \)'s in \( W \).

### 6 Conclusions

We investigated a variant of the Starobinsky inflation, which can be embedded in a moderate extension of MSSM supplemented by three \( N_{c_i} \)'s and two more superfields, the inflaton and an accompanied field. Key role in our proposal plays a continuous \( R \) symmetry, which is reduced to the well-known \( R \)-parity of MSSM, a \( \mathbb{Z}_2 \) discrete symmetry and a no-scale-type symmetry imposed on the Kähler manifold. The adopted symmetries have a number of ensuing consequences: (i) The inflaton appears quadratically in the super- and Kähler potentials; (ii) it couples to \( N_{c_i} \) via SUGRA-induced interactions ensuring low \( T_{rh} \) and no important contributions to the one-loop radiative corrections; (iii) the \( \mu \) problem of MSSM can be elegantly resolved provided that a related parameter in superpotential is somehow suppressed. The last issue can be naturally incorporated in various schemes of SSB with relatively large — of the order \((10^4–10^6) \) GeV — \( m_{3/2} \)'s which facilitate the explanation of the recently observed mass of the electroweak Higgs and the satisfaction of the \( \tilde{G} \) constraint.

The next important modification of our set-up compared to other incarnations — cf. refs. [7–9] — of the Starobinsky inflation in SUGRA is the introduction of a variable scale
(M) — besides the existing one in refs. [7–9] — in the superpotential and a parameter (cT) in the Kähler potential which was ultimately confined in the range \( 76 \leq c_T \leq 4500 \). One of these parameters (M and cT) can be eliminated demanding that the gravitational strength takes its conventional value at the SUSY vacuum of the theory. Actually our inflationary model interpolates between the Starobinsky [3–5] and the induced-gravity [48–54] inflation. Variation of the free model parameters (\( \lambda \) and cT) gives us the necessary flexibility in order to obtain inflation for subplanckian values of the inflaton. Consequently, our proposal is stable against possible corrections from higher order terms in the super- and/or Kähler potentials. Moreover, we showed that the one-loop radiative corrections remain subdominant during inflation and the corresponding effective theory is trustable up to \( m_p \).

Despite the addition of the extra parameters, our scheme remains very predictive since all the possible sets (\( \lambda, c_T \)) which are compatible with the two inflationary requirements, concerning the number of the e-foldings and the normalization of the curvature perturbation, yield almost constant values of \( r \) and \( n_s \) and a unique inflaton mass, \( \hat{m}_{\delta \phi} \). In particular, we find \( n_s \simeq 0.963 \), \( a_{as} \simeq -0.00068 \) and \( r \simeq 0.0038 \), which are in excellent agreement with the current data, and \( \hat{m}_{\delta \phi} = 3 \cdot 10^{13} \text{ GeV} \). Moreover, the post-inflationary evolution within our model remains intact from the variation of the inflationary parameters (\( \lambda \) and cT). Implementing the (type I) seesaw mechanism for the generation of the light neutrino masses, we restricted their Dirac masses, \( m_{1D} \), and the masses of \( N_i^c \)'s, \( M_{1N^c} \), fulfilling a number of requirements, which originate from the BAU, the (unstable) \( \tilde{G} \) abundance and the neutrino oscillation parameters. Namely, we found \( m_{1D} \geq 0.5 \text{ GeV} \) and \( m_{2D} \simeq 10 \text{ GeV} \) resulting mostly to \( M_{1N^c} \simeq 10^{11} \text{ GeV} \) and \( M_{2N^c} \simeq 10^{12} \text{ GeV} \).

As a bottom line, we would like to emphasize that the Starobinsky-type inflation in no-scale SUGRA can be linked to the phenomenology of MSSM, even if it is not realized by a matter-like inflaton as in refs. [24, 25]. In our framework, this type of inflation, driven by a modulus-like field, suggests a resolution of the \( \mu \) problem of MSSM, compatible with large values of \( m_3/2 \) and it is followed by a robust cosmological scenario — already applied in many inflationary settings [39, 40, 43, 45, 46, 80–83, 113] — ensuring spontaneous nTL reconcilable with the \( \tilde{G} \) constraint and the neutrino oscillation parameters.

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References


