Soft SUSY Breaking Grand Unification: Leptons vs Quarks on the Flavor Playground


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We systematically analyze the correlations between the various leptonic and hadronic flavor violating processes arising in SUSY Grand Unified Theories. Using the GUT-symmetric relations between the soft SUSY breaking parameters, we assess the impact of hadronic and leptonic flavor observables on the SUSY sources of flavor violation.

I. INTRODUCTION

Supersymmetry (SUSY) Breaking (SB) remains one of the biggest issues in physics beyond the Standard Model (SM). In spite of various proposals [1], we still miss a realistic and theoretically satisfactory model of SB.

Flavor violating processes have been instrumental in guiding us towards consistent SB models. Indeed, even in the absence of a well-defined SB mechanism and, hence, without a precise knowledge of the SUSY Lagrangian at the electroweak scale, it is still possible to make use of the Flavour Changing Neutral Current (FCNC) bounds to infer relevant constraints on the part of the SUSY soft breaking sector related to the sfermion mass matrices [2].

The model-independent method which is adopted is the so-called Mass-Insertion Approximation

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In this approach, the experimental limits lead to upper bounds on the parameters (or combinations of) \( \delta_{ij}^f \equiv \Delta_{ij}^f / m_{\tilde{f}}^2 \), where \( \Delta_{ij}^f \) is the flavor-violating off-diagonal entry appearing in the \( f = (u, d, l) \) sfermion mass matrices and \( m_{\tilde{f}}^2 \) is the average sfermion mass. The mass insertions include the LL/LR/RL/RR types, according to the chirality of the corresponding SM fermions. Detailed bounds on the individual \( \delta \)'s have been derived by considering limits from various FCNC processes [4]-[10].

As long as one remains within the simple picture of the Minimal Supersymmetric Standard Model (MSSM), where quarks and leptons are unrelated, the hadronic and leptonic FCNC processes yield separate bounds on the corresponding \( \delta \)'s and \( \delta \)'s.

The situation changes when one embeds the MSSM within a Grand Unified Theory (GUT). In a SUSY GUT, quarks and leptons sit in the same multiplets and are transformed into each other through GUT symmetry transformations. In supergravity theories, where the supersymmetry breaking (SB) mediation to the visible sector is gravitational, the structure of the soft terms is essentially dictated by the Kähler potential. If the effective supergravity Lagrangian is defined at a scale higher than the Grand Unification scale, the matter fields present in the Kähler function have to respect the underlying gauge symmetry which is the GUT group itself. Subsequently, the SB mediation would give rise to the usual soft terms which however now follow the GUT symmetry. Hence, we expect quark-lepton correlations among entries of the sfermion mass matrices [11, 12].

In other words, the quark-lepton unification seeps also into the SUSY breaking soft sector.

Imposition of a GUT symmetry on the soft SUSY breaking Lagrangian \( \mathcal{L}_{\text{soft}} \) entails relevant implications at the weak scale. This is because the flavor violating (FV) mass-insertions do not get strongly renormalized through Renormalization Group (RG) scaling from the GUT scale to the weak scale in the absence of new sources of flavor violation. On the other hand, if such new sources are present, for instance due to the presence of new neutrino Yukawa couplings in SUSY GUTs with a seesaw mechanism for neutrino masses, then one can compute the RG-induced effects in terms of these new parameters. As has been noted earlier [11], even in such cases, the correlations between hadronic and leptonic flavor violating MIs survive at the weak scale as a function of these parameters. As for the flavor conserving (FC) mass insertions (i.e., the diagonal entries of the sfermion mass matrices), they get strongly renormalized but in a way which is RG computable.

In conclusion, in SUSY GUTs where the soft SUSY breaking terms respect boundary conditions which are subject to the GUT symmetry to start with, we generally expect the presence of relations among the (bilinear and trilinear) scalar terms in the hadronic and leptonic sectors. Such relations hold true at the (superlarge) energy scale where the correct symmetry of the theory is...
the GUT symmetry. After its breaking, the mentioned relations will undergo corrections which
are computable through the appropriate RGE’s which are related to the specific structure of the
theory between the GUT and the electroweak scale (for instance, new Yukawa couplings due to the
presence of right-handed (RH) neutrinos acting down to the RH neutrino mass scale, presence of
a symmetry breaking chain with the appearance of new symmetries at intermediate scales, etc.).
As a result of such a computable running, we can infer the correlations between the softly SUSY
breaking hadronic and leptonic δ terms at the low scale where we perform our FCNC tests. Ex-

dplicit examples of such correlations in the context of an SU(5) SUSY GUT will be provided in next
Section.

Given that a common SUSY soft breaking scalar term of $L_{\text{soft}}$ at scales close to $M_{\text{Planck}}$ can give
rise to RG-induced $\delta q$’s and $\delta l$’s at the weak scale, one may envisage the possibility to make use of
the FCNC constraints on such low-energy δ’s to infer bounds on the soft breaking parameters of
the original supergravity Lagrangian ($L_{\text{sugra}}$). Indeed, for each scalar soft parameter of $L_{\text{sugra}}$ one
can ascertain whether the hadronic or the corresponding leptonic bound at the weak scale yields
the stronger constraint at the large scale. This constitutes the major goal of this work: we intend
to go through an exhaustive list of the low-energy constraints on the various $\delta q$’s and $\delta l$’s and, then, after RG evolving such δ’s up to $M_{\text{Planck}}$, we will establish for each δ of $L_{\text{sugra}}$ which one
between the hadronic and leptonic constraints is going to win, namely which provides the strongest
constraint on the corresponding $\delta_{\text{sugra}}$.

We will show that there exists a very interesting complementarity in the sensitivity of the soft
breaking sector of $L_{\text{sugra}}$ to the FCNC constraints provided by hadronic and leptonic physics.

The second, related purpose of this paper is to fully exploit the common origin from single $\delta_{\text{sugra}}$’s
of hadronic and leptonic weak scale δ’s to make use of leptonic (hadronic) FCNC constraints to
limit $\delta q$’s ($\delta l$’s). In other words, for instance in a SUSY SU(5) context, one can use a leptonic
FCNC process like $\tau \rightarrow \mu \gamma$ to impose constraints on the $(\delta_{23}^d)_{RR}$ term of the hadronic sector which
are stronger than those which are derived from genuine hadronic FCNC processes like $b \rightarrow s \gamma$.

In this respect, the present work intends to provide a particularly striking example of the
correlation between “low-energy” (i.e. weak scale) experiments and “high-energy” (i.e. GUT or
Planck scale) SUSY Lagrangian. The effort in the coming years, provided LHC yields some SUSY
evidence, will be to “reconstruct” the original supergravity Lagrangian from which our weak scale
testable SUSY descend. As we know, such work of reconstruction will be rather painful if we
have to rely only on LHC results; on the other hand, accompanying the SUSY direct searches at
LHC with the powerful FCNC tests will prove to be quite efficient in our effort of tracing back the
original parameters entering the underlying supergravity Lagrangian. In the case of SUSY GUTs such role of FCNC processes is further enhanced because of the above mentioned hadron–lepton correlations.

The paper is organized as follows. In the next Section we will provide an example of relations between hadronic and leptonic FC $\delta'$s at the weak scale within SUSY SU(5). In Section III we discuss the impact of GUT breaking effects (both at the tree level and at 1-loop) and the subsequent basis dependence which seeps in to the GUT-symmetric relations. In Section IV we discuss our procedure for obtaining bounds detailing various constraints we have imposed on our supersymmetric spectrum. In Sections V and VI we proceed with a separate analysis of the hadronic and leptonic $\delta'$s, to conclude with a study of their correlations in Section VII. In Section VIII the conclusions include some kind of final “score” on the tightness of the bounds in GUT related $\delta'$s coming from the various FCNC processes and the outlook for the interplay of the kind of flavor physics we consider here with the possible LHC outcome.

II. ON HADRON–LEPTON FCNC RELATIONS IN SUSY GUTS

In this Section we provide an example of the correlations between hadronic and leptonic $\delta'$s entering the weak scale MSSM Lagrangian when the underlying theory at the large scale, where supergravity lives, is restricted by a grand unified symmetry. Let us consider the scalar soft breaking sector of the MSSM:

\[
-L_{\text{soft}} = m_{\tilde{Q}_i}^2 \tilde{Q}_i \tilde{Q}_i + m_{\tilde{u}_i}^2 \tilde{u}_i \tilde{u}_i^* + m_{\tilde{d}_i}^2 \tilde{d}_i \tilde{d}_i^* + m_{\tilde{e}_i}^2 \tilde{e}_i \tilde{e}_i^* + m_{\tilde{l}_i}^2 \tilde{l}_i \tilde{l}_i^* + m_{H_1}^2 H_1 H_1 + m_{H_2}^2 H_2 H_2
\]

\[
+ A_{ij}^{\alpha} \tilde{Q}_i \tilde{u}_j H_2 + A_{ij}^{\alpha} \tilde{Q}_i \tilde{d}_j H_1 + A_{ij}^{\alpha} \tilde{l}_i \tilde{e}_j H_1 + (\Delta_{ij}^{L})_{LL} \tilde{l}_i \tilde{l}_j + (\Delta_{ij}^{L})_{LR} \tilde{l}_i \tilde{d}_j
\]

\[
+ (\Delta_{ij}^{d})_{RR} \tilde{u}_i \tilde{d}_j + \Delta_{ij}^{e} \tilde{e}_i \tilde{e}_j + (\Delta_{ij}^{e})_{LR} \tilde{l}_i \tilde{e}_j + (\Delta_{ij}^{e})_{LR} \tilde{u}_i \tilde{e}_j + \ldots
\]

(1)

where we have used the standard notation for the MSSM fields and have explicitly written down the various $\Delta$ parameters. Notice that the different $\Delta_{LR}$ include the contributions from the trilinear terms with the corresponding Higgs vev. Therefore the trilinear couplings in the second line represent only the real Higgs couplings and not their vacuum expectation values.

Consider $SU(5)$ to be the relevant symmetry at the scale where the soft terms are generated. Then, taking into account that matter is organized into the $SU(5)$ representations $10 = (q, u^c, e^c)$
Relations at the weak scale

\[
(\delta_{ij}^{u})_{RR} \approx (m_{e/c}^{2}/m_{e/c}^{2}) \ (\delta_{ij}^{l})_{RR}
\]

\[
(\delta_{ij}^{d})_{LL} \approx (m_{e/c}^{2}/m_{Q}^{2}) \ (\delta_{ij}^{l})_{RR}
\]

\[
(\delta_{ij}^{d})_{RR} \approx (m_{L}^{2}/m_{Q}^{2}) \ (\delta_{ij}^{l})_{LL}
\]

\[
(\delta_{ij}^{d})_{LR} \approx (m_{L}^{2}/m_{Q}^{2}) \ (\delta_{ij}^{l})_{LR}
\]

\[
m_{\tilde{u}} = m_{\tilde{c}} = m_{\tilde{e}} = m_{\tilde{10}}
\]

\[
m_{\tilde{d}} = m_{\tilde{L}} = m_{\tilde{5}}
\]

\[
A_{ij}^{e} = A_{ji}^{d}
\]

TABLE I: Links between various transitions between up-type, down-type quarks and charged leptons for SU(5). The suffix ‘0’ implies GUT scale parameters.

and \(\tilde{5} = (l, d^c)\), one obtains the following relations

\[
m_{Q}^{2} = m_{\tilde{e}}^{2} = m_{\tilde{u}}^{2} = m_{\tilde{10}}^{2}
\]

(2)

(3)

(4)

Eqs. (2)–(4) are matrices in flavor space. These equations lead to relations between the slepton and squark flavor violating off-diagonal entries \(\Delta_{ij}\). These are:

\[
(\Delta_{ij}^{u})_{LL} = (\Delta_{ij}^{u})_{RR} = (\Delta_{ij}^{d})_{LL} = (\Delta_{ij}^{l})_{RR}
\]

(5)

(6)

(7)

These GUT correlations among hadronic and leptonic scalar soft terms are summarized in the second column of Table I. Assuming that no new sources of flavor structure are present from the \(SU(5)\) scale down to the electroweak scale, apart from the usual SM CKM one, one infers the relations in the first column of Table I at the low-energy scale. A comment is in order when looking at Table I. Mass insertions for down-type quarks and leptons in Table I always exhibit opposite “chiralities”, i.e. LL insertions are related to RR ones and vice-versa. This stems from the arrangement of the different fermion chiralities in \(SU(5)\) multiplets (as it clearly appears from the last column in Table I). This restriction can easily be overcome if we move from \(SU(5)\) to left-right symmetric unified models like SO(10) or the Pati-Salam (PS) case, with gauge group \(SU(4)_c \times SU(2)_L \times SU(2)_R\).
III. WEAK-SCALE QUARK-LEPTON RELATIONS

The exact equality of the mass and trilinear matrices of the different components of a GUT multiplet is only true when the GUT symmetry is valid. At low scales, where we perform our experiments, these relations are modified. In this section, we analyze how these relations are modified after the breaking of the GUT symmetry and the RGE running from $M_{GUT}$ to $M_W$.

A. Definition of the SCKM basis

In low-energy phenomenology we obtain the different mass insertion bounds in the so-called super-CKM (SCKM) basis, i.e. the basis where through a rotation of the whole superfield (fermion + sfermion) we obtain diagonal Yukawa couplings for the corresponding fermion field [13]. At low energies the SCKM bases for quarks and leptons are unrelated. However in a GUT theory quarks and leptons unify and thus we would expect the quark and lepton Yukawa couplings to be equal at the GUT scale and hence their SCKM bases to unify. Unfortunately this is not true when we evolve the Yukawa couplings using the SM (or MSSM) Renormalization Group Equations (RGEs). This discrepancy, that could be solved with some GUT breaking effects or new non-renormalizable contributions, implies a relative rotation between the quark and leptonic SCKM basis. While in principle this is an obstacle for a model-independent analysis, we argue below that its effects are not expected to change the order of magnitude of our bounds.

Let us consider for example the case in which we obtain a MI bound from a leptonic process at low energies. This bound is given in the basis of diagonal charged lepton masses. The next step in our scheme consists in using the RGE equations to evolve this bound to the GUT scale. The resultant bound is a bound on the element in the sfermion mass matrix common for the GUT multiplet including both quark and leptons at $M_{GUT}$ in the basis of diagonal charged lepton Yukawa matrices.\(^1\) However, if we want to convert this bound into a bound on the corresponding squark MI we must take into account a possible misalignment between the charged-lepton and the down-quark Yukawa matrices just below the GUT scale. GUT symmetry breaking effects could introduce different corrections to the quark and lepton Yukawa matrices. For example, it is well-known that the minimal $SU(5)$ Lagrangian with a $\mathbf{5}$ and a $\overline{\mathbf{5}}$ Higgs representations is not realistic as the first two generations of the down quark and charged lepton fermion masses do not follow

\(^1\) An additional (small) RGE-induced rotation can be necessary in the presence of neutrino Yukawa couplings. We neglect its effect in our discussion.
the GUT relations. This means that quark and lepton Yukawa matrices are not the same below $M_{\text{GUT}}$ and they are not simultaneously diagonalizable. Taking into account these facts, we must rotate our bounds from the leptonic sector to the basis of diagonal quark Yukawa matrices before we compare them with the bounds obtained from quark processes at low energies.

In a complete GUT model we would be able to obtain both the quark and lepton Yukawa matrices after the breaking of the GUT symmetry. In such a theory it would be possible to compute the relative rotation between the basis of diagonal quark and the basis of diagonal lepton Yukawa matrices exactly. On more general grounds, as we do not have a complete GUT theory yet, we have a new uncertainty introduced in the translation of the bounds between quark and leptonic MIs, due to this relative rotation between the leptonic and quark Yukawa couplings. To see the effect of this ‘new’ rotation matrix, which can be represented by a unitary matrix, $V^{(ql)}$, while translating the bounds from one sector to another sector (leptonic to hadronic or vice-versa), we consider one particular case. Let $\Delta^l_{i\neq j}$, of any ‘chirality’, have a bound at the GUT scale, given by $\Delta_{\text{max}}$. This element $\Delta^l_{i\neq j}$ corresponds to a combination of matrix elements in the basis of diagonal quark Yukawas, $\Delta^{(q)}_{ij}$, given as

$$|\Delta^l_{i\neq j}| = |\sum_{k,l} V^{(ql)*}_{ki} \Delta^{(q)}_{kl} V^{(ql)}_{lj}| \leq \Delta_{\text{max}}, \quad (8)$$

where $V^{(ql)}$ would represent the rotation between the basis of diagonal quark and lepton Yukawas. Given that $\Delta^{(f)}$ and $V^{(ql)}$ have different origins, we do not expect large cancellations between different terms in this sum and therefore, “barring accidental cancellations”, we can apply the bound on $\Delta^l_{i\neq j}$ to each of the individual elements in this sum

$$|V^{(ql)*}_{ki} \Delta^{(q)}_{kl} V^{(ql)}_{lj}| \leq \Delta_{\text{max}}, \quad (9)$$

Once more, in principle it is necessary to know the relative rotation $V^{(ql)}$ to apply the bound on $\Delta^{(q)}_{kl}$. In SU(5) GUT, requiring that the successful $b-\tau$ unification is not accidental, we can expect that $|V^{(ql)}_{33}| \simeq 1$ and $|V^{(ql)}_{3i}|, |V^{(ql)}_{i3}| \ll 1$. The relative misalignment for the first two generations is much more uncertain. However, we consider reasonable to expect that the different families are approximately arranged in the usual way and $(c, s)$ quarks are mostly associated with $(\mu, \nu_\mu)$ and $(u, d)$ with $(e, \nu_e)$. So, we make the “reasonable assumption” that diagonal elements in these rotations, $V^{(ql)}_{ii}$, are $O(1)$, where $O(1)$ may mean $1/\sqrt{2}$ or even $1/2$ but not much smaller. Therefore, under these conditions we have that

$$|V^{(ql)*}_{ii} \Delta^{(q)}_{i\neq j} V^{(ql)}_{jj}| \simeq |\Delta^{(q)}_{ij}| \leq \Delta_{\text{max}}, \quad (10)$$
and thus, order of magnitude MI bounds in the leptonic sector can be safely translated into order of magnitude MI bounds in the quark sector, taking into account that these bounds are not precise within factors of two.

In the following we present a realistic example to show the possible effects of these rotations. A popular solution to the first two generations Yukawa unification problem in SU(5) is provided by the Georgi-Jarlskog mechanism \[14\]. There one introduces an additional Higgs representation which is 45 dimensional contributing mainly to the second generation Yukawa couplings. This Higgs gets a vev along the \((B - L)\) direction and therefore breaks the charged lepton-down quark symmetry. Assuming simple symmetric Yukawa textures, the down quark and charged lepton Yukawa matrices are given as follows:

\[
Y_d^{M_{\text{GUT}}} \propto \begin{pmatrix}
0 & \lambda^3 & \lambda^3 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix},
\]

\[
Y_e^{M_{\text{GUT}}} \propto \begin{pmatrix}
0 & \lambda^3 & \lambda^3 \\
\lambda^3 & 3\lambda^2 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix},
\]

(11)

where \(\lambda\) is a parameter of the order of the Cabibbo angle. In this way, we preserve the successful \(b-\tau\) unification and correct the bad GUT relations for the second and first generations. Clearly, if the Yukawa matrices have this structure, the basis of diagonal down-quark Yukawas is different from the basis of diagonal charged-lepton Yukawas. Therefore the rotation \(V^{(q\ell)}\), due to the misalignment between quarks and leptons is non-trivial. However, it is very easy to check that up to terms of \(O(\lambda^3)\) this rotation is given by

\[
V^{(q\ell)} \simeq \begin{pmatrix}
1 - \frac{2\lambda^2}{9} & \frac{2\lambda}{3} & 0 \\
-\frac{2\lambda}{3} & 1 - \frac{2\lambda^2}{9} & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

(12)

Using this rotation and barring accidental cancellations in Eq. (9), it is evident that Eq. (10) is an excellent approximation in this case and we can safely relate off-diagonal elements in the squark and slepton mass matrices.

B. RG evolution

The SU(5) GUT relations between squark and slepton masses and trilinear couplings in Eqs. (2)-(7) are only valid at the SU(5) scale. As it is well-known, one could expect the long RG running from the GUT scale down to the weak scale to modify these relations significantly. However, the crucial aspect is that the flavor violating entries \((i \neq j)\) are not significantly modified due to this
running. In fact, even in the presence of right handed neutrinos in a seesaw mechanism, this statement remains true up to factors order one. This approximate non-modification of the $\Delta_{i\neq j}$’s, allows us to recast them at the weak scale in terms of $\delta_{i\neq j}$. In the present section, we will try to quantify our statements using semi-numerical solutions of the RGE within the SU(5) model we have been discussing so far. We will consider the effects of adding right-handed neutrinos a bit later on.

In the RG evolution of diagonal elements of sfermion mass matrices we neglect small Yukawa couplings and keep only $Y_b$, $Y_\tau$ and $Y_t$. Then, for the first two generations of squarks and sleptons, the following simple expressions hold in terms of the high scale soft parameters, $m_{10}^2 = m_{\tilde{b}}^2 = m_{\tilde{b}}^2$ and $M_{1/2}$ (for a general expression with different $m_{10}^2$ and $m_{\tilde{b}}^2$ see Table I in [15]):

\[
\begin{align*}
(m_Q^2)_{1,2}(M_W) &\simeq m_{10}^2 + 6.5 M_{1/2}^2 \\
(m_{\tilde{D}}^2)_{1,2}(M_W) &\simeq m_{10}^2 + 6.1 M_{1/2}^2 \\
(m_{\tilde{E}}^2)_{1,2}(M_W) &\simeq m_{10}^2 + 0.15 M_{1/2}^2 \\
(m_{\tilde{L}}^2)_{1,2}(M_W) &\simeq m_{10}^2 + 0.5 M_{1/2}^2
\end{align*}
\]

The third generation masses receive contributions proportional to Yukawa couplings which depend on the value of $\tan \beta$. For low $\tan \beta \simeq 5$ we have

\[
\begin{align*}
(m_Q^2)_3(M_W) &\simeq 0.6 m_{10}^2 + 5.5 M_{1/2}^2 \\
(m_{\tilde{D}}^2)_3(M_W) &\simeq 0.2 m_{10}^2 + 4.1 M_{1/2}^2
\end{align*}
\]

while for $\tan \beta \simeq 30$ we have

\[
\begin{align*}
(m_Q^2)_3(M_W) &\simeq 0.5 m_{10}^2 + 5.2 M_{1/2}^2 \\
(m_{\tilde{D}}^2)_3(M_W) &\simeq 0.8 m_{10}^2 + 5.14 M_{1/2}^2 \\
(m_{\tilde{E}}^2)_3(M_W) &\simeq 0.8 m_{10}^2 + 0.12 M_{1/2}^2 \\
(m_{\tilde{L}}^2)_3(M_W) &\simeq 0.92 m_{10}^2 + 0.5 M_{1/2}^2
\end{align*}
\]

RGE evolution for off-diagonal elements is solely dependent on Yukawa couplings and does not include gaugino contributions. Now, if we neglect first two generations Yukawa couplings, in the basis of diagonal down-quark Yukawa couplings we have:

\[
\begin{align*}
(16\pi^2) \frac{d(m_{\tilde{D},E}^2)_{i\neq j}}{dt} &= -2 Y_{b,\tau}^2 (m_{\tilde{D},E}^2)_{3j} \delta_{i3} - 2 Y_{b,\tau}^2 (m_{\tilde{D},E}^2)_{i3} \delta_{j3} \\
(16\pi^2) \frac{d(m_Q^2)_{i\neq j}}{dt} &= - \left[ Y_b^2 (m_Q^2)_{3j} \delta_{i3} + Y_b^2 (m_Q^2)_{i3} \delta_{j3} + Y_t^2 V_{ti} V_{k\tau} (m_{Q}^2)_{kj} + Y_t^2 V_{tk} V_{ij} (m_{Q}^2)_{ik} + 2 (Y_A Y_A^+)_{ij} \right] \\
(16\pi^2) \frac{d(m_{\tilde{L}}^2)_{i\neq j}}{dt} &= - Y_{\tau}^2 (m_{\tilde{L}}^2)_{3j} \delta_{i3} - Y_{\tau}^2 (m_{\tilde{L}}^2)_{i3} \delta_{j3}
\end{align*}
\]
where \( t = \log \frac{M_{\text{GUT}}}{Q} \). We are interested in two different aspects of these equations. First we want to determine how a non-vanishing off-diagonal entry in the sfermion mass matrices at \( M_{\text{GUT}} \) is modified due to RGE evolution from \( M_{\text{GUT}} \) to \( M_W \). Then, we are also interested in the size of the off-diagonal entries generated by the running from \( M_{\text{GUT}} \) to \( M_W \) in the case of exactly vanishing off-diagonal entries at \( M_{\text{GUT}} \).

Regarding the evolution of GUT elements, we can neglect the non-diagonal CKM elements and then, we see from Eq. (16) that the \((\Delta^\tilde{f}_{12})_{ij}\) are not modified at the leading order. Furthermore, in the low \( \tan \beta \) limit, off-diagonal elements in \( m^2_D \) or \( m^2_E \) in the SCKM basis are not significantly changed through RG evolution. Thus, one can safely assume Eq. (6) to be valid at any scale at low \( \tan \beta \). For large \( \tan \beta \simeq 40 \), the effect of \( Y^2_{tb} \) is only relatively important and in the leading log approximation we obtain

\[
(m^2_D)_{i=3\neq j}(M_W) \simeq (m^2_L)_{i=3\neq j}(M_W) \exp \left( -\frac{1}{16\pi^2}(2Y^2_b - Y^2_\tau) \log \frac{M_{\text{GUT}}}{M_W} \right) \simeq 0.84 (m^2_L)_{i=3\neq j}(M_W)
\]

and therefore it can account at most for a \((15-20)\% \) relative change. Something analogous happens in the left-handed squark mass matrix. For low \( \tan \beta \) the effect of the top Yukawa coupling in the third row elements would be

\[
(m^2_Q)_{i=3\neq j}(M_W) \simeq (m^2_E)_{i=3\neq j}(M_W) \exp \left( -\frac{1}{16\pi^2}Y^2_t \log \frac{M_{\text{GUT}}}{M_W} \right) \simeq 0.86(m^2_E)_{i=3\neq j}(M_W),
\]

In the large \( \tan \beta \) region, we have to multiply the argument in the exponential by a factor \((Y^2_t + Y^2_b - 2Y^2_\tau)/Y^2_t\), which for \( \tan \beta = 40 \) amounts only to a factor of 1.16. Therefore, the relative change between \((m^2_Q)_{i=3\neq j}\) and \((m^2_E)_{i=3\neq j}\) is practically the same as the previous case.

A second feature of these RGE equations is that, even if we start from a flavor blind situation at the GUT (or Plank) scale, flavor off-diagonal entries in the LL squark mass matrix at the weak scale (via the CKM matrix) are unavoidable. The running from \( M_{\text{GUT}} \) to the weak scale \( M_W \) gives rise to the following FV effects

\[
(\delta^d_{LL})_{ij} \simeq -\frac{1}{8\pi^2}Y^2_t V^*_{ti} V^*_{tj} \frac{3m^2_0 + a^2_0}{m^2_\tilde{q}} \ln \frac{M_{\text{GUT}}}{M_W}.
\]

If we take \( M_{\text{GUT}} \simeq 2 \times 10^{16} \) GeV, \( Y_t \simeq 1 \), \( m^2_\tilde{q} \simeq 6M^2_{1/2} + m^2_0 \) and in the limit \( M_{1/2} \simeq m_0 \simeq a_0 \), we have \((\delta^d_{LL})_{ij} \simeq -0.2 V^*_{ti} V_{tj}\). More generally we can say that \((\delta^d_{LL})_{ij} \simeq -c V^*_{ti} V_{tj}\) with \( c \) a numerical coefficient depending on \((m_0, M_{1/2}, a_0)\) taking values between 0.1 and 1. As we will discuss in following sections, these RGE induced FV effects have an important phenomenological impact, specially when FV entries in the RR squark sector are also present. This is due to the fact that,
as we will see, $\Delta M_{B,K}$ mass differences are much more sensitive to contributions proportional to $\delta_{LL} \times \delta_{RR}$.

Finally, for the last equation in \((7)\), the RGE equations for off-diagonal down quark and charged lepton trilinear couplings are

\[
(16\pi^2) \frac{d(Y^A_d)_{i\neq j}}{dt} = \left( \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{7}{9} g_1^2 \right) (Y^A_d)_{ij} - 5 Y^2_b (Y^A_d)_{3j} \delta_{i3} - 4 Y^2_b (Y^A_d)_{i3} \delta_{j3} \quad (20) \\
-3 Y^2_b (Y^A_d)_{ij} - Y^2 (Y^A_d)_{3j} \delta_{i3} - Y^2 (Y^A_d)_{i3} \delta_{j3}
\]

\[
(16\pi^2) \frac{d(Y^A_e)_{i\neq j}}{dt} = \left( 3 g_2^2 + 3 g_1^2 \right) (Y^A_e)_{ij} - 5 Y^2_\tau (Y^A_e)_{3j} \delta_{i3} - 4 Y^2_\tau (Y^A_e)_{i3} \delta_{j3} - Y^2_\tau (Y^A_e)_{ij} - 3 Y^2_b (Y^A_e)_{ij}.
\]

From the equations above, it’s clear that the trilinear couplings scale almost in the same manner as the corresponding Yukawa couplings. Thus the corresponding relation given in Table I holds well especially in the limit of small $\tan \beta \lesssim 25$, where we can neglect the bottom and $\tau$ Yukawas. For large $\tan \beta$ and for 13 and 23 MI’s, we expect similar corrections $\sim (15 - 20)\%$ as we have mentioned above. Given that the MI bounds are intrinsically order of magnitude bounds, we can safely consider these off-diagonal elements to evolve analogously to the Yukawa couplings from $M_W$ to $M_{\text{GUT}}$.

Let us now briefly mention the implication of adding right-handed neutrinos as in a seesaw mechanism. The RG effects and subsequent flavour violation have been studied in detail in the literature \cite{16,17}. The implication of these effects on the relations Eqs. \((5-7)\) depends crucially on the strength of the neutrino Dirac yukawa couplings $Y_\nu$ and the “mixing” they carry. For example, a rough estimate of the weak scale $\left(\Delta^l_{LL}\right)_{ij}$ is given by

\[
(m^2_\tilde{L})^2_{i\neq j}(M_W) \approx (m^2_\tilde{L})^2_{i\neq j}(M_{\text{GUT}}) - \frac{1}{8\pi^2} \sum_k (Y_\nu Y^\dagger_\nu)_{ij}(3m^2_0 + A^2_0) \log \left( \frac{M_{\text{GUT}}}{M_{R_k}} \right) \quad (21)
\]

where $Y_\nu$ represents the neutrino Dirac Yukawa couplings. From the equation above we see that RG effects could “generate” large $\Delta$s if the mixing in $Y_\nu$ is large. In the general case, it is very difficult to separate the RGE-induced $\Delta$s from the RGE-modified GUT off-diagonal elements in $m^2_\tilde{L}$ and $Y^A_e$. Changes in the original GUT off-diagonal entries and the RGE-induced $\Delta$s depend on the neutrino Yukawa couplings. These neutrino Yukawa matrices are present in the RGE between the scales $M_{\text{GUT}}$ and the mass of the corresponding right handed neutrino. Typically one of the eigenvalues in the neutrino Yukawa matrices can be of the same order as the top Yukawa coupling, but this large eigenvalue is necessarily associated with the heaviest right-handed neutrino \cite{18,19}.

\footnote{We do not consider here the possibility of having several large eigenvalues in $Y_\nu$.}
and therefore it decouples early. The main problem regarding the neutrino Yukawa couplings is that the mixing diagonalizing $Y_\nu Y_\nu^T$ in the basis of diagonal $Y_\nu Y_\nu^T$ is unknown and could be large. Nevertheless, we can discuss some cases.

The simplest situation corresponds to the case where the mixing diagonalizing the charged lepton and neutrino Yukawa matrices are both small. In this case, the large neutrino mixings are generated through the seesaw mechanism and they play no role in the RGE evolution of slepton matrices. The RGE evolution of $m_L^2$ is then very similar to the previous cases without the right handed neutrinos. In the low tan $\beta$ region we can completely neglect the effects of $Y_e Y_e^T$ in the RGEs. Then in the basis of diagonal neutrino Yukawas (approximately equal to the basis of diagonal charged-lepton Yukawas) the off-diagonal elements would be

$$\left( m_L^2 \right)_{i=3\neq j} (M_W) \simeq \left( m_L^2 \right)_{i=3\neq j} (M_W) \operatorname{Exp} \left( \frac{1}{16\pi^2} (Y_\nu^2) \log \frac{M_{\text{GUT}}}{M_{R_3}} \right) \simeq 1.1 \left( m_L^2 \right)_{i=3\neq j} (M_W),$$ (22)

taking $M_{R_3}$ as low as $10^{10}$ GeV.

In the large tan $\beta$ region, the effects of Eq. (22) and Eq. (17) have different signs and partially cancel out. So, we can also expect these off-diagonal elements to remain of similar magnitude between $M_{\text{GUT}}$ and $M_W$. Notice also that the $(1, 2)$ mass insertions remain unchanged if we have only one large neutrino-Yukawa coupling corresponding to the third generation.

The second situation corresponds to large mixings in the neutrino Yukawa matrices in the basis of diagonal charged-lepton Yukawas. This case depends strongly on the mixings and size of the neutrino Yukawa couplings. There are two simultaneous effects, the creation of a new MI due to the RGE and the change from RG evolution of the GUT delta. These effects cannot be sorted out in general and thus we cannot describe the evolution of the GUT delta without specifying completely the Yukawa matrices. In this situation, it is difficult to correlate quark and leptonic deltas. Finally, the effects of neutrino Yukawas on the relations between charged-lepton A-parameters is expected to be similarly small even for large enough (top-quark like) Yukawa couplings.

We conclude noting that, in general, the RG effects do not significantly modify the GUT relations for off-diagonal elements already present at $M_{\text{GUT}}$. However in the presence of right-handed neutrinos and large mixings in the neutrino Yukawa couplings, the new contributions to the off-diagonal entries in the slepton mass matrices can destroy the quark-lepton correlations and the analysis becomes model dependent.
IV. CONSTRAINTS

In this section, we list the constraints we have imposed on the SUSY parameter space before starting the analysis of flavor physics in the leptonic and hadronic sectors.

A. Direct SUSY search

The model we use in this work is a minimal departure from the usual Constrained MSSM (CMSSM) where we add a single flavor changing entry in the sfermion couplings at $M_{\text{GUT}}$. The usual CMSSM contains (assuming vanishing flavor blind SUSY phases) five parameters: $M_{1/2}$, $m_0$, $A_0$, $\tan \beta$ and $\text{sign}(\mu)$.$^3$ In addition we include the single MI relevant to the process(ies) of interest at the GUT scale. Here, we take the $\mu$ sign positive as required by the $b \to s\gamma$ and muon anomalous magnetic moment constraints. Therefore the SUSY parameters that enter our analysis for a given MI are only $(m_0, m_{1/2}, A_0, \tan \beta, \Delta_f)$.

We scan the values of these parameters in the following ranges: $M_{1/2} \leq 160$ GeV, $m_0 \leq 380$ GeV, $|A_0| \leq 3m_0$ and $5 < \tan \beta < 15$. The bound on the $A_0$ parameters is set to avoid charge and/or colour breaking minima [20]. This would typically correspond to the following highest mass scales: slepton masses as large as $m_{\tilde{\ell}} \approx 400$ GeV and squark masses as large as $m_{\tilde{q}} \approx 550$ GeV; both possibly observable at LHC.

At the low scale, we impose the following constraints on each point:

- Lower bound on the light and pseudo-scalar Higgs masses [21];
- The LEP constraints on the lightest chargino and sfermion masses [22];
- The LEP and Tevatron constraints on squark and gluino masses [22].

We consider the MSSM with conserved R-parity and thus the lightest neutralino (that coincides with the lightest supersymmetric particle - LSP) provides an excellent dark matter candidate.

B. $b \to s\gamma$

$\mathcal{B}(B \to X_s\gamma)$ is particularly sensitive to possible non-standard contributions and it provides a non-trivial constraint on the SUSY mass spectrum given its precise experimental determination and the very accurate SM calculation at the NNLO [23]. When witnessing such a light SUSY

$^3$ Notice, we are not enforcing here the minimal supergravity relation between the $A$ and $B$ parameters, $B = A - 1$
spectrum (we remind that we take $M_{1/2} \leq 160$ GeV, $m_0 \leq 380$ GeV) a legitimate worry is whether the bounds on $B(B \to X_s \gamma)$ are respected. In this work we choose $\mu > 0$ which implies destructive interference between chargino and charged Higgs contributions (and is also preferred by the $(g-2)_{\mu}$ constraints). We have explicitly checked that the combined chargino and charged Higgs contributions satisfy the $B(B \to X_s \gamma)$ constraints. For these points, we checked simultaneously that gluino contributions satisfy by themselves the $B(B \to X_s \gamma)$ constraints. Then, these gluino contributions set a bound on the $\delta_{ij}^{\nu}$ MIs. Notice that, in this way, we are not allowing the possibility of an accidental cancellation of charged-Higgs and chargino contributions with gluino ones.

C. The lightest Higgs boson mass

The non-observation of the lightest neutral Higgs boson ($h^0$) at present colliders is already a stringent constraint on the MSSM parameter space [21]. Even if the $h^0$ mass depends on the whole set of MSSM parameters (after the inclusions of loop corrections), $m_{h^0}$ mainly depends on (and, indeed, grows with) the left-right mixing term in the stop mass matrix $\tilde{M}_t^{LR} = m_t(A_U - \mu/\tan \beta)$, on the average stop mass $\tilde{M}_t$ and on $\tan \beta$. In particular, it is well known that values of $\tan \beta \leq 2$ are strongly disfavored. Taking into account that the lower bound on the $h^0$ mass changes with the SUSY parameters, we have explicitly checked that the experimental limits [21] are fulfilled in our parameter space.

D. $(g-2)_{\mu}$

The possibility that the anomalous magnetic moment of the muon [$a_{\mu} = (g-2)_{\mu}/2$], which has been measured very precisely in the last few years [24], provides a first hint of physics beyond the SM has been widely discussed in the recent literature. Despite substantial progress both on the experimental and on the theoretical sides, the situation is not completely clear yet (see Ref. [25] for an updated discussion).

Most recent analyses converge towards a $2\sigma$ discrepancy in the $10^{-9}$ range [25]:

$$\Delta a_{\mu} = a_{\mu}^{\exp} - a_{\mu}^{\text{SM}} \approx (2 \pm 1) \times 10^{-9}.$$ (23)

The main SUSY contribution to $a_{\mu}^{\text{MSSM}}$ is usually provided by the loop exchange of charginos and sneutrinos. The basic features of the supersymmetric contribution to $a_{\mu}$ are correctly reproduced
by the following approximate expression:

\[
\frac{a_{\mu}^{\text{MSSM}}}{1 \times 10^{-9}} \approx 1.5 \left( \frac{\tan \beta}{10} \right) \left( \frac{300 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 \left( \frac{\mu M_2}{m_{\tilde{\nu}}^2} \right),
\]

which provides a good approximation to the full one-loop result \([26]\).

The most relevant feature of Eqs. \((24)\) is that the sign of \(a_{\mu}^{\text{MSSM}}\) is fixed by the sign of the \(\mu\) term so that the \(\mu > 0\) region is strongly favored. This is specially true for the Standard Model prediction which uses the data from \(e^+e^-\) collisions to compute the hadronic vacuum polarization (HVP). This predicts a smaller value than the experimental result by about 3 \(\sigma\). In case one uses the \(\tau\) data to compute the HVP, the discrepancy with SM is reduced to about 1 \(\sigma\), but it still favors a positive correction and disfavors strongly a sizable negative contribution. Thus, taking \(\mu > 0\), the region of parameter space considered in this analysis satisfies the constraint of Eq. \((23)\).

E. Electroweak Precision Observables (EWPO)

The good agreement between the SM predictions and the electroweak precision observables (EWPO) points to a decoupling of new physics contributions to these precision observables. As we also consider light superpartners, we need to take into account the tight constraints on the supersymmetric spectrum emerging from this agreement.

Several recent and thorough analyses for the MSSM are available in the literature \([27]\). The most relevant effect is due to the mass splitting of the superpartners, and in particular of the third generation squarks. Indeed, large splitting between \(\tilde{m}_{b_L}\) and \(\tilde{m}_{t_L}\) would induce large contributions to the electroweak \(\rho\) parameter. This universal contribution enters the \(Z^0\) boson couplings and the relation between \(M_W, G_\mu\) and \(\alpha\) and is therefore significantly constrained by present data. In the cases of the \(W\)-boson mass and of the effective weak mixing angle \(\sin^2 \Theta^\text{eff}_W\), for example, a doublet of heavy squarks would induce shifts proportional to its contribution to \(\rho\): \(\delta M_W/M_W \approx 0.72 \Delta \rho\) and \(\delta \sin^2 \Theta^\text{eff}_W \approx -0.33 \Delta \rho\).

In our analysis, we have required that \(\Delta \tilde{q}_\rho^{(0)} < 1.5 \times 10^{-3}\) and we have checked that no relevant constraints arise from EWPO, as it is confirmed by the thorough analysis (relative to the CMMSM framework) in Ref. \([27]\).

V. MASS INSERTION BOUNDS FROM HADRONIC PROCESSES

The comparison of several hadronic flavor-changing processes to their experimental values can be used to bound the MIs in the different sectors \([4]-[10]\). In these analyses it is customary to
<table>
<thead>
<tr>
<th>Observable</th>
<th>Measurement/Bound</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1–2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta M_K$</td>
<td>$(0.0 - 5.3) \times 10^{-3}$ GeV</td>
<td>[30]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>$(2.232 \pm 0.007) \times 10^{-3}$</td>
<td>[30]</td>
</tr>
<tr>
<td>$</td>
<td>\langle \varepsilon'/\varepsilon \rangle_{SUSY}</td>
<td>$</td>
</tr>
<tr>
<td>Sector 1–3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta M_{B_d}$</td>
<td>$(0.507 \pm 0.005)$ ps$^{-1}$</td>
<td>[31]</td>
</tr>
<tr>
<td>$\sin 2\beta$</td>
<td>$0.675 \pm 0.026$</td>
<td>[31]</td>
</tr>
<tr>
<td>$\cos 2\beta$</td>
<td>$&gt; -0.4$</td>
<td>[32]</td>
</tr>
<tr>
<td>Sector 2–3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{BR}(b \to (s+d)\gamma)(E_\gamma &gt; 2.0$ GeV)</td>
<td>$(3.06 \pm 0.49) \times 10^{-4}$</td>
<td>[33]</td>
</tr>
<tr>
<td>$\text{BR}(b \to (s+d)\gamma)(E_\gamma &gt; 1.8$ GeV)</td>
<td>$(3.51 \pm 0.43) \times 10^{-4}$</td>
<td>[34]</td>
</tr>
<tr>
<td>$\text{BR}(b \to s\gamma)(E_\gamma &gt; 1.9$ GeV)</td>
<td>$(3.34 \pm 0.18 \pm 0.48) \times 10^{-4}$</td>
<td>[35]</td>
</tr>
<tr>
<td>$A_{CP}(b \to s\gamma)$</td>
<td>$0.004 \pm 0.036$</td>
<td>[31]</td>
</tr>
<tr>
<td>$\text{BR}(b \to s^+l^-)(0.04$ GeV $&lt; q^2 &lt; 1$ GeV)</td>
<td>$(11.34 \pm 5.96) \times 10^{-7}$</td>
<td>[36, 37]</td>
</tr>
<tr>
<td>$\text{BR}(b \to s^+l^-)(1$ GeV $&lt; q^2 &lt; 6$ GeV)</td>
<td>$(15.9 \pm 4.9) \times 10^{-7}$</td>
<td>[36, 37]</td>
</tr>
<tr>
<td>$\text{BR}(b \to s^+l^-)(14.4$ GeV $&lt; q^2 &lt; 25$ GeV)</td>
<td>$(4.34 \pm 1.15) \times 10^{-7}$</td>
<td>[36, 37]</td>
</tr>
<tr>
<td>$A_{CP}(b \to s^+l^-)$</td>
<td>$-0.22 \pm 0.26$</td>
<td>[30]</td>
</tr>
<tr>
<td>$\Delta M_{B_s}$</td>
<td>$(17.77 \pm 0.12)$ ps$^{-1}$</td>
<td>[38]</td>
</tr>
</tbody>
</table>

**TABLE II:** Measurements and bounds used to constrain the hadronic $\delta^d$'s.

consider only the dominant contributions due to gluino exchange which give a good approximation of the full amplitude, barring accidental cancellations. In the same spirit, the bounds are usually obtained taking only one non-vanishing MI at a time, neglecting the interference among MIs. This procedure is justified *a posteriori* by observing that the MI bounds have typically a strong hierarchy, making the destructive interference among different MIs very unlikely.

The effective Hamiltonians for $\Delta F = 1$ and $\Delta F = 2$ transitions including gluino contributions computed in the MI approximation can be found in the literature together with the formulae of several observables [4]. Even the full NLO calculation is available for the $\Delta F = 2$ effective Hamiltonian [28, 29].

In our study we use the phenomenological constraints collected in Table III. We use the same set of SUSY parameters described in the previous Section, so that hadronic and leptonic MIs are related as discussed in Section II. In particular:
**Sector 1–2** The measurements of $\Delta M_K$, $\varepsilon$ and $\varepsilon'/\varepsilon$ are used to constrain the $(\delta^d_{12})_{AB}$ with $(A,B) = (L,R)$. The first two measurements, $\Delta M_K$ and $\varepsilon$ respectively bound the real and imaginary part of the product $(\delta^d_{12}) (\delta^d_{12})$. In the case of $\Delta M_K$, given the uncertainty coming from the long-distance contribution, we use the conservative range in Table III. The measurement of $\varepsilon'/\varepsilon$, on the other hand, puts a bound on $\text{Im}(\delta^d_{12})$. This bound, however, is effective in the case of the LR MI only. Notice that, given the large hadronic uncertainties in the SM calculation of $\varepsilon'/\varepsilon$, we use the very loose bound on the SUSY contribution shown in Table III. The bounds coming from the combined constraints are shown in Table III. Notice that, here and in the other sectors, the bound on the RR MI is obtained in the presence of the radiatively-induced LL MI given in Eq. (19). The product $(\delta^d_{12})_{LL} (\delta^d_{12})_{RR}$ generates left-right operators that are enhanced both by the QCD evolution and by the matrix element (for kaons only). Therefore, the bounds on RR MIs are more stringent than the ones on LL MIs.

**Sector 1–3** The measurements of $\Delta M_{B_d}$ and $2\beta$ respectively constrain the modulus and the phase of the mixing amplitude bounding the products $(\delta^d_{13}) (\delta^d_{13})$. For the sake of simplicity, in Table III we show the bounds on the modulus of $(\delta^d_{13})$ only.

**Sector 2–3** This sector enjoys the largest number of constraints. The recent measurement of $\Delta M_{B_s}$ constrains the modulus of the mixing amplitude, thus bounding the products $| (\delta^d_{23}) (\delta^d_{23}) |$. Additional strong constraints come from $\Delta B = 1$ branching ratios, such as $b \to s\gamma$ and $b \to s l^+ l^-$. Also for this sector, we present the bounds on the modulus of $(\delta^d_{23})$ in Table III.

All the bounds in Table III have been obtained using the NLO expressions for SM contributions and for SUSY where available. Hadronic matrix elements of $\Delta F = 2$ operators are taken from lattice calculations [39, 40, 41, 42]. The values of the CKM parameters $\tilde{\rho}$ and $\tilde{\eta}$ are taken from the UTfit analysis in the presence of arbitrary loop-mediated NP contributions [43]. This conservative choice allows us to decouple the determination of SUSY parameters from the CKM matrix. For $b \to s\gamma$ we use NLO expressions with the value of the charm quark mass suggested by the recent NNLO calculation [23]. For the chromomagnetic contribution to $\varepsilon'/\varepsilon$ we have used the matrix element as estimated in Ref. [44]. The 95% probability bounds are computed using the statistical method described in Refs. [7, 45].

Concerning the dependence on the SUSY parameters, the bounds mainly depend on the gluino mass and on the “average squark mass”. A mild dependence on $\tan \beta$ is introduced by the presence
TABLE III: 95% probability bounds on $|\left(\delta_{ij}^d\right)_{AB}|$ obtained using the data set described in Section [IV]. See the text for details.

<table>
<thead>
<tr>
<th>$ij$ \ $AB$</th>
<th>$LL$</th>
<th>$LR$</th>
<th>$RL$</th>
<th>$RR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>$1.4 \times 10^{-2}$</td>
<td>$9.0 \times 10^{-5}$</td>
<td>$9.0 \times 10^{-5}$</td>
<td>$9.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>13</td>
<td>$9.0 \times 10^{-2}$</td>
<td>$1.7 \times 10^{-2}$</td>
<td>$1.7 \times 10^{-2}$</td>
<td>$7.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>23</td>
<td>$1.6 \times 10^{-1}$</td>
<td>$4.5 \times 10^{-3}$</td>
<td>$6.0 \times 10^{-3}$</td>
<td>$2.2 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

TABLE IV: Present and Upcoming experimental limits on various leptonic processes at 90% C.L.

<table>
<thead>
<tr>
<th>Process</th>
<th>Present Bounds</th>
<th>Expected Future Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{BR}(\mu \rightarrow e \gamma)$</td>
<td>$1.2 \times 10^{-11}$</td>
<td>$\mathcal{O}(10^{-13} - 10^{-14})$</td>
</tr>
<tr>
<td>$\text{BR}(\mu \rightarrow e e e)$</td>
<td>$1.1 \times 10^{-12}$</td>
<td>$\mathcal{O}(10^{-13} - 10^{-14})$</td>
</tr>
<tr>
<td>$\text{BR}(\mu \rightarrow e \text{ in Nuclei (Ti)})$</td>
<td>$1.1 \times 10^{-12}$</td>
<td>$\mathcal{O}(10^{-18})$</td>
</tr>
<tr>
<td>$\text{BR}(\tau \rightarrow e \gamma)$</td>
<td>$1.1 \times 10^{-7}$</td>
<td>$\mathcal{O}(10^{-8})$</td>
</tr>
<tr>
<td>$\text{BR}(\tau \rightarrow e e e)$</td>
<td>$2.7 \times 10^{-7}$</td>
<td>$\mathcal{O}(10^{-8})$</td>
</tr>
<tr>
<td>$\text{BR}(\tau \rightarrow e \mu \mu)$</td>
<td>$2. \times 10^{-7}$</td>
<td>$\mathcal{O}(10^{-8})$</td>
</tr>
<tr>
<td>$\text{BR}(\tau \rightarrow \mu \gamma)$</td>
<td>$6.8 \times 10^{-8}$</td>
<td>$\mathcal{O}(10^{-8})$</td>
</tr>
<tr>
<td>$\text{BR}(\tau \rightarrow \mu \mu \mu)$</td>
<td>$2 \times 10^{-7}$</td>
<td>$\mathcal{O}(10^{-8})$</td>
</tr>
<tr>
<td>$\text{BR}(\tau \rightarrow \mu e e)$</td>
<td>$2.4 \times 10^{-7}$</td>
<td>$\mathcal{O}(10^{-8})$</td>
</tr>
</tbody>
</table>

VI. MASS INSERTION BOUNDS FROM LEPTONIC PROCESSES

In this section, we study the constraints on slepton mass matrices in low energy SUSY imposed by several LFV transitions, namely $l_i \rightarrow l_j \gamma$, $l_i \rightarrow l_j l_k l_k$ and $\mu-e$ transitions in nuclei [46]. The present and projected bounds on these processes are summarized in Table [IV]. These processes are mediated by chargino and neutralino loops and therefore they depend on all the parameters entering chargino and neutralino mass matrices. In order to constrain the leptonic MIs $\delta_{ij}^d$, we will first obtain the spectrum at the weak scale for our SU(5) GUT theory as has been mentioned in detail in section [IV]. Furthermore, we take all the flavor off-diagonal entries in the slepton mass matrices equal to zero except for the entry corresponding to the MI we want to bound. To calculate the branching ratios of the different processes, we work in the mass eigenstates basis of double MIs $\left(\delta_{ij}^d\right)_{LL}$, $\left(\delta_{ij}^d\right)_{LR}$ in chromomagnetic operators. This dependence however becomes sizable only for very large values of $\tan \beta$. 
TABLE V: Bounds on leptonic $\delta_{12}^l$ from various $\mu \rightarrow e$ processes. The bounds are obtained by making a scan of $m_0$ and $M_{1/2}$ over the ranges $m_0 < 380$ GeV and $M_{1/2} < 160$ GeV and varying $\tan \beta$ within $5 < \tan \beta < 15$. The bounds are rather insensitive to the sign of the $\mu$ mass term.

<table>
<thead>
<tr>
<th>Type of $\delta_{12}^l$</th>
<th>$\mu \rightarrow e \gamma$</th>
<th>$\mu \rightarrow e e e$</th>
<th>$\mu \rightarrow e$ conversion in $T_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>$6 \times 10^{-4}$</td>
<td>$2 \times 10^{-3}$</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>RR</td>
<td>-</td>
<td>0.09</td>
<td>-</td>
</tr>
<tr>
<td>LR/RL</td>
<td>$1 \times 10^{-5}$</td>
<td>$3.5 \times 10^{-5}$</td>
<td>$3.5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

The process that sets the most stringent bounds is the $l_i \rightarrow l_j \gamma$ decay, whose amplitude has the form

$$T = m_i e^{\lambda_{j}(p - q)}[iq^{\nu}\sigma_{\lambda\nu}(A_{L}P_{L} + A_{R}P_{R})]u_{i}(p),$$

where $p$ and $q$ are momenta of the leptons $l_k$ and of the photon respectively, $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ and $A_{L,R}$ are the two possible amplitudes entering the process. The lepton mass factor $m_i$ is associated to the chirality flip present in this transition. In a supersymmetric framework, we can implement the chirality flip in three ways: in the external fermion line (as in the SM with massive neutrinos), at the vertex through a higgsino Yukawa coupling or in the internal gaugino line together with a chirality change in the sfermion line. The branching ratio of $l_i \rightarrow l_j \gamma$ can be written as

$$BR(l_i \rightarrow l_j \gamma) = \frac{48\pi^3 \alpha}{G_F^2}(|A_{L}^{ij}|^2 + |A_{R}^{ij}|^2),$$

with the SUSY contribution to each amplitude given by the sum of two terms $A_{L,R} = A_{L,R}^{n} + A_{L,R}^{c}$. Here $A_{L,R}^{n}$ and $A_{L,R}^{c}$ denote the contributions from the neutralino and chargino loops respectively.

Even though all our numerical results presented in Tables V, VII are obtained performing an exact diagonalization of sfermion and gaugino mass matrices, it is more convenient for the discussion to use the expressions for the $l_i \rightarrow l_j \gamma$ amplitudes in the MI approximation. In particular, we treat both the slepton mass matrix and the chargino and neutralino mass matrix off-diagonal elements...
Hand, the only term proportional to $\beta$ independent of $\tan^2 \theta$ where $\theta$ is the weak mixing angle, $\alpha = \frac{\beta}{2}$, $\delta_{ij}^l = \frac{\delta_{ij}}{m_i^2}$, $f_{ij}(a_1) + \mu M_1 \tan \beta \left( \frac{f_{3n}(a_1)}{m_i^2} - \frac{f_{2n}(a_1, b)}{(\mu^2 - M_i^2)} \right)$, $A_{ij}^L = \frac{\alpha_2}{4\pi} \left( \frac{\delta_{ij}^l}{m_i^2} \right)_{LL} \left[ f_{1n}(a_2) + f_{1c}(a_2) + \frac{\mu M_2 \tan \beta}{(M_i^2 - \mu^2)} \left( f_{2n}(a_2, b) + f_{2c}(a_2, b) \right) \right] + \tan^2 \theta_W \left( f_{1n}(a_1) + \mu M_1 \tan \beta \left( \frac{f_{3n}(a_1)}{m_i^2} + \frac{f_{2n}(a_1, b)}{(\mu^2 - M_i^2)} \right) \right)$, $A_{ij}^R = \frac{\alpha_1}{4\pi} \left( \frac{\delta_{ij}^l}{m_i^2} \right)_{RR} \left[ 4 f_{1n}(a_1) + \mu M_1 \tan \beta \left( \frac{f_{3n}(a_1)}{m_i^2} - \frac{2 f_{2n}(a_1, b)}{(\mu^2 - M_i^2)} \right) \right] + \left( \frac{\delta_{ij}^l}{m_i^2} \right)_{LR} \left( \frac{M_1}{m_i^2} \right) \left( 2 f_{2n}(a_1) \right)$, where $\theta_W$ is the weak mixing angle, $\alpha_{1,2} = M_1^2/\tilde{m}^2$, $b = \mu^2/m_i^2$, and $f_{i(c,n)}(x, y) = f_{i(c,n)}(x) - f_{i(c,n)}(y)$. The loop functions $f_i$ are given as

\[
\begin{align*}
f_{1n}(x) &= (-17 x^3 + 9 x^2 + 9 x - 1 + 6 x (x + 3) \ln x)/(24(1 - x)^5), \\
f_{2n}(x) &= (-5 x^2 + 4 x + 1 + 2 x (x + 2) \ln x)/(4(1 - x)^4), \\
f_{3n}(x) &= (1 + 9 x - 9 x^2 - x^3 + 6 x (x + 1) \ln x)/(3(1 - x)^5), \\
f_{1c}(x) &= (-x^3 - 9 x^2 + 9 x + 1 + 6 x (x + 1) \ln x)/(6(1 - x)^5), \\
f_{2c}(x) &= (-x^2 - 4 x + 5 + 2 (2 x + 1) \ln x)/(2(1 - x)^4). \end{align*}
\]  

We note that all $\left( \frac{\delta_{ij}^l}{m_i^2} \right)_{LL}$ contributions with internal chirality flip are $\tan \beta$-enhanced. On the other hand, the only term proportional to $\left( \frac{\delta_{ij}^l}{m_i^2} \right)_{LR}$ arises from pure $\tilde{B}$ exchange and it is completely independent of $\tan \beta$, as can be seen from Eqs. (26) and (27). Therefore the phenomenological

<table>
<thead>
<tr>
<th>Type of $\delta_{ij}^l$</th>
<th>$\tau \rightarrow e\gamma$</th>
<th>$\tau \rightarrow eee$</th>
<th>$\tau \rightarrow e\mu\mu$</th>
</tr>
</thead>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RR</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>LR/RL</td>
<td>0.04</td>
<td>0.5</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE VI: Bounds on leptonic $\delta_{ij}^l$ from various $\tau \rightarrow e$ processes obtained using the same values of SUSY parameters as in Table [V]
TABLE VII: Bounds on leptonic $\delta_{23}^l$ from various $\tau \to \mu$ processes obtained using the same values of SUSY parameters as in Table V.

<table>
<thead>
<tr>
<th>Type of $\delta_{23}^l$</th>
<th>$\tau \to \mu \gamma$</th>
<th>$\tau \to \mu\mu\mu$</th>
<th>$\tau \to \mu e\mu$</th>
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</thead>
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<td>-</td>
</tr>
<tr>
<td>RR</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>LR/RL</td>
<td>0.03</td>
<td>-</td>
<td>0.5</td>
</tr>
</tbody>
</table>

bounds on $\left(\delta_{ij}^l\right)_{LL}$ depend on $\tan \beta$ to some extent, while those on $\left(\delta_{ij}^l\right)_{LR}$ do not. The bounds on LL and RL MIs are expected to approximately fulfill the relation

$$\left(\delta_{ij}^l\right)_{LR} \simeq \frac{m_i}{m} \tan \beta \left(\delta_{ij}^l\right)_{LL}.$$ 

This is confirmed by our numerical study.

The $\delta_{RR}^d$ sector requires some care because of the presence of cancellations among different contributions to the amplitudes in regions of the parameter space. The origin of these cancellations is the destructive interference between the dominant contributions coming from the $\tilde{B}$ (with internal chirality flip and a flavor-conserving LR mass insertion) and $\tilde{B}\tilde{H}^0$ exchange [46, 47]. We can see this in the MI approximation if we compare the $\tan \beta$ enhanced terms in the second line of Eq. (26) with the $\tan \beta$ enhanced terms in Eq. (27). Here the loop function $f_3(a_1)$ corresponds to the pure $\tilde{B}$ contribution while $f_{2n}(a_1, b)$ represents the $\tilde{B}\tilde{H}^0$ exchange. These contributions have different relative signs in Eq. (26) and Eq. (27) due to the opposite sign in the hypercharge of $SU(2)$ doublets and singlets. Thus, the decay $l_i \to l_j \gamma$ does not allow to put a bound on the RR sector. We can still take into account other LFV processes such as $l_i \to l_j l_k l_k$ and $\mu-e$ in nuclei. These processes get contributions not only from penguin diagrams (with both photon and Z-boson exchange) but also from box diagrams. Still the contribution of dipole operators, being also $\tan \beta$-enhanced, is dominant. Disregarding other contributions, one finds the relations

$$\frac{Br(l_i \to l_j l_k l_k)}{Br(l_i \to l_j \gamma)} \simeq \frac{\alpha_e}{3\pi} \left(\log \frac{m_i^2}{m_{l_k}^2} - 3\right),$$

$$Br(\mu-e \text{ in Ti}) \simeq \alpha_e BR(\mu \to e\gamma),$$

which clearly shows that $l_i \to l_j \gamma$ is the strongest constraint and gives the more stringent bounds on the different $\delta_{ij}$'s. As we have mentioned above, however, in the case of $\delta_{RR}^d$ the dominant dipole contributions interfere destructively in regions of parameters, so that $Br(l_i \to l_j \gamma)$ is strongly suppressed while $Br(\mu-e \text{ in nuclei})$ and $Br(l_i \to l_j l_k l_k)$ are dominated by monopole penguin
(both $\gamma^*$ and Z-mediated) and box diagrams. The formulae for these contributions can be found in Ref. [48]. However, given that non-dipole contributions are typically much smaller than dipole ones outside the cancellation region, it follows that the bound on $\delta_{LR}^l$ from $\mu \rightarrow eee$ are expected to be less stringent than the one on $\delta_{LL}^l$ from $\mu \rightarrow e\gamma$ by a factor $\sqrt{\alpha/(8\pi)} \tan \beta \simeq 0.02/\tan \beta$, if the experimental upper bounds on the two BRs were the same. This is partly compensated by the fact that the present experimental upper bound on the $BR(\mu \rightarrow eee)$ is one order of magnitude smaller than that on $BR(\mu \rightarrow e\gamma)$, as shown in Tab. IV. On the other hand, the process $BR(\mu - e$ in nuclei) suffers from cancellations through the interference of dipole and non-dipole amplitudes as well. These cancellations prevent us from getting a bound in the RR sector from the $\mu-e$ conversion in nuclei now as well as in the future when their experimental sensitivity will be improved. However, the $\mu \rightarrow e\gamma$ and $\mu-e$ in nuclei amplitudes exhibit cancellations in different regions of the parameter space so that the combined use of these two constraints produces a competitive or even stronger bound than the one we get from $BR(\mu \rightarrow eee)$ alone [46].

We summarize the different leptonic bounds in tables V-VII. All these bounds are obtained making a scan of $m_0$ and $M_{1/2}$ over the ranges $m_0 < 380$ GeV and $M_{1/2} < 160$ GeV and therefore correspond to the heaviest possible sfermions. As expected, the strongest bounds for $\delta_{LL}^l$ and $\delta_{LR}^l$ come always from $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ processes. In the case of $\delta_{RR}^l$ we can only obtain a mild bound for $(\delta_{12}^l)_{RR}$ from $\mu \rightarrow eee$ and there are no bounds for $(\delta_{23}^l)_{RR}$ and $(\delta_{13}^l)_{RR}$. Notice, however, that does not mean that these LFV processes are not effective to constrain the SUSY parameter space in the presence of RR MIs. For most of the values of $m_0$ and $M_{1/2}$ there is no cancellation and the values of these MI are required to be of the order of the LL bounds. Only for those values of $m_0$ and $M_{1/2}$ satisfying the cancellation conditions a large value of the RR MI is allowed. Therefore, we must check individually these constraints for fixed values of the SUSY parameters.

VII. QUARK-LEPTON MI RELATIONS IN GUT SCENARIOS

In the previous two sections we have collected the MI bounds obtained from the hadronic and leptonic processes. In the present section, let us consider a GUT theory, with the corresponding GUT symmetric relations holding at the GUT scale. We will focus on the SU(5) case as summarized by the relations in Table I. To make a comparison between leptonic and hadronic MI bounds we have to take into account that these bounds have a different dependence on the low-energy parameters of the theory. On the one hand, the hadronic processes are dominated by gluino contributions and
therefore they mainly depend on the gluino mass and the “average squark mass”.

On the contrary, as we saw in section VI, the leptonic bounds depend basically on three parameters: gaugino mass, “average slepton mass” and $\tan \beta$. In our model squark and slepton masses originate from a common scalar mass $m_0$ at the GUT scale and therefore we can relate the average squark and slepton masses.

As we have discussed in section III B, the off-diagonal elements of sfermion mass matrices are not significantly modified in the RGE evolution from $M_{\text{GUT}}$ to $M_W$. However, under some conditions, as for example in the presence of large neutrino Yukawa couplings, the RG evolution can generate sizable off-diagonal elements in the slepton mass matrices even starting from a vanishing value at $M_{\text{GUT}}$. Clearly these effects are never present in the squark mass matrices, thus breaking the GUT symmetric relations. This implies that, given our ignorance on the structure of neutrino Yukawa couplings, we have to be careful when applying the MI bounds obtained from quarks to leptons or vice-versa.

In fact, if we obtain a bound on a $\delta^l_{ij}$ MI from a leptonic process at low scales, we can say that, barring accidental cancellations, this bound applies both to the mass insertions already present at $M_{\text{GUT}}$ and to the mass insertions generated radiatively between $M_{\text{GUT}}$ and $M_{\nu_R}$. Therefore we can translate this low-scale bound into a bound on the MI at the GUT scale. This bound applies also to the squark MI at $M_{\text{GUT}}$ and using RGEs we can transport this bound to the electroweak scale. For example, in $SU(5)$, we find:

\[
|\langle \delta^d_{ij}\rangle_{RR}| \leq \frac{m^2}{m^2_{\tilde{d}}} |\langle \delta^l_{ij}\rangle_{LL}| .
\]

The situation is different if we try to translate the bound from quark to lepton MIs. An hadronic MI bound at low energy leads, after RGE evolution, to a bound on the corresponding grand-unified MI at $M_{\text{GUT}}$, applying both to slepton and squark mass matrices. However, if the neutrino Yukawa couplings have sizable off-diagonal entries, the RG running from $M_{\text{GUT}}$ to $M_W$ could still generate a new contribution to the slepton MI that exceeds this GUT bound. Therefore hadronic bounds cannot be translated to leptons unless we make some additional assumptions on the neutrino Yukawa matrices.

On general grounds, given that SM contributions in the lepton sector are absent and that the branching ratios of leptonic processes constrain only the modulus of the MIs, it turns out that all the MI bounds arising from the lepton sector are circles in the $\text{Re} (\delta^1_{ij})_{AB} - \text{Im} (\delta^1_{ij})_{AB}$ plane and

\[\text{(30)}\]

Note that the $\tan \beta$ dependence seeps in once we consider the double MIs $\langle \delta_{ij}\rangle_{LL,RR} \langle \delta_{ij}\rangle_{LR,RL}$. 

[5]
are centered at the origin. In some cases, the hadronic bounds from $B$ physics constraints are too loose and, in principle, this would allow the presence of MIs larger than one. The Mass Insertion approximation cannot be trusted when the bounds on the $\delta$ approach values $O(1)$. Therefore, in our analysis we always consider MI values smaller than one.\(^6\) In the following we will analyze

\(^6\) Even in the case we consider the possibility of $O(1)$ $\delta$s, the requirement of absence of tachyonic scalar masses in the slepton sector, i.e. $(\delta_{ij}^l)_{AB} \leq 1$, provides a bound on the squark MIs through the GUT-symmetric relations among leptonic and hadronic MIs (see Table I)
the effect of leptonic bounds on the quark mass insertions. For instance, if we had a $\left(\Delta_{23}^d\right)_{LR}$ at the GUT scale, this would have effects both in the $\tau \rightarrow \mu \gamma$ and $b \rightarrow s \gamma$ decays. Neglecting the effects of neutrino Yukawas that, if present, could generate an additional $\left(\delta_{23}^d\right)_{LR}$ in the RGE evolution, and using $\left(\delta_{23}^d\right)_{LR} \simeq (m_b/m_\tau) \left(m_2^2/m_3^2\right) \left(\delta_{23}^l\right)_{RL}$, a bound on $\left(\delta_{23}^l\right)_{RL}$ from the $\tau \rightarrow \mu \gamma$ decay translates into a bound on $\left(\Delta_{23}^d\right)_{LR}$ thus, into a bound on the SUSY contributions to $\text{BR}(B \rightarrow X_s \gamma)$. Similarly, the bound on $\left(\delta_{23}^d\right)_{LR}$ would translate into an upper bound for the $\tau \rightarrow \mu \gamma$ branching ratio.

We present the effect of this GUT correlation in our numerical analysis in Fig. 1. In the top row, we show the allowed region in the $\text{Re}\left(\delta_{23}^d\right)_{LR}$ - $\text{Im}\left(\delta_{23}^d\right)_{LR}$ plane (larger boxes correspond to higher probability densities), using hadronic (left) or leptonic (right) constraints only. We see that the present leptonic bounds have no effect on the $\left(\delta_{23}^d\right)_{LR}$ couplings. This is due both to the existence of strong hadronic bounds from $b \rightarrow s \gamma$ and CP asymmetries and to the relatively weak leptonic bounds here. Even assuming a future bound on $\text{BR}(\tau \rightarrow \mu \gamma)$ at the level of $10^{-8}$, attainable at B factories, leptonic bounds would marginally improve the hadronic constraints. We remind the reader that the LR bounds are basically independent of $\tan \beta$ and hence this fact does not change for different $\tan \beta$ values. In Fig. 2, we present the results of the same analysis for $\left(\delta_{23}^d\right)_{RL}$. While the leptonic bounds do not change with respect to the previous case, the hadronic ones are different as SM and SUSY $b \rightarrow s \gamma$ and $b \rightarrow s \ell^+ \ell^-$ amplitudes interfere in the LR case only.

Similarly, if we have a $\left(\Delta_{23}^d\right)_{RR}$, the corresponding MIs at the electroweak scale are $\left(\delta_{23}^d\right)_{RR}$ and $\left(\delta_{23}^l\right)_{LL}$ that contribute to $\Delta M_{Bs}$ and $\tau \rightarrow \mu \gamma$ respectively (the impact of $\left(\Delta_{23}^d\right)_{RR}$ on $b \rightarrow s \gamma$ and $b \rightarrow s \ell^+ \ell^-$ is not relevant because of the absence of interference between SUSY and SM contributions). In Fig. 3, we present the allowed values of $\text{Re}\left(\delta_{23}^d\right)_{RR}$ and $\text{Im}\left(\delta_{23}^d\right)_{RR}$ with the different constraints. The leptonic constraints are quite effective as the bound on the $\text{BR}(\tau \rightarrow \mu \gamma)$ from B-factories is already very stringent, while the recent measurement of $\Delta M_{Bs}$ is less constraining. The plots correspond to $5 < \tan \beta < 15$, thus, the absolute bound on $\left(\delta_{23}^l\right)_{LL}$ is set by $\tan \beta = 5$ and it scales with $\tan \beta$ as $\left(\delta_{23}^l\right)_{LL} \sim (5/\tan \beta)^7$.

In Fig. 4, we show the results of our analysis for $\left(\delta_{23}^d\right)_{LL}$. In this case, there is no appreciable improvement from the inclusion of leptonic constraints. In fact, we remind that $\tau \rightarrow \mu \gamma$ is not effective to constrain $\left(\delta_{23}^d\right)_{RR}$, i.e. the leptonic MI related to $\left(\delta_{23}^d\right)_{LL}$ in our SUSY-GUT’s scheme, in large portions of the parameter space because of strong cancellations among amplitudes.

7 Sizable SUSY contributions to $\Delta M_{Bs}$ are still possible from the Higgs sector in the large $\tan \beta$ regime both within [49, 50] and also beyond [9] the Minimal Flavor Violating (MFV) framework. However, for our parameter space, the above effects are completely negligible.
The analysis of the constraints on the different \( (\delta^{d}_{13}) \) MIs is similar to that of the \( (\delta^{d}_{23}) \) MIs. In this case, the hadronic constraints come mainly from \( \Delta M_{B_d} \) and the different CP asymmetries measured at B-factories. The leptonic bounds are due to the decay \( \tau \rightarrow e\gamma \). We present the numerical results in Figs. 5, 6 and 7. Notice that, in spite of comparable experimental resolutions on \( \Delta M_{B_s} \) and \( \Delta M_{B_d} \), the constraints on \( (\delta^{d}_{13})_{RR} \) are stronger than those on \( (\delta^{d}_{23})_{RR} \) (see Figs. 8, 9). The reason is that, in the 13 sector, we can make use also of the constraints from \( 2\beta \), in addition to those relative to \( \Delta M_{B_d} \). Moreover, from Fig. 6 we see that the constraints arising from a combined analysis of leptonic and hadronic processes are much more effective than the bounds obtained from
the hadronic and leptonic processes alone. This is due to the fact that the maximal allowed values for the hadronic and leptonic deltas in the upper row of Fig. 6 correspond to different values of \((m_0, M_{1/2})\) in the two cases. So, the different \(m_0\) dependence of \(\Delta M_{B_s}\) and \(\tau \to e\gamma\) provide the explanation of their interplay in constraining \((\delta_{23}^d)_{RR}\) (as it is clearly shown in the lower plot on the left of Fig. 6). On the other hand, the above interesting interplay is not effective in the \(\Delta M_{B_s}\).

---

\[\text{From Eq. (19) we see that the maximal value of the radiatively induced } \delta_{LL}^d \text{ corresponds to small } M_{1/2} \text{ (small } m_{\tilde{q}}) \text{ and large } m_0. \text{ The largest allowed value for } \delta_{RR}^d \text{ is set by the minimum value of this radiatively induced } \delta_{LL}^d, \text{ i.e. large } M_{1/2} \text{ and small } m_0/M_{1/2}. \text{ On the other hand the maximal allowed values from the leptonic delta correspond to large } M_{1/2} \text{ and large } m_0.\]
case due to the better bound on the decay $\tau \to \mu \gamma$ and the absence of analog $2\beta$ constraints in the 23 sector.

Moreover, the leptonic bounds do not have a sizable impact in the RL and LL cases, as clearly shown in Figs. [5] and [7] respectively. The LR case is identical to the RL one.

Finally we analyze the 1–2 sector. In Fig. [8] we can see the allowed values of $\text{Re} \left( \delta_{12}^d \right)_{\text{LL}}$ and $\text{Im} \left( \delta_{12}^d \right)_{\text{LL}}$. As before, the plot on the left of the upper row corresponds to the allowed values of these parameters from hadronic constraints. The dominant hadronic bound comes from $\varepsilon_K$ which however is ineffective along the $\text{Re} \left( \delta_{12}^d \right)_{\text{LL}}$ and $\text{Im} \left( \delta_{12}^d \right)_{\text{LL}}$ axes. These directions are eventually
bounded by the milder $\Delta M_K$ constraint. The upper right plot represents the values allowed taking into account the limits on the branching ratios of the processes $\mu \to e\gamma$, $\mu \to eee$ and $\mu-e$ conversion in nuclei as per the SU(5) relations between $(\delta_{12})_{LL}$ and $(\delta_{12})_{RR}$. As we saw in the previous section, the $\mu \to e\gamma$ decay does not provide a bound to this MI due to the presence of cancellations between different contributions. We can only obtain a relatively mild bound, $(\delta_{12})_{RR} \leq 0.09$, if we take simultaneously into account all the leptonic processes. However, we see in the lower left plot that, once rescaled by the factor $\frac{\bar{m}_e^2}{\bar{m}_\mu^2}$, this bound is more stringent than $\Delta M_K$, so that it cuts the tails along the axes. Using the expected bounds for these decays from the proposed experiments,
we obtain the lower right plot in the figure. In these plots the leptonic constraints come from the monopole and box contributions to $\mu \to eee$ and $\mu-e$ conversion in nuclei and therefore these bounds are independent of $\tan \beta$. There is a modest improvement of the bounds on $(\delta_{12}^{d})_{LL}$ which however do not take into account possible improvements of the $\epsilon_K$ constraint.

In Fig. 9 we present the allowed values of Re $(\delta_{12}^{d})_{RR}$ and Im $(\delta_{12}^{d})_{RR}$. In this case, leptonic constraints, already using the present upper bound, are competitive and constrain the direction in which the constraint from $\epsilon_K$ is not effective (see the upper left plot). Notice that this direction is rotated with respect to the LL case because of the presence of LL × RR double MIs.
In Figure 10 we can see the bounds on $\text{Re}(\delta_{12}^d)_{\text{RL}}$ and $\text{Im}(\delta_{12}^d)_{\text{RL}}$. The same bounds apply also to the LR case. For these MIs, the hadronic bounds come also from $\epsilon'/\epsilon$ and are quite stringent. However, the bounds from $\mu \rightarrow e\gamma$ are even more effective. Also in this case the bounds are independent of $\tan \beta$.

**VIII. CONCLUSIONS**

While there exists a huge literature dealing, separately, with FCNC constraints on the hadronic and leptonic SUSY soft breaking terms, much less attention has been devoted to the intriguing...
possibility that the two sectors may find correlated bounds in SUSY theories with an underlying grand unified symmetry.

We have pursued such an analysis in the context of a broad class of theories which are based on two appealing assumptions: i) local SUSY is broken in the observable sector through gravity mediation with the corresponding soft breaking terms arising (as momentum-independent hard terms) at an energy scale close to the Planck mass; ii) the fundamental gauge symmetry of the theory includes a grand unification of quarks and leptons which is present down to the typical GUT scale and hence constrains the form of the supergravity Lagrangian, in particular its Kähler potential.
The presence of the general conditions i) and ii) entails some correlation between hadronic and leptonic soft terms at the superlarge scale where they first arise.

Obviously, the extent to which such correlation survives when performing the superlarge running of the soft breaking terms from a scale close to the Planck mass down to the electroweak scale depends on the new physics present in such long interval. Here we adopted the two simplest possibilities: just a big desert or a new intermediate scale, below the GUT scale, where the right-handed neutrinos acquire a mass in a SUSY see-saw framework. Moreover, to make the problem treatable in a model independent way, we made two relevant simplifications on the possible pattern of the
soft breaking terms: we considered that only one of the FC MIs in Eqs. (5)–(7) is switched on at a time and in the discussion of the hadronic FCNC constraints we took the gluino exchange to be the representative source of the SUSY FCNC contributions. On the other hand, in the leptonic sector where there is no analog of the gluino dominance, we performed a full computation (in the slepton mass eigenstate basis) pointing out the possible cancellations which may arise when the various SUSY contributions are taken into account. Indeed, the first part of the present work provides a renewed, thorough and comprehensive assessment of the bounds on the hadronic and leptonic MIs taking into account all the relevant pieces of information on the FCNC phenomenology we have
been accumulating in these last years. Obviously this constitutes the basis for the subsequent work of correlating the hadronic and leptonic FC MIs which is the main goal of the paper.

The extent of the impact of such correlation on the present upper bounds on the FC $\delta$ parameters has been exemplified in the study of the role of LFV processes in constraining the hadronic $\delta$ parameters in the down-squark sector. The relevant hadronic processes (in kaon and beauty physics) which are involved in bounding the $\left( \delta_{ij}^d \right)_{AB}$ (AB=LL,RR,LR,RL) already provide rather stringent limits (see Table III) on most of them. Yet, the inclusion of the correlated constraints arising from $l_i \to l_j + \gamma$ proves to be extremely powerful for quite a number of such $\delta$’s. This is the case for $\left( \delta_{23}^d \right)_{RR}$ (Fig. 3) as well as for $\left( \delta_{12}^d \right)_{RL}$ and $\left( \delta_{12}^d \right)_{LR}$ (Fig. 10).

Leptonic bounds are competitive for $\left( \delta_{12}^d \right)_{LL}$ (Fig. 8), $\left( \delta_{12}^d \right)_{RR}$ (Fig. 9) and $\left( \delta_{13}^d \right)_{RR}$ (Fig. 6). Interestingly enough, this holds true even when we consider the present upper bounds on the relevant LFV processes and, at least in some cases, it becomes dramatic when we take into account the future experimental sensitivities to LFV (namely, the third column of Table IV). On the other hand, most of $\left( \delta_{ij}^d \right)$ FC insertions are essentially unscathed by the inclusion of the related bounds from LFV. This is the case for LL, LR and RL MIs in the 13 and 23 sectors.

The above considerations provide a precious tool in the effort to disentangle the underlying SUSY theory in case some SUSY particles should show up in LHC physics. It will be very difficult to have some “direct” signal of the presence of a grand unified supergravity (for instance, through the observation and study of proton decay modes). Looking at correlated SUSY contributions

<table>
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<tr>
<th>$\left( \delta_{ij}^d \right)_{AB}$</th>
<th>Value</th>
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</tr>
<tr>
<td>$\left( \delta_{12}^d \right)_{RR}$</td>
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<td>$\left( \delta_{13}^d \right)_{RL}$</td>
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<td>$\left( \delta_{23}^d \right)_{RL}$</td>
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</table>

TABLE VIII: The final score of the quarks vs leptons match showing the dominance of hadronic bounds.
in hadronic and leptonic FCNC processes, together with some information on the scale of the soft breaking sector from LHC, we could have some hints on whether there exists a grand unified underlying symmetry. Admittedly, even if we are particularly lucky (for instance we observe SUSY particles and some LFV processes), this is going to be a long term project which requires a lot of sweat and educated guesses. But, at least, our paper indicates possible paths to follow to achieve some result in the difficult task of “reconstructing” the correct fundamental SUSY theory.

In this sense, our work is yet another relevant proof of the complementarity of flavor and LHC physics in shedding light on such an underlying new physics beyond the SM.

On a more phenomenological ground, our results can find an important application in individuating for each FC $\delta$ MI which process (either hadronic or leptonic) is more suitable to constrain it. If it is true that in some cases hadronic $\delta$’s find a better limit when LFV processes are taken into account (as we discussed above), also the reverse turns out to hold in several circumstances. For instance, we pointed out that there are cases when no bound emerges from LFV for some leptonic $\delta$. This happens for $\left(\delta_{23}^l\right)_{RR}$ (see Table VII) and in this case we have to make use of FCNC in $B$ physics to extract a bound on such leptonic FC quantity. There is a healthy competition between hadronic and leptonic FCNC physics in limiting the SUSY MIs. A comprehensive score of such hadron versus lepton “match” is provided in Table VIII which shows the final ranking: quarks win with 21 points and leptons follow with 12 points.9

In conclusion, we hope that this work may display the richness which is present in flavor physics once we assume a grand unified supergravity framework with gravity mediated SUSY breaking. It could be that, at the end, flavor physics is one of the very few handles we have to understand from low-energy physics whether Nature has chosen to possess supersymmetry and grand unification at the root of its symmetries.

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9 Scores in Table VIII and points assignment follow the rules of the Italian (world champion) football league.
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