D–¯D MIXING AND NEW PHYSICS:
GENERAL CONSIDERATIONS AND CONSTRAINTS ON THE MSSM

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Combining the recent experimental evidence of D–¯D mixing, we extract model-independent information on the mixing amplitude and on its CP-violating phase. Using this information, we present new constraints on the flavour structure of up-type squark mass matrices in supersymmetric extensions of the Standard Model.

The study of meson oscillations represents one of the most powerful probes of New Physics (NP) currently available. The K and B_q systems are very well studied experimentally and all the measurements performed up to now are compatible with the Standard Model (SM) expectation, although there is still room for NP which could be revealed with improved theoretical tools and experimental facilities hopefully available in the future 1, 2.

As far as the B_u is concerned, the experimental evidence of oscillation was found only recently at the TeVatron 3. While the oscillation frequency is already very well known, information on the phase of the mixing amplitude is still quite vague, leaving ample room for experimental improvements expected from hadronic colliders.

All this experimental information allows to put model-independent constraints on NP contributions to the mixing amplitudes involving down-type quarks 1. These constraints already induce highly non-trivial bounds on the flavour structure of many extensions of the SM. In particular, considering the Minimal Supersymmetric Standard Model (MSSM), the flavour properties of the down-type squark mass matrices have already been thoroughly analyzed 4.

On the other hand, up to now no evidence was found of oscillations of mesons involving up-type quarks. Correspondingly, the off-diagonal entries of up-type squark mass matrices were only weakly bounded 5, 6. It is remarkable that one of the proposed mechanisms to explain the flavour structure of the MSSM and to suppress the unwanted SUSY contributions to Flavour-Changing Neutral Current (FCNC) processes, namely alignment of quark and squark mass matrices 7, naturally produces sizable effects in the up-type sector. In the absence of stringent experimental information, these models were not tightly constrained 6.

Very recently, BaBar 8 and Belle 9, 10 independently reported evidence for D–¯D mixing. In this letter we use this information, combined with previous constraints on D mixing 11, 12, 13, 14, to put model-independent bounds on the mixing amplitude and to constrain the relevant entries of the up-type squark mass matrices. To fulfill this task we use the mass-insertion approximation. Treating off-diagonal sfermion mass terms as interactions, we perform a perturbative expansion of FCNC amplitudes in terms of mass insertions. The lowest non-vanishing order of this expansion gives an excellent approximation to the full result, given the tight experimental constraints on flavour changing mass insertions. It is most convenient to work in the super-CKM basis, in which all gauge interactions carry the same flavour dependence as SM ones. In this basis, we define the mass insertions (δ_{i2}^u)_{AB} as the off-diagonal mass terms connecting up-type squarks of flavour u and c and helicity A and B, divided by the average squark mass.

Let us first discuss the recent experimental novelties. BaBar studied D^0 → K^+π^- and D^0 → K^-π^+ decays as a function of the proper time of the D mesons. Assuming no CP violation in mixing, which is safe in the SM, this analysis allows to measure the parameters x' and y', defined in terms of the mixing parameters x and y through the relations:

x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi}, \quad y' = -x \sin \delta_{K\pi} + y \cos \delta_{K\pi},

where δ_{K\pi} is the relative strong phase between the Cabibbo-suppressed D^0 → K^+π^- decays and the Cabibbo-favoured D^0 → K^-π^+ ones. This phase has been recently measured by CLEO-c 11. From a fit to D^0 and D^0̅ decays, BaBar is able to exclude the point x'^2 = y'^2 = 0 (which corresponds to the no-mixing scenario) with a 3.9σ significance (including systematic effects). In addition, the BaBar collaboration fitted separately the parameters x'^2 and y'^2 of D^0 → K^±π^∓ decays allowing for CP violation.

Belle directly determines x and y, studying the D^0 → K^0_Sπ^+π^- Dalitz plot. In this way, one can separately measure the mixing parameters and the strong phase. Even though this analysis is not precise enough to claim the observation of D–¯D mixing, it allows to disentangle mixing parameters from the strong phase δ_{K\pi}, when D^0 → K^0_Sπ^+π^- and D^0 → Kπ results are combined.
Belle also found evidence of $D \to \bar{D}$ mixing, observing a deviation from zero (at 3.2σ including the systematic error) of $y_{\text{CP}} = \frac{\tau(D^+ \to K^+\pi^-)}{\tau(D^0 \to f_{CP}^-)} - 1$ and in addition measured the CP asymmetry $A_{\Gamma} = (\Gamma(D \to KK) - \Gamma(\bar{D} \to KK)))/\left(\Gamma(D \to KK) + \Gamma(\bar{D} \to KK)\right)$.

We assume that CP violation can occur in mixing but not in decay amplitudes, since the latter are dominated by SM tree-level contributions. Therefore, we assume that $\Gamma_{12}$ is real. Our aim is to determine the parameters $|M_{12}|e^{-i\Phi_{12}}$ and $\Gamma_{12}$ from the available experimental data. One can write the following relations \[15]\:

\[|M_{12}| = \frac{1}{\tau_D} \sqrt{\frac{x^2 + \delta^2 y^2}{4(1 - \delta^2)}}, \quad |\Gamma_{12}| = \frac{1}{\tau_D} \sqrt{\frac{y^2 + \delta^2 x^2}{1 - \delta^2}},\]

\[
\sin \Phi_{12} = \frac{|\Gamma_{12}|^2 + 4|M_{12}|^2 - (x^2 + y^2)|q/p|^2/\tau_D^2}{4|M_{12}|\Gamma_{12}},
\]

\[
\phi = \arg(y + i\delta x), \quad y'_{\pm} = \left|\frac{q}{p}\right| \left(y' \cos \phi \mp x' \sin \phi\right),
\]

\[
x'^2_{\pm} = \left|\frac{q}{p}\right|^2 \left(x' \cos \phi \pm y' \sin \phi\right)^2, \quad R_M = x^2 + y^2/2,
\]

\[
y_{\text{CP}} = \left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\frac{y}{2} \cos \phi - \left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\frac{x}{2} \sin \phi,
\]

\[
A_{\Gamma} = \left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\frac{y}{2} \cos \phi - \left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\frac{x}{2} \sin \phi,
\]

where $\delta = |p|^2 - |q|^2$ and $\phi$ is the phase of the mixing parameter $q/p$. We fit for $|M_{12}|$, $|\Gamma_{12}|$ and $\Phi_{12}$ using the experimental inputs listed in Table I, taking into account the correlations between $y_{\pm}$ and $x_{\pm}$ in the BaBar results. Notice that all observables can be written in terms of $|M_{12}|$, $|\Gamma_{12}|$ and $\Phi_{12}$.

The results of the simultaneous fit are quoted in Table II, and shown in Fig. We fit for $|M_{12}|$, $|\Gamma_{12}|$ and $\Phi_{12}$ using the experimental inputs listed in Table I, taking into account the correlations between $y_{\pm}$ and $x_{\pm}$ in the BaBar results. Notice that all observables can be written in terms of $|M_{12}|$, $|\Gamma_{12}|$ and $\Phi_{12}$.

Concerning the upper bound on $|M_{12}|$, we find an improvement of almost an order of magnitude with respect to the analysis of ref. \[15\], while for $x$ the improvement with respect to ref. \[17\] is about a factor of three.

The calculation of $|M_{12}|$ is plagued by long-distance contributions \[18\]. To take them into account, we proceed in the following way. We assume that the full amplitude $M_{12}$ is the sum of the NP amplitude $A_{\text{NP}}e^{i\phi_{\text{NP}}}$ and of a SM real amplitude containing both short- and long-distance contributions, $A_{\text{SM}}$. We take $A_{\text{SM}}$ to be flatly distributed in the range $[-0.02, 0.02]$ ps$^{-1}$, so that it can saturate the experimental bound in Table III and derive from the $\Phi_{12}$ vs $|M_{12}|$ distribution the p.d.f. for $A_{\text{NP}}$ vs $\phi_{\text{NP}}$, barring accidental order-of-magnitude cancellations between SM and NP contributions. The results, reported in Table III and shown in Fig. provide a new constraint that should be fulfilled by any extension of the SM. We see that the lack of knowledge of the SM contribution causes a dilution of the bound on $\phi_{\text{NP}}$. Clearly, if a reliable estimate of $A_{\text{SM}}$ were available, the constraint would be much more effective. Notice also that if $|M_{12}|$ is dominated by NP, then $\phi_{\text{NP}} \sim \Phi_{12}$ and the NP phase can be experimentally accessed.

### Table I: Experimental results used in our analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$6.2 \pm 2.0 \times 10^{-3}$</td>
<td>$0.0022, 0.0105$</td>
</tr>
<tr>
<td>$y$</td>
<td>$5.5 \pm 1.4 \times 10^{-3}$</td>
<td>$0.0027, 0.0084$</td>
</tr>
<tr>
<td>$\delta_{K_S}$</td>
<td>$-31 \pm 39 \times 10^{-2}$</td>
<td>$-103^\circ, 28^\circ$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$(1 \pm 7) \times 10^{-3}$</td>
<td>$-15^\circ, 17^\circ$</td>
</tr>
<tr>
<td>$</td>
<td>\delta</td>
<td>- 1$</td>
</tr>
<tr>
<td>$</td>
<td>M_{12}</td>
<td>$ (ps$^{-1}$)</td>
</tr>
<tr>
<td>$\Phi_{12}$ ($^\circ$)</td>
<td>$(2 \pm 14) \cup (179 \pm 14)$</td>
<td>$-30, 36 \cup [144, 210]$</td>
</tr>
<tr>
<td>$</td>
<td>\Gamma_{12}</td>
<td>$ (ps$^{-1}$)</td>
</tr>
</tbody>
</table>

### Table II: Results on mixing and CP violation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>68% prob.</th>
<th>95% prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{NP}}$ (ps$^{-1}$)</td>
<td>$[0, 0.006]$</td>
<td>$[0, 0.02]$</td>
</tr>
<tr>
<td>$\phi_{\text{NP}}$ ($^\circ$)</td>
<td>$[-180, -149] \cup [-180, -112] \cup [-68, 180]$</td>
<td>$[-31, 39] \cup [141, 180]$</td>
</tr>
</tbody>
</table>

### Table III: Allowed ranges for the NP amplitude.
FIG. 1: Probability density functions of the combined fit from Tab. II projected on $y$ vs $x$ (top left), $\phi$ vs $\phi$ vs $y$ (center left), $A_{NP}$ vs $\phi_{NP}$ (center right), $A_{NP}$ (bottom left) and $\phi_{NP}$ (bottom right). Dark (light) regions correspond to 68% (95%) probability.

FIG. 2: Probability density functions of the combined fit from Tab. II. Dark (light) regions correspond to 68% (95%) probability.

$B_1 = 0.87 \pm 0.03 \quad B_2 = 0.82 \pm 0.03 \quad B_3 = 1.07 \pm 0.09$

$B_4 = 1.08 \pm 0.03 \quad B_5 = 1.46 \pm 0.09$

TABLE IV: B parameters defined as in ref. [21] interpolated at the physical $D$ meson mass, renormalized at the scale $\mu = 2.8$ GeV in the Landau-RI scheme.

We now turn to the MSSM and consider the bounds on $(\delta_{12})_{AB}$ that can be obtained from the determination of $A_{NP}$ and $\phi_{NP}$ discussed above. To this aim, we focus on gluino exchange and use the full Next-to-Leading expression for the Wilson coefficients [19] and for the renormalization group evolution down to the hadronic scale of 2.8 GeV [20]. For the matrix elements, we extrapolate the results of ref. [21] as given in Table IV (see also ref. [22] for another recent calculation of $B_1$).

To select the allowed regions on the $\text{Re}(\delta_{12})_{AB}$--$\text{Im}(\delta_{12})_{AB}$ planes, we use the method described in ref. [22]. The results are presented in Fig. 3 for a reference value of 350 GeV for squark and gluino masses. We consider three cases. First, a dominant $LL$ mass inser-
tion. The case of a dominant \( RR \) insertion is completely identical. Second, a dominant \( LR \) insertion. In this case, chirality-flipping four-fermion operators are generated. These operators are strongly enhanced by the renormalization group evolution \[24\], so that these mass insertions are more strongly constrained than \( LL \) or \( RR \) ones. Constraints on \( RL \) insertions are identical. Finally, we can switch on simultaneously \( (\delta u_{12})_{LL} = (\delta u_{12})_{RR} \). In this case, we also generate chirality-flipping operators, so that the constraint is much stronger than the case in which \( (\delta u_{12})_{LL} \gg (\delta u_{12})_{RR} \).

In Table IV we report the bounds on the absolute value of the mass insertions for several values of gluino and squark masses. Our bounds are typically a factor of \(~3\) more stringent than those of ref. \[6\].

It is very interesting that SUSY models with quark-squark alignment generically predict \( (\delta u_{12})_{LL} \sim 0.2 \) \[6\]. We conclude that, to be phenomenologically viable, they need squark and gluino masses to be above \(~2\) TeV. Therefore, they probably lie beyond the reach of the LHC.

In this Letter, we have analyzed the first experimental evidence of \( D-\bar{D} \) mixing recently obtained by the BaBar and Belle collaborations. Combining the experimental results we obtained new constraints on the mixing amplitude and on NP contributions. We have then considered the MSSM and derived new bounds on off-diagonal squark mass terms connecting up and charm squarks. Finally, we have briefly commented on the impact of these new constraints on SUSY models with quark-squark alignment.

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