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# On the geometrical design of segmented annular arrays

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**Abstract** Segmented annular arrays (SAA) can be a good option to generate 3-D ultrasonic images. Design this kind of aperture involves a variety of geometrical parameters that are decisive for the quality of the image. For instance, it is possible to use regular or non-regular SAA configurations in which the number of annuli and the elements per annulus can be varied. The objective of this work is to analyze these configurations in order to define the most important design parameters. In this sense, our study is centered in several points: the alignment between rings, the array size and the element aspect ratio. The evaluation of these parameters is focused in obtaining the best trade-off between the number of elements and the image quality.

The arrays under analysis are equivalent in the sense that their size, number of active elements and active areas are similar and they emit the same ultrasonic pulse. In order to achieve a clear analysis of the grating lobes, the array factor approach, which considers the array formed by vibrating points, is applied. Also a more accurate analysis, based on the computation of the spatial impulse response approach, is made to reinforce our conclusions in both excitation modes, continuous wave and wide band.

## 1. INTRODUCTION

The development of ultrasonic volumetric imaging is strongly tied to the development of the 2D transducer array systems. Those systems are able to control the 2D transducer array emission and reception beam formation, allowing improve the image quality and the image formation velocity by focusing and steering the main beam[1]. Unfortunately due to the transducer distribution, image artifacts that are generated by array secondary lobes can reduce the signal to noise ratio. These lobes are known as grating lobes and avoid its formation is nowadays the key of the array design. For matrix and linear arrays, with a regular element distribution and consequently a high degree of periodicity, the composition of these lobes can only be avoided using an element distance limited to

$\lambda/2$. Nevertheless, due to the fact that the lateral resolution is determined by the total aperture of the transducer this condition forces several problems:

- The requirement of thousen of channels that increases the cost and complexity of the imaging system.
- The small size of elements which is associated to low signal-to-noise ratios (SNR).
- The electrical connections that have severe fabrication difficulties.

Some of these problems are now under research[2]. One way is to break the periodicity in the aperture using different thinning strategies to reduce the number of active elements maintaining at the same time good field characteristics[3]. However, SNR for such solutions can become low in excess and the element size still is maintained very small.

Our alternative assumes that grating lobes are intrinsic to any element distribution. So instead of suppress them our work is addressed to control its formation in order to ensure a SNR suitable for 3D imaging. What we are looking for is to find 2D distributions that allow applying inter-element spacing higher than $\lambda/2$ in the array, and consequently to reduce the number of arrays elements.

For this reason segmented annular arrays have been proposed[4, 5]. These apertures have two advantages: first, their axial symmetry which provides regularity in the radiated field and; second, compared with squared arrays their geometry entails a reduction of the periodicity level and therefore of the intensity of the grating lobes.

In this work we will show the how this periodicity reduction is achieved and what are the keys of segmented annular arrays design.

## 2. Tools for array design and analysis

The pressure waveform radiated over a field point $\vec{x}$ by an array can be computed thorough the equation[6]:

$$p(\vec{x},t) = \rho_o \frac{\partial v(t)}{\partial t} * h_R(\vec{x}, t - T_R) \tag{1}$$

where $v(t)$ is the velocity wave and the $h_R(\vec{x},t)$ is the array spatial impulse response.

$$h_R(\vec{x}, t - T_R) = \sum_{i=1}^{N} a_i h_i(\vec{x}, t - T_i) \tag{2}$$

Where N is the number of elements, $T_i, \quad i = 1, \cdots, N$ are the delays of the focus law and $a_i, \quad i = 1, \cdots, N$ are the gains of the apodization law, for each element. If elements are considered as emitting points and it is applied continuous wave excitation and far field condition conditions,

$$h_i(\vec{x},t) = \frac{\delta(t - T_i - |\vec{x} - \vec{x}_i|/c)}{2\pi|\vec{x} - \vec{x}_i|} \tag{3}$$

$$v(t) = \exp\left(-jwkt\right) \tag{4}$$

The equation 1 can be reduced to the Array Factor equation:

$$FA(\theta, \phi) = \sum_{i=1}^{N} a_i \exp\left(jk\left(cT_i + \left(x_{xi}\cos\phi + x_{yi}\sin\phi\right)\sin\theta\right)\right) \tag{5}$$

Where $x_{xi}$ and $x_{yi}$ are the cartesian coordinates of the elements in the aperture and $k = \frac{2\pi}{\lambda}$.

The Array Factor (AF) has several advantages[7]: first the element influence is suppressed so it is possible to analyze the element distribution isolated in the composition of the main lobe and the grating lobes; second it is very fast to compute so it is possible to examine a large set of cases.

In this work the AF is used to evaluate what we have call Periodicity Degree (PD) that is a measurement of the regularity in the element distribution. To achieve this objective the procedure followed is based in the fact that if we increase the excitation frequency there is a moment in which the GL achieves its maximum value, which is maintained for any other higher frequencies becoming then an absolute value for all frequencies. At that moment the ratio between the main lobe and the maximum grating lobe is a measurement of the regularity in the element distribution.

$$PD = \frac{FA(\theta = \theta_{GL}, \phi = \phi_{GL})}{FA(\theta = \theta_o, \phi = \phi_o)} \tag{6}$$

where $(\theta_{GL}, \phi_{GL})$ is the location of the maximum grating lobe and $(\theta_o, \phi_o)$ is the main beam location.

For instance it means that all matrix squared based apertures achieves its periodicity degree when the distance between elements is $\lambda$ and it value is $1 (0dB)$. What we suggest through this work is that operating with apertures with PD¡ 1 it is possible to use element distances higher than $\lambda$ and consequently reduce the number of elements in the array.

## 3. ANNULAR SEGMENTED ARRAY DISTRIBUTION

### 3.1 Array configuration

The annular distribution introduce several design factors that helps to reduce the distribution regularity. For instance, it is possible to use different ring widths or to use elements of different dimensions, so the analysis of this apertures could be very complex. To reduce the number of variables this study is centered in what we have called the Regular Distribution. This set of apertures is defined by one condition: all the elements and the distance between them show similar dimensions. From this simple condition two consequences can be formulated:

- All the rings that compose the aperture are of equal width.
- The number of elements in each ring is increased with the radius.

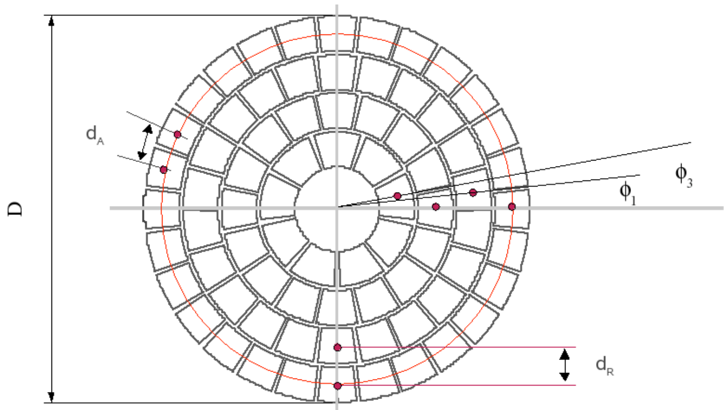


*Figure 1: Annular Segmented Array.*

Figure 1 shows a 4-ring model of the Regular Distribution, where the principal design parameters are presented:

- The radial distance ($d_R$).
- The angular distance ($d_A$).
- The phase alignment per ring ($\phi_i$).
- The aperture size ($D$).

### 3.2 Grating lobe formation

It is well known that the factor which determines the generation of grating lobes for any array is the distance between elements and it is not different for the annular segmented apertures. For any azimuth direction it can be shown, through the linear equivalent array, that angular and radial distances are the main responsible of the grating lobes formation and its location are determined by these values[7]. Then the position of ASA grating lobes can be computed by the equations:

$$RL = \arcsin\left(\frac{\lambda}{d_R}\right) \quad RA = \arcsin\left(\frac{\lambda}{d_A}\right) \tag{7}$$

From the manipulation of these distances we can conclude that there are two different kind of grating lobes. In figure 2 there are two examples of AF where one of these distances have been chosen to avoid GL formation (it means $d < 0.5\lambda$). So the two kind of lobes can be distinguished easily:

- The Radial Lobe (RL) that has a narrow circular shape and shows continuity for any azimuth direction (left image). Its position is determined by $d_R$.

- The Angular Lobes (AL) that have beam shape and are distributed in different elevation position for each azimuth direction (right image). The nearest lobes are determined by the $d_A$ distance and the rest are determined by the projection of $d_A$ in the linear equivalent array, which depend on the azimuth angle.

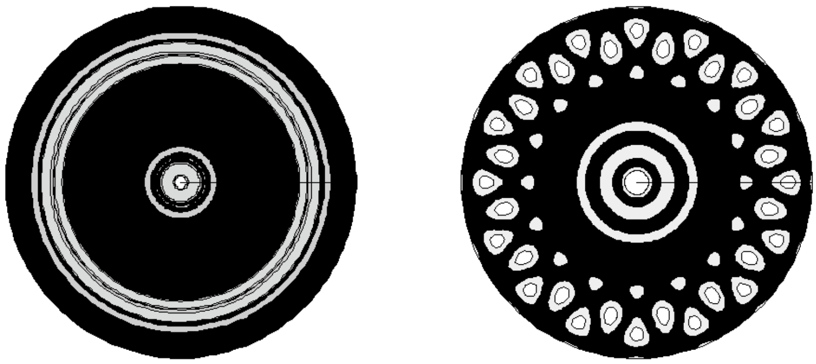

*Figure 2: Grating lobes composed by radial distance (left), and by angular distance (right)*

Nevertheless the amplitude of this lobes for any particular azimuth direction is a relationship with the number of element coincidences for that direction, which is not always the same. The first option to reduce the coincidences in the linear equivalent array is to change the element Aspect Ratio (AR), which is the ratio between the angular distance $d_A$ and the radial distance $d_R$. The objective of this technique is to avoid the grating lobe reinforcement that occurs when both dimensions are equal.

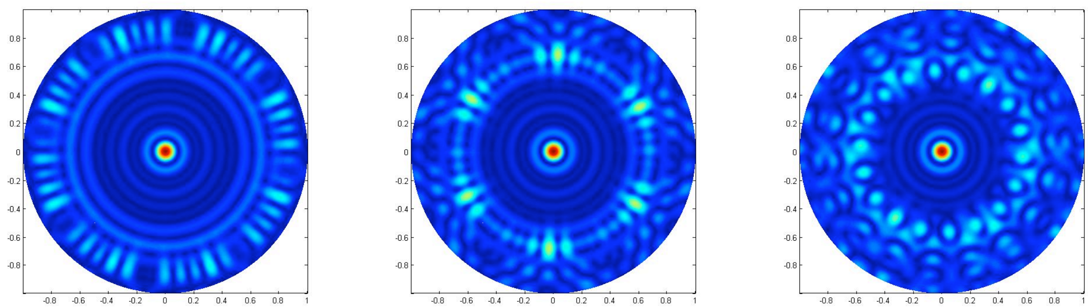

*Figure 3: Grating Lobe Formation.*

Figure 3 presents the array factor for three different cases, Aspect Ratio: 0.8, 1.0 and 1.2. All the arrays were designed as demonstrators, with $d_R = 1.5\lambda$ and $D = 12\lambda$, and the AF was computed to show how grating lobes can be combined.

- For $AR = 0.8$ both lobes are completely uncoupled and it is easy to remark its properties. The Radial Lobe has a narrow circular shape; the Angular Lobes have a straight shape and are distributed in different azimuth directions, two important consideration are: they show higher values than RL and they are spread a wider region.

- For $AR = 1$ Both lobes coincide in the same position and there is a reinforce specially for those directions were the angular lobes are higher.
- For $AR = 1.2$, due to the fact that Angular Lobes spread their influence in a wide region there still is a coincidence in position with the Radial Lobes, but with lower intensity than in the $AR = 1$ case.

There are two more design parameters can be use to reduce the number of element coincidences in the equivalent linear.

- The number of elements helps to reinforce the main beam an at the same time increases the diversity in the aperture reducing the grating lobes. This parameter is determined by the aperture size, which is determined by the application.
- The Phase alignment is used shift the elements from one ring to the other in order to avoid privileged directions in the aperture. It helps mainly to reduce angular lobes.

Phase alignment introduce a random factor in the array design. Figure 4 shows the histogram of the GL mean and the GL maximum computed in one thousen random phase-alignment cases for the same aperture ($AR = 1, D = 33, 3\lambda, N = 314$). As can be seen the mean GL values are concentrated between -28dB and -27dB, so we can say that all array produce similar grating lobes energy. For GL peak values, although the mean is centered around -12dB, it can be shown important deviations from -10dB to -14dB. So we can say that for any configuration there is a phase alignment that improves the PD.

## 4. GEOMETRICAL DESIGN

### 4.1 Evaluation procedure

In the annular segmented apertures case, due to the diversity, it is more difficult to determine the maximum GL position so we have to operate in a more conservative way. Grating lobes start around the position determined by the equation3 and the maximum is composed around this elevation area, but it is difficult to guarantee its azimuth position so in our computations we have considerer a wide area. Table 1 sumarizes the maximum elevation position for each aspect ratio configuration, these values remain constant for all the size aperture.

*Table 1: Maximum grating lobe elevation position*

| GL | AR = 0.8 | AR = 1.0 | AR = 1.2 |
|---|---|---|---|
| RL | $36.8^o$ | $36.8^o$ | $36.8^o$ |
| AL | $48.6^o$ | $36^o$ | $30^o$ |

In order to study the design parameters we have analyzed three different aspect ratio cases. For each case ten different arrays apertures, increasing the diameter size, and for

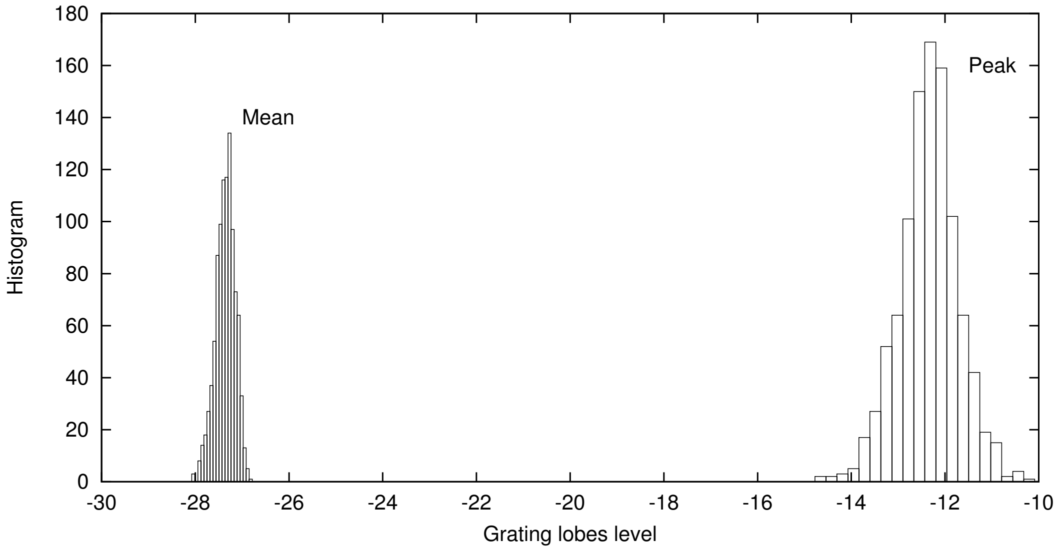


Figure 4: Phase alignment in grating lobe formation. One thousen apertures histogram of Mean grating lobe and maximum grating lobes.

each diameter we have evaluated twenty different random phase alignment. In total we have studied the PD of six hundred arrays.

The $d_R$ has been fixed to $1.5\lambda$ and $d_A$ has been changed by $AR = 0.8, 1, 1.2$. The diameter has been changed from $40\lambda$ to $144\lambda$.

### 4.2 Results

The PD dB values of this six hundred apertures had been computed and a summary of the results are shown in the figure 5. For each Aspect Ratio, the minimum, the maximum and the mean of the PD has been presented against the number of elements. The main curve joins the mean values showing the trend of the PD with the number of elements, and the straight lines at a fixed element position show the variation of PD with the phase alignment. Also, because expanding the aperture with different AR does not provide an equivalent number of elements, the table 2 has been composed to enable a more accurate analysis.

**AR = 0.8** The mean values curve goes from -14dB for 300 elements to almost -20dB for 4000 elements. This descending trend show two different behaviors. The first section arrives through 1500 elements, in this region the PD loses around 4dB. From this point it seems to be a second section where only 2dB are loosed.

**AR = 1** The mean values curve goes from -12dB for 250 elements to almost -17dB for 3400 elements. This descending trend is continuous, although in some cases

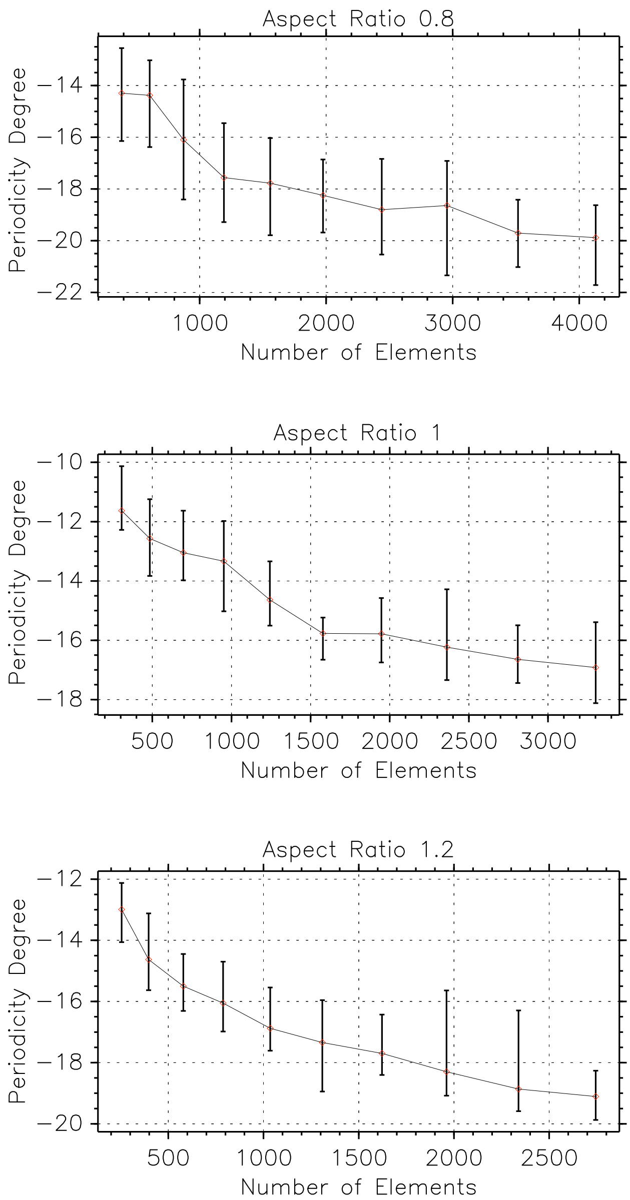


*Figure 5: Evolution of the PD with the aperture size. Aspect ratio* 0.8, 1.0 *and* 1, 2.

the mean value has an insignificant change form one value to the other. Again the first 1500 elements are specially significant because the PD loses around 4dB. From this point doubling the number of elements only achieves a decrease 1dB in the PD.

**AR = 1.2** The mean values curve goes from -13dB for 250 elements to almost -19dB for 2700 elements. This descending trend is continuous. In the first 1500 elements a reduction of 4.5dB is achieved in the PD value

The table 2 allows to compare the results between all configurations. Although more or less all configurations show similar evolution with the number of elements it is obvious that the unity Aspect Ratio shows the worst results. The $AR = 0.8$ and $AR = 1.2$ show very similar results, although it seems that $AR = 1.2$ are slightly better than $AR = 0.8$.

Table 2: *Mean Periodicity Degree (dB) for different number of elements and Aspect Ratio*

| Elements | AR = 0.8 | AR = 1.0 | AR = 1.2 |
|---|---|---|---|
| 500 | -14.5 | -12.5 | -15 |
| 1000 | -16.6 | -13.5 | -16.7 |
| 1500 | -17.8 | -15.5 | -17.5 |
| 2000 | -18.4 | -15.5 | -18.4 |
| 2500 | -18.8 | -16.5 | -19 |

## Conclusions

From the obtained results it can be summarized that:

- The number of elements is the most important factor to increase the diversity in the aperture and reduce the PD.
- The element aspect ratio can be used to improve the element PD curve by a constant factor. Values near 1 produce the worst results.
- The Phase alignment is a critical factor in the array design that can be use to improve a solution with a fix number of elements.

Then, it has been shown how the number of elements, the element aspect ratio and the phase alignment can be used to improve the PD in the annular segmented configuration. This configuration can arrive easily to PD values under $-10dB$ and. And using the adequate aspect ratio and phase alignment with a the number of elements near 1500 it can be possible find solutions near $-20dB$. Form this point it is possible to design operating wide-band array for 3D imaging with distances between elements larger than $\lambda/2$, reducing then the number of elements.

Our future work will be addressed to analyze the element influence on Aspect Ratio results and in the optimization of Phase Alignment.

- The element radiation pattern is a factor that should be considered in future woks, values of AR different than one could locates the GL very near to the main lobes reducing the steering capabilities.
- In this work we have used random values but the Array Factor can be easily combined with optimization tools to search the best configuration.

## ACKNOWLEDGEMENTS

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