

Initial drop velocity in a fixed spray plate sprinkler

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Keywords: Sprinkler irrigation, ballistic model, fixed spray plate, pivot, lateral move

Abstract

Ballistic simulation has been successfully applied to impact sprinklers. However, ballistic simulation of center pivot sprinkler irrigation has been limited by the difficulty in estimating the initial drop velocity vector in fixed and rotating spray plate sprinklers. Initial velocity is severely affected by the impact of the jet on the sprinkler deflecting plate (or plates). In this work, experimental techniques based on drop photography have been employed to obtain the droplet velocity and angle in the vicinity of a fixed spray plate sprinkler, using three different nozzle diameters. Furthermore, simulation techniques based on the inverse solution of drop trajectory were combined to determine the initial velocity vector and energy loss at the spray. Our analysis suggests that the ballistic model can be used to simulate drop inverse trajectory in these sprinklers, although the ballistic model can benefit from 5-10% effective drag force screening. The ratio of initial drop velocity to jet velocity ranged between 0.67 and 0.82, while the kinetic energy losses in the spray sprinklers amounted to 33-55%.

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INTRODUCTION

The ballistic simulation of sprinkler irrigation started five decades ago, with the formulation by Seginer (1965). Since that time, several developments have been proposed (Fukui et al. 1980; Vories et al. 1987; Kincaid 1996). Carrión et al. (2001) and Montero et al. (2001) introduced the SIRIAS model, which focused on the effect of wind on the movement of individual drops, and applied it to different simulation problems. Playán et al. (2006) presented a complete calibration-validation exercise for two sprinkler models and two nozzle diameters under the effect of variable wind. Dechmi et al. (2004) presented the combination of a sprinkler ballistic model with a soil-water-crop model, which permitted assessing the complex interactions between the environment, the irrigation system and the crop in a solid-set irrigated field. Recently, Dechmi et al. (2010) presented the coupling of a ballistic model to a full crop model.

All these simulation developments targeted impact sprinklers. These are characterized by the fact that drops emitted by the main nozzle form a jet which disintegrates along its path to the soil surface. This jet is affected by the impact arm of the sprinkler, which facilitates the jet break-up into packages of drops. The impact arm also affects a few individual drops, which deviate from the general sprinkler trajectory. The drops emitted by the auxiliary nozzle (if present) usually follow different trajectories on their emitting from the nozzle.

Assuming a dynamic movement similar to that of the drops formed at the nozzle, the initial velocity of the drops emitted by impact sprinklers can be numerically calculated or experimentally measured. Numerical calculation is performed using Bernoulli theorem. The method requires a measurement of pressure at the nozzle and the determination of a head loss coefficient whose numerical value is very close to one. The experimental determination of velocity requires a measurement of discharge which is then divided by the nozzle area:

$$v = \frac{q}{s} \quad (1)$$

42 where v is the jet velocity exactly after nozzle, q is the sprinkler discharge and s is the
43 nozzle cross sectional area. Initial drop velocity is one of the initial conditions of a ballistic
44 simulation of sprinkler irrigation.

45 The case of the spray sprinklers commonly used in pivot or linear move irrigation machines
46 differs from impact sprinklers. In this case, the jet produced at the nozzle immediately
47 undergoes an inelastic shock as it frontally hits a plate. Although most spray sprinkler
48 models include certain curvature in the plate and grooves designed to create a number of
49 small jets, the energy lost at the plate is sufficiently large to create uncertainty about the
50 initial velocity of the drops. As a consequence, ballistic models have rarely been applied to
51 the two main designs of spray plate sprinklers: the classical Fixed Spray Plate Sprinklers
52 (FSPS) and the more modern Rotating Spray Plate Sprinklers (RSPS). While in FSPS the
53 spray plate is fixed, in RSPS the spray plate rotates thanks to the energy of the jet. Kincaid
54 (1996) applied a ballistic model to both types of sprinklers using measured data as input.

55 The objectives of this work are:

- 56 1. to measure the instantaneous drop velocity vector at a bounded region close to the
57 sprinkler;
- 58 2. to estimate the initial drop velocity; and
- 59 3. to estimate head losses at the spray sprinkler. These result from small losses at the
60 nozzle and large losses at the spray plate.

61 Experiments were performed for three commercial FSPS of different diameters and configu-
62 rations.

63 **THE EXPERIMENTAL SETUP**

64 The FSPS used in this paper were manufactured by Senninger Irrigation (Clermont,
65 Florida, USA) for Valmont Irrigation (Valley, Nebraska, USA), and corresponded to the
66 series LDN (citing the manufacturer and model does not imply endorsement). These were
67 of the same series as the ones reported by Faci et al. (2001). Please note the error in the

68 sprinkler manufacturer and model at the original reference. The same nozzles and plate
69 combinations were used in both research works.

70 The diameter of the experimental nozzles was 3.75, 6.75 and 7.97 mm. The sprinklers
71 had one, two and three plates, respectively. They were coded 1P, 2P and 3P. In the case of
72 sprinkler 2P, the jet first impacts the upper plate (2P-U), which has a central orifice of 4.90
73 mm in diameter (see Fig. 1). As a consequence, the outer part of the jet breaks down into a
74 number of small jets which exit the plate with a angle of 11.3° above the horizontal plane.
75 The inner part of the jet passes to the lower plate (2P-L), located 26 mm below the first
76 one. Plate 2P-L has no central orifice: all the jet breaks down into small jets which leave the
77 sprinkler at a horizontal angle of 0° . Table 1 presents this information for the three different
78 nozzle diameters. The Upper and Intermediate plates of sprinkler 3P (3P-U and 3P-I) have
79 central orifices. Drops leave plate 3P-U at an angle of 13° respect the horizontal plane, while
80 the exit velocity vectors from plates 3P-I and 3P-L are horizontal.

81 All experimental measurements were performed under windless field conditions. The
82 three FSPS were sequentially installed at one extreme of the experimental center pivot
83 system described by Playán et al. (2004). A pressure regulator was used to maintain a
84 constant pressure of 138 kPa just upstream from the FSPS (Fig. 1). The nozzle discharge
85 was volumetrically measured. In the case of FSPS 2P and 3P, the discharge emitted by each
86 plate was determined by subtracting from the discharge reaching the plate the discharge
87 volumetrically measured below the plate orifice. This permitted determination of discharge
88 emitted by each plate (Table 1).

89 **CHARACTERIZING DROP DIAMETER, VELOCITY AND ANGLE**

90 The simultaneous determination of drop diameter, velocity and angle was performed
91 using the photographic method proposed by Salvador et al. (2009). Laser precipitation
92 monitors (King et al. 2010) constitute an interesting methodological alternative, although
93 this technique does not measure drop angle. The photographic method is based on low
94 speed photography (1/100 s) of the sprinkler droplets as they travel from the sprinkler to

95 the soil surface. This technique requires intense illumination, which may be easily obtained
96 in the local conditions by outdoor operation near solar midday. In these circumstances
97 the drops are photographed as cylinders whose diameter corresponds to the drop diameter
98 and whose length is equivalent to the drop displacement during 1/100 s. This technique
99 permits determination of cross-sectional diameter, tangential velocity and vertical angle of
100 individual drops located at different distances from the FSPS. As each drop moves across
101 the air it gets deformed, offering a cross-sectional diameter slightly wider than that of the
102 static drop (Pruppacher and Pitter 1971). Although the cross-sectional diameter measured
103 with this technique is correct for the determination of the drag-force, it is larger than that
104 of the static drop. Consequently, the drop mass may be overestimated.

105 The experimental procedures were designed to limit the sources of experimental error,
106 such as sprinkler vibration, sprinkler and camera levelling, and wind speed. Visual inspection
107 of the spray jets indicated complete formation of individual drops at a distance of 0.7-0.8 m
108 from the FSPS. For each FSPS a measurement point was located at a horizontal measurement
109 distance from the sprinkler of about 1.5 m. This distance permitted adequate drop separation
110 in the photographs.

111 Three hundred drops were analyzed in photos (50 drops for every sprinkler-plate combi-
112 nation). The complete data set has been published on-line by Delirhasannia et al. (2012).
113 Table 1 presents the average measurement distance from the FSPS to the drop measure-
114 ment point and the average elevation of the measurement point (measurement elevation)
115 for each sprinkler nozzle and plate configuration. The average and standard deviation (SD)
116 of drop cross-sectional diameter, velocity and angle are also presented in Table 1. Average
117 drop cross-sectional diameters ranged from 1.93 to 2.11 mm. Average drop velocities ranged
118 from 6.78 to 10.45 m/s. Differences in average velocity among the plates of spray sprin-
119 klers 2P and 3P derive from differences in jet diameter (plate orifice) and deflecting plate
120 geometry. These velocities may cause a relevant drop deformation, such that the measured
121 cross-sectional diameter can be up to 10% higher than the static diameter for the largest

122 drops (Pruppacher and Pitter 1971). However, the aim of this work is to estimate the initial
 123 velocity of the drops, rather than their precise size or shape. The measured velocities are
 124 substantially lower than the velocities determined at the nozzles of the experimental FSPS
 125 (16.1 - 17.7 m/s). Main differences can be attributed to the energy loss at the plates and
 126 to friction in the drop trajectory from the FSPS to the measurement point. The measured
 127 velocities showed an average SD of 0.9 m/s, and a coefficient of variation of 9.8%. The
 128 average coefficient of variation in drop angle was 1.3°, suggesting that the drop trajectory
 129 was quite uniform at the measurement point. The correlation coefficient between measured
 130 drop diameter and velocity was 0.37, indicating that part of the variability in velocity was
 131 due to differences in drag force.

132 **RESOLUTION OF THE INVERSE TRAJECTORIES OF THE DROPS**

133 **Ballistic model**

134 The main hypothesis of the ballistic model is that the drops emitted by the sprinkler
 135 move as independent spheres in the surrounding air (Fukui et al. 1980; Carrión et al. 2001).
 136 The drag force of a sphere in turbulent flow can be expressed as:

$$137 \quad \vec{F}_r = -\frac{1}{2}\lambda\rho_a A|\dot{\vec{r}} - \vec{w}|(\dot{\vec{r}} - \vec{w}) \quad (2)$$

138 where ρ_a is air density, A is the effective section, \vec{r} is the position vector, \vec{w} is the wind
 139 velocity vector, and λ is a drag coefficient depending on the Reynolds number. The ballistic
 140 dynamic equations of a drop constitute a set of three ordinary differential equations. In
 141 vector notation these equations can be expressed as:

$$142 \quad m\ddot{\vec{r}} = \vec{F}_r + m\vec{g} = -\frac{1}{2}\lambda\rho_a A|\dot{\vec{r}} - \vec{w}|(\dot{\vec{r}} - \vec{w}) + m\vec{g} \quad (3)$$

143 with m the drop mass and $\vec{g} = (0, 0, -g)^T$ the gravitational field, with g the gravitational
 144 constant. Dividing this equation by the mass, and considering a spherical drop with diameter

145 d :

$$146 \quad \ddot{\vec{r}} = -\frac{3\lambda\rho_a}{4\rho_w d}|\dot{\vec{r}} - \vec{w}|(\dot{\vec{r}} - \vec{w}) + \vec{g} \quad (4)$$

147 with ρ_w the water density. λ can be approximated, following Fukui et al. (1980) or Seginer
148 et al. (1991), as:

$$149 \quad \lambda = \lambda_b = \begin{cases} 1.2 - \frac{3.3Re}{1000} + \frac{33.3}{Re}; & Re \in [0, 128) \\ 0.48 - \frac{5.56Re}{100000} + \frac{72.2}{Re}; & Re \in [128, 1440) \\ 0.45; & Re \in [1440, \infty) \end{cases} \quad (5)$$

150 with λ_b the drag coefficient of the ballistic model, $Re = d|\dot{\vec{r}}|/\nu$ the Reynolds number and ν
151 the kinematic viscosity of the air.

152 **Parabolic model**

153 The hypothesis of this model is that, in the initial part of the trajectory, individual drops
154 are not formed or they move very grouped. Under this hypothesis, the drag force is much
155 lower than that corresponding to a single drop, and can be neglected. In the absence of a
156 drag force, the drop trajectory can be directly solved and the solution takes the form of a
157 parabola:

$$158 \quad \vec{r} = \vec{r}_0 + \dot{\vec{r}}_0 t + \frac{1}{2}\vec{g}t^2, \quad \dot{\vec{r}} = \dot{\vec{r}}_0 + \vec{g}t \quad (6)$$

159 with \vec{r}_0 the initial position and $\dot{\vec{r}}_0$ the initial drop velocity.

160 **Screening model**

161 This third model assumes that the drag force acting in the initial part of the drop
162 trajectory differs from that corresponding to a spherical drop. Then, this model corrects the
163 drag-coefficient by a "screening factor" S :

$$164 \quad \lambda = (1 - S)\lambda_b \quad (7)$$

165 The measurement technique used in this work, as described in last section, could result

166 in overestimation of the drop mass. This would in turn lead to an underestimation of the
 167 drag-force acceleration. In such a case, a negative value of S could be advisable.

168 On the other hand, a screening effect created by drops travelling very close to each other
 169 implies an effective decrease in drag force. In such a case, S should lay in the range $[0, 1]$ to
 170 account for this effect.

171 The sign of S depends on which one of these opposite effects dominates in a particular
 172 case. Note that in the $S \rightarrow 1$ limit the model converges to the parabolic one and in the
 173 $S \rightarrow 0$ limit to the ballistic one.

174 A similar screening approach was proposed by Kincaid (1996) where, for complete trajec-
 175 tories, S is distance dependent. Since the drop trajectories analyzed in this work are much
 176 shorter, we considered an average value of S .

177 Numerical resolution of the inverse trajectory

178 The ballistic and the screening models require the numerical solution of the inverse trajec-
 179 tory from the measurement point to the sprinkler position (to the center of the spray plate).
 180 Solution can be obtained using the classical Runge-Kutta method of fourth order applied
 181 to (4). For initial values of time t_0 , of the position \vec{r}_0 and of the velocity $\dot{\vec{r}}_0$, Runge-Kutta
 182 methods approximate the position and velocity in the next time step as:

$$183 \quad \ddot{\vec{r}}_0 = \ddot{\vec{r}}(t_0, \dot{\vec{r}}_0), \quad \ddot{\vec{r}}_i = \ddot{\vec{r}}\left(t_0 + c_i \Delta t, \dot{\vec{r}}_0 + \Delta t \sum_{j=0}^{i-1} a_{ij} \ddot{\vec{r}}_j\right),$$

$$184 \quad \dot{\vec{r}}_i = \dot{\vec{r}}_0 + \Delta t \sum_{j=0}^{i-1} a_{ij} \ddot{\vec{r}}_j, \quad \dot{\vec{r}}(t_0 + \Delta t) \approx \dot{\vec{r}}_0 + \Delta t \sum_{i=0}^{p-1} b_i \ddot{\vec{r}}_i,$$

$$185 \quad \vec{r}(t_0 + \Delta t) \approx \vec{r}_0 + \Delta t \sum_{i=0}^{p-1} b_i \dot{\vec{r}}_i \quad (8)$$

188 with p being the number of steps. The classical Runge-Kutta method of fourth order is a
 189 four step method ($p = 4$) defined by the coefficients:

$$\begin{aligned}
 190 \quad c_1 = c_2 = a_{10} = a_{21} = \frac{1}{2}, \quad c_3 = a_{32} = 1, \\
 191 \\
 192 \quad a_{20} = a_{30} = a_{31} = 0, \quad b_0 = b_3 = \frac{1}{6}, \quad b_1 = b_2 = \frac{1}{3} \quad (9)
 \end{aligned}$$

193 Solution of the inverse trajectory needs to proceed backwards in time. This can be achieved
 194 by using negative time steps ($\Delta t < 0$). A scheme of this process is represented in Fig. 2.

195 A very small numerical time step was used: $\Delta t = -0.001$ s. Numerical errors in analytical
 196 test trajectories have proved to be in the order of 10^{-6} m, being much lower than the
 197 experimental uncertainty derived from the field conditions.

198 Numerical results

199 Two parameters can be used to estimate the quality of the numerical solution, as shown
 200 in Fig. 2: the z coordinate and the angle of the velocity vector obtained at the sprinkler
 201 position. We hypothesize that all drop trajectories have the same kinetic properties at the
 202 initial position. Such hypothesis is supported by the coherence observed in the emitted jets,
 203 as shown in Fig. 3. As a consequence, the estimated initial z coordinate and velocity vector
 204 should not show any dependence on drop size.

205 Fig. 4 presents the initial z coordinate of the drops, obtained by numerical solution
 206 of the inverse trajectory with ballistic and parabolic (friction-less) models, and the plate
 207 elevation. These variables were found to vary with the drop diameter. The initial drop
 208 angle, obtained with the ballistic and parabolic models, and measured at the sprinkler jet,
 209 are compared at Fig. 5 as a function of drop diameter. These figures do not permit to assess
 210 the best simulation model because of the experimental uncertainties. Uncertainty could
 211 indeed exceed the differences in simulation results derived from the use of both (parabolic
 212 and ballistic) models. Nevertheless, the ballistic model shows lower dependence of the results
 213 on the drop diameter, as evidenced by lower regression slopes.

214 In Fig. 6, the initial drop velocity, estimated through numerical solution of the inverse
215 trajectory with ballistic and parabolic models and estimated at the nozzle, are compared
216 as a function of drop diameter. A noticeable velocity decrease can be appreciated in all
217 cases: drop velocity results much smaller than the velocity measured at the nozzle. This
218 difference can be directly attributed to the inelastic shock at the sprinkler plate causing a
219 strong kinetic energy loss. Estimated drop velocity always resulted lower with the parabolic
220 model than with the ballistic model. Furthermore, the ballistic and parabolic models show
221 opposite trends as a function of drop diameter. This suggests that a combination between
222 those models, as with the screening model with $S \in [0, 1]$, may lead to more satisfactory
223 results. It can be inferred from Fig. 6 that the screening effect is stronger than the drop
224 deformation effect.

225 Figures 7-9 present the evolution with drop diameter of: deviations from expected initial
226 z coordinate; deviations from expected initial velocity angle; and ratio between simulated
227 initial drop velocity and jet velocity at the nozzle, respectively. Results are presented in
228 all figures for different values of the screening factor (from 0 to 1). Linear regression lines
229 are included in all subfigures, since all of them were significant at the 95% probability level.
230 The correlation coefficient was very low, ranging from 0.04 and 0.35. A statistical analysis
231 was performed to assess if the regression slope statistically differed from zero. At the 95%
232 probability level, four slopes did not differ from zero: for z coordinate $S = 0$; for angle $S = 0$;
233 and for velocity $S = 0.2$ and $S = 0.4$. The uncertainty associated to the measurements
234 makes it difficult to select the most adequate value of the screening factor. Nevertheless, the
235 regressions show a lower dependence on the drop diameter for the ballistic model than for
236 the parabolic model. Low values of the screening factor result in slopes not different from
237 zero.

238 In Fig. 10 the mean values and the standard deviations of the errors in the estimation
239 of initial z coordinate and angle of the velocity vector are shown. Fig. 10 also presents the
240 slope of the regressions presented in Figs. 7 to 9, along with its standard error. Regarding

241 the plate elevation, the best results can be obtained for $S = 0$. Regarding the initial velocity
 242 angle, the best results can be obtained for $S = 0.9$. In any case, all values of S are in the
 243 range of uncertainty. The results of the regression slope analysis are clearer, and confirm
 244 the results of the previous paragraph. Only for low values of S (0.05–0.10) the results of the
 245 regression slope fall within the adequate range in all cases. Nevertheless, the different trends
 246 shown by Fig. 10 indicate that this range of values of S might not be optimum.

247 Finally, in Table 2 the losses in velocity and kinetic energy, estimated with the ballistic,
 248 the parabolic, the screening ($S = 0.10$) and Kincaid’s models are presented. Kincaid (1996)
 249 proposed a model for the energy loss in FSPS based in the ratio between nozzle and plate
 250 diameters:

$$251 \quad \frac{|\dot{\vec{r}}_0|}{v} = \begin{cases} \frac{d_n}{0.3d_p}, & \text{if } \frac{d_n}{d_p} < 0.3; \\ 0.97, & \text{if } \frac{d_n}{d_p} \geq 0.3; \end{cases} \quad (10)$$

252 with d_n the nozzle diameter and d_p the plate diameter. The results of introducing a low
 253 screening factor of $S = 0.10$ (as the former analysis suggested), are very similar to those
 254 obtained with the ballistic model. Following the results of the screening model, initial drop
 255 velocity amounts to 0.85, 0.69 and 0.71 of the nozzle velocity for FSPS 1P, 2P and 3P,
 256 respectively. Regarding energy losses at the FSPS, these amount to 33, 55 and 52% for
 257 FSPS 1P, 2P and 3P, respectively. Differences between the ballistic and parabolic models are
 258 generally large, but the differences between the screening and ballistic models are moderate.
 259 Kincaid’s model predictions and our numerical results (ballistic, screening and parabolic
 260 models) considerably differ. Kincaid’s model does not take into account elements that can
 261 strongly affect the energy loss, such as plate shape or deflection angle. As a consequence, it
 262 represents a pioneering approximation. We believe that this paper represents a contribution
 263 to this issue, though further research will be required to confirm the reported results and to
 264 extend them to other spray sprinkler models and nozzle diameters.

265 CONCLUSIONS

266 Despite the efforts to control experimental error, data uncertainty resulted in a variability

267 which was often larger than the differences between the mathematical models. It is the
268 opinion of the authors that minor wind spells, including air turbulence induced by the jets
269 themselves could have been the main sources of error. Levelling and vibration stand as
270 relevant error sources too. Consequently, it was not possible to select a model based on the
271 average results. However, a statistical analysis based on the dependence of the model results
272 on drop diameter suggests that the results of the ballistic model are more realistic than those
273 of the parabolic model. Only for low values of S (0.05 - 0.10) the results of the regression
274 slope fall within the adequate range in all cases. As a consequence, once the exit velocity is
275 adequately characterized for each sprinkler, the screening model (with $S \approx 0.05 - 0.10$) or
276 even the ballistic model can be adequately applied to its simulation.

277 The initial drop velocity was characterized for the three sprinkler models and for the three
278 mathematical models. The use of the screening model revealed that the ratio of initial drop
279 velocity to jet velocity emitted from nozzle ranged between 0.67-0.82, and that the losses in
280 kinetic energy amounted to 33-55%. This research is based on the hypothesis that all drop
281 trajectories have the same kinetic properties at the initial position. Sensible results have been
282 reported in this research, but the validation of this hypothesis remains an issue. Droplet
283 initial velocity and angle could be significantly affected by stochastic variations requiring
284 specific modelling developments. Further research efforts should be devoted to clarify this
285 issue.

286 This research represents a contribution to the characterization of the energy efficiency of
287 FSPS. More importantly, estimation of the initial droplet velocity in spray sprinklers may
288 lead to the development of a new generation of improved ballistic pivot irrigation models.
289 Current pivot models are based on the overlapping of experimental sprinkler application
290 patterns (Omary and Sumner 2001; Delirhasannia et al. 2010). The calibration of a ballistic
291 model to FSPS irrigation under the effect of wind speed will permit to develop simulation
292 models taking the time evolution of the wind vector as a simulation input. Simulation will
293 be possible for untested wind speeds. Such models should be extended to use RSPS, for

294 which the determination of initial droplet velocity will require additional efforts, due to the
295 high variability in the initial drop angle.

296 **ACKNOWLEDGEMENTS**

297 This research was funded by the National Research, Development and Innovation Plan of
298 the Government of Spain (Plan Nacional de I+D+I) through grant AGL2010-21681-C03-01,
299 and by the Special Intramural Grant 200940I041 of CSIC (funding G. Sánchez Burillo). R.
300 Delirhasannia received a scholarship from the Ministry of Science, Research and Technology
301 of Iran to perform a scientific stay at the Aula Dei Experimental Station of CSIC, Zaragoza,
302 Spain.

303 **NOTATION**

304 λ = drag coefficient,

305 λ_b = drag coefficient of the ballistic model,

306 ν = air kinematic viscosity,

307 ρ_a = air density,

308 ρ_w = drop density,

309 A = drop effective section,

310 a_{ij}, b_i, c_i = Runge-Kutta coefficients,

311 d = drop diameter,

312 d_n = nozzle diameter,

313 d_p = plate diameter,

314 g = gravitational constant,

315 \vec{g} = gravitational field vector,
316 \vec{F}_r = drop drag force vector,
317 m = drop mass,
318 p = steps number of a Runge-Kutta method,
319 q = nozzle discharge,
320 \vec{r} = drop position vector,
321 $\dot{\vec{r}}$ = drop velocity vector,
322 $\ddot{\vec{r}}$ = drop acceleration vector,
323 \vec{r}_0 = drop position vector exactly after the plate,
324 $\dot{\vec{r}}_0$ = drop velocity vector exactly after the plate,
325 Re = Reynolds number,
326 S = screening factor,
327 s = nozzle cross sectional area,
328 t = time,
329 v = jet velocity exactly after the nozzle,
330 \vec{w} = wind velocity vector,
331 x = horizontal coordinate.
332 z = vertical coordinate.

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Table 1. Characteristics of the FSPS used for drop characterization, along with the experimental conditions, the average coordinates of the drop measurement point and basic statistics of the measured drop cross-sectional diameter, velocity and angle.

FSPS code	1P	2P		3P		
Nozzle diameter (mm)	3.75	6.75		7.97		
Nozzle discharge (L/s)	0.180	0.577		0.882		
Nozzle velocity (m/s)	16.5	16.1		17.7		
Plate code	1P	2P-U	2P-L	3P-U	3P-I	3P-L
Plate location	-	Upper	Lower	Upper	Intermediate	Lower
Color	Black	Blue	Black	Blue	Black	Black
Plate orifice diameter (mm)	-	4.90	-	7.25	4.96	-
Initial angle (°)	0.0	11.3	0.0	13.0	0.0	0.0
Plate elevation (m)	0.747	0.744	0.718	0.738	0.713	0.689
Plate discharge (L/s)	0.180	0.228	0.349	0.176	0.361	0.345
Plate diameter (mm)	30.7	30.7	30.7	30.7	30.7	30.7
Measurement distance (m)	1.52	1.58	1.54	1.38	1.42	1.46
Measurement elevation (m)	0.614	0.944	0.650	0.739	0.548	0.508
Average cross-sectional diameter (mm)	1.97	1.93	2.00	2.00	2.03	2.11
SD cross-sectional diameter (mm)	0.41	0.33	0.57	0.38	0.47	0.65
Average velocity (m/s)	10.07	8.16	8.75	6.78	9.41	10.45
SD Velocity (m/s)	0.65	0.77	1.34	0.57	0.99	0.92
Average angle (°)	-8.87	1.39	-5.33	-7.28	-9.04	-10.97
SD angle (°)	0.76	1.79	1.21	1.40	1.32	1.24

Table 2. Relative velocity and kinetic energy losses as estimated with the ballistic, parabolic and screening with $S = 0.10$ models for the Senninger LDN sprinkler model. Results are presented for the different plates and for each of the three FSPS.

		1P	2P		3P				
		Total	2P-U	2P-L	Total	3P-U	3P-I	3P-L	Total
Nozzle discharge (L/s)		0.180	0.228	0.349	0.577	0.176	0.361	0.345	0.882
Nozzle discharge (%)		100	40	60	100	20	41	39	100
Ballistic model	Relative velocity	0.85	0.73	0.67	0.69	0.51	0.73	0.78	0.71
	Kinetic energy loss (%)	28	47	55	52	74	47	39	49
Screening model ($S = 0.10$)	Relative velocity	0.82	0.70	0.65	0.67	0.50	0.71	0.76	0.69
	Kinetic energy loss (%)	33	51	58	55	75	50	42	52
Parabolic model	Relative velocity	0.60	0.52	0.48	0.50	0.39	0.54	0.58	0.53
	Kinetic energy loss (%)	64	73	77	75	85	71	66	72
Kincaid's model	Relative velocity	0.41	0.73	0.73	0.73	0.87	0.87	0.87	0.87
	Kinetic energy loss (%)	83	46	46	46	25	25	25	25

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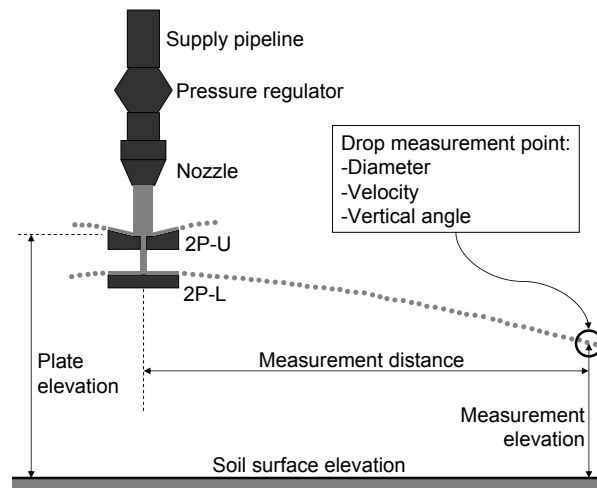


Figure 1. Scheme of FSPS 2P in operation, specifying its different pieces and the main distances and elevations. The data obtained through drop measurement are also indicated.

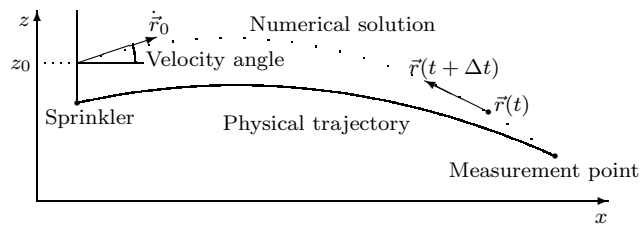


Figure 2. Scheme of the numerical resolution of the inverse drop trajectories beginning at the measurement point.



Figure 3. Three coherent jets emitted by three grooves of the FSPS 1P. A plastic screen has been installed to allow visualization of the jet shapes. The observed splash droplets, produced by the plastic screen, never reached the photographic area.

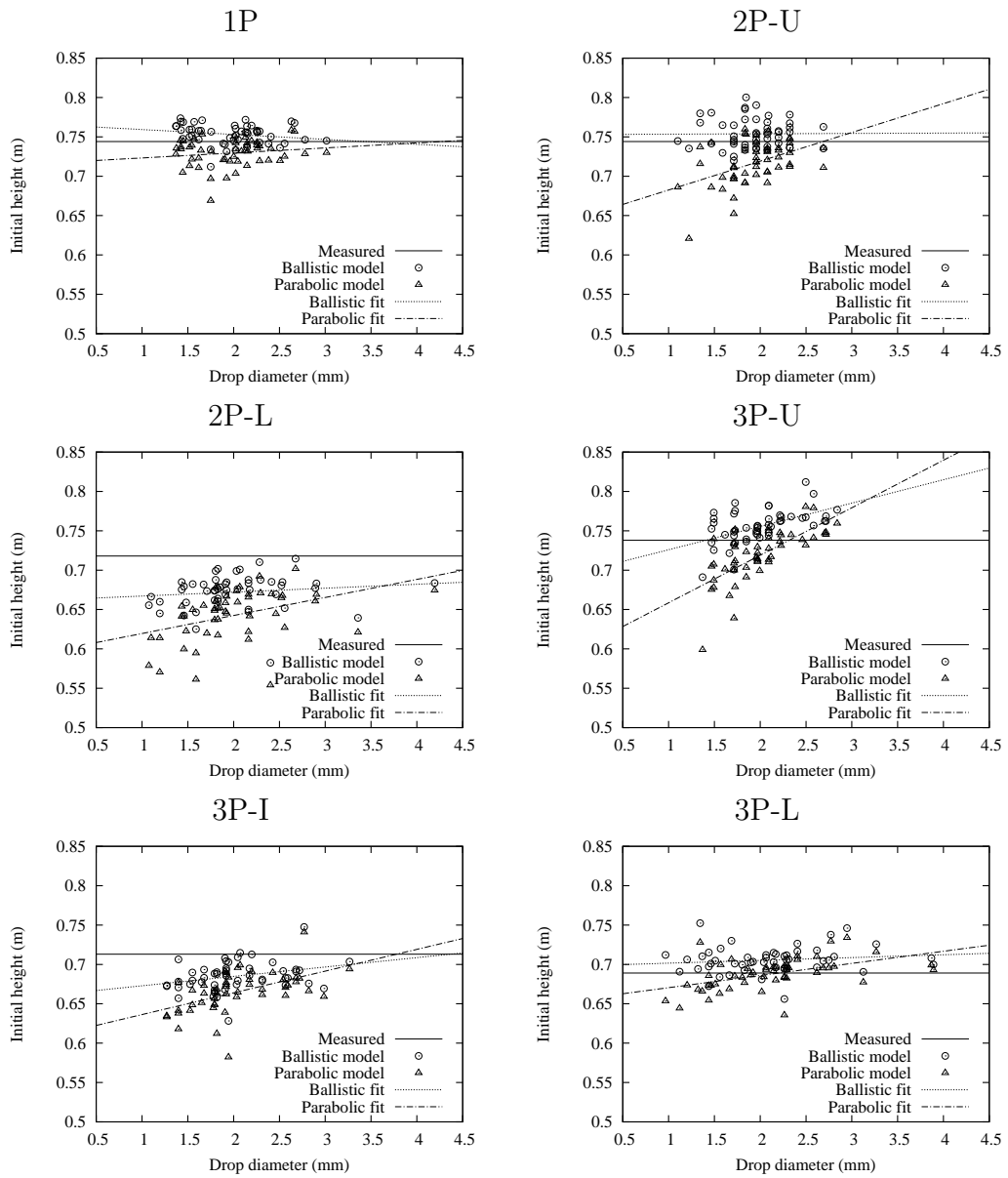


Figure 4. Initial z coordinate of each drop trajectory obtained with the ballistic and the parabolic models compared with the plate elevation for the Senninger LDN sprinkler model. Results are presented as a function of drop diameter.

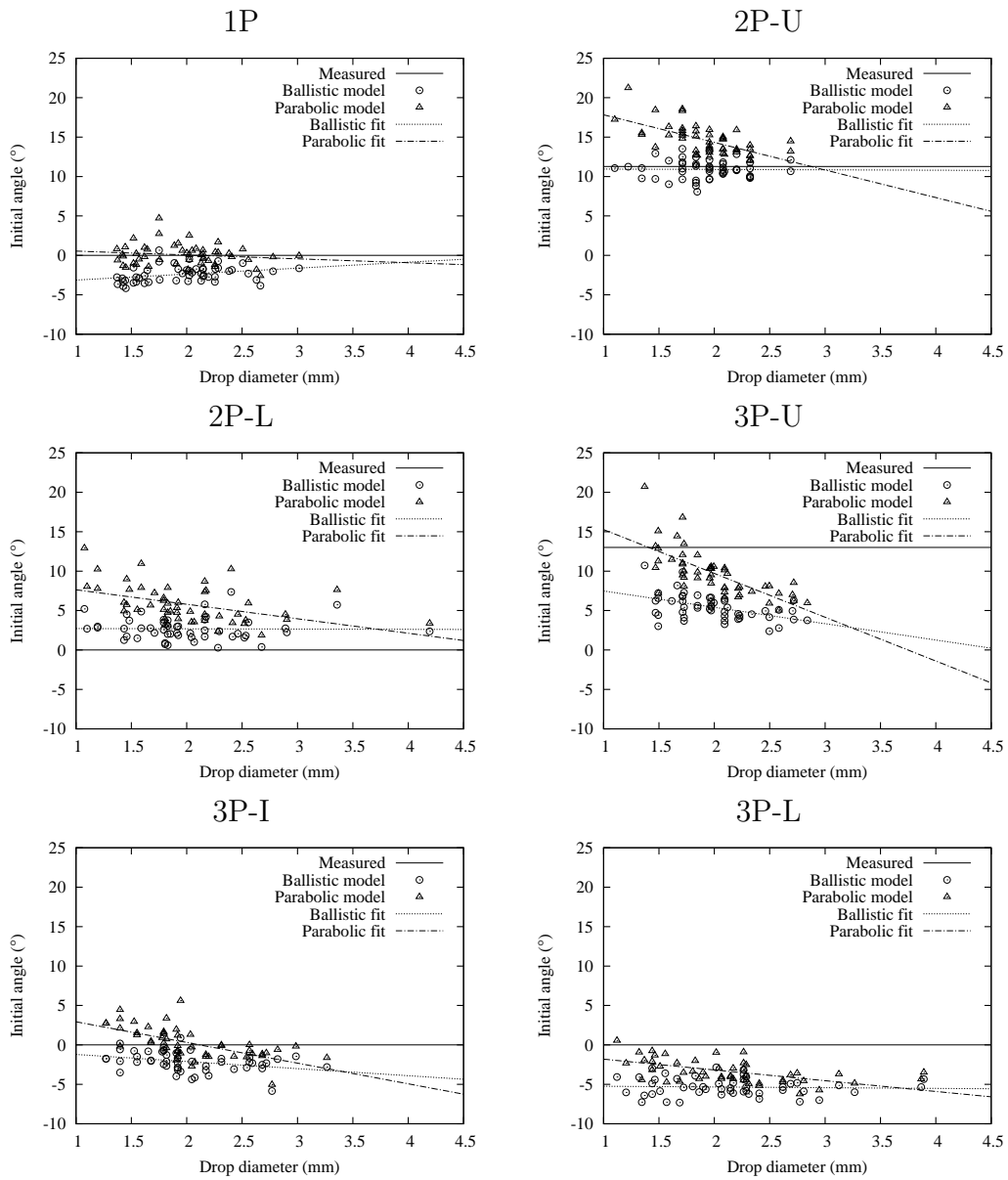


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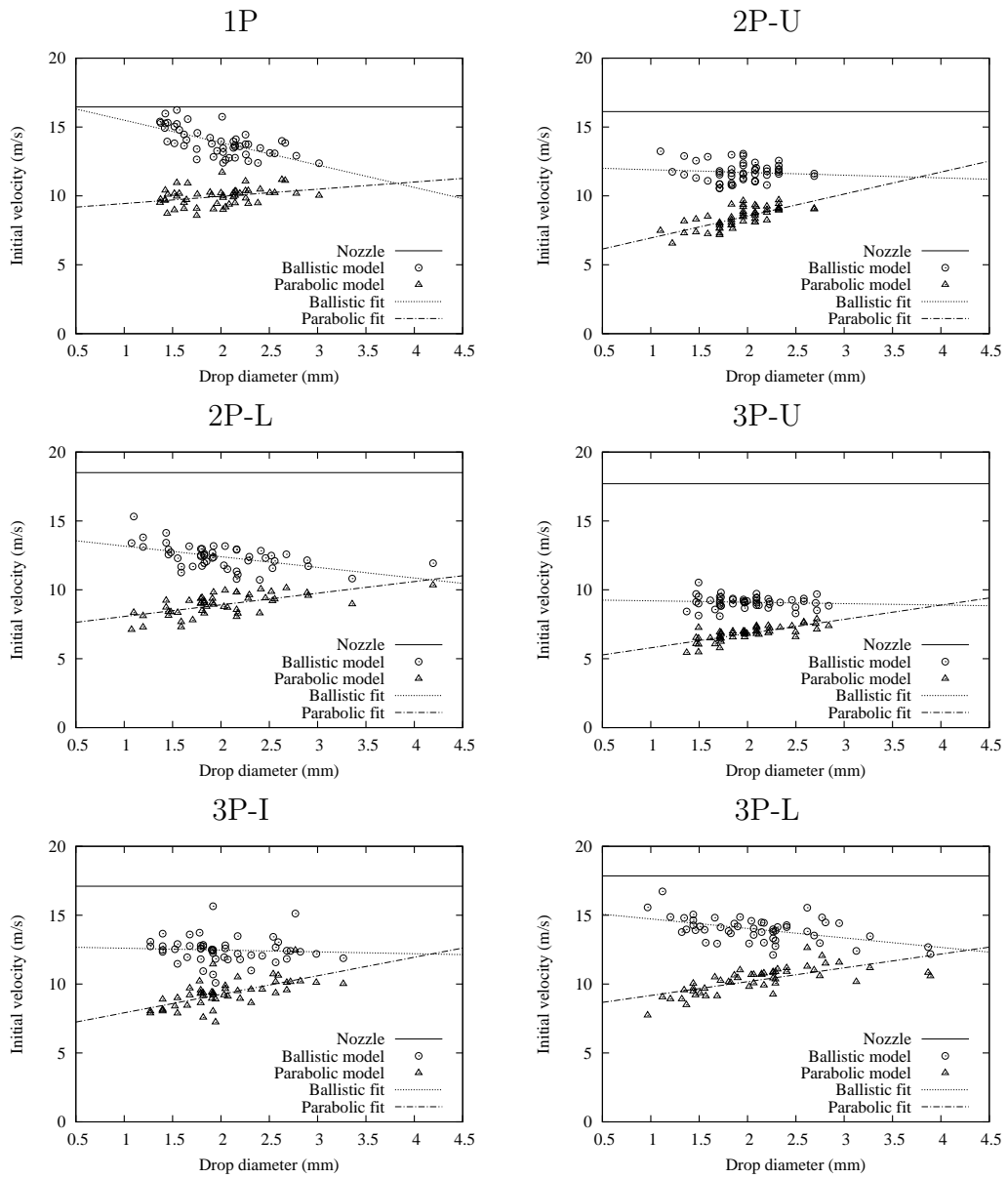


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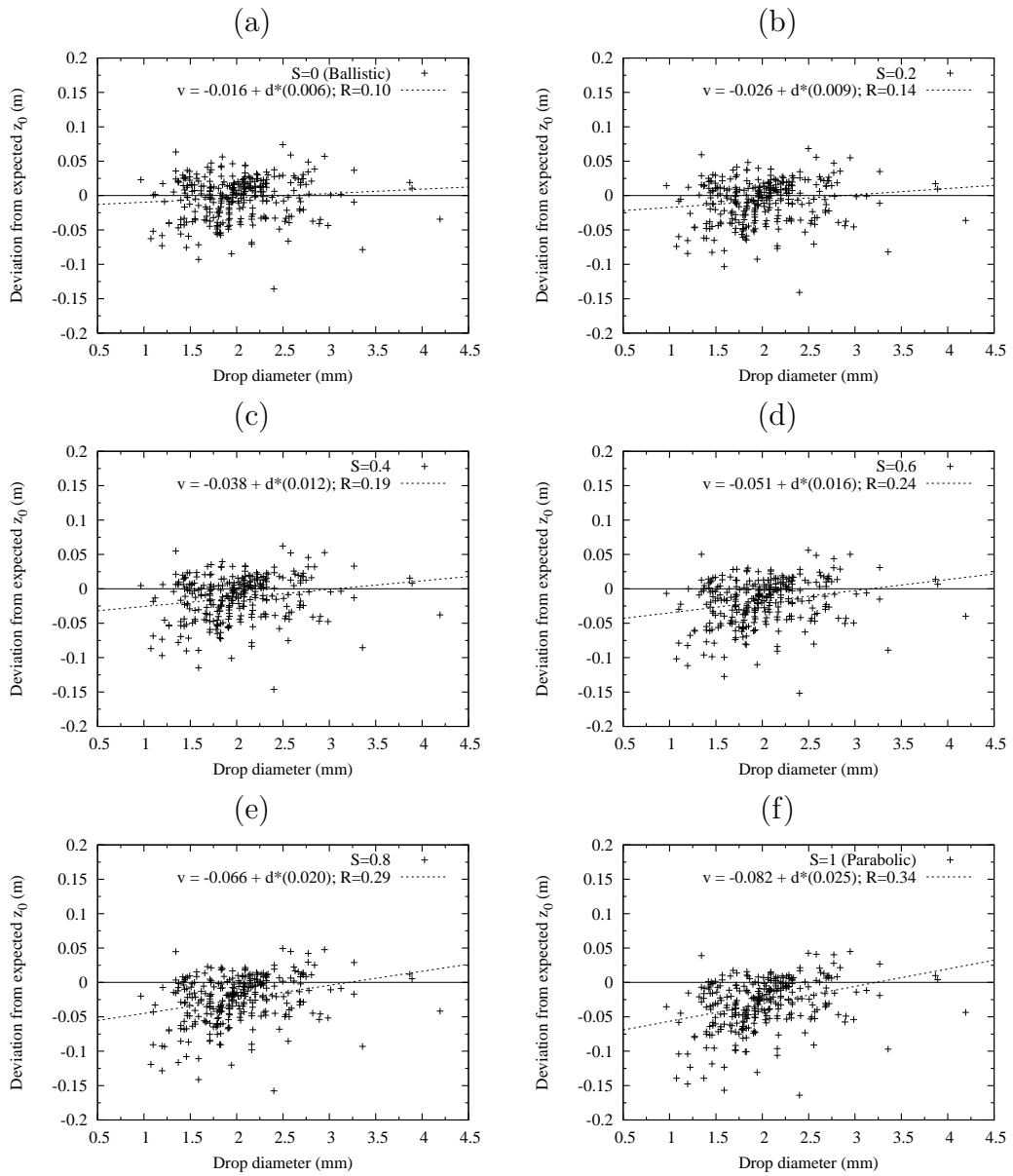


Figure 7. Deviation from the measured value of the initial z coordinate for different values of S . Results are presented as a function of drop diameter.

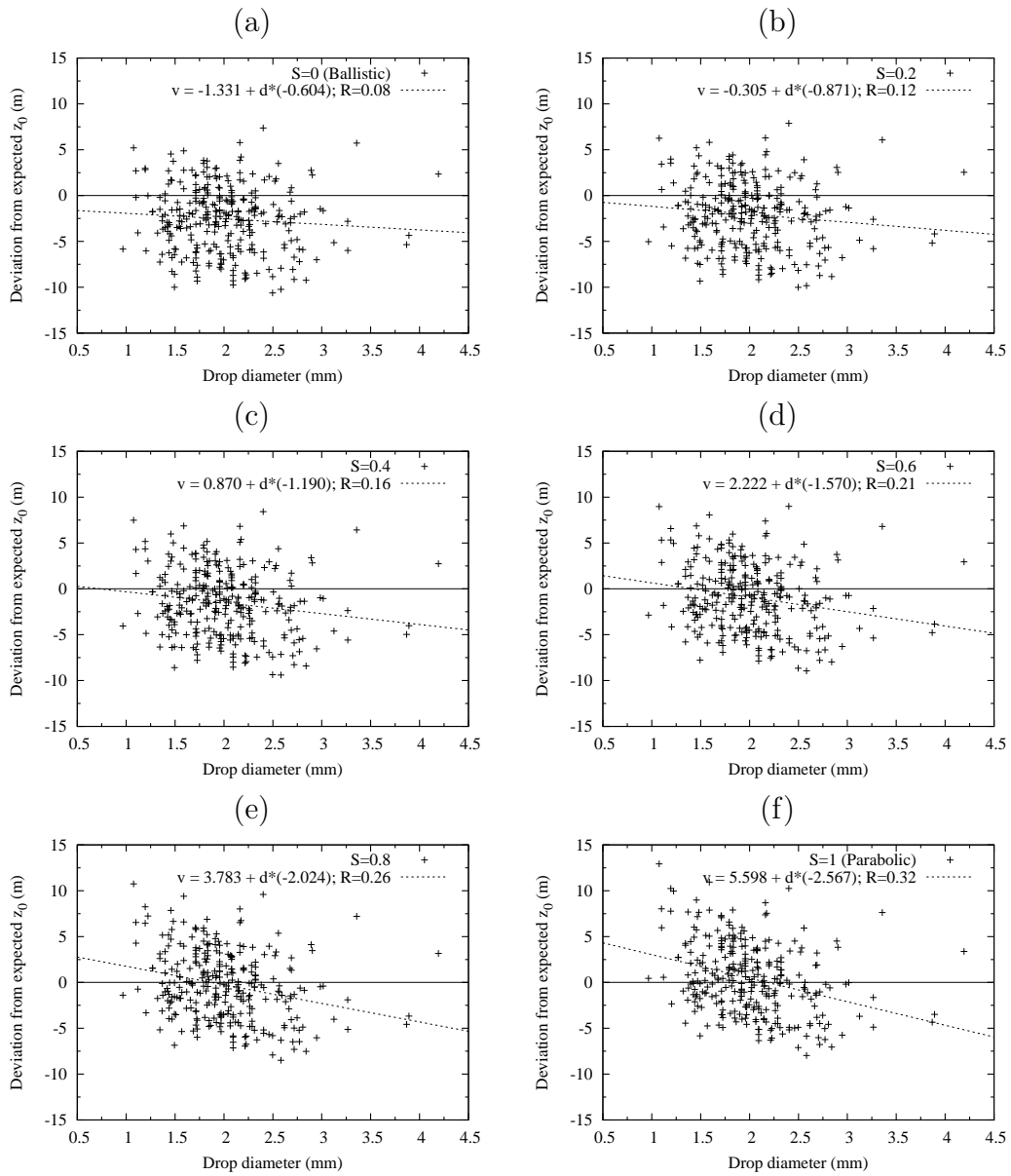


Figure 8. Deviation from the manufacturer initial angle of the velocity for different values of S . Results are presented as a function of drop diameter.

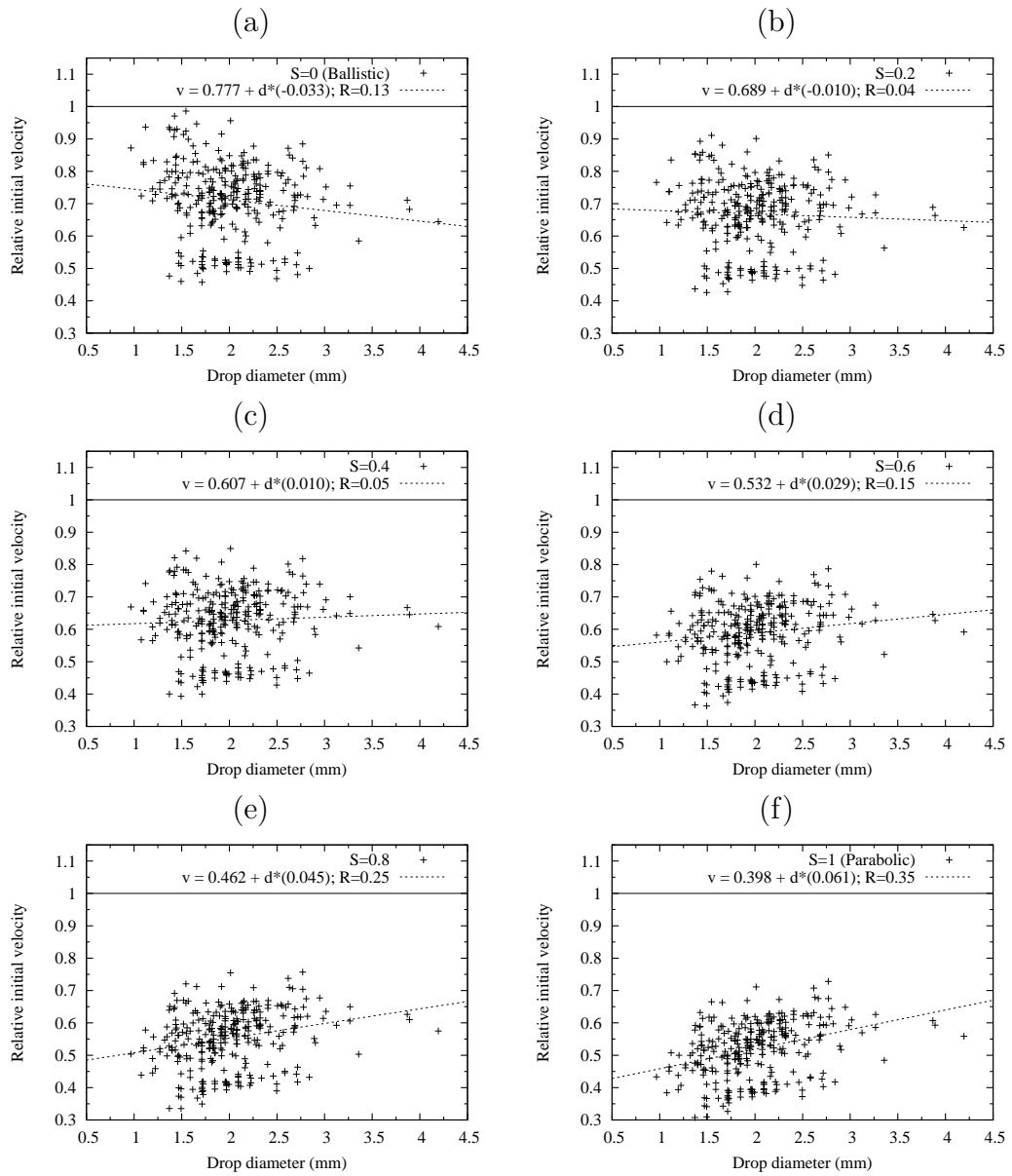


Figure 9. Initial velocity, relative to the velocity at the nozzle, for different values of S . Results are presented as a function of drop diameter.

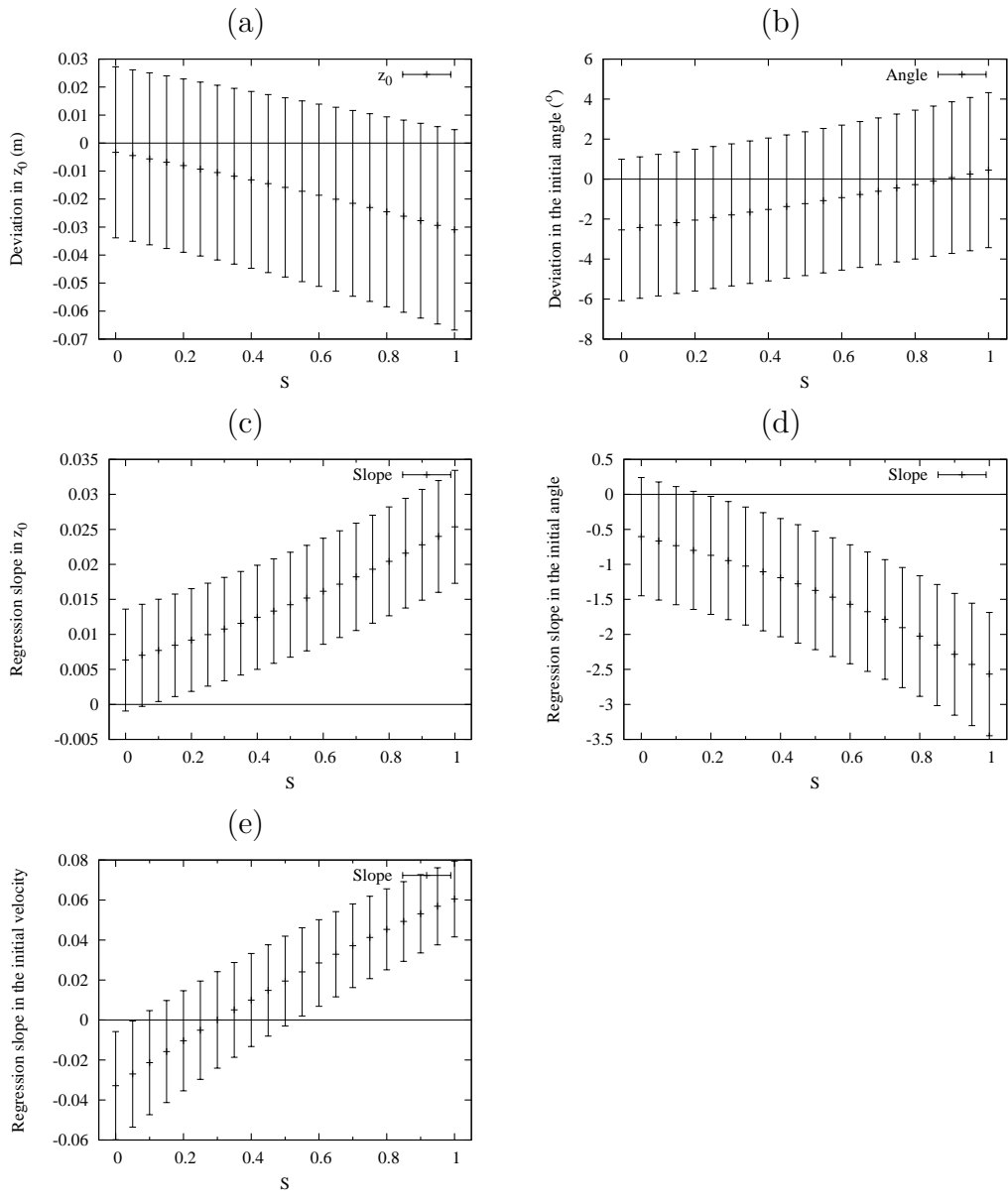


Figure 10. Mean values of the deviation (a) from the measured value of the initial z coordinate and (b) from the initial angle for different values of S . Furthermore, average regression slope (c) in the initial z coordinate, (d) in the initial angle and (e) in the initial velocity for different values of S . Values are presented plus minus one standard deviation.