Understanding reactions induced by $^6$He at energies around the Coulomb barrier

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The $^6\text{He}$ nucleus

- **Radioactive:**
  \[ ^6\text{He} \xrightarrow{\beta^-} ^6\text{Li} \quad (t_{1/2} \approx 807 \text{ ms}) \]

- **Weakly bound:**
  \[ \epsilon_b = -0.973 \text{ MeV} \]

- **Neutron halo**

- **Borromean system:**
  n-n and $\alpha$-n unbound

- **$\sim$ 3 body system:**
  $\alpha$ almost inert
Some recent experimental data

SIMILAR EXPERIMENTS AT NOTRE DAME: $^6\text{He} + ^{209}\text{Bi}$

- Low statistics
- Small angular resolution
- More focused to fusion studies
Elastic scattering

- $E_{\text{beam}} = 12 - 27$ MeV ($V_b \sim 20$ MeV)
- Good angular resolution
- Clear evidence of disappearance of diffraction peak

![Graph showing elastic scattering data for $^6\text{He} + ^{208}\text{Pb} @ 22$ MeV]
**Sevilla-Huelva-LLN** $^6$He$^+^{208}$Pb experiment at LLN

**Breakup:** angular and energy distributions of $\alpha$ particles (neutrons not recorded)
- Large yield (transfer/breakup?)
- $\alpha$ particles post-accelerated
SOME QUESTIONS TO BE ADDRESSED:

1. How does the $^6\text{He}$ continuum affect the elastic distributions?

2. Is the optical model valid to describe the elastic data?

3. Is it possible to describe the elastic and breakup data within a few-body model?

4. What are the mechanisms responsible for the production of $\alpha$’s?

5. Why are the $\alpha$ particles accelerated with respect to beam velocity?
Optical model calculations for $^6\text{He}+^{208}\text{Pb}$
How does the halo structure affect the elastic scattering?

- $^4\text{He}+^{208}\text{Pb}$ shows typical Fresnel pattern → strong absorption
- $^6\text{He}+^{208}\text{Pb}$ shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
- $^6\text{He}+^{208}\text{Pb}$ requires a large imaginary diffuseness → long-range absorption
Optical model calculations for $^6\text{He}^+\text{^{208}Pb}$


\[ U(r) = V(r) + iW(r) + V_C(r) + V_{pol}(r) \]

\( V(r), W(r) \): Phenomenological (WS) OMP
Radius of sensitivity of $V(r)$ and $W(r)$

Imaginary part is sensitive to distances well beyond the strong absorption radius
Some conclusions from the analysis of the elastic data with OMP:

- Elastic data can be indeed well reproduced with OMP, but the imaginary part requires a large diffuseness ⇒ long-range absorption effect

- Imaginary part is sensitive to distances well beyond the strong absorption radius ($r_{ra} \approx 12.5$ fm).

- The Dynamic Dipole Polarization Potential can account, but only partially, for the long range absorption.
$^6$He+$^{208}$Pb within a few-body model
Assumptions of the dineutron model:

- Assume $^6\text{He}=\alpha + ^2\text{n}$
  (inspired in $\alpha + d$ model for $^6\text{Li}$)
- Ignore n-n dynamics
- $2n-\alpha$ ground state described by 2S wavefunction with $\varepsilon_{\text{bind}} = S_{2n} = 0.97$ MeV.
- Theoretically very appealing, because permits the application of the standard CDCC method
Standard CDCC formalism: coupled-channel equations

- Hamiltonian: \( H = T_R + T_r + V_{pn}(r) + V_{pt}(r_{pt}) + V_{nt}(r_{nt}) \)

- Internal states: \( \{ \phi_{gs}, \phi_n \} \)

- Model wavefunction:
  \[
  \Psi(R, r) = \phi_{gs}(r)\chi_0(R) + \sum_{n>0}^{N} \phi_n(r)\chi_n(R)
  \]

- Coupled equations: \([H - E]\Psi(R, r)\)

\[
[E - \epsilon_{\alpha} - T_R - V_{\alpha,\alpha}(R)]\chi_{\alpha}(R) = \sum_{\alpha' \neq \alpha} V_{\alpha,\alpha'}(R)\chi_{\alpha'}(R)
\]

- Transition potentials:
  \[
  V_{\alpha;\alpha'}(R) = \int dr \phi_{\alpha}(r)^* \left[ V_{pt}(R + \frac{r}{2}) + V_{nt}(R - \frac{r}{2}) \right] \phi_{\alpha'}(r)
  \]
Continuum discretization: 2-body case

- Truncate the continuum spectrum in $\varepsilon$ and $\ell$:
  - $0 < \varepsilon < \varepsilon_{\text{max}}$
  - $0 < \ell < \ell_{\text{max}}$

- Division into energy intervals (bins)
  $\Delta \varepsilon_i ; \quad i, \ldots, N$

- For each bin, a representative function is constructed by superposition of the continuum states

\[
\phi_{\ell,i}^{\text{bin}}(r) = N_{\ell,i} \int_{k_i}^{k_{i+1}} w(k) \phi_\ell(k, r) dk \quad i = 1, \ldots, N
\]
The *naive* dineutron model fails to describe the elastic data
The dineutron model tends to overestimate the coupling to the continuum: we need a more sophisticated model for $^6\text{He}$.
● Internal states: $\phi_n(x, y)$

● Model wavefunction:

$$\Psi(R, x, y) = \phi_{gs}(x, y)\chi_0(R) + \sum_{n>0} \phi_n(x, y)\chi_n(R)$$

● $[H - E]\Psi_{JM} = 0 \Rightarrow$ System of coupled equations:

$$[E - \epsilon_\alpha - T_R - V_{\alpha,\alpha}(R)]\chi_\alpha(R) = \sum_{\alpha' \neq \alpha} V_{\alpha,\alpha'}(R)\chi_{\alpha'}(R)$$

● Coupling potentials:

$$V_{n,n'}(R) = \langle \phi_n|\hat{V}_{nT} + \hat{V}_{nT} + \hat{V}_{\alpha,T}|\phi_{n'}\rangle$$
Coupling potentials

\[
V_{Lnj,L' n' j'}^J(R) = \langle LnjJM | \sum_{k=1}^{3} \hat{V}_{kt}(\vec{r}_k) | L' n' j' JM \rangle
\]

where

\[
\Phi_{Lnj}^{JM}(\vec{R}, \vec{x}, \vec{y}) = \sum_{\mu M L} \psi_{j \mu n}^{\mu L} (\vec{x}, \vec{y}) \langle LM L j \mu | JM \rangle Y_{L M L} (\vec{R})
\]

multipolar expansion

\[
V_{Lnj,L' n' j'}^J(R) = \sum_{Q} (-1)^{J-J'} L \hat{L} \hat{L}' \left( \begin{array}{ccc} L & Q & L' \\ 0 & 0 & 0 \end{array} \right)
\]

\[
\times W(LL'jj', QJ) F_{n_j, n'_j}^Q(R)
\]
Form factors

\[
F_{n\beta n'j'}^Q(R) = (-1)^{Q+j-j'}\hat{j}\hat{j}'(2Q + 1) \\
\times \sum_{\beta'\beta'} \sum_{\kappa=1}^{3} \sum_{\beta_k\beta'_k} N_{\beta\beta_k} N_{\beta'\beta'_k} \\
\times (-1)^{l_{x\kappa} + S_{x\kappa} + j'_{abk} - j_{abk} - I_{k}} \delta_{l_{x\kappa} l'_{x\kappa}} \delta_{S_{x\kappa} S'_{x\kappa}} \\
\times \hat{l}_{y\kappa} \hat{l}'_{y\kappa} \hat{l}_{y_k} \hat{l}'_{y_k} \hat{j}_{abk} \hat{j}'_{abk} \left( \begin{array}{ccc}
    l_{y\kappa} & Q & l'_{y\kappa} \\
    0 & 0 & 0
  \end{array} \right) \\
\times W(l_{k} l'_{k} l_{y\kappa} l'_{y\kappa} ; Ql_{x\kappa}) W(j_{abk} j'_{abk} l_{k} l'_{k} ; QS_{x\kappa}) \\
\times W(j j' j_{abk} j'_{abk} ; QI_{k}) \int \int (\sin \alpha_{k})^2 (\cos \alpha_{k})^2 \rho^5 d\alpha_{k} d\rho \\
\times R_{\beta j n}^l (\rho) \varphi_{k_k}^{l_{x\kappa} l_{y\kappa}} (\alpha_{k}) V_{Q}^{l_{x\kappa} l'_{y\kappa}} (R, y_{k}) \varphi_{k'_k}^{l_{x\kappa} l'_{y\kappa}} (\alpha_{k}) R_{\beta' j' n'}^l (\rho)
\]
Multipolar expansion

\[ V^k_Q(R, y_k) = \frac{1}{2} \int_{-1}^{+1} \hat{V}_{kt}(\vec{r}_k) P_Q(z_k) dz_k \]

where

\[ \hat{V}_{kt}(\vec{r}_k) = \sum_Q (2Q + 1) V_Q(R, y_k) P_Q(z_k) \]

\[ z_k = \hat{y}_k \cdot \hat{R} \]

\[ V^J_{Lnj, L'n'j'}(R) = \sum_Q (-1)^{J-j} \hat{L}\hat{L}' \begin{pmatrix} L & Q & L' \\ 0 & 0 & 0 \end{pmatrix} \]

\[ \times W(LL'jj', QQ) F^Q_{n_j, n'_j}(R) \]
2-body case:

- 1 single degree of freedom (inter-cluster relative coordinate $r$)
- 2-body Hamiltonian: $H = T_r + V_{pn}(r)$
- 2-body wavefunction:

$$\phi_{n,\beta}(r) = R_{n,\beta}(r) [Y_{\ell}(\Omega) \otimes \chi_{S,jm}]_{\beta} = \{\ell, S, j\}$$
3-body case:

- 2 degrees of freedom (6 coordinates)

- 3-body Hamiltonian:

\[ H = T_R + T_r + V_{nn} + V_{n\alpha} + V_{n\alpha} + V_{nn\alpha} \]

- The 3-body wavefunction can be expressed in different coordinate systems:
  
  - Jacobi coordinates: \( \{x, y\} \)
  
  - Hyperspherical coordinates: \( \{\rho, \Omega_x, \Omega_y, \alpha\} \quad \rho^2 \equiv x^2 + y^2 \quad \tan \alpha = \frac{y}{x} \)

\[
\phi_{n,jm}(\rho, \Omega) = \frac{1}{\rho^{5/2}} \sum_{\beta=1}^{N\beta} R_{n\beta}(\rho) \left[ \Upsilon_{Kl}^{l_x l_y}(\Omega_x, \Omega_y, \alpha) \otimes X_{S}\right]_{jm} \\
\beta \equiv \{K, l_x, l_y, l, S_x, j\}
\]
Two methods for continuum discretization:

1. Pseudo-state method: 2-body and 3-body projectiles
2. Binning method: so far, only 2-body projectiles.
1. Choose a complete basis for the degree of freedom under consideration

Eg: HO basis: \{\phi_{\ell,n}^{HO}(r)\}; \quad n = 0, \ldots, \infty

2. Truncate the basis: \(n = 0, \ldots, N\)

3. Diagonalize the Hamiltonian in the truncated basis

\[
\begin{align*}
\left\{\varphi_{\ell,n}(r)\right\}_{n=0}^{N} \quad \text{Diagonalize H} \quad \Rightarrow \\
\left\{\varphi_{\ell,n}(r)\right\}_{n=0}^{N} \quad \Rightarrow \\
\begin{cases}
\epsilon_0 \simeq \epsilon_{gs} < 0 & \Rightarrow \text{g.s.} \\
\epsilon_n < 0 \quad (n \neq 0) & \Rightarrow \text{Bound excited states} \\
\epsilon_n > 0 & \Rightarrow \text{Continuum states}
\end{cases}
\end{align*}
\]
Transformed Harmonic Oscillator Basis


- The HO basis has incorrect asymptotic behaviour for bound states of finite potentials

\[ \phi_{i,\ell}^{HO}(s) \propto \exp(-s^2) \]

- Apply Local Scale Transformation \( s(r) \) to the HO basis such that:

\[
s(r) \rightarrow \begin{cases} 
  r & r \rightarrow 0 \\
  \sqrt{r} & r \rightarrow \infty 
\end{cases}
\]

- THO basis:

\[ \phi_{i,\ell}^{THO}(r) \propto \sqrt{\frac{ds}{dr}} \phi_{i,\ell}^{HO}[s(r)]; \quad i = 1, \ldots \]

- Advantages:
  - Proper asymptotic behaviour for bound states
  - Preserve simplicity and analytic properties of HO basis.
Two alternative THO prescriptions

1. **Determine the LST $s(r)$ from the g.s. of the system:**
   \[
   \int_0^r |\phi_{gs}(r')|^2 dr' = \int_0^s |\phi_{HO}^{s}(s')|^2 ds'
   \]
   ☛ **Advantage:** the g.s. wavefunction is exactly reproduced for any $N$

2. **Analytic $s(r)$ :**
   \[
   s(r) = \left[ \frac{1}{\left(\frac{1}{r}\right)^m + \left(\frac{1}{\gamma \sqrt{r}}\right)^m} \right]^{\frac{1}{m}}
   \]
   ☛ **Advantage:** simpler to calculate, flexibility
**Bin method: 3-body system**

- Continuum states can be expanded in HH as

\[
\Psi_{\kappa j \mu}(\rho, \Omega, \Omega_{\kappa}) = \sum_{\beta \beta'} R_{\beta \beta' j}(\kappa, \rho) \mathcal{U}_{\beta \beta' j \mu}(\Omega, \Omega_{\kappa})
\]

\(\beta' \leftrightarrow\) incoming channels
\(\beta \leftrightarrow\) outgoing channels
\(\kappa = \sqrt{2m\varepsilon/\hbar}\)

- For each incoming channel, the radial part of the bin WF is calculated as

\[
R_{n j \beta}^{bin}(\rho) = \sqrt{\frac{2}{\pi N_{\beta' j}}} \int_{\kappa_1}^{\kappa_2} d\kappa e^{-i\delta_{\beta' j}(\kappa)} R_{\beta \beta' j}(\kappa, \rho)
\]

\(n \equiv \{\beta' \varepsilon_{av}\}\)
Bin method: 3-body system

- Continuum states can be expanded in HH as

\[ \Psi_{\kappa,j\mu}(\rho, \Omega, \Omega_\kappa) = \sum_{\beta \beta'} R_{\beta \beta'}(\kappa, \rho) \mathcal{Y}_{\beta' j \mu}(\Omega, \Omega_\kappa) \]

\( \beta' \leftrightarrow \text{incoming channels} \)
\( \beta \leftrightarrow \text{outgoing channels} \)
\( \kappa = \sqrt{2m\varepsilon/\hbar} \)

- For each incoming channel, the radial part of the *bin* WF is calculated as

\[ R_{n,j\beta}^{bin}(\rho) = \sqrt{\frac{2}{\pi N_{\beta' j}}} \int_{\kappa_1}^{\kappa_2} d\kappa e^{-i\delta_{\beta' j}(\kappa)} R_{\beta \beta'}(\kappa, \rho) \]

\( n \equiv \{ \beta' \varepsilon_{av} \} \)

- The basis space for 3-body case is huge!!
Bin method: eigenchannels

How to establish a hierarchy among the states for each energy $\varepsilon$ and total angular momentum $j$?

① Define eigenstates of the S-matrix for each $\varepsilon \Rightarrow$ eigenchannels (EC)

② Order the EC according to the magnitude of their phase-shifts

③ Truncate in the number of EC
B(E1) and B(E2) in a three-body model

![Graph showing dB(E1)/dε and eigenphase vs. ε (MeV)]
$B(E1)$ and $B(E2)$ in a three-body model
Applications of the four-body CDCC formalism

$^6\text{He} + ^{12}\text{C} @ 230 \text{ MeV}$

\[ \frac{\sigma}{\sigma_{\text{Ruth}}} \]

\[ \theta_{\text{c.m.}} (\text{deg}) \]

- V. Lapoux et al. PRC 66 034608
- no continuum
- $n_b=2 \epsilon_{\text{max}}=30 \text{ MeV}$
- $n_b=4 \epsilon_{\text{max}}=30 \text{ MeV}$
Applications of the four-body CDCC formalism

$^{6}\text{He}+^{64}\text{Zn}$

$E=10\text{ MeV}$

$E=13.6\text{ MeV}$
Applications of the four-body CDCC formalism


\[ ^{6}\text{He}^{+}^{208}\text{Pb} \]

\( E = 22 \text{ MeV} \)

\( E = 27 \text{ MeV} \)
Breakup cross sections for $^6\text{He}^+^{208}\text{Pb}$
- **Real part:** long range behaviour
  - Repulsive around the strong absorption radius.
  - Attractive at large distances (mainly from dipole Coulomb breakup).
- **Imaginary part:** shows long range absorption obtained with phenomenological OMP and dineutron model.
- **Real part**: long range behaviour
  - Repulsive around the strong absorption radius.
  - Attractive at large distances (mainly from dipole Coulomb breakup).

- **Imaginary part**: shows long range absorption obtained with phenomenological OMP and dineutron model.
Open problems and work under development:

- Pseudo-state vs Binning procedure
- Is there a more efficient method to represent the 3-body continuum?
- Inclusion of Coulomb in 3-body structure (e.g., $^6\text{Be}$)
- Calculation of neutron-neutron and neutron-$\alpha$ correlations (energy, angle) to compare with new experiments.
- Transfer within four-body model.
The dineutron model of $^6$He revisited
The dineutron model for $^6\text{He}$ revisited

Density distribution for $\alpha$-$2n$ relative motion:

- 3-body: $\langle r_{\alpha-2n} \rangle = 3.25$ fm
- 2-body: $\langle r_{\alpha-2n} \rangle = 4.10$ fm

$(R_0=1.90$ fm; $a=0.25$ fm)
The dineutron model for $^6$He revisited

Density distribution for $\alpha$-$2n$ relative motion:

- 3-body: $\langle r_{\alpha-2n} \rangle = 3.25$ fm
- 2-body: $\langle r_{\alpha-2n} \rangle = 4.10$ fm
Improved dineutron model for $^6\text{He}$

$^6\text{Li}=\alpha + d$

$\varepsilon_{\alpha-d} = -1.47 \text{ MeV}$

$\varepsilon_{n-p} = \varepsilon_d = -2.22 \text{ MeV}$

$^6\text{He}=\alpha + ^2n$

$\varepsilon_{\alpha-2n} = S_{2n} = -0.97 \text{ MeV}$

$\varepsilon_{n-n} \sim 0$ ?

In reality, we expect $\varepsilon_{n-n} > 0$ !!!
An improved dineutron model for $^6$He

Our model: determine an effective $\alpha^{-2}n$ binding energy: $\varepsilon_b \simeq -1.6$ MeV

- 3-body: $\langle r_{\alpha^{-2}n} \rangle = 3.25$ fm
- 2-body: $\langle r_{\alpha^{-2}n} \rangle = 3.45$ fm
An improved dineutron model for $^6$He

Our model: determine an effective $\alpha^-2n$ binding energy: $\varepsilon_b \sim -1.6$ MeV

- Three-body: $\langle r_{\alpha-2n} \rangle = 3.25$ fm
- 2-body: $\langle r_{\alpha-2n} \rangle = 3.45$ fm
Application to $^6\text{He}^+\,^{64}\text{Zn}$

Elastic scattering in the improved dineutron model

$^6\text{He} + ^{208}\text{Pb}$ at $E = 27$ MeV

$^6\text{He} + ^{208}\text{Pb}$ @ $E = 27$ MeV

- 1 channel: $\varepsilon_b = -1.6$ MeV
- CDCC: $\varepsilon_b = -0.97$ MeV
- CDCC: $\varepsilon_b = -1.5$ MeV
Elastic scattering in the improved dineutron model

$^6\text{He}^+^{197}\text{Au}$ at $E = 20\text{-}40\text{ MeV}$

Discussion session at CEA/Saclay workshop

- $^6\text{He}$
- $^{197}\text{Au}$
- $E = 20\text{-}40\text{ MeV}$

**Graphs:**
- $E = 27\text{ MeV}$
- $E = 29\text{ MeV}$
- $E = 40\text{ MeV}$

Graph legend:
- 1 channel
- CDCC: $\varepsilon_b = -0.97\text{ MeV}$
- CDCC: $\varepsilon_b = -1.6\text{ MeV}$

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Discussion session at CEA/Saclay workshop

CEA Saclay February 2009 – 42 / 50
Inclusive breakup in the improved dineutron model

![Graphs showing inclusive breakup data for different energies and angles.](#)
What is the mechanism responsible for the production of $\alpha$'s?

$^6\text{He} + ^{208}\text{Pb} \rightarrow ^4\text{He} + X$
What is the mechanism responsible for the production of $\alpha$'s?

$^6\text{He} + ^{208}\text{Pb} \rightarrow ^4\text{He} + X$

$E_\alpha \approx \frac{4}{6} E_{\text{beam}}$
What is the mechanism responsible for the production of $\alpha$'s?

$^{6}\text{He}^{+}^{208}\text{Pb} \rightarrow ^{4}\text{He} + X$

**Direct Breakup (BU)**

**Transfer to the Continuum (TC)**

$E_\alpha \approx \frac{4}{6} E_{\text{beam}}$

$E_\alpha \approx E_{\text{beam}}$
What is the mechanism responsible for the production of $\alpha$'s?

$^6$He$+^{208}$Pb $\rightarrow ^4$He + X

- Direct Breakup (BU)
  
  $E_\alpha \approx \frac{4}{6} E_{\text{beam}}$

- Transfer to the Continuum (TC)
  
  $E_\alpha \approx E_{\text{beam}}$

- 1-neutron transfer (1NT)
  
  $E_\alpha \approx \frac{4}{5} E_{\text{beam}}$
The TC model explains satisfactorily the magnitude and energy distribution of the measured $\alpha$'s.

DBU fails to explain the yield and energy of $\alpha$'s.
Our calculations suggest a picture in which the $\alpha$ particles are repelled by the Coulomb field, while the neutrons are transferred to highly excited states of the target (transfer to the continuum).
Conclusions and medium-term wishes

- $^6$He experiments at Coulomb barrier energies provide a beautiful as well as challenging probe to test our understanding of the structure and the reaction mechanisms that arise in the collision of weakly bound nuclei.

- The CDCC framework can be applied to describe the scattering of three-body systems using an extension of the binning method.

- Unlike the $\alpha$-d model of $^6$Li, the *naive* dineutron model is not appropriate to describe the scattering of $^6$He. However, a semi-quantitative understanding of elastic and transfer/breakup can be achieved using a modified dineutron model provided the (positive) n-n relative energy is taken into account in the model.

- In close future we would like to:
  - Calculate more complicated observables, such as neutron-neutron or neutron-$\alpha$ correlations in breakup experiments.
  - Extend these analyses to the scattering of other Borromean nuclei.
  
  *E.g.*: $^{11}$Li$^{+208}$Pb measured in July 2008 at TRIUMF (Vancouver, Canada).
Collaborators:

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- L. Acosta, I. Martel, F. Pérez-Bernal, A. Sánchez-Benítez (Univ. de Huelva)
- I. Thompson, J. Tostevin (Univ. de Surrey, Reino Unido)
- D. Escrig, M.J. Borge (CSIC, Madrid)
Some recent experimental data

$^6$He experiments at LLN

$^6$He+$^{64}$Zn @ $E_{lab}=13.6$ MeV

$E_{lab}=10$ MeV

Optical model calculations for $^6$He$^{+}{}^{208}$Pb


\[ U(r) = V(r) + iW(r) + V_C(r) + V_{\text{pol}}(r) \]