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## Abstract

This working paper analyzes the endogenous creation of financial intermediaries. We construct an occupational choice model where agents differ amongst themselves in their accumulated assets and skill level. Every period these agents have to choose among three occupations: a worker, an entrepreneur or a banker. Financial intermediaries (bankers) have access to a transaction technology through which they can first attract funds from depositors and then assign these funds to borrowers. In the process of intermediation, bankers and entrepreneurs face financial constraints, so that the allocation of capital is not fully efficient. In our model, intermediating assets consumes productive resources, and any agent is free to choose that occupation. In this manner intermediating activities improve on the allocation of resources but at a cost. We analyze to what extent this activity approximates the economy to the constraint-efficient equilibrium.

## Key words

Financial intermediation, occupational choice, wealth distribution, financial constraint.

## Resumen

Este documento de trabajo analiza la creación endógena de intermediarios financieros. Para ello, hemos elaborado un modelo de elección de ocupación en el que los agentes se diferencian entre sí por sus niveles de activos acumulados y productividad. En cada período estos agentes pueden elegir entre tres ocupaciones: trabajador, productor o intermediario financiero. Los intermediarios financieros tienen acceso a una tecnología de transacciones por la que pueden atraer depósitos que canalizan a los prestatarios de la economía. En este proceso de intermediación, tanto los empresarios como los banqueros se enfrentan a restricciones financieras, de forma que la asignación de capital no es totalmente eficiente. En nuestro modelo intermediar activos consume recursos productivos y cualquier agente es libre de elegir esa ocupación. De esta forma, el sector financiero mejora la asignación de recursos pero a un coste. Se analiza en qué medida la actividad financiera aproxima la economía al equilibrio eficiente.

## Palabras clave

Intermediación financiera, elección de ocupación, distribución de la riqueza, restricciones financieras.

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***Endogenous Financial Intermediation***


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# 1. Introduction

THIS working paper analyzes the endogenous creation of financial intermediaries. We construct an occupational choice model where agents differ amongst themselves in their accumulated assets and skill level. Every period these agents have to choose among three occupations: a worker, an entrepreneur, or a banker. Financial intermediaries (bankers) have access to a transaction technology to obtain funds from depositors and to assign these funds to borrowers.

So far the literature has developed models in which financial intermediaries solve problems of information, transaction costs or lack of commitment devices. Usually in these models the intermediation technology presents increasing returns to scale, so it is efficient to concentrate all intermediation activities in a single agent. In our model, intermediating assets consumes productive resources, and any agent is free to choose that occupation. In this manner intermediating activities improve on the allocation of resources but at a cost. We analyze to what extent this activity approximates the economy to the constraint-efficient equilibrium.

# 2. The Economy

In the economy we analyze, there is a continuum of agents with mass one. These individuals may differ in their amount of accumulated assets and in their productivity. Each agent has access to a decision on whether to allocate his talent to be an entrepreneur *(E)* and produce goods, to be a banker *(B)* and intermediate resources between borrowers and lenders, or to be a worker *(W)* and work either for a banker or for an entrepreneur.

Each agent is endowed with a unit of time and evaluates streams of consumption *(c)* with the utility function

$$E\left[\sum_{t=0}^{\infty}\beta^{t}\, u\,(c_t)\right],$$

where $\beta \in (0, 1)$, and $u : \Re_{+} \rightarrow \Re$ is a bounded, strictly increasing, strictly concave, and twice differentiable continuous function that satisfies the Inada conditions. At the beginning of every period, agents are identified by a level of accumulated assets $a \in A = [0, \infty)$ and by an idiosyncratic productivity shock $z \in Z = (\underline{z}, \bar{z})$. This productivity level is carried from the previous period and represents a signal for the effective productivity the agent will have later in the period when production takes place, $z' \in Z$.

The timing of events is as follows. Each agent starts each period with accumulated assets, $a$, and a signal $z$ of the productivity he will have later in the period. Given this information, first agents make an occupational choice and decide whether to become a worker, an entrepreneur or a banker. Workers deposit their assets at the financial intermediaries and offer their labor services in the market. Entrepreneurs decide how much capital and labor to demand. Bankers are in the business of intermediating between suppliers of capital, that is, all workers as well as the entrepreneurs with assets in excess of their capital needs, and demanders of capital, that is, entrepreneurs who want to expand their business above the level of their accumulated assets. At the end of the period, productivities for the period $(z')$ are realized, and production takes place. Then workers are paid their wages, and entrepreneurs and bankers realize profits. Finally agents decide how much to consume and how much to save to accumulate assets.

If an agent decides to be a worker, he will draw his effective skill, $z'$, from a fixed distribution $\psi(z')$. The revenue from working is $z'w$ where $w$ is the wage per unit of productivity. Each agent who decides to be an entrepreneur will draw his productivity level from a Markov process with transition function $Q(z, z')$. The productivity shocks for workers and entrepreneurs are drawn from the same set $Z = (\underline{z}, \bar{z})$. Agents who decide to be a banker will draw their productivity level from a Markov process with transition function $\mu(z, \bar{z})$. We assume $Q$ and $\mu$ to be monotone and to satisfy the Feller property. As explained below, bankers draw their productivity shocks from a different set $Z^B = (\underline{z}^B, \bar{z}^B)$. All assets, independently of whether used or not as capital, depreciate at the rate $\delta \in (0, 1)$.

After labor income and profits are realized, agents decide on how much to consume $(c)$ and the amount of physical assets $(a')$ that they will take to next period. This period's shock $z'$ is also carried as next period's signal for future productivity shocks. At the stationary equilibrium, the problem of an agent who enters the period with the pair $(a, z)$ can be summarized by the value function

$$v(a, z) = \max \Big\{ \int v^W(a, z') \psi(dz'),$$

$$\max_{k, n} \int v^E(a, z') Q(z, dz'), \max_{d} \int v^B(a, z') \mu(z, dz') \Big\}, \tag{2.1}$$

$$v^i(a, z') = \max_{c, a'} \big\{ u(c) + \beta v(a', z') \big\}, \tag{2.2}$$

with $i = W, E, B$. Entrepreneurs and the bankers must commit capital and labor inputs. Entrepreneur hire workers $(n)$ and can rent capital $(k)$, bankers must use their own capital and hire workers to produce deposits $(d)$. Workers and nonborrowing entrepreneurs deposit at bankers. All agents take prices as given. These are wages $(w)$, deposit rates $(r^D)$ and lending rates $(r^L)$.

## 2.1. The worker

The worker's budget constraint is

$$c + a' \leq (1 - \delta)\ a + \pi^W(a, z'), \tag{2.3}$$

where

$$\pi^W(a, z') = (r^D a + \delta)\ a + wz'. \tag{2.4}$$

## 2.2. The entrepreneur

The entrepreneur's budget constraint is

$$c + a' \leq (1 - \delta)\ a + \pi^E (a, z' \mid z). \tag{2.5}$$

Entrepreneurs hire capital *(k)* and labor *(n)* in a competitive fashion and produce according to the production function

$$y = z' f(k, n) = z'(k^{\alpha^E} n^{1-\alpha^E})^{\theta^E}, \tag{2.6}$$

where $\alpha^E \in (0,1)$ and $\theta^E < 1$. The production function exhibits decreasing returns to scale which, as in Lucas (1978), can be thought of as capturing the presence of decreasing returns to managerial control. Finally we assume hired labor $n$ consists of a pool of perfectly diversified workers so that it always realizes the average worker's skill, which we normalize to be equal to 1. Thus the productivity of the firm only depends on the realization of the entrepreneur's shock, $z'$.

The profit function for entrepreneurs equals

$$\pi^E (a, z' \mid z) = z' f(k, n) - wn + r^D \max\{0, a - k\} - r^L \max\{0, k - a\}. \tag{2.7}$$

### 2.2.1. Financing constraint

We assume there is no possibility of default on bank loans $k - a$ [1]. The specification of the Inada-type utility function together with the uncertainty in entrepreneurial profits imply that agents with a low level of accumulated assets may be constrained with respect to their size of the entrepreneurial project. In particular, the total entrepreneurial income must guarantee a nonnegative consumption for all possible realizations of profits, that is, entrepreneurs have to satisfy the financial constraint

$$(1 - \delta)\ a + \pi^E (a, z' \mid z) \geq 0 \text{ for all } z' \in Z. \tag{2.8}$$

Since in each period $Q(z, \{\underline{z}\}) > 0$ for all $z \in Z$, this financing constraint must be satisfied for the lowest effective ability shock $\underline{z}$. As this constraint depends only on the asset level and not on the signal of the effective entrepreneurial activity $z$, an agent with an initially low level of assets $(a)$ but with the

1. This could be motivated by the existence of limited commitment on the part of borrowers in repaying the loans as in Cooley, Marimón and Quadrini (2004).

prospects of having good skills (a high expected $z'$) may not be able to reach the optimal size of the firm due to the possibility of drawing a low realization of his productivity and generating negative profits he will not be able to cover with his accumulated assets. Nevertheless because of the properties assumed for $Q$, becoming an entrepreneur has a future value. That is, an entrepreneur is willing to sacrifice current consumption for the possibility of starting a career that increases the firm's return over time. The way to obtain this return profile is by investing a large share of his income and wealth so as to relax the credit constraint in order to run the firm at its optimal size.

### 2.3. The banker

To produce the amount of deposits $d$, the banker has to use capital ($k$) and labor ($n$). We assume that $k$ must be self-financed, so that $k \leq a$. Also, as with entrepreneurs, labor $n$ consists of a pool of perfectly diversified workers with the labor's average skill level. The production function for deposits presents decreasing returns to scale,

$$d\,(a, z) = d\,(k\,(a, z),\, n\,(a, z)) = A^B\,(k^{\alpha^B}\, n^{1-\alpha^B})^{\theta^B}, \tag{2.9}$$

where $a^B \in (0,1)$ and $\theta^B < 1$. Bankers also face idiosyncratic shocks. This shock represents the ability of the banker to transform already created deposits into loans, i.e., to attract borrowers. Once the bankers make these loans they are pooled into a risk-free pool of loans from which each draws a repayment rate of $r^L$. Thus each banker has the same portfolio out of the loans they make but only a part $z' \in [0, 1]$ of the deposits and assets available for loans is lent out. For the rest of funds $1 - z'$ the banker does not find a match or borrower. As mentioned before, these shocks follow a Markov process with transition function $\mu\,(z, z')$.

So the banker returns $(1 + r^D)$ of deposits to all depositors but he gets back $(1 + r^L)$ only on the fraction $z'$ of loans made. Thus the banker's budget constraint is

$$c + a' \leq (1 - \delta)\; a + \pi^B\,(a, z' \mid z), \tag{2.10}$$

where

$$\pi^B\,(a, z' \mid z) = z'\, r^L\, d - r^D\, d - wn, \tag{2.11}$$

and

$$k \leq a.$$

Bankers, as entrepreneurs, face a financing constraint guaranteeing positive consumption for all possible values of $z'$.

$$(1-\delta)\ a + \pi^B (a, z' \mid z) \geq 0 \text{ for all } z' \in Z. \tag{2.12}$$

This constraint has the same kind of implications with respect to the choice of inputs as with entrepreneurs.

### 2.4. Stationary equilibrium

Summarizing, each period agents first make decisions based on their individual state $(a, z)$ and the prices of the economy. These decisions are whether to become a worker, an entrepreneur or a banker and the amount of labor, $n\,(a, z)$, and capital, $k\,(a, z)$, to hire. Then, after productivity for the period $(z')$ is revealed, they decide how much to consume, $c\,(a, z')$, and how many assets to transfer to the next period, $a'\,(a, z')$. At the aggregate level, the equilibrium outcome of these decisions is a probability measure l that determines the density of agents with each combination of productivities $z$ and capital $a$. This measure evolves as

$$\lambda'\,(A', Z') = \int_s \Delta\ (z, dz')\,\lambda\ (da \times dz),$$

where $S = \{(a, z') : a'\,(a, z') \in A' \text{ and } z' \in Z'\}$ and $\Delta$ is a transition selector

$$\Delta\ (z, dz') \equiv \psi\ (dz')\,|_{W} + Q\,(z, dz')\,|_{E} + \mu\ (z, dz')\,|_{B},$$

that determines the end of period productivities from the beginning of period productivities. This transition selector is endogenous because the way productivities change within the period is different among workers, entrepreneurs and bankers, and such occupational choice is the agents' decision. So, from the law of motion of $\lambda$, the measure of agents with next period's state in the set $(A', Z')$ is those whose skills evolve to the set $Z'$ and whose optimal decision is to accumulate assets that belong to the set $A'$.

The concept of stationary equilibria requires that assets supplied by all agents equal the amount of capital demanded by the entrepreneurs and bankers; that labor supply by workers equals the labor hired by entrepreneurs and bankers; and that all allocations are feasible for a time invariant probability measure $\lambda$ that determines the density of agents with each combination of productivities $z$ and capital $a$. So this measure should satisfy

$$\lambda\,(A',\,Z') = \int_s \Delta\,(z,\,dz')\,\lambda\,(da \times dz).$$

With these elements, we can define the steady state of this economy.

**Definition 1.** A stationary recursive competitive equilibrium is constant prices $(r^D,\, r^L,\, w)$, value functions $v\,(a,\,z)$, $v^W\,(a,\,z)$, $v^E\,(a,\,z)$, $v^B\,(a,\,z)$, policy functions $k\,(a,\,z)$, $n\,(a,\,z)$, $d\,(a,\,z)$, $c\,(a,\,z')$, $a'\,(a,\,z')$, a probability measure $\lambda$, transition selector $\Delta\,(z,\,dz')$, and aggregate levels $(D,\,L,\,N,\,K,\,A)$, such that:

1. At given prices the policy functions solve the optimization problem of each agent $(a,\,z)$.
2. The probability measure $\lambda$ is time invariant.
3. Prices are such that markets clear: these are the market for deposits

$$D^s = \int_{E\times W} \max\,\{0,\; a - k\}\,\lambda\,(da \times dz) = \int_B d\,(a,\,z)\,\lambda\,(da \times dz) = D^d,$$

the loans market

$$L^s = \int_B z'\,d\mu\,(z,\,dz')\,\lambda\,(da \times dz) = \int_E \max\,\{0,\; k - a\}\,\lambda\,(da \times dz) = L^d,$$

and the labor market

$$N^s = \int_W z'\,\psi\,(dz')\,\lambda\,(da \times dz) = \int_{E\times B} n\,(a,\,z)\,\lambda\,(da \times dz) = N^d,$$

with

$$\int_W \lambda\,(da \times dz) + \int_E \lambda\,(da \times dz) + \int_B \lambda\,(da \times dz) = 1.$$

4. The aggregate feasibility constraint holds at equality

$$\begin{aligned} &\int \left\{c\,(a,\,z') + \delta a'\,(a,\,z')\right\} \Delta\,(z,\,dz')\,\lambda\,(da \times dz) = \\ &= \int z' f\,(k\,(a,\,z),\; n\,(a,\,z))\;\; Q\,(z,\,dz')\,\lambda\,(da \times dz)\; Y. \end{aligned}$$

These imply that there is no aggregate uncertainty, that all assets are used as capital

$$A = \int a\lambda\,(da \times dz) \geq \int_{E\times B} k\,(a,\,z)\,\lambda\,(da \times dz) = K.$$

# 3. Four Banking Cases

WE solve the model for four different credit institutions. In all cases the labor market exists and clears at the equilibrium wage *w*, which may be different for each case. These financial institutions are:

1. *Autarky.* Agents can only choose to be either workers or entrepreneurs. There is no credit market, so no borrowing or lending is allowed. Entrepreneurs must self-finance and use their own assets as capital.
2. *Exogenous banking.* In this case, agents can also only chooseto be either worker or entrepreneur. However credit goes through a perfectly competitive and costless market for which $r^D = r^L = r > 0$.
3. *Pure lending.* This is a particular case of the model described in the previous section. Loan markets exist, and bankers choose their occupation but do not have access to the deposit technology (2.9). Instead, bankers can only loan out their assets and are not able to obtain external funds in the form of deposits.
4. *Endogenous banks.* This is the model described in the previous section. Both deposit and loan markets exist, bankers choose their occupation and have access to the intermediation technology described above.

## 3.1. Calibration

The model contains eight parameters to calibrate. These parameters are the discount factor $(\beta)$, the depreciation rate $(\delta)$, the capital elasticity of output for entrepreneurs and bankers ($\alpha^E$ and $\alpha^B$), the manager's span of control for entrepreneurs and bankers ($\theta^E$ and $\theta^B$), the total factor productivity (TFP) in the goods production sector $(A^E)$ and in the banking sector $(A^B)$. Also we need to provide particular forms for the transition matrices associated with the Markov process for skills of workers $(\psi)$, entrepreneurs $(Q)$, and bankers $(\mu)$.

To calibrate these parameters and evaluate the accuracy of the model we use data for output, labor, capital, assets, wages and interest rates. This data is taken from the Flow of Fund Accounts (FFA) of the Federal Reserve System (FED), the National Income and Product Accounts (NIPA) of the Bureau of Economic Analysis (BEA), and the Current Employment Statistic survey (CES) of the Bureau of Labor Statistics (BLS).

We take each period in the model to be two years. This is so because this is the frequency of the data used to calibrate the transition matrix for entrepreneurial skills. Table 3.1 presents the values associated with the parameters in the model.

**TABLE 3.1: Parameters in the model**

| | | | |
|---|---|---|---|
| | *Utility* | | |
| $\beta$ | | **Discount factor** | **0.955** |
| | *Technology* | | |
| $\delta$ | | Depreciation rate | 0.046 |
| $\alpha^E$ | | Capital elasticity of output | 0.316 |
| $\alpha^B$ | | Capital elasticity of deposits | 0.247 |
| $\theta^E$ | | Entrepreneur's span of control | 0.908 |
| $\theta^B$ | | Banker's span of control | 0.764 |
| $A^E$ | | TFP of goods production | 1.000 |
| $A^B$ | | TFP of deposit's production | 100.0 |

These numbers are computed to match several observations such as the fraction of capital that depreciates each period or the shares of labor and capital income as well as profits. The number for the TFP of goods' production is used as a numeraire while $A^B$ is obtained so that banking activities were attractive for a sizable measure of agents. Appendix 1 includes the values for transition matrices $\psi$, $Q$, and $\mu$, while appendix 2 describes the data sources and the details of the calibration process.

Table 3.2 presents a collection of ratios and compares them with the equivalent measure in the banks model, which we use as reference.

**TABLE 3.2: Comparing the data with the banks model**

| Variable | Data | Model |
|---|---|---|
| Aggregate capital to output ratio | 3.423 | 3.463 |
| Aggregate consumption to output ratio | 0.756 | 0.811 |
| Capital to output ratio in goods production sector | 3.479 | 3.429 |
| Fraction of aggregate capital in banking sector | 0.014 | 0.010 |
| Fraction of aggregate assets in banking sector | 0.070 | 0.069 |
| Workers as a fraction of labor force | 0.947 | 0.919 |
| Entrepreneurs as a fraction of labor force | 0.049 | 0.073 |
| Bankers as a fraction of labor force | 0.004 | 0.008 |
| Fraction of workers in financial sector | 0.041 | 0.014 |
| Lending rate | 0.034 | 0.033 |
| Deposit rate | 0.012 | 0.016 |

Overall the model closely reproduces most of these ratios.

## 3.2. Results

The results of the four cases are presented in table 3.3 through 3.6. Table 3.3 presents the aggregated values for the whole economy on an annual basis.

**TABLE 3.3: Aggregated values**

| | Autarky | Exogenous | Lending | Banks |
|---|---|---|---|---|
| Assets | 6.6144 | 4.8091 | 4.9227 | 4.4382 |
| Capital | 4.7860 | 4.8501 | 4.3538 | 4.1040 |
| Labor | 1.0233 | 0.9698 | 1.0639 | 0.9668 |
| Output | 1.9914 | 2.5179 | 2.5054 | 2.3699 |
| TFP | 0.8833 | 1.4357 | 1.2351 | 1.4093 |
| Consumption | 1.5042 | 2.0077 | 1.9305 | 1.9235 |
| Wage | 0.8625 | 1.3563 | 1.2607 | 1.3220 |
| Deposit rate | — | 0.0209 | — | 0.0167 |
| Lending rate | — | 0.0209 | 0.0297 | 0.0329 |

Tables 3.4 to 3.6 present results disaggregated by occupation. Table 3.3 presents data for workers.

**TABLE 3.4: Average values for workers**

| | Autarky | Exogenous | Lending | Banks |
|---|---|---|---|---|
| Fraction of population | 0.9596 | 0.9190 | 0.9366 | 0.9189 |
| Assets | 1.7457 | 1.8669 | 0.4806 | 1.3355 |
| Consumption | 1.0170 | 1.6148 | 1.3630 | 1.5093 |
| Income | 1.0668 | 1.6777 | 1.3732 | 1.5597 |

Table 3.5 presents data for entrepreneurs.

**TABLE 3.5: Average values for entrepreneurs**

| | Autarky | Exogenous | Lending | Banks |
|---|---|---|---|---|
| Fraction of population | 0.0404 | 0.0806 | 0.0445 | 0.0729 |
| Assets | 122.39 | 38.380 | 60.859 | 39.842 |
| Consumption | 13.090 | 6.4978 | 9.8016 | 6.7054 |
| Income | 26.613 | 11.142 | 18.949 | 11.664 |

Table 3.6 presents data for bankers.

**TABLE 3.6: Average values for bankers**

| | Autarky | Exogenous | Lending | Banks |
|---|---|---|---|---|
| Fraction of population | — | — | 0.0185 | 0.0079 |
| Assets | — | — | 95.224 | 39.154 |
| Consumption | — | — | 11.749 | 6.0963 |
| Income | — | — | 13.130 | 4.9954 |

We observe how autarky is the worst equilibrium of the four economies. Because entrepreneurs must self-finance, agents accumulate way too many assets. The inefficient distribution of resources is reflected in low levels of output and consumption. Also as an indicator of how efficient each economy is, table 3.3 computes measured TFP [2]. Autarky presents the lowest

2. TFP is measured using a constant-returns-to scale Cobb-Douglas production function in capital and labor, where the elasticity output with respect to labor equals the share of income remunerating that input.

level for this indicator. On the other extreme there is the constraint efficient exogenous banking. In this case, consumption, output and measured TFP reach the highest levels. Measured TFP is 63%higher than in the autarkic economy. In between but close to the exogenous banking, there is the economy with banking as an occupational choice. The spread between borrowing and lending represents one of the inefficiencies of the economy reflecting the financial constraints of bankers, which force them to obtain profits on average. Finally pure lending improves measured TFP with respect to autarky. However the pure lending economy suffers from the inefficiencies associated with not being able to raise deposits. The inability to transfer assets for productive purposes in the most efficient hands is also reflected in the level wages and interest rates. As the economy is more efficient in the intermediation process wages increase, improving worker's welfare, and interest rates decrease.

We see how on average workers are the poorest agents of the economy in terms of assets, consumption and income, while bankers and entrepreneurs are the richest agents of the economy. In this sense, entrepreneurs are richer on average in the least efficient economies, namely autarky and pure lending. This is because in these economies there are fewer entrepreneurs than in the exogenous banking economy and in the economy banking as an occupational choice (about half as many entrepreneurs). Thus only the very rich entrepreneurs are able to run firms in the least efficient economies.

# 4. Further Research

OUR financial intermediation model could be used for a variety of purposes to be developed in future applications. First it offers a unified theoretical framework to test different capital market regulations and frictions. These regulations and frictions vary from country to country, and the model can shed some light into the extent that they slow down development as they inhibit the generation of financial intermediaries. The regulations and frictions we can think of are:

— Cost of intermediation: intermediation may be subject to fixed costs,
— Regulations of entry in capital markets: governments may require minimal assets (that is, to be a banker one must possess a level of assets larger than a minimal level $a$), minimum deposit-to-asset ratio $d(k; n)/a$, a minimum skills $z > \underline{z}$, etc.,
— Regulation of market share: this means imposing a maximum share of loans or deposits for one banker, $d + a - k < kL^s$,
— Regulation of interest rates: some countries put interest rate ceiling on lending to prevent usury or to guarantee certain deposit rate.

On the other hand, different regulations, frictions or parameters across different countries imply different distributions of capital across agents. This means that the same aggregate amount of capital may imply different levels of gross domestic product (GDP) because assets are in different hands with different entrepreneurial skills. Thus this model can also provide a theory of measured total factor productivity (TFP).

# Appendices

# Appendix 1. Transition Matrices

THE calibrated transition matrices are as follows. For workers, shocks are independent and identicaly distributed (i.i.d.)with the following probabilities

**TABLE A.1.1: Transition matrix for workers' skills: $\psi$ *(z)***

| $z$ | 0.50 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 |
|---|---|---|---|---|---|---|---|---|
| $\psi$ $(z)$ | 0.10 | 0.70 | 0.08 | 0.06 | 0.03 | 0.02 | 0.01 | 0.00 |

With respect to entrepreneurs, the evolution of their skills is governed by the following transition matrix

**TABLE A.1.2: Transition matrix for entrepreneurs' skills: $Q$ *(z', z)***

| | | $z'$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.00 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 |
| | 0.00 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1.00 | 0.109 | 0.698 | 0.192 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 1.25 | 0.059 | 0.095 | 0.713 | 0.131 | 0.002 | 0.000 | 0.000 | 0.000 |
| | 1.50 | 0.053 | 0.001 | 0.111 | 0.736 | 0.098 | 0.001 | 0.000 | 0.000 |
| $z$ | 1.75 | 0.044 | 0.004 | 0.015 | 0.107 | 0.755 | 0.074 | 0.001 | 0.000 |
| | 2.00 | 0.039 | 0.000 | 0.000 | 0.001 | 0.162 | 0.756 | 0.041 | 0.001 |
| | 2.25 | 0.025 | 0.000 | 0.000 | 0.000 | 0.008 | 0.172 | 0.758 | 0.037 |
| | 2.50 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.339 | 0.642 |

TABLE A.1.3: **Transition matrix for bankers' skills: *Q (z', z)***

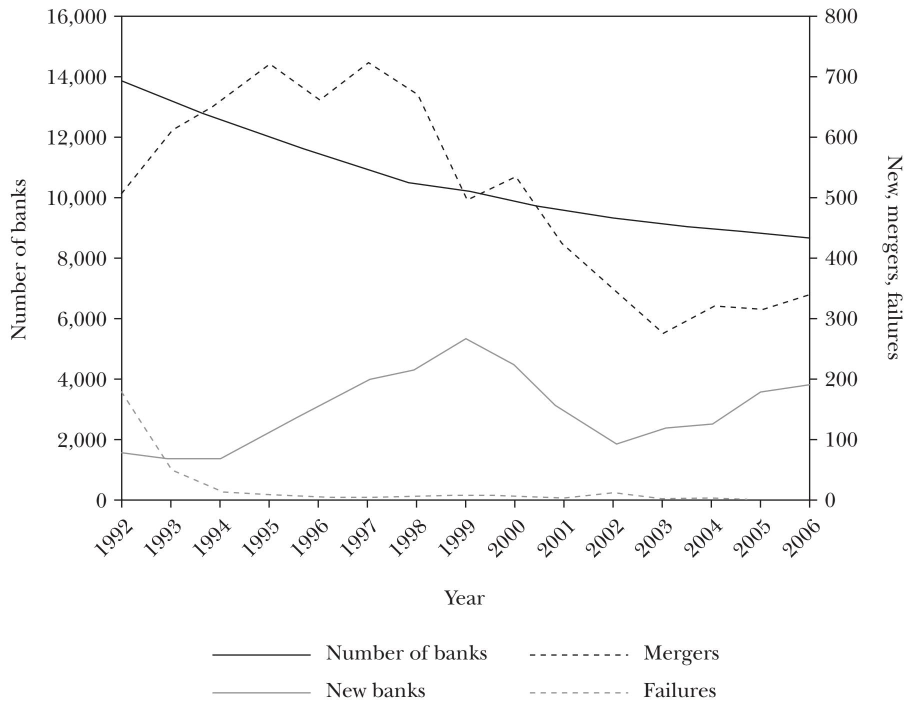
| | | z' | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.00 | 0.85 | 0.88 | 0.90 | 0.93 | 0.95 | 0.98 | 1.00 |
| z | 0.00 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.85 | 0.006 | 0.809 | 0.184 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.88 | 0.005 | 0.084 | 0.705 | 0.205 | 0.001 | 0.000 | 0.000 | 0.000 |
| | 0.90 | 0.005 | 0.042 | 0.144 | 0.499 | 0.308 | 0.002 | 0.000 | 0.000 |
| | 0.93 | 0.008 | 0.025 | 0.029 | 0.099 | 0.592 | 0.244 | 0.003 | 0.000 |
| | 0.95 | 0.018 | 0.004 | 0.005 | 0.015 | 0.143 | 0.558 | 0.256 | 0.001 |
| | 0.98 | 0.016 | 0.010 | 0.002 | 0.009 | 0.033 | 0.137 | 0.528 | 0.265 |
| | 1.00 | 0.006 | 0.006 | 0.001 | 0.002 | 0.010 | 0.040 | 0.228 | 0.707 |

The transition matrices for workers and entrepreneurs skills are specified so as to reproduce table A.1.1 in Evans (1987). Evans' study collected data on age and number of employees for 27,046 firms in the U.S. for 1976, 1978, 1980 and 1982. For each age-size category, Evans reported the number of surviving firms, the average annual growth rate of employment between 1976 and 1982 for these surviving firms, and the fraction of firms that survived between 1976 and 1982.

To generate the transition matrix for bankers skills, we followed a similar procedure using data for banks. The data are collected from the Federal Deposit Insurance Corporation (FDIC). For each bank, the database includes information on the age of the institution in a particular year, the sum of all assets owned by the institution, the sum of all deposits, and the number of full-time employees on the payroll of the bank and its subsidiaries.

There are several differences with respect to the sample in Evans. First he had data for 1976, 1978, 1980 and 1982. Our sample spans from 1992 until 2006 with an annual frequency. Observations were taken on the 31st of December of each year. Second Evans measured size by employment in the firm. We have three measures of size: employment, assets and deposits. To make the results consistent with the production sector, we measured size of each bank by employment. Third, unlike Evans, we control for mergers and acquisitions. In his study, Evans claimed mergers were not important in his sample. On the contrary, mergers seem to be an important phenomenon in the banking industry. Graph A.1.1 plots data on total number of banks, failures, mergers and new banks. As we can see, mergers account for the vast majority of exits. Also the total number of exits (failures plus mergers) is much larger than the number of entering banks. Thus the total number of banks has been significantly decreasing over time.

**GRAPH A.1.1: Number of banks, new banks, mergers and failures**




The way we control for mergers is as follows. We start at the end of the sample and work backwards creating virtual banks by adding the employment (as well as assets and deposits) of banks that merge later in the sample. With these computations, banks exit the sample only because they fail, and growth of the remaining banks is only the result of a rise in business.

Just to get an idea of the demographics, table A.1.4 gives the number of banks in each age-size category in 1992.

**TABLE A.1.4: Number of banks in each category in 1992**

| | Age | | | | | |
|---|---|---|---|---|---|---|
| **Employment (1992)** | **0-6** | **7-20** | **21-45** | **46-95** | **96 +** | **Total** |
| 1-19 | 190 | 267 | 194 | 1,748 | 453 | 2,852 |
| 20-49 | 162 | 298 | 249 | 1,218 | 475 | 2,402 |
| 50-99 | 40 | 140 | 146 | 483 | 229 | 1,038 |
| 100-249 | 21 | 73 | 94 | 234 | 148 | 570 |
| 250-499 | 16 | 21 | 28 | 62 | 42 | 169 |
| 500-999 | 9 | 13 | 12 | 34 | 17 | 85 |
| 1,000 + | 10 | 11 | 10 | 28 | 59 | 118 |
| Total | 448 | 823 | 733 | 3,807 | 1,423 | 7,234 |

Tables A.1.5 to A.1.7 reproduce table 2.1 in Evans (1987).

**TABLE A.1.5: Growth rate**

| | Age | | | | | |
|---|---|---|---|---|---|---|
| **Employment (1992)** | **0-6** | **7-20** | **21-45** | **46-95** | **96 +** | **Mean rate** |
| 1-19 | 14.06 | 7.01 | 5.14 | 4.57 | 2.86 | 5.85 |
| 20-49 | 9.33 | 6.64 | 3.50 | 3.02 | 3.29 | 4.18 |
| 50-99 | 7.50 | 4.86 | 3.63 | 4.33 | 3.05 | 4.16 |
| 100-249 | 3.77 | 6.36 | 3.10 | 3.81 | 2.51 | 3.75 |
| 250-499 | 1.58 | 4.86 | 5.06 | 2.38 | 1.67 | 3.04 |
| 500-999 | 3.74 | 7.20 | 1.67 | 0.69 | 1.81 | 2.73 |
| 1000 + | 3.67 | 4.02 | 2.34 | –1.72 | 1.80 | 1.15 |
| Mean rate | 4.71 | 5.17 | 3.19 | 0.30 | 1.89 | 1.91 |

**TABLE A.1.6: Exit rate**

| | Age | | | | | |
|---|---|---|---|---|---|---|
| **Employment (1992)** | **0-6** | **7-20** | **21-45** | **46-95** | **96 +** | **Mean rate** |
| 1-19 | 13.68 | 10.48 | 4.12 | 1.77 | 1.54 | 3.50 |
| 20-49 | 8.02 | 10.40 | 5.62 | 1.23 | 0.63 | 3.16 |
| 50-99 | 12.50 | 12.14 | 4.10 | 0.82 | 0.87 | 3.27 |
| 100-249 | 28.57 | 12.32 | 3.19 | 2.56 | 3.37 | 5.08 |
| 250-499 | 43.75 | 23.80 | 0.00 | 9.67 | 4.76 | 11.83 |
| 500-999 | 22.22 | 15.38 | 0.00 | 11.76 | 5.88 | 10.58 |
| 1,000 + | 10.00 | 0.00 | 10.00 | 3.57 | 1.69 | 3.38 |
| Mean rate | 13.39 | 11.17 | 4.36 | 1.75 | 1.47 | 3.76 |

**TABLE A.1.7: Number of survivors**

| | Age | | | | | |
|---|---|---|---|---|---|---|
| **Employment (1992)** | **0-6** | **7-20** | **21-45** | **46-95** | **96 +** | **Survivors** |
| 1-19 | 164 | 239 | 186 | 1,717 | 446 | 2,752 |
| 20-49 | 149 | 267 | 235 | 1,203 | 472 | 2,326 |
| 50-99 | 35 | 123 | 140 | 479 | 227 | 1,004 |
| 100-249 | 15 | 64 | 91 | 228 | 143 | 541 |
| 250-499 | 9 | 16 | 28 | 56 | 40 | 149 |
| 500-999 | 7 | 11 | 12 | 30 | 16 | 76 |
| 1,000 + | 9 | 11 | 9 | 27 | 58 | 114 |
| Survivors | 388 | 731 | 701 | 3,740 | 1,402 | 6,962 |

In these tables, the growth rate is the average annual growth rate of employment (denoted as $N_t$) in the age-size category between 1992 and 2006, that is,

$$\text{Growth rate} = 100 \times \frac{\ln\,(N_{2006}) - \ln\,(N_{1992})}{14}.$$

Furthermore the exit rate is the percentage of banks in the age-size category that were in the sample in 1992 but not in 2006. The percentage is with respect to the number of banks in 1992 in each age-size category. The number of survivors is the number of banks that were in the sample in both 1992 and 2006.

# Appendix 2. Data Sources

THE data used in this working paper is taken from the Flow of Fund Accounts (FFA) of the Federal Reserve System (FED), the National Income and Product Accounts (NIPA) as well as the Industry Accounts (IND) of the Bureau of Economic Analysis (BEA), the Current Employment Statistic survey (CES) of the Bureau of Labor Statistics (BLS), and the Historical Statistics on Banking (HSB) of the Federal Deposit Insurance Corporation (FDIC).

### A.2.1. Aggregate output and income

Our base measure of income is gross national product (GNP). However our model splits output between consumption and investment. Thus it does not contain a government sector, nor a household production sector, nor a foreign sector and neither an explicit treatment of inventories, so we have to make some adjustments to our measure of income in the data to adjust it to the model. Some of the adjustments imply computing the service flow of durables and government capital. In doing so, we follow Cooley and Prescott (1995) and Chen, Imrohoroglu and Imrohoroglu (2008) who show how to compute these services as well as the output elasticities with respect to capital and labor.

Also our measure of income needs to be decomposed between a nonfinancial and a financial sector. For that, we can use the Industry Accounts (IND) of the BEA. In particular, there is information about value added by industry. This value added does not sum up to GNP but to gross domestic product (GDP). We could use the fraction of the financial sector in GDP and apply that fraction to GNP. To complete the computation of income, the service flows from government capital, and durable goods (which are included in our measure of income) are considered part of nonfinancial output.

### A.2.2. Capital

For all the computations of service flows, it is important to have measures of capital, both for the total economy as well as for the nonfinancial and financial sectors. We will also need a value for capital in the nonfinancial entrepreneurial sector. We define total capital of the economy, $K$, as the sum of the fixed assets (both private and government), consumer durables, inventory stock, land, and net foreign assets. Fixed assets include structures, equipment and software. The data collected by the BEA does not include land, though. Land is included in the balance tables of the FFA. From FFA.B102, line 2, we get data on *tangible assets* of the Nonfarm Nonfinancial Corporate Business. We add this figure to line 2 of FFA.B103 which includes data on *tangible assets* of the Nonfarm Nonfinancial Noncorporate Business. As mentioned before, the FFA figures include estimations of land in the real estate. They also include durable goods when using data for households (FFA.B100).

The FFA tables do not include data on capital for financial firms nor on public capital. To get data on capital for financial firms and the government, we use the *fixed asset* tables of the BEA. The problem with this data is that it does not include the value of land, only of structures (FAT4.1 line 27 and FAT7.1 line 3) and equipment (FAT4.1 line 26 and FAT7.1 line 2). To get a figure for the value of land, we use data for the nonfinancial corporate sector. The difference between the data on real estate of the FFA (FFA.B102, line 3) and the value of structures of the *fixed asset* table of BEA (FAT4.1 line 30) must be the value of land for the nonfinancial corporate sector. Thus for the nonfinancial corporate sector, we compute the ratio of real estate from the FFA (which includes land) to structures from BEA. This ratio gives us how much we have to increase the BEA figure of structures to include the value of land. We apply this ratio to the structures of the financial sector and add it to the figure of equipment to get a value of real estate for financial firms and the government. Notice we do not include inventories in the financial sector or the government because the BEA assigns all inventories to the nonfinancial sector.

### A.2.3. Labor statistics

#### A.2.3.1. Amount of labor

First we can compute the fraction of total labor in financial sector. We have two sources of data. First there is the data from BEA as collected by NIPA. Tables 6.5 of the NIPA include *Full-Time Equivalent Employees by Industry*. We can get employment for total nonfarm private industries and for financial services. The fraction of labor in financial sector averages 0.041 between 1952 and 2006. On the other hand, tables 6.9 include *Hours Worked* *by Full-Time and Part-Time Employees by Industry*. Doing the same computation with this data implies that the fraction of labor in the financial sector averages 0.051 between 1952 and 2006.

The second source is the BLS where we can get data for employment and hours by industries. Thus we can construct total hours in nonfarm private industries and in financial firms. The problem is that the financial sector is aggregated with *real estate, rental and leasing*. Here hours in the financial sector average around 5.3 percent of total hours between 1952 and 2006. For the years 1999 until 2006, this data separates the financial sector from *real estate, rental and leasing*. For these years, 4.4% of total hours are used in the financial sector.

#### A.2.3.2. Managers

The BLS provides with estimates of employment by occupational categories and by industry. One of the occupations (management) can be interpreted as our entrepreneurs and bankers. There is data disaggregated at this level between 1999 and 2006.

### A.2.4. Assets

Data for assets is taken from Flow of Fund Accounts (FFA) of the Federal Reserve System (FED). We construct measures of long term assets that are used to finance capital and consumer durables in the U.S. economy. These can be measured by long term Treasury (table L209) and municipal (L211) securities, corporate bonds (L212), corporate equity (L213), mortgages (L217), consumer credit (L222) and proprietors' equity in noncorporate business (L227). The series in levels are not seasonally adjusted. We construct a series of seasonally-adjusted levels by starting at the level of the first observation (first quarter of 1952) and adding seasonally adjusted flows.

# Appendix 3. Solution

## A.3.1. Exogenous and endogenous banking

The general program is

$$v(a, z) = \max \Big\{ \int v^W (a, z' \psi) (dz'), \max_{k, n} \int v^E (a, z') Q(z, dz'), \max_{k, n} \int v^B (a, z') \mu (z, dz') \Big\}, \tag{A.3.1}$$

$$v^i (a, z') = \max_{c, a'} \{ u(c) + \beta v(a', z') \}, \tag{A.3.2}$$

with $i = W, E, B$. The workers face the budget constraint

$$c + a' \leq (1 - \delta) \; a + r^D a + wz'). \tag{A.3.3}$$

The entrepreneurs face the budget constraint

$$c + a' \leq (1 - \delta) \; a + z' (k^{\alpha^E} n^{1 - \alpha^E})^{\theta^E} - wn + r^D \max \{0, a - k\} - - r^L \max \{0, k - a\}, \tag{A.3.4}$$

and the financing constraint

$$(1 - \delta) \; a + z' (k^{\alpha^E} n^{1 - \alpha^E})^{\theta^E} + r^D \max \{0, a - k\} \geq r^L \max \{0, k - a\} + wn.$$

The bankers face the budget constraint

$$c + a' \leq (1 - \delta) \; a + z' r^L d - r^D d - wn, \tag{A.3.5}$$

with

$$d(a, z) = A^B (k^{\alpha^E} n^{1 - \alpha^B})^{\theta^B},$$

and the capital constraint

$$k \leq a.$$

In the exogenous banking there are no bankers and a single interest rate $r = r^L = r^D$. Otherwise the two cases are identical.

Let $\eta$ and $\xi$ be the multipliers associated with the budget constraint and the capital constraint, respectively. The first order conditions (FOCs) with respect to consumption and assets are the same for all three types of agents. These FOCs are

$$u'(c) = \eta,$$

and

$$\eta = \frac{\beta\, \partial v\,(a', z')}{\partial a'}.$$

The choice of inputs for the entrepreneurs is governed by the following FOCs. The FOC with respect to capital is [3]

$$\theta\alpha k^{\alpha\theta - 1}\; n^{(1-\alpha)\theta} \int \eta z' Q\,(z, dz') = r^E \int \eta Q\,(z, dz'), \qquad \text{(A.3.6)}$$

where $r^E$ is the interest rate faced by the agent. This interest rate is $r^D$ for depositing entrepreneurs and $r^L$ for borrowing entrepreneurs for the endogenous banking case and $r$ for the exogenous banking case. The FOC and with respect to labor is

$$\theta\,(1 - \alpha)\; k^{\alpha\theta} n^{(1-\alpha)\theta - 1} \int \eta z' Q\,(z, dz') = w \int \eta Q\,(z, dz'). \qquad \text{(A.3.7)}$$

Dividing (A.3.6) by (A.3.7) we obtain the capital-to-labor ratio

$$k \equiv \frac{k}{n} = \left(\frac{\alpha}{1 - \alpha}\right) \frac{w}{r},$$

which is independent on whether the entrepreneur is constrained or not. For a financially-unconstrained entrepreneur, the scale of production is obtained substituting back the optimal capital-to-labor ratio in either (A.3.6) or (A.3.7). This produces

$$n^u = \left[\theta \left(\frac{\alpha}{r^E}\right)^{\alpha\theta} \left(\frac{1-\alpha}{w}\right)^{1-\alpha\theta} \frac{\int \eta z' Q\,(z, dz')}{\int \eta Q\,(z, dz')}\right]^{\frac{1}{1-\theta}}.$$

For any entrepreneur the profit function is

3. Superindeces in parameters $\alpha$ and $\theta$ are supressed to simplify expressions.

$$\begin{aligned}\pi^E (a, z' \mid z) &= z' (k^{\alpha} n^{1-\alpha})^{\theta} - wn - r^E k + r^E a \\ &= z' k^{\alpha\theta} n^{\theta} - wn - r^E kn + r^E a \\ &= z' \left(\frac{\alpha}{1-\alpha}\right)^{\alpha\theta} \left(\frac{w}{r^E}\right)^{\alpha\theta} n^{\theta} - wn - r^E \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{w}{r^E}\right) n + r^E a \\ &= z' \left(\frac{\alpha}{1-\alpha}\right)^{\alpha\theta} \left(\frac{w}{r^E}\right)^{\alpha\theta} n^{\theta} - \frac{wn}{1-\alpha} + r^E a,\end{aligned}$$

so that the financing constraint becomes

$$(1 - \delta + r^E)\ a + \underline{z} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha\theta} \left(\frac{w}{r^E}\right)^{\alpha\theta} n^{\theta} \geq \frac{w}{1-\alpha} n.$$

This equation evaluated at equality defines the maximum amount of labor for financially constrained entrepreneurs, call it $n^c_{\max}$, conditioned on asset level $a$. This number exists and is positive because of the concavity of the production function. Then the constrained amount of labor has to satisfy $n^c < n^c_{\max}$. For the special case $\underline{z} = 0$, the constraint amount of labor must satisfy

$$n^c < \frac{(1 - \delta + r^E)\ (1 - \alpha)}{w} a \equiv n^c_{\max}\,.$$

The choice of inputs for the bankers is governed by the following FOCs: with respect to capital

$$\theta A \alpha k^{\alpha\theta - 1} n^{(1-\alpha)\theta} \int \eta\ (r^L z' - r^D) \mu\ (z, dz') = \xi, \qquad \text{(A.3.8)}$$

and with respect to labor

$$\theta A\ (1 - \alpha)\ k^{\alpha\theta} n^{(1-\alpha)\theta - 1} \int \eta\ (r^L z' - r^D) \mu\ (z, dz') = w \int \eta\mu\ (z, dz'). \qquad \text{(A.3.9)}$$

Now assume there is a financial intermediary that does not use all its assets as capital, so that $k < a$ and $\xi = 0$. Then (A.3.8) becomes

$$r^L \int \eta z' \mu\ (z, dz') = r^D \int \eta\mu\ (z, dz'),$$

which means the demand of capital by that financial intermediary becomes perfectly elastic at that level of interest rates. However substituting this expression in (A.3.9) we obtain

$$w \int \eta\mu\ (z, dz') = 0,$$

which is not possible because $\eta > 0$ for all $z'$. This means banks use all their assets as capital, so that $k = a$ and $\xi > 0$.

For a financially-unconstrained banker, the scale of production is obtained substituting back the optimal capital-to-labor ratio in either (A.3.8) or (A.3.9). This produces

$$n^u = \left[\frac{A\theta\,(1-\alpha)\,a^{\alpha\theta}\int \eta z'\mu\;(z,\,dz')}{w\int \eta\mu\;(z,\,dz')}\right]^{\frac{1}{1-(1-\alpha)\theta}}.$$

The financing constraint for bankers becomes (assuming $\underline{z} = 0$)

$$(1-\delta)\;a - r^D A\,(a^{\alpha}\,n^{1-\alpha})^{\theta} - wn > 0.$$

This equation evaluated at equality defines the maximum amount of labor for financially-constrained bankers, call it $n^c_{\max}$, conditioned on asset level $a$. This number exists and is positive because of the concavity of the production function. Then the constrained amount of labor has to satisfy $n^c < n^c_{\max}$.

### A.3.2. Autarky

The general program is

$$v\,(a,\,z) = \max\,\{\int v^W(a,\,z')\,\psi\;(dz');\;\max_{k,\,n}\int v^E\,(a,\,z')\;Q\,(z,\,dz')\},\quad \text{(A.3.10)}$$

$$v^i\,(a,\,z') = \max_{c,\,a'}\{u\,(c) + \beta v\,(a',\,z')\},\qquad \text{(A.3.11)}$$

with $i = W, E$. The workers face the budget constraint

$$c + a' \leq (1-\delta)\;a + wz'.\qquad \text{(A.3.12)}$$

The entrepreneurs face the budget constraint

$$c + a' \leq (1-\delta)\;a + z'\,(k^{\alpha^E}\,n^{1-\alpha^E})^{\theta^E} - wn,\qquad \text{(A.3.13)}$$

the financing constraint

$$(1-\delta)\;a + z'\,(k^{\alpha^E}\,n^{1-\alpha^E})^{\theta^E} - wn \geq 0,$$

and the capital constraint

$$k \leq a.$$

There are no bankers in this economy.

Let $\eta$ and $\xi$ be the multipliers associated with the budget constraint and the capital constraint, respectively. The FOCs with respect to consumption and assets are the same for all types of agents. These FOCs are

$$u'(c) = n,$$

and

$$\eta = \beta \frac{\partial v(a', z')}{\partial a'}.$$

The choice of inputs for the entrepreneurs is governed by the following FOCs. The FOC with respect to capital is

$$\theta \alpha k^{\alpha\theta - 1} \; n^{(1-\alpha)\theta} \int \eta z' Q(z, dz') = \xi. \tag{A.3.14}$$

The FOC with respect to labor is

$$\theta(1-\alpha) \; k^{\alpha\theta} n^{(1-\alpha)\theta - 1} \int \eta z' Q(z, dz') = w \int \eta Q(z, dz'). \tag{A.3.15}$$

Now assume there is an entrepreneur that does not use all his assets as capital, i.e., $k < a$ and $\xi = 0$. Then (A.3.14) becomes

$$\int \eta z' Q(z, dz') = 0,$$

which is not possible because $\eta > 0$ for all $z'$. This means entrepreneurs in the autarkic case use all their assets as capital, so that $k = a$ and $\xi > 0$. For a financially-unconstrained entrepreneur, the scale of production is obtained substituting back $k = a$ in (A.3.9). This produces

$$n^u = \left[\frac{\theta(1-\alpha) a^{\alpha\theta} \int \eta z' \mu(z, dz')}{w \int \eta \mu(z, dz')}\right]^{\frac{1}{1-(1-\alpha)\theta}}.$$

On the other hand, the financing constraint is (assuming $\underline{z} = 0$)

$$(1-\delta) \; a \geq wn.$$

This equation evaluated at equality defines the maximum amount of labor for financially constrained entrepreneurs, call it $n^c_{\max}$, conditioned on asset level $a$. Thus the constrained amount of labor must satisfy

$$\eta^c < \frac{(1-\alpha)\ a}{\mathrm{w}} \equiv n^c_{\max}.$$

### A.3.3. Lenders

The general program is

$$v\,(a, z) = \max\left\{\int v^W (a, z')\,\psi\,(dz'),\ \max_{k,\,n}\int v^E (a, z')\,Q\,(z, dz'),\ \max_{l}\int v^B (a, z')\,\mu\,(z, dz')\right\}, \tag{A.3.16}$$

$$v^i\,(a, z') = \max_{c,\,a'}\left\{u\,(c) + \beta v\,(a', z')\right\}, \tag{A.3.17}$$

with $i = W, E, B$. The workers face the budget constraint

$$c + a' \leq (1-\delta)\ a + wz', \tag{A.3.18}$$

so it is like the autarkic case. The entrepreneurs face the budget constraint

$$c + a' \leq (1-\delta)\ a + z'\,(k^{\alpha^E} n^{1-\alpha^E})^{\theta^E} - wn - r^L \max\{0, k-a\}, \tag{A.3.19}$$

and the financing constraint

$$(1-\delta)\ a + z'\,(k^{\alpha^E} n^{1-\alpha^E})^{\theta^E} - wn - r^L \max\{0, k-a\} \geq 0.$$

The lenders face the budget constraint

$$c + a' \leq (1-\delta)\ a + z'\,r^L l, \tag{A.3.20}$$

the financing constraint

$$(1-\delta)\ a + z'\,r^L l \geq 0,$$

and the capital constraint

$$l \leq a.$$

Let $\eta$ and $\xi$ be the multipliers associated with the budget constraint and the capital constraint, respectively. The FOCs with respect to consumption and assets are the same for all three types of agents. These FOCs are

$$u'(c) = \eta,$$

and

$$\eta = \frac{\beta \, \partial v(a', z')}{\partial a'}.$$

The choice of inputs for the entrepreneurs is governed by the following FOCs. The FOC with respect to capital is

$$\theta \alpha k^{\alpha\theta - 1} \, n^{(1-\alpha)\theta} \int \eta z' Q(z, dz') = r^L I(k > a) \int nQ(z, dz'), \qquad \text{(A.3.21)}$$

where $I(k > a)$ is an indicator function taking value 1 if the statement in parenthesis is true. The FOC and with respect to labor is

$$\theta(1-\alpha) \, k^{\alpha\theta} n^{(1-\alpha)\theta - 1} \int \eta z' Q(z, dz') = w \int \eta Q(z, dz'). \qquad \text{(A.3.22)}$$

If $k > a$, dividing (A.3.21) by (A.3.22) we obtain the capital-to-labor ratio

$$k \equiv \frac{k}{n} = \left(\frac{\alpha}{1-\alpha}\right) \frac{w}{r^L}.$$

For a financially unconstrained entrepreneur, the scale of production is obtained substituting back the optimal capital-to-labor ratio in either (A.3.21) or (A.3.22). This produces

$$n^u = \left[\theta \left(\frac{\alpha}{r^L}\right)^{\alpha\theta} \left(\frac{1-\alpha}{w}\right)^{1-\alpha\theta} \frac{\int \eta z' Q(z, dz')}{\int \eta Q(z, dz')}\right]^{\frac{1}{1-\theta}}.$$

Thus this agent behaves as the borrowing entrepreneur in the endogenous banking case. However if entrepreneurs do not borrow, they use up all their capital so that $k = a$. Then labor is determined as in the autarkic case.

The choice of inputs for the lenders is governed by the following FOCs:
with respect to lending funds

$$r^L \int \eta z' \mu(z, dz') = \xi.$$

Again lenders lend all their assets, so that $l = a$ and $\xi > 0$.

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