Following early ideas based on exotic p-wave superconductors, it has been recently predicted that Majorana quasiparticles should appear in topological insulators and semiconductors with strong spin-orbit (SO) coupling. In proximity to s-wave superconductors, these systems behave as topological superconductors (TSS) when the excitation gap is closed and reopened again: as the gap crosses zero, Majorana bound states (MBSs) appear wherever the system gap is closed and reopened again: as the gap crosses zero, Majorana bound states (MBSs) appear wherever the system interfaces with a nontopological insulator (see Refs. 9 and 10 for reviews).

The TS transition occurs when an external Zeeman field $B$ exceeds a critical value $B_c = \sqrt{\mu^2 + \Delta^2}$ defined in terms of the Fermi energy $\mu$ and the induced s-wave pairing $\Delta$. This prediction has spurred a great deal of experimental activity towards detecting MBSs in hybrid superconductor-semiconductor systems. Indeed, signatures of Majorana detection have been recently reported in Ref. 11. We find that for this class of devices, a number of distinct transport regimes arise as the various sections of the wire transition to different electronic phases. We characterize these regimes and the rich phenomenology that results beyond the simplest picture. In particular, we address the question of whether the ZBAs are related to Majorana physics, and confirm that this is indeed the case for long depletion regions. We also analyze the development of ABSs close to zero energy when the pinch-off gate lies at a finite distance from the NS junction. Our main results are summarized in Fig. 4(e) where we demonstrate that the $dI/dV$ of realistic junctions with inhomogeneous depletion and multisubband filling may not show a distinct closing of the gap and yet exhibit ZBAs of Majorana origin. In most cases, these ZBAs show a residual splitting and may coexist with ABSs, features also observed in Ref. 11.

Model. We first consider a one-dimensional NS junction [see Fig. 1(a)], with a BCS-type Hamiltonian $H = H_0 + H_{\text{pairing}}$, where

$$H_0 = \int dx \psi^\dagger(x) \left[ -\frac{\partial^2}{2m} + i\alpha \sigma_\tau \partial_x + B \sigma_z + U(x) - \mu \right] \psi(x)$$

and

$$H_{\text{pairing}} = \int dx \psi^\dagger(x) \Delta(x) \sigma_z \psi^\dagger(x) + \text{H.c.}$$

Here $\alpha$ is the SO coupling and $B$ is the Zeeman splitting (given by $B = g\mu_B B/2$, where $B$ is an in-plane magnetic field, $\mu_B$ is the Bohr magneton, and $g$ is the nanowire $g$ factor). We assume a position-dependent pairing $\Delta(x)$ induced by the superconducting electrode such that $\Delta(x \to \infty) = \Delta$ and $\Delta(x \to -\infty) = 0$. The term $U(x) = U_d(x) + U_p(x)$ is composed of two parts: $U_p(x)$ comes from the pinch-off gate $V_p$ in the normal region at a distance $L_N$ from the NS interface, and $U_d(x)$ models the potential induced by the depletion gate $V_d$. Gate $V_d$ may extend all the way into the normal side of the NS interface [case $N^dS^dS$, with a depleted length $L_d = L_{NS} + L_{S'}$, Fig. 1(a)], or be limited to the end of the superconducting side [case $N^dS^dS$, $L_d = L_{S'}$, Fig. 1(b)]. We will consider the former case first, where we cover different parametric regimes, and then turn to the second one, which is closer to the experimental setup. Realistic experimental parameters are $\Delta = 250\mu eV$ is the induced gap that, for an InSb effective mass $m = 0.015m_e$, corresponds to...
a length scale $L_\Delta \equiv \hbar / \sqrt{m \Delta} \approx 142 \text{ nm}$. Strong SO coupling, representative of InSb wires, is $\alpha = 20 \text{ meV nm}$, with SO length $L_{SO} = \hbar^2 / (m \alpha) = 200 \text{ nm} = 1.4 L_\Delta$.

Scales. A localized MBS is formed at the boundary of a superconducting portion of the wire. At a point $x$ the wire will be in the TS phase if $\Delta(x) > 0$ and

$$B > \sqrt{[\mu - U(x)]^2 + \Delta(x)^2}. \quad (1)$$

The asymptotic value of the critical field is the proper (bulk) critical field $B_c$. Apart from $B_c$, several other Zeeman scales dictate the junction’s transport properties. The first one is the TS critical field in the depleted part of the superconducting wire, $B_c^d \equiv \sqrt{(\mu - U_d)^2 + \Delta^2}$, which is smaller than $B_c$, as is the purpose of the depletion gate. It should be noted, however, that the depleted $S^d$ region has a finite length, which crucially affects Majorana modes for $B_c^d < B < B_c$, as discussed later, while the $S$ portion is assumed infinite. Second, there is the field above which the normal side of the wire becomes a length scale $L_{Nd} \equiv \sqrt{\mu - U_d}$.

**FIG. 1.** (Color online) Schematics of the nanowire junction in the $N^d S^d S$ (a) and $N S^d S$ (b) setups, and spatial variation of superconducting gap and potential profiles (c). Gate $V_p$ depletes the wire, while $V_d$ creates a tunnel contact ($I$) to the left (normal) reservoir. One (red) or two (red and yellow spheres) Majorana bound states may appear at the edges of the depleted region depending on the Zeeman field and gate voltage $V_d$. (d) Transport regimes for a transparent $NS$ junction ($V_{d,p} = 0, \mu = 4 \Delta$) in the Zeeman-field–bias plane.

**FIG. 2.** (Color online) Density plots of the $dI/dV$ in the $N^d S^d S$ junction ($\mu = 4 \Delta, U_d = 3.25 \Delta, U_p = 25 \Delta, S = 0$) for $L_{SO} = 1.4 L_\Delta$ as a function of bias voltage $V$ and Zeeman field $B$ with a tunnel pinch-off barrier and a depletion region of length $L_{Nd} + L_{S^d}$, Fig. 1(a). Different columns feature increasing values of $L_{S^d}$ from left to right, whereas different rows feature increasing length $L_{Nd}$ from top to bottom.

Here, $N$ is the number of propagating channels in the normal side at energy $\epsilon = V$, and $r_{ee}$ and $r_{eh}$ are their normal and Andreev reflection matrices. These matrices can be computed in a number of ways. The most flexible is the recursive Nambu Green’s function approach, employed here (for full details, see Ref. 23).

Before considering the effect of $U(x)$, we show the transport phase diagram [see Fig. 1(c)] in the simple $NS$ transparent limit, i.e., in a regime where the concepts of MBSs and ZBAs no longer hold. We observe different transport regions in the $B$-$V$ plane characterized by an integer $dI/dV \approx n e^2 / h$, with $n = 0, 1, 2, 3, 4$. Such is the case of Cooper pair transport (region I, $n = 4$) or single quasiparticle transport (region III, $n = 2$). The latter is a TS regime, whose topology becomes evident in the $dI/dV$ despite the fact that the associated Majorana fermion is completely smeared out due to the gapless spectrum for $x < 0.24$–27. Between these two regions, the helical regime is characterized by a fully suppressed zero-bias conductance (region II, $n = 0$). These results extend the concept of half-integer conductance quantization beyond linear response.

We now consider the $N^d S^d S$ junction with the full $U(x)$ response of the $dI/dV$ (with $L_{SO} = 1.4 L_\Delta$) is plotted in Fig. 2. Different panels cover different ratios $L_{Nd} / L_\Delta$ and $L_{S^d} / L_\Delta$. The tunnel barrier $U_p$ is tuned in each case to yield spectroscopic resolution in the transport response. A wide range of behaviors becomes apparent, which reflects the local density of states (DOS) at the pinch-off gate. The most paradigmatic one is probably the one in the top-left panel. It reflects the closing of the effective superconducting gap (marked by the gap-edge...
conductance peaks) as $B$ increases. The gap-edge DOS peak transforms into a Majorana mode at zero energy for $B > B_c$ [pictured in Fig. 1(a) as a red sphere]. The gap reopens in the TS phase (see solid blue line in Fig. 3), but the local DOS at the contact is no longer peaked because the spectral weight is transferred to the Majorana mode. Hence, a prototypical three-pronged structure arises in the $B$-$V$ plane. However, the relevant phenomenology is by no means exhausted by this. Different scenarios arise at large $L_{N'}$ (Fig. 2, bottom row), with the development of ABSs, or large $L_{S'}$ (right column), with the development of ZBAs below $B_c$. Further phenomenology is obtained by varying $L_{SO}$.\textsuperscript{23}

**Short $N^dS^dS^d$ junction.** We now analyze in more detail the short junction case, $L_{N'}, L_{S'} \ll L_S$ (top-left panel in Fig. 2). The scale $B^d$ has little meaning in this case, since any pair of Majorana modes forming at the ends of a short TS wire will strongly hybridize into two conventional states with energies $\sim \pm \Delta$. The effect of increasing $L_{SO}$ is to flatten the gap-edge $dI/dV$ peak (bright yellow) to lower energies, as shown in Fig. 3(a) ($L_{SO} = 5L_S$). At these large $L_{SO}$, spin becomes a good quantum number and the Zeeman field splits the particle and hole bands of the $S$ region into spin-up and spin-down subbands (white arrows in Fig. 3). Particle and hole gap edges anticross at zero energy for $B > B_A$ (green dot), resulting in a minigap (which vanishes for $L_{SO} \rightarrow \infty$) with almost flat edges near zero. This structure appears as a (split) ZBA below $B_c$, that is unrelated to Majorana mode formation.\textsuperscript{28}

**Long $N^dS^dS^d$ junction.** Increasing the length $L_{Nd}$ of the depleted normal wire gives rise to ABSs in the $N^d$ region that are probed by the tunnel barrier $U_p$ (see bottom row of Fig. 2). Increasing $L_{SO}$ we find these ABS resonances approaching a degenerate zero energy crossing, see Fig. 3(b). These low-energy ABSs, however, cease after the helical transition at $B^h_d$ (red dot). While the ABSs move up and down in energy for $B < B^h_d$ depending on their spin character, as soon as the $N^d$ region becomes helical, all ABSs disperse away from zero energy with increasing $B$, since incoming states do not have spin partners anymore. Beyond the second threshold $B^d$ (vertical dashed white line), the local band gap closes in the $S^d$ region (dashed blue line) and a ZBA builds up, which is caused by Majorana fermion pairs forming in the $S^d$ region of length $L_{S'}$, also assumed large. The finite $L_{S'}$, however, produces a residual splitting of the two Majoranas, visible in the ZBA that oscillates with $B$ (Refs. 29–32) as long as $B < B_c$. Above the bulk $B_c$, the rightmost Majorana escapes to $x \rightarrow \infty$ (see Fig. 1), where it no longer overlaps with the one at the $N^dS^d$ interface, and the splitting vanishes [see inset of Fig. 3(b)].

**$N^dS^dS$ junction.** We now turn to the type of setup explored in Ref. 11, the $N^dS^dS$ case. The crucial difference with the $N^dS^dS$ setup is that the ABSs zero-energy anticrossings may coexist with Majorana ZBAs in the $S^d$ region, since the $N$ region becomes helical after the $S^d$ region becomes topological. When that happens, the zero-energy ABSs repel the MBS wave function (usually decolocalized into the $N$ region) back into the $S^d$ region, hence decoupling it from the lead. As a result, the ZBA, as measured by the $dI/dV$, is suppressed (see arrows in Fig. 4, where $L_{SO} = 1.4L_D$). In a single-mode wire (Fig. 4, top row), this may happen only for $B < B_d < B_c$, since it requires an interface between a nonhelical $N$ and a topological $S^d$.\textsuperscript{33} Interestingly, the above behavior agrees with the experimental observation of an intermittent ZBA that disappears and reappears as an ABS anticrosses at zero energy. Constant-$B$ sweeps of $U_p$ are also found to closely correlate with the experiment.\textsuperscript{23}

A further feature apparent in the experiments is the absence of gap-edge singularities closing just before the formation of the ZBA. Up till now, all potential profiles have been...
assumed spatially abrupt [decay length \( \delta \approx 0 \) in all profiles of Fig. 1(c)]. Imperfect screening, however, will lead to gate-induced potentials that decay slowly along the wire, specially at low electron densities. When a smooth \( U_p(x) \) profile is taken into account, the gap-edge peaks are quickly washed out, and the \( dI/dV \) in the tunneling regime is no longer a perfect measure of the local density of states.\(^{35}\) Unlike for sharp pinch-off barriers, transmission through smooth barriers mostly preserves longitudinal momentum. Since moreover barrier transmission is lower for smaller momenta, the gap-edge resonance closing around zero momentum at \( B_d \) is poorly probed by a smooth barrier. Visibility is restored as \( B \) increases beyond \( B_d \), since then the Zeeman splitting increases the momentum components of all resonances, including the ZBA. Figure 4(b) shows this effect. Note on the other hand that the gap-edge signal at higher energies \( \sim \Delta \) is roughly constant in \( B \). This corresponds to the large-momentum band edge, not the true gap edge at small momentum. The former band edge shows no sign of the different transitions (the TS transition at \( B_t \), in particular), and never closes. Therefore, all zero-energy structure appears disconnected from any closing of the gap, as measured by the \( dI/dV \).

Transport features connected to large momenta should be \textit{enhanced} in multisubband systems. The pinch-off condition for tunneling spectroscopy requires a higher \( U_p \) barrier in this case to shut off the additional open modes, which will necessarily contribute with a stronger signal relative to the shallower, lower momentum mode. This is likely the case, \( \sim 1 \). From this we find that the ZBA splitting is large.

Conclusions. We have identified various transport regimes in depleted \( NS \) nanowire junctions with SO coupling and Zeeman field. Depending on the Zeeman field, the wire as a whole may be in any given combination of helical/nonhelical and trivial/topological phases for its normal and superconducting portions, respectively. These include helical depletion \((B_r^1 < B < B_c^1)\) and topological depletion \((B_r^2 < B < B_c^2)\) phases that arise at fields below those of the proper helical bulk \((B_r < B < B_c)\) and topological bulk \((B > B_c)\) regimes. The different phases have distinct subgap signatures in transport, particularly if the depleted \( S^d \) region is long enough. In this case, ZBAs appear that are caused by the formation of either a single (in the topological bulk phase) or a pair (topological depletion phase) of Majorana modes in the junction. The latter is characterized by a residual ZBA splitting. In the case of short junctions, Majoranas cannot develop below \( B_c \), although one may still distinguish between the conventional and minigap transport regimes (unrelated to Majorana physics) if \( L_{SO} \) is large.

Apart from this general analysis, we have discussed a configuration similar to the one in the experiment of Ref. 11. We argue that at least a number of nontrivial features observed in this experiment are consistent with most of our results corresponding to transport through a multimode nonhelical normal/topological depleted superconductor/trivial bulk superconductor junction, hosting Majorana fermion pairs within the central region [Figs. 4(d) and 4(e)]. While a qualitative correspondence can be traced between our observations and the experiment, a more quantitative agreement is beyond the goal of our study. This is in part due to a considerable number of unknown parameters (precise \( L_{SO} \), screened gate potential profiles, localization and pair-breaking effects in the bulk superconductor,\(^{35}\) etc.). Moreover, other features that we find, such as the ZBA splitting oscillations in \( B \), have not been observed. Thus, a different physical origin of the measured ZBAs (reflectionless tunneling,\(^{56}\) Kondo, etc.) cannot be completely excluded. The latter, however, should be accompanied (at least in its most conventional form) with even-odd effects as a function of gate voltages,\(^{37}\) which are not apparent in Ref. 11. Disorder (not considered here) might also be of relevance. However, its precise role on Majorana physics is currently under active investigation.\(^{38–40}\)

Note added in proof. During the review process of our paper, two manuscripts\(^{41,42}\) appeared addressing similar questions.

Acknowledgments. We are grateful to S. Frolov and L. Kouwenhoven for fruitful discussions. We acknowledge the support of the CSIC JAE-Doc program and the Spanish Ministry of Science and Innovation through Grants No. FIS2008-00124/FIS (P.S.-J) and No. FIS2009-08744 (E.P. and R.A.). This research was supported in part by the National Science Foundation under Grant No. NSF PHY05-51164.
19. This gate has turned out to be crucial for the observation of ZBAs in the Delft experiment.
21. These parameters, while probably being a good estimation, are not directly measured in the experiment but rather inferred from different samples in a different geometry. Thus significant changes in $L_{SO}$ cannot be excluded.
23. See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.86.180503 for a description of the recursive Green’s function algorithm for computing transport, including finite width and finite temperature. Also included is an extensive parametric study of differential conductance for varying geometric parameters and spin-orbit lengths, as well as differential conductance maps as a function of bias and depletion gate voltage.
28. Note that this structure of the $dI/dV$ remains essentially unchanged in the absence of a depleted region.
33. Therefore, for single-mode wires, ABSs cannot coexist with the ZBA near zero energy for $B > B_c$. Although this is possible in multimode wires, the ZBA suppression does not occur in such cases.
34. This is in stark contrast with local probes such as the tunneling current from a metallic tip into the end of the nanowire, see e.g. Ref. 18. Note moreover that a smooth $\Delta(x)$ leaves the $dI/dV$ largely unaffected, which allows us to bypass a self-consistent calculation of the pairing profile.