Angle-Dependent Ultrasonic Transmission through Plates with Subwavelength Hole Arrays

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We study the angle and frequency dependence of sound transmission through water-immersed perforated aluminum plates. Three types of resonances are found to govern the acoustic properties of the plates: lattice resonances in periodic arrays, Fabry-Perot modes of the hole cavities, and elastic Lamb modes. The last two of them are still present in random arrays and have no parallel in optical transmission through holes. These modes are identified by comparing experiment with various levels of theoretical analysis, including full solution of the elasto-acoustic wave equations. We observe strong mixture of different transmission mechanisms, giving rise to unique acoustic behavior and opening new perspectives for exotic wave phenomena.

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The similarities and discrepancies between mechanical waves, such as sound, and electromagnetic waves have puzzled scientists for a long time [1]. This is much more than a semantic question. It deals with the essential nature of matter, because sound is basically a classical-physics phenomenon, while light is deeply rooted into the quantum-mechanical description of reality. Light and sound have been confronted in numerous experiments and theories, ranging from band gap effects in photonic [2] and phononic [3] crystals to negative refraction [4] and invisibility [5] phenomena.

The discovery of extraordinary optical transmission in metallic membranes perforated by subwavelength apertures [6] has raised again the question of the similarity between sound and light in the context of wave transmission. Very recently, several groups have reported on the transmission properties of sound through plates with slits [7,8] and holes [9–11]. Some groups [7–9] claim that extraordinary transmission of sound is similar to its optical counterpart in corrugated metal films. However, Hou et al. [10] have shown that, despite some similarities, intrinsic differences separate light and sound. Holes in membranes cannot sustain optical modes in the subwavelength regime. This is not the case for sound. Moreover, we have recently shown that sound has specific properties unforeseen from the perspective of optical transmission in metallic films [11]. First, extraordinary acoustic transmission is not solely determined by hole periodicity, since it is also observed in plates pierced by a random distribution of holes. More importantly, periodically perforated plates are capable of shielding sound in a wide region near the onset of diffraction much better than what is predicted by the well-known mass law [11]. This behavior, with no parallel in optics, should have important applications in soundproofing engineering. However, a detailed study is still missing regarding the role of the wave vector parallel to the plate in sound transmission for oblique incidence in both periodic and random distributions of holes.

In this Letter, we investigate the angular dependence of sound transmission through water-immersed aluminum plates perforated by subwavelength apertures. Our results reinforce previous findings for normal-incidence transmission [11] but otherwise show a remarkable interplay between Lamb waves and lattice modes induced by periodic distributions of holes, which is unveiled only under oblique incidence. A rich structure is observed in the measured angle and frequency distribution of transmission intensity, which is correctly described by numerical solution of the full elasto-acoustic wave equations. Partial understanding of the observed features comes from comparison of the full theory with (i) a description of the plate as a hard solid, leading to lattice resonances and the resulting extraordinary transmission features, and (ii) a real-solid homogeneous-plate description yielding the Lamb modes of the plate. However, both experiment and full real-solid theory for perforated plates reveal complex hybridization of different resonance mechanisms, leading to numerous mode anticrossings, enhancement of some of the lattice resonances, and a significant shift of the Lamb modes.

Our experimental setup is based on the well-known ultrasonic immersion transmission technique [11]. The angle of incidence θ is varied by rotating the sample plate from 0° to 60° from normal incidence in steps of 1°. The plates used in this work are made of aluminum (density ρ = 2.7 g/cm³, longitudinal wave velocity c_1 = 6500 m/s, and transversal wave velocity c_2 = 3130 m/s [12]), mechanically drilled (holes of diameter d = 3 mm, periodically distributed in a squared array of period a = 5 mm), and immersed in water. The measured transmission spectra are collected as a function of frequency ω and wave...
vector $k_\parallel$ parallel to the $\Gamma X$ direction of the plate [see Fig. 1(a)]. The frequency, angle, and parallel wave vector are related through $k_\parallel = k \sin \theta$, where $k = \omega/c$ and $c = 1480 \text{ m/s}$ is the speed of sound in water.

Figures 1(b)–1(d) show transmission intensity maps as a function of normalized parallel wave vector $k_\parallel a/\pi$ and reduced frequency $\omega a/2\pi c$ for perforated plates of thickness $t = 1, 2, \text{ and } 5 \text{ mm}$, respectively. A complex structure of transmission features can be observed. Their evolution with $t$ allows us to classify them into two distinct categories: (i) The position of some of the features are relatively insensitive to $t$, and these are connected to lattice resonances, as we show below; (ii) other features exhibit dramatic variations with $t$, and these are related to intrinsic elastic modes of the film. The presence of mode anticrossings in the intersections between the two types of features indicates strong mutual coupling.

We have solved the full elasto-acoustic wave equations [13] under the same conditions as in the actual sampled plates, for an infinite number of holes. The calculated results are presented in Figs. 1(e)–1(g). The method of solution involves the following steps: (i) The displacement field $u$ and the Lamé coefficients are Fourier transformed along directions parallel to the periodic plate; (ii) the eigenstates of a 2D crystal formed by infinitely long holes with the same periodicity as the plate are obtained by inserting the solutions $u_q(r) = \sum_\mathbf{G} u_{q,G} \exp[i(\mathbf{q} + \mathbf{G}) \cdot \mathbf{r}]$ into the wave equations, where $\mathbf{q} = (k_\parallel, q_z)$, the sum runs over 2D reciprocal-lattice vectors $\mathbf{G}$, and the wave vector along the holes $q_z$ is an eigenvalue of the resulting quadratic-algebra equations for fixed frequency [14]; (iii) Rayleigh expansions are used for the pressure in the water outside the plate, whereas the displacement field inside is expanded in terms of $u_q$ eigenstates; (iv) the continuity of the displacement and the stress in the plate boundaries leads to a set of linear equations that are solved to yield the coefficients of these expansions. This method of solution gives a rigorous expansion for finite plates, in which the thickness enters through the boundary conditions matching the internal 2D modes to the Rayleigh expansions outside the film. The calculated transmission maps [Figs. 1(e)–1(g)] are in reasonable agreement with experiment. However, it should be mentioned that some spurious modes appear when a finite number of $G$’s is used ($\sim 1000$ in Fig. 1). These modes are mainly localized in the water-aluminum interface and originate in unphysical values of the Fourier-expanded Lamé coefficients because of the large mismatch in both media. Spurious modes produce noise that can be partially removed by eliminating them from the expansion of the displacement $u$, although part of their effects are still discernible in Fig. 1 and increase with $t$.

Further insight into the physical origin of the transmission features is provided by two simpler models, as shown in Fig. 2 for $t = 2 \text{ mm}$. (i) First, we have performed calculations in the hard-solid limit [Fig. 2(b)], which is equivalent to the perfect conductor idealization in electromagnetism. In this limit, the wave equations become scalar in terms of the pressure field $\phi$, which satisfies Helmholz’s equation $\nabla^2 \phi = 0$ in the water and is subject to the condition that its normal derivative vanishes at the interface with the solid. This approximation leads to excellent results when the sound velocity mismatch between solid and water is large (for example, in brass [11]). This model produces transmission maxima at slightly lower frequencies with respect to the onset of diffraction (i.e., when a Bragg beam $G$ becomes grazing [15]), which under the conditions of Fig. 1(a) leads to

$$|k_\parallel + G| = \sqrt{(k_\parallel + 2\pi n/a)^2 + (2\pi t/a)^2} = \omega/c. \quad (1)$$

FIG. 1 (color online). Angle dependence of sound transmission through aluminum plates immersed in water and perforated by holes of fixed diameter $d = 3 \text{ mm}$, arranged in a square array of period $a = 5 \text{ mm}$. (a) Schematic representation of a drilled plate, the incident sound wave vector $k$, and its component parallel to the film $k_\parallel$. (b)–(d) Measured sound transmission as a function of $k_\parallel$ and frequency $\omega$ for three different film thicknesses $t$, as indicated by text insets. (e)–(g) Theoretical counterpart of (b)–(d), obtained by solving the full elasto-acoustic wave equations. The transmission is represented in linear gray scale.
meets the crossing of the (0, 1) and (−1, 1) features at the boundary of the first Brillouin zone, where it has zero group velocity, as predicted by the two-band model [17], and then falls down with negative group velocity outside that zone. A similar behavior is observed for thinner plates [Figs. 1(b) and 1(e)]. As a rule, the low-frequency S₀ mode of the perforated plates is significantly less steep than in nonperforated plates [cf. Figs. 2(a) and 2(c)], implying smaller group velocity (i.e., the effective holey-plate parameters correspond to smaller Lamé coefficients compared to the homogeneous aluminum plate and therefore lower elastic wave velocities).

The interaction between lattice resonances and Lamb modes becomes more involved when the thickness increases, as shown in Figs. 1(d) and 1(g) for t = 5 mm. In homogeneous aluminum plates of the same thickness (see auxiliary material [18]), the A₀ Lamb mode moves to lower wave vector values, away from the Scholte-Stoneley mode, while the A₁ Lamb mode appears in our range of measurement. As a result, the S₀ mode is much fainter in the t = 5 mm perforated plate compared to plates of smaller thickness, but we observe instead a rich elastic-mode structure with high transmission values above the (−1, 0) lattice-sum singularity. Furthermore, the near-normal-incidence measured transmission in the pierced thick plate [Fig. 1(d)] exhibits a peak originating in a Fabry-Perot resonance, flanked by a lower lattice resonance, as inferred by comparison with hard-solid theory [11]. Also, the transmission features become broader in thicker plates.

The transmission vanishes right when this condition is satisfied. Different values of the Miller indices \((n, m)\) produce the dashed curves of Figs. 2(a), 2(b), and 2(d). Like in the optical case, these transmission resonances are driven by lattice-sum singularities originating in cumulative in-phase scattering among the holes of the array [15,16]. For sufficiently thick samples, Fabry-Perot resonances are also predicted to play a dominant role [11].

(ii) Another insightful approach consists in examining homogeneous films [Fig. 2(c)], the transmission properties of which are dominated by the presence of Lamb modes [12]. These resonances reduce to \(S₀\) and \(A₀\) for \(t = 2\) mm, and, in particular, the \(A₀\) mode is mixed with a Scholte-Stoneley mode near grazing incidence. This is reasonable because this mode (unlike Lamb oscillations) is confined to the surface in a similar way as surface-plasmon polaritons in metals.

The measurement in the \(t = 2\) mm drilled plate [Fig. 2(a)] resembles the hard-solid theory [Fig. 2(b)]. Both of them show transmission dips at the onset of diffraction [Eq. (1)] and transmission maxima at slightly lower frequency, similar to what happens in optical transmission as a result of the interplay between finite hole polarization and divergent interhole interaction [16].

However, a new mode consistent with \(S₀\) appears and strongly interacts with the lattice resonances at \(kₜ₀a/\pi \approx 0.6\) and 1.0 in the experiment. This interaction is well reproduced by the full wave calculation [Fig. 2(d)]. The Lamb mode crosses the (−1, 0) lattice-sum singularity,
The dramatic influence of hole ordering on the transmission performance of perforated plates is demonstrated in Fig. 3, which shows measurements for 2 mm thick plates pierced by periodic [Fig. 3(a), period $a = 5$ mm] and random [Fig. 3(b)] arrangements of holes (diameter $d = 3$ mm) with the same average filling fraction. The rich interplay between lattice modes and intrinsic plate modes in the ordered array [Fig. 3(a)] is completely absent in the random sample. However, the latter displays a feature resembling the $S_0$ Lamb mode of the homogeneous plate [cf. Figs. 2(c) and 3(b)]. Interestingly, the Fourier transform of the 2D distribution of geometrical openings in random arrays shows a broad annular maximum with a radius close to $a/\pi$, which gives rise to a broad dark region near normal incidence, close to the lattice resonance of the ordered array [19]. The phase velocity of the low-frequency $S_0$ mode is found to be $\sim 3150$ m/s in both perforated plates, which seems to respond to the average elastic parameters of the water-aluminum holey sample as noted above, quite different from the faster $S_0$ mode of the pure aluminum plate [5486 m/s, Fig. 2(c)]. In contrast to random arrays, the phononic crystal generated for the Lamb waves in periodic arrangements [20] could interact with Fabry-Perot resonances and lattice-driven modes, although further research is needed to address this issue.

In summary, the angle-resolved acoustic transmission through perforated plates exhibits both lattice resonances, which are similar in nature to their extraordinary optical transmission counterpart, and excitation of intrinsic elastic Lamb modes of the plates. The latter constitute a genuine aspect of sound transmission. These two types of modes interact with each other and produce the complex transmission patterns reported here through both measurements and full solution of the elasto-acoustic wave equations (Fig. 1). The nature of the modes becomes clear when comparing these results with the calculated transmission of either a homogeneous plate, dominated by Lamb modes, or a hard-solid drilled plate, showing lattice resonances (Fig. 2). The interplay between lattice and intrinsic resonances, their interaction with Fabry-Perot modes of the hole cavities, and the partial transparency of real materials to sound define altogether a new scenario in wave transmission through subwavelength apertures and open up a source of novel phenomena and applications of the field of sound transmission through real materials structured at subwavelength scales.

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[14] The wave equation contains both linear and quadratic terms in $q_z$, and we solve it as a quadratic eigenvalue problem.
[19] Like in optics, the strength of the hole polarization is proportional to $p = 1/(\alpha^{-1} - G)$, where $\alpha$ is the hole polarizability and $G$ is the sum of the interaction of each hole with all other holes in the array [16]. The real part of the sum $G$ is usually divergent under the conditions of Eq. (1) in ordered arrays, and this is directly related to the divergence of the Fourier transform of the hole distribution [Fig. 3(c)], whereas the imaginary part is partially compensated by $\alpha^{-1}$. In random arrays, $G$ has broader, smaller maxima, accompanied by a significant imaginary part, and still leading to small values of $p$ [i.e., depleted transmission in Fig. 3(b)] but not to enhanced transmission.