Improved Limit on Direct $\alpha$ Decay of the Hoyle State


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The current evaluation of the triple-$\alpha$ reaction rate assumes that the $\alpha$ decay of the 7.65 MeV, $0^+$ state in $^{12}$C, commonly known as the Hoyle state, proceeds sequentially via the ground state of $^8$Be. This assumption is challenged by the recent identification of two direct $\alpha$-decay branches with a combined branching ratio of 17(5)%. If correct, this would imply a corresponding reduction in the triple-$\alpha$ reaction rate with important astrophysical consequences. We have used the $^{11}$B($^3$He, $d$) reaction to populate the Hoyle state and measured the decay to three $\alpha$ particles in complete kinematics. We find no evidence for direct $\alpha$-decay branches, and hence our data do not support a revision of the triple-$\alpha$ reaction rate. We obtain an upper limit of $5 \times 10^{-3}$ on the direct $\alpha$ decay of the Hoyle state at 95% C.L., which is 1 order of magnitude better than a previous upper limit.

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In 1952, Edwin Salpeter suggested that the synthesis of $^{12}$C in stars is accomplished by a two-step process—now known as the triple-$\alpha$ reaction—where two $\alpha$ particles first form the unbound ground state of $^8$Be, which subsequently captures a third $\alpha$ particle [1,2]. The efficiency of the triple-$\alpha$ reaction is limited by the short lifetime of $^8$Be of the order of $10^{-16}$ s, which results in an equilibrium fraction of $^8$Be in the stellar plasma of the order of $10^{-9}$. In 1953, Fred Hoyle realized that, for the triple-$\alpha$ reaction to produce the observed abundance of $^{12}$C, the second step of the process, $\alpha + ^8$Be $\rightarrow ^{12}$C + $\gamma$, like the first step, $\alpha + ^4$He $\rightarrow ^8$Be, had to be enhanced by an $s$-wave resonance [3]. This led him to predict the existence of a $0^+$ state in $^{12}$C close to the $\alpha + ^8$Be threshold. Soon after, a $0^+$ state was indeed discovered experimentally [4] at an excitation energy of 7.65 MeV, close to the excitation energy predicted by Hoyle. Subsequently, this state has become known as the Hoyle state.

In the temperature range where stellar helium burning takes place, $T = 10^8$–$10^9$ K, the triple-$\alpha$ reaction is fully dominated by the $s$-wave resonances in $\alpha + \alpha$ and $\alpha + ^8$Be, and—precisely for this reason—the reaction rate is known rather accurately even though it cannot be measured directly in the laboratory. The rate depends on the properties of the Hoyle state in the following way [5]:

$$R \propto T^{-3/2} \frac{\Gamma_{\alpha_0} \Gamma_{\text{rad}}}{\Gamma} \exp \left( - \frac{E}{kT} \right)$$

where $k$ is the Boltzmann constant, $E$ is the energy of the Hoyle state relative to the triple-$\alpha$ threshold, $\Gamma_{\alpha_0}$ is the $\alpha + ^8$Be width, $\Gamma_{\text{rad}}$ is the radiative width, and $\Gamma = \Gamma_{\alpha} + \Gamma_{\text{rad}}$ is the total width, $\Gamma_{\alpha}$ being the total $\alpha$-decay width. The radiative branching ratio has been determined to be $\Gamma_{\text{rad}}/\Gamma = 4.12(11) \times 10^{-4}$ [6], so, to a good approximation, $\Gamma_{\alpha}$ $\approx$ $\Gamma_{\alpha_0}$. The largest source of uncertainty in the evaluation of the reaction rate is the radiative width, $\Gamma_{\text{rad}}$, which is determined from the combined measurement of the radiative branching ratio, $\Gamma_{\text{rad}}/\Gamma$; the $e^+e^-$ pair-decay branching ratio, $\Gamma_{e^+e^-}/\Gamma$; and the $e^+\nu e^-$ pair-decay width, $\Gamma_{e^+\nu}$. The current estimate of the overall uncertainty on the reaction rate is 10% [7].

In the evaluation of the reaction rate, it is assumed that the total $\alpha$-decay width is well approximated by the $\alpha + ^8$Be width, i.e., $\Gamma_{\alpha} \approx \Gamma_{\alpha_0}$. In other words, it is assumed that the $\alpha$ decay of the Hoyle state proceeds exclusively as a sequential two-step process via the ground state of $^8$Be. This central assumption in our understanding of the triple-$\alpha$ reaction has received relatively little scrutiny. In 1994, an upper limit of $(\Gamma_{\alpha} - \Gamma_{\alpha_0})/\Gamma_{\alpha} < 0.04$ on direct $\alpha$-decay branches bypassing the ground state of $^8$Be was obtained by Freer et al. [8]. However, in a recent work, Raduta et al. identified two direct $\alpha$-decay branches with a combined branching ratio of $(\Gamma_{\alpha} - \Gamma_{\alpha_0})/\Gamma_{\alpha} = 0.17(5)$ [9]. If correct, this would imply $\Gamma_{\alpha_0}/\Gamma = 0.83(5)$ and, as seen from Eq. (1), a corresponding reduction in the reaction rate in the temperature range $T = 10^8$–$10^9$ K. The consequences would be even greater at lower temperatures ($T < 10^8$ K) where the $s$-wave resonances in $\alpha + \alpha$ and $\alpha + ^8$Be cease to dominate the triple-$\alpha$ reaction (at $T < 0.3 \times 10^8$ K for $\alpha + \alpha$ and $T < 0.7 \times 10^8$ K for $\alpha + ^8$Be). In this temperature range, direct reactions become dominant and the findings of Raduta et al. [9] imply an increase of several orders of magnitude in the reaction rate [10–12].
The 17(5)\% reduction in the reaction rate in the temperature range \( T = 10^8-10^9 \) K is larger than the current estimate of the uncertainty on the reaction rate and would have consequences for a number of astrophysical systems where the triple-\( \alpha \) reaction plays a role; see, e.g., Refs. [13–17]. It is therefore essential to verify the findings of Raduta et al. [9]. The existence of direct \( \alpha \)-decay branches would also have implications for our understanding of the structure of the Hoyle state, a topic which has received much theoretical attention recently (lattice effective field theory, Refs. [18,19]; no-core shell model, Ref. [20]; fermionic molecular dynamics, Ref. [21]; and \( \alpha \)-cluster models, Refs. [22–24]).

Here, we report on a new measurement of the \( \alpha \) decay of the Hoyle state using the experimental method of low-energy, high \( Q \)-value reactions measured in complete kinematics [25]. The advantages of this method are high resolution, high efficiency, and high signal-to-background ratio. Specifically, we use the \( ^{11}\text{B}(^{3}\text{He}, d) \) reaction at 8.5 MeV to populate the Hoyle state, and we use a compact detection system consisting of four segmented telescopes to measure the momenta of the deuteron and the three \( \alpha \) particles resulting from the decay of the Hoyle state; see Refs. [25,26] for details.

The spectrum of \( ^{12}\text{C} \) excitation energies populated in the \( ^{11}\text{B}(^{3}\text{He}, d) \) reaction, as determined from the momentum of the deuteron, is shown in Fig. 1(a). Only events of multiplicity four, i.e., events where all three \( \alpha \) particles are detected along with the deuteron, are used in the analysis. By gating on the peak corresponding to the Hoyle state, we select the events of interest. The angular distribution of the Hoyle state, shown in Fig. 1(b), has the characteristic shape of a direct reaction with nonzero angular momentum transfer.

The \( \alpha \)-particle data are best visualized using the symmetric Dalitz plot shown in Fig. 2(a), which is particularly adapted to the case of three particles of equal mass [27]. Because the decay of the Hoyle state is isotropic (the Hoyle state has spin zero and hence no directional memory) and because energy and momentum conservation must be obeyed, knowledge of two \( \alpha \)-particle energies fully specifies the kinematics of the decay. The Dalitz plot thus extracts the maximum amount of information from the data. The radial coordinate of the Dalitz plot, \( \rho \), is given by

\[
(3\rho)^2 = (3E_i - 1)^2 + 3(2E_i + 2E_j - 1)^2,
\]

where \( E_{i,j,k} = E_i + E_j + E_k \) are the \( \alpha \)-particle energies in the \( ^{12}\text{C}^* \) rest frame normalized to the total decay energy. Different decay mechanisms result in different Dalitz-plot distributions. In addition to sequential decay (SD) via the ground state of \( ^{8}\text{Be} \), we consider three direct decay mechanisms: three \( \alpha \) particles of equal energy (DDE), one \( \alpha \) particle at rest and the other two with equal energy (DDL), and phase-space decay (DD\( \Phi \)). The Dalitz-plot distributions corresponding to SD, DDE, and DDL are shown in Fig. 2(b). DD\( \Phi \) results in a flat distribution. It so happens that the ratio of the \( \alpha+\alpha \) and \( \alpha+^{8}\text{Be} \) resonance energies (91.8 and 287.6 keV, respectively) nearly equals \( 1/3 \), which causes approximate kinematic overlap between SD and DDL. A resolution better than 1.5 keV would be required to clearly separate the two. The Dalitz-plot distribution obtained from the present measurement comprises \( 5 \times 10^3 \) multiplicity-four events and is shown in Figs. 2(c) and 2(d), before and after kinematic fitting. The background level is below 0.1\%. In comparison, the background level in the work of Raduta et al. was 40\% [9].

Kinematic fitting is a mathematical procedure commonly used in particle physics [28]. The underlying idea is to use the laws (constraints) that govern a physical process to improve the resolution of the measurement of the process. The physical process considered here is

\[
^{11}\text{B} + ^{3}\text{He} \rightarrow d + ^{12}\text{C}^* \rightarrow d + \alpha + \alpha + \alpha,
\]

which is constrained by the requirement of energy and momentum conservation and the requirement that the invariant mass of the three \( \alpha \) particles correspond to the
excitation energy of the Hoyle state. Owing to finite experimental resolution (or systematical effects that are not accounted for), the final-state momenta deduced from the measured quantities only satisfy the above constraints approximately. Kinematic fitting is applied on an event-by-event basis to ensure that the constraints are satisfied exactly. This is achieved by modifying the measured data. Based on a careful estimate of the uncertainty on each measured quantity, a $\chi^2$ function is written down that measures the magnitude of such a modification. Kinematic fitting essentially consists in minimizing $\chi^2$, with the measured quantities as variable parameters but subject to the constraints; see Ref. [28] for details. We would like to stress that no assumption on the decay mechanism enters into this procedure.

Figure 3 shows the radial projection of Fig. 2(d) compared to the predictions of the four decay mechanisms, determined through Monte Carlo simulations taking into account experimental effects. (The experimental acceptance is fairly uniform, except a slight increase at the periphery of the Dalitz plot.) The simulation program [26] has been validated in several previous studies [25,29,30]. The SD mechanism provides an excellent fit to the experimental data on its own. With the normalization fixed by the requirement of equal areas under the graphs, the fit quality is $\chi^2$/d.o.f. = 0.92. The branching ratios reported by Raduta et al. for the direct decay branches, $DDE = 0.075(40)$ and $DDL = 0.095(40)$, are clearly incompatible with the experimental data. We place the following upper limits at 95% C.L.:

\[
DDE < 0.9 \times 10^{-3}, \quad DDL < 0.9 \times 10^{-3}, \quad DD\Phi < 5 \times 10^{-3}.
\]

The upper limit on $DD\Phi$ is 1 order of magnitude better than the previous upper limit of 0.04 obtained by Freer et al. We note that the ghost of the $^8$Be ground state [31–34] only contributes at the level of $6 \times 10^{-3}$, which is below the present sensitivity.

Raduta et al. make the premise that observation of a certain distribution of $\alpha$-particle energies provides evidence for corresponding structural features of the Hoyle state. Specifically, $DDE$ is linked to $\alpha$-condensate structure [35,36] and $DDL$ is linked to the long-discussed linear-chain structure [37]. Raduta et al. thus conclude that the observation of a $DDE$ branch constitutes direct evidence for the $\alpha$-condensate structure of the Hoyle state. (On the other hand, in Ref. [38], the observation of a SD branch is advanced as evidence for $\alpha$-condensate structure.)
While we dispute the suggested direct link between energy distribution and structure, because the former must be strongly influenced by the tunneling through the Coulomb barrier, it seems natural to expect some connection between energy distribution and structure, and therefore a precise measurement of the energy distribution, viz., the Dalitz-plot distribution, should provide a sensitive test of structure models if the latter is combined with a sophisticated decay model such as that described in Refs. [22,23].

To summarize, we have measured the $\alpha$ decay of the 7.65 MeV, $0^+$ state in $^{12}$C, commonly known as the Hoyle state, and find that the decay proceeds exclusively as a sequential two-step process via the ground state of $^8$Be, contradicting the recent work of Raduta et al. [9]. We obtain an upper limit of $5 \times 10^{-3}$ on direct $\alpha$-decay branches bypassing the ground state of $^8$Be, which is an improvement of 1 order of magnitude over the previous upper limit of 0.04 obtained by Freer et al. [8]. The contribution from $\Gamma_{0\alpha}/\Gamma$ to the uncertainty on the rate of the triple-$\alpha$ reaction in the temperature range $T = 10^8$–$10^9$ K is correspondingly reduced.

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Note added in proof.—Results recently published by Manfredi et al. [39] also contradict the findings of Raduta et al.