Light bullets and dynamic pattern formation in nonlinear dissipative systems

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Abstract: In the search for suitable new media for the propagation of (3+1)D optical light bullets, we show that nonlinear dissipation provides interesting possibilities. Using the complex cubic-quintic Ginzburg-Landau equation model with localized initial conditions, we are able to observe stable light bullet propagation or higher-order transverse pattern formation. The type of evolution depends on the model parameters.

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References and links

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1. Introduction

The concept of “light bullet” refers to a spatio-temporal soliton which is confined due to nonlinearities in the three spatial dimensions, in addition to localization in the temporal domain [1]. The self-focusing nonlinearity of the medium should balance the natural trend of localized light fields to spread with propagation. This occurs transversally due to spatial diffraction, and longitudinally (temporally) due to group velocity dispersion. For optical bullets, achieving confinement in three dimensions is a major task. Specific study should be made for two and three-dimensional cases of confinement. For example, a pure $\chi^{(3)}$ Kerr medium, which is appropriate for transverse confinement in the 1D case, does not provide stable confinement for higher dimensions [1].

Investigations of spatio-temporal formations at the beginning of the 90’s were mainly done for conservative systems [2]. The central issue for media with Kerr nonlinearities is the problem of collapse. A cubic-quintic medium that features saturation of the Kerr effect could avoid collapse and provide confinement in several dimensions [3, 4]. Criteria for the experimental observation of multidimensional optical solitons in saturable media have been discussed in [5]. However, such media are difficult to find in practice. Photorefractive materials [6, 7] have strong nonlinearity with saturation but the nonlinearity is slow. Hence, it is hard to imagine that optical bullets could be observed in such materials. Quadratic materials [8-12] are possible alternative media, in which spatial solitons consist of mutually-trapped fundamental and second-harmonic waves. Experimentally, (2+1)D solitons were observed in this arrangement and reported in Ref. 9. Soliton collisions in quadratic media have also been studied in detail [10, 11]. Temporal phase-matching conditions could also be found in the frame of nonlinear conical waves [12].

During the past ten years, “cavity solitons” have attracted much attention from researchers [13-17]. Cavity solitons are stable transverse structures (spatial solitons) that are formed in the dissipative environment of a large aperture laser cavity. In particular, the use of semiconductor cavities allows external control of several cavity solitons, and seems potentially attractive for the development of all-optical processing devices [15]. Knowledge, both theoretical and practical, accumulated in recent years in the field of dissipative solitons [16] has provided more understanding of the unique features of cavity solitons and their interactions [14, 17]. Cavity solitons are essentially (2+1) D formations.

For dissipative systems, (1+1) D and (2+1) D problems have been studied extensively [16, 18]. However, less systematic work was done in the case of (3+1) D systems. In Ref. 19, Rosanov considered theoretically (m+1) D solitons with m=1, 2 and 3 in the case of a saturable nonlinearity bearing spherical symmetry in all three (space-time) variables. The symmetry allows simplifying the problem but is likely to be broken in reality, since the spatial and time variables cannot be completely equivalent. Our aim is to extend these investigations, using space-time asymmetry in the statement of the problem. We hope that optical bullets, that can be called “dissipative light bullets”, exist in this latter case too. Dissipative light
bullets can also be considered as extensions of (2+1) D cavity solitons to the temporal domain [20], that could be implemented with the use of ultra-short pulses. Dissipation is also likely to play an important role in the formation of stationary conical waves in real systems [21].

In this paper, we present numerical results that are based on the complex cubic-quintic Ginzburg-Landau equation (CCQGLE) model, which includes both spatial diffraction and temporal dispersion. The CCQGLE model has been used successfully in a number of studies to provide a realistic description of temporal dissipative solitons and their interactions observed in passively mode-locked laser cavities or transmission lines [16]. With the inclusion of transverse diffraction effects, our purpose is to provide theoretical and numerical grounds for the development of “dissipative light bullet” experiments.

The work is planned as follows. Section 2 presents the model and numerical algorithm used for simulations. Section 3 provides numerical results that demonstrate stationary propagation of a light bullet, after it is formed from an initial condition which is far from the stationary solution. Optical bullets can be stable and unstable, as shown in Section 4. The influence of initial conditions on the excitation of optical bullets is discussed in Section 5. Altering the parameters in the CCQGLE model can result in drastic changes in the propagation regime. We describe, in section 6, the formation of dynamic transverse higher-order patterns.

2. Model of the dissipative cubic-quintic medium

Our numerical simulations are based on an extended complex cubic-quintic Ginzburg-Landau equation (CCQGLE) model. This model includes cubic and quintic nonlinearities of dispersive and dissipative types, and we have added transverse operators to take into account spatial diffraction. The normalized propagation equation reads:

$$i \psi_c + \frac{D}{2} \psi_{tt} + \frac{1}{2} \psi_{xx} + \frac{1}{2} \psi_{yy} + |\psi|^2 \psi + \nu |\psi|^4 \psi = i \delta \psi + i \epsilon |\psi|^2 \psi + i \beta \psi_{tt} + i \mu |\psi|^4 \psi. \quad (1)$$

The optical envelope $\psi$ is a function of four variables $\psi = \psi(t,x,y,z)$, where $t$ is the retarded time in the frame moving with the pulse, $z$ is the propagation distance, $x$ and $y$ are the two transverse coordinates. Equation (1) is written in normalized form. The left-hand-side contains the conservative terms, viz. $D = +I(-1)$ is for the anomalous (normal) dispersion propagation regime and $\nu$ is saturation coefficient of the Kerr nonlinearity. In the following, the dispersion is anomalous, and the saturation of the Kerr nonlinearity is kept relatively small. The right-hand-side includes all dissipative terms: $\delta$, $\epsilon$, $\beta$ and $\mu$ are the coefficients for linear loss (if negative), nonlinear gain (if positive), spectral filtering and saturation of nonlinear gain (if negative), respectively. In contrast to the model considered in Ref. 19, the spectral filtering is related only to the term with the $t$-variable, and any sign of the dispersion can be considered.

This distributed model could be applied to the modeling of a wide aperture laser cavity in the short pulse regime of operation. The model provides for two-dimensional transverse diffraction of the beam (pulse), longitudinal dispersion of the pulse and its evolution along the cavity. Dissipative terms describe the gain and loss of the pulse in the cavity. Higher-order dissipative terms are responsible for the nonlinear transmission characteristics of the cavity (passive mode-locking). This equation is a natural extension of the complex cubic-quintic Ginzburg-Landau equation (CCQGLE). The first question to answer is whether this equation admits 3D dissipative solitons, i.e. optical bullets. We will also be interested in the general evolution of localized initial conditions.

We have solved Eq. (1) using a split-step Fourier method. Thus, the second-order derivative terms in $x,y$ and $t$ are solved in Fourier space. Consequently, we apply periodic boundary conditions in $x$, $y$ and $t$. All other linear and nonlinear terms in the equation are solved in real space using a fourth-order Runge-Kutta method. Simulations presented in the paper were calculated using a numerical grid of 512 points in $x$ and $y$ directions, and 256 points in $t$. We used various values of step sizes along the spatial and temporal domains to
check that the results do not depend on the mesh intervals, thus avoiding any numerical artifacts. A typical numerical run lasts from several hours to several days on a standard PC. When the solution in the form of stable dissipative bullets is achieved, the numerical grid can be changed to accelerate the calculations.

3. Evidence of stationary light bullets

In the case of the (1+1) D CCQGLE, the equation admits soliton solutions [22]. Moreover, several solutions can exist for the same set of parameters. These solitons are not necessarily stable. Stability is controlled by the parameters of the equation and by the choice of the soliton branch. For the present (3+1) D simulations with Eq. (1), we choose the parameters to be: \( D = 1 \), \( \mu = -0.1 \), \( \nu = -0.01 \), \( \delta = -0.4 \), \( \beta = 0.3 \). This set is found to admit solutions in the form of bullets. We have chosen the cubic gain as a variable parameter and changed it in the range from 1.0 to 1.8. As the initial condition, we used pulses localized in all three dimensions, \( t \), \( x \) and \( y \), with a Gaussian profile in each dimension. To obtain cylindrically symmetric solutions in the \( x \) and \( y \) dimensions, it seems natural to choose a cylindrically-symmetric initial condition. Thus, we used one with the form:

\[
\psi(t, x, y, 0) = 1.8 \exp\left(-t^2 - x^2 - y^2\right).
\]  

The natural control parameter of the solution would be the total energy \( Q \), given by the three dimensional integral of \( |\psi|^2 \) over \( x \), \( y \) and \( t \):

\[
Q(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dt |\psi(t, x, y, z)|^2.
\]  

For a dissipative system, the energy is not conserved but evolves in accordance with the so-called balance equation [22]. If the solution stays localized, the energy will evolve but remains finite. Furthermore, if a stationary solution is reached, the energy \( Q \) will converge. When the optical field spreads out, the energy tends to infinity. Another possibility is that the solution dissipates, and then the energy evolves to zero. Various examples of the evolution of \( Q \) are shown in Figs. 1(a) and 1(b).

![Fig. 1. (a) Pulse energy \( Q \) versus propagation distance \( z \) for cylindrically-symmetric initial condition (2). (b) The evolution for the initial condition (3) with elliptic symmetry. The parameter \( \varepsilon \) in the equation (1) is different for each curve. Other parameters are fixed for all simulations. Their values are shown inside each plot.](image)

The initial condition (2) is not a stationary solution, but a stationary localized solution exists in its neighborhood. Therefore, the field evolves until it converges to an optical bullet. Figure 1(a) shows the initial variations of \( Q(z) \). After several oscillations, \( Q \) converges to a
fixed value that corresponds to the energy of the stationary solution. The stationary solution has a cylindrical symmetry that is conserved until the numerical error breaks it at approximately $z = 30$. Up to this value, the solution propagates as an optical bullet without any visible changes. The soliton profile in all three dimensions is shown in Fig. 2. Although this optical bullet is a stationary solution of (1), it is unstable, and perturbations that lift cylindrical symmetry destroy the solution.

![Fig. 2. (a) The shape of the optical bullet (stationary solution) in the $(x,y)$ plane (b) The profile of the same bullet along the $t$-axis.](image)

We stress that the initial condition (2) is not a solution, but it does converge to the shape of the bullet and remains in that form for an extended traveling distance. Specifically, the energy of the initial solution is 6.38, whereas the energy of the stationary bullet is nearly 20. This numerical example shows that even unstable bullets can be excited under proper conditions.

### 4. Stable stationary optical bullets

When we decrease the parameter $\varepsilon$ while leaving the other parameters untouched, an unstable optical bullet can be transformed into a stable one. The plot in Fig. 1(b) shows the evolution of the energy $Q$ along the $z$ axis for five different values of $\varepsilon$. The initial condition in all five cases is the same, having an elliptic shape in the $(x,y)$ plane:

$$
\psi(t,x,y) = 4.0 \exp \left\{ -\frac{(t - 0.4)^2}{1.3} - x^2 - \left( \frac{y}{0.9} \right)^2 \right\}.
$$

Elliptic initial condition is a useful means to avoid convergence of the solution into an unstable bullet, as occurred in the previous example. We can see that the three lower values of $\varepsilon$ (1.0, 1.2 and 1.4) provide convergence of this initial condition to stable optical bullets. The curves for these cases are coded by amber, light blue and green colors. The energy becomes a constant after $z$ reaches the value of approximately 10 in the simulations. Thus, we can see evidence of excitation of stable optical bullets. This happens for a wide range of the parameter $\varepsilon$. The shape of optical bullets is shown in Fig. 3 for each above-mentioned case. Each profile has a plain bell-shape.
5. Dissipative optical bullet has a basin of attraction

Not every initial condition converges to an optical bullet. For the same set of equation parameters, the evolution depends on initial conditions. The initial conditions must be reasonably close to the shape of the optical bullet for the solution to converge to it. Figure 4(a) shows the evolution of the energy $Q$ for two different initial conditions. The red line is obtained using initial condition (3), while the blue line is obtained using the following initial condition:

$$
\psi(t, x, y, 0) = 4.0 \exp\left(-t^2 - x^2 - \left(\frac{y}{0.8}\right)^2\right).
$$

In the first case, the initial condition is outside the basin of attraction. This results in an infinite increase of the width of the pulse and subsequent different pattern formation. The movie linked to Fig. 4(b) shows this complicated evolution. Although the solution initially oscillates around the cylindrically-symmetric configuration, after a few oscillations it deviates from it, resulting in more complicated structures. The $Q(z)$ curve in Fig. 4(a) also shows this tendency of apparent early convergence. However, at $z = 1.1$, the red curve diverges dramatically from the blue curve and never approaches any constant value. In the second case, initial conditions (4) result in excitation of an optical bullet. The blue curve finally converges to a constant value indicating this convergence.
Fig. 4. On the left (a), we show pulse energy $Q$ versus propagation distance $z$ for two different elliptically-symmetric initial conditions. The equation parameters are the same in each case. The initial condition for the blue curve provides convergence to an optical bullet, while that for the red curve does not. The movie on the right demonstrates the field evolution for the case of the red line in the left diagram. It should be noted that when the field expansion reaches the boundaries, the simulation corresponds to a collision between the central light pulse and ghost neighbors.

As far as we know, the present results are the first numerical demonstration of (3+1) D spatio-temporal optical bullets in a cubic-quintic dissipative medium. The main difficulty in obtaining these solutions is the great amount of computer time needed for the simulations. Our main task was to find a set of parameters that admits solutions in the form of optical bullets. To obtain more information about the regions in parameter space where these solutions exist is a task that requires an appreciable amount of time. We hope that this can be done in future.

6. Delocalization and transverse pattern formation

For higher values of $\varepsilon$, namely $\varepsilon=1.6$ and $1.8$, stable optical bullets do not exist and the localization of the solution is lost. The fields spread indefinitely in $z$ and the energy also increases indefinitely. The transverse evolutions of the fields in each of these two cases are shown in the movies linked to Fig. 5. Initially, the elliptic shape of the pulse is transformed into a pulse which is elongated in the orthogonal direction. Then it splits into several strips and the pattern evolves continuously into a wider structure, never converging to a stationary solution. Thus, the evolution basically leads to dynamic pattern formation. The size of the localization increases, both in the transverse direction and in the $t$-direction. The latter can be seen from Figs. 6(a) and 6(b). The width of the structure increases with a linear trend in $z$. 

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Fig. 5. Movies of pulse evolution and pattern formation when stable optical bullets do not exist. The cubic gain is $\varepsilon = 1.6$ (left), $\varepsilon = 1.8$ (right). As in Fig. 4(b), the simulations describe the evolution of an isolated pulse, valid as long as the field does not reach significantly the boundaries.

Fig. 6. Pulse evolution in the t-domain, in a parameter domain where stable optical bullets do not exist. The two cases are for $\varepsilon = 1.6$ (left) and $\varepsilon = 1.8$ (right).

7. Conclusion

In conclusion, based on numerical simulations, we have presented the first evidence for the existence of stable stationary optical bullets in a dissipative medium described by the (3+1) D complex cubic-quintic Ginzburg-Landau equation, in which asymmetry between space-time variables is included. For the domains of parameters that we explored, we have shown that when the dissipative optical bullet exists, it possesses a finite basin of attraction. The choice of initial condition is thus important.

Varying the model parameters, we studied the three following situations: existence of unstable light bullets, existence of stable light bullets and non-existence of confined light fields. In the latter case, transverse pattern formation seems to occur naturally.
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